G52ACE 2017-18
Simple Sorting Algorithms

Lecturer: Andrew Parkes

http://www.cs.nott.ac.uk/~pszajp/

“Sorted! dude!

Simple sorting algorithms and their complexity

Consider an array of integers, and the goal is to sort into non-decreasing order (put the largest on the right)

1. Bubble sort
2. Selection sort
3. Insertion sort

Bubble Sort: Basic Idea

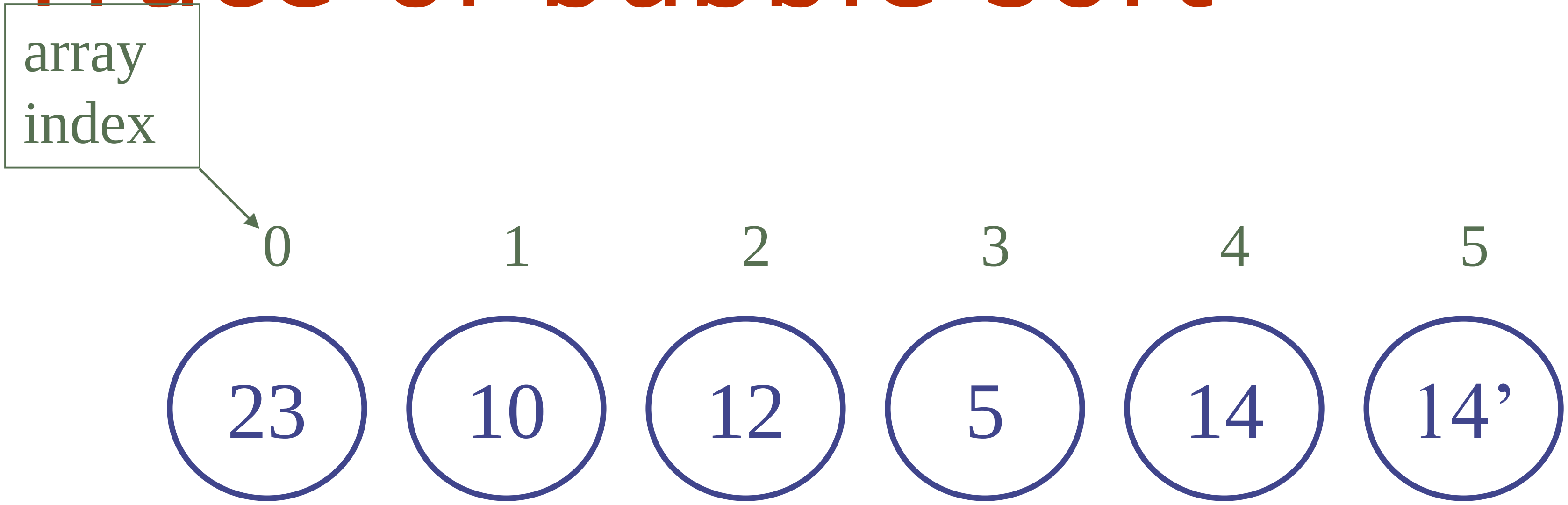
- Outer loop:
  Repeated scans through array
- Inner loop: on each scan do comparison with immediate neighbour
  - think of air bubbles rising in water
  - do swaps to make sure that the largest number “bubbles up” to the end of the array

Bubble sort

```
void  bubbleSort(int arr[]){
   int i;
   int j;
   int temp;
   for(i = arr.length-1; i > 0; i--){
      for(j = 0; j < i; j++){
         if(arr[j] > arr[j+1]){
            temp = arr[j];
            arr[j] = arr[j+1];
            arr[j+1] = temp;
         }//
      }// end inner loop
   }//end outer loop}// end bubble sort
```

swap adjacent elements, if in the wrong order

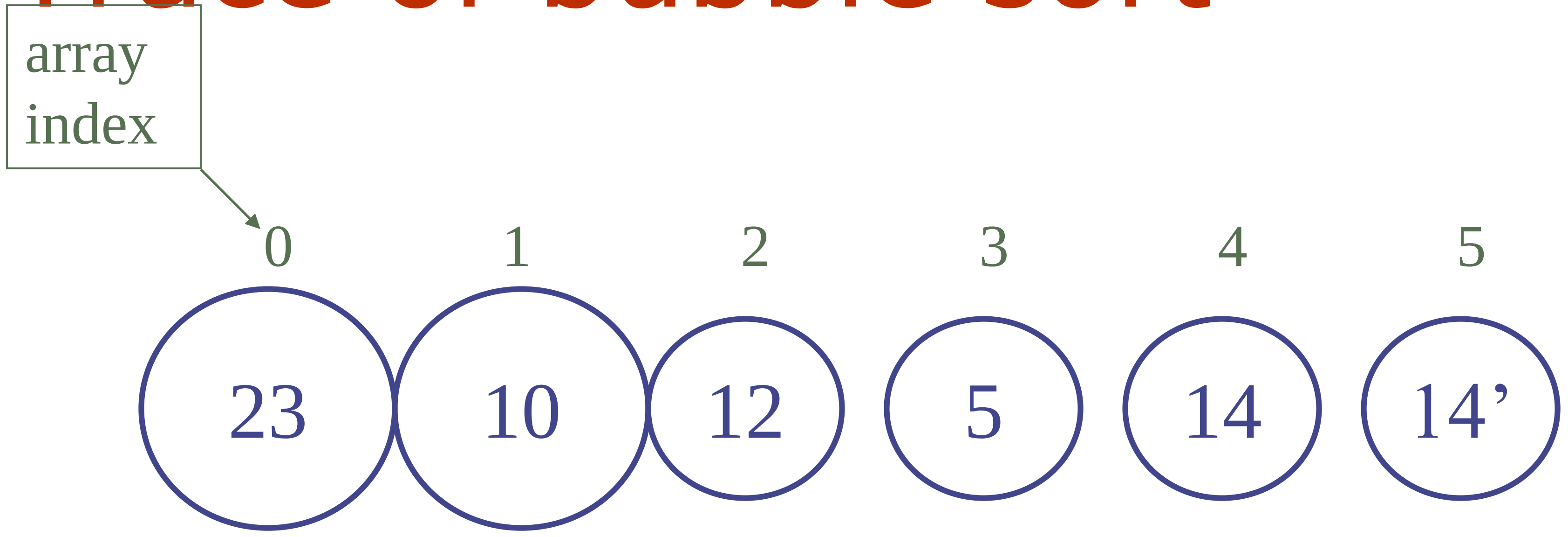
Trace of bubble sort



i = 5, first iteration of the outer loop

“Stability”: 14 and 14’ are two copies of the same number but keep track of which copy ends up where. (Explained why later)

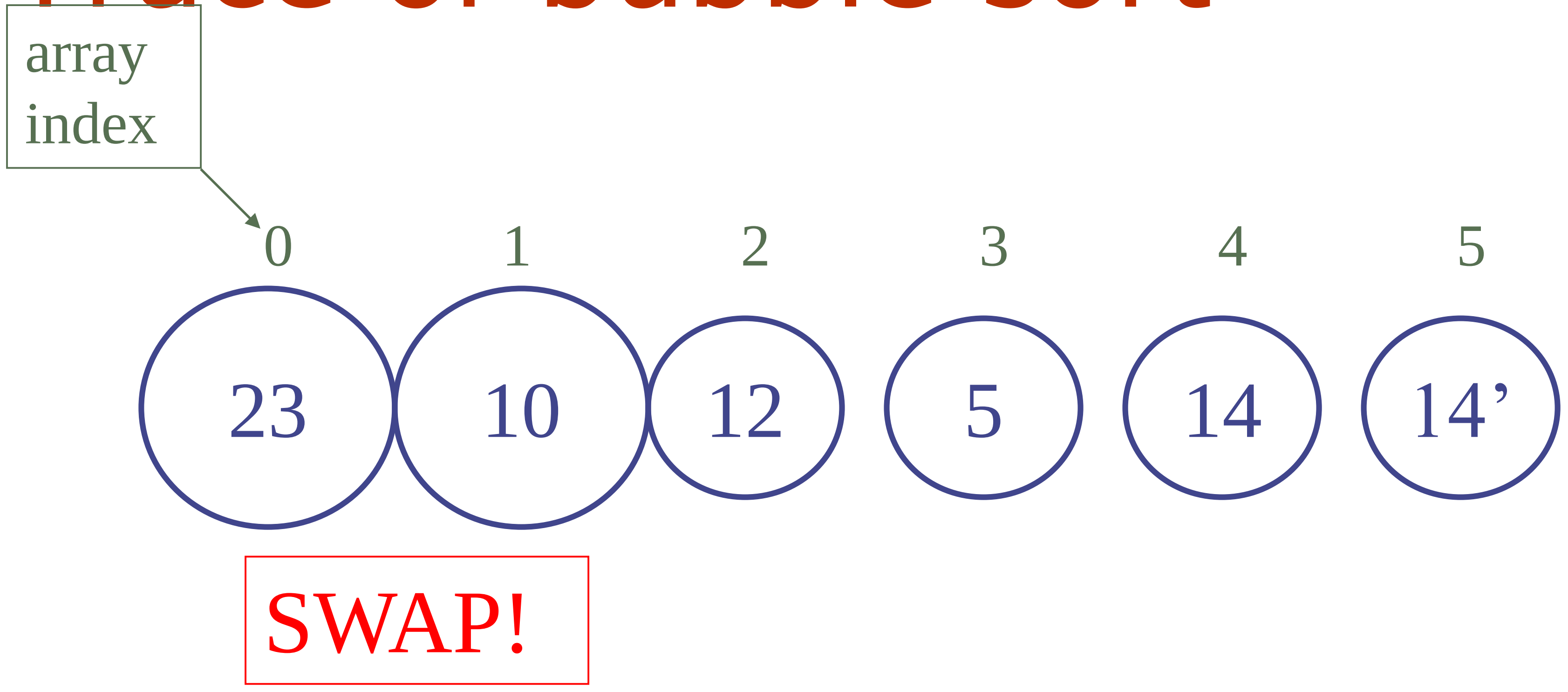
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 0, comparing arr[0] and arr[1]

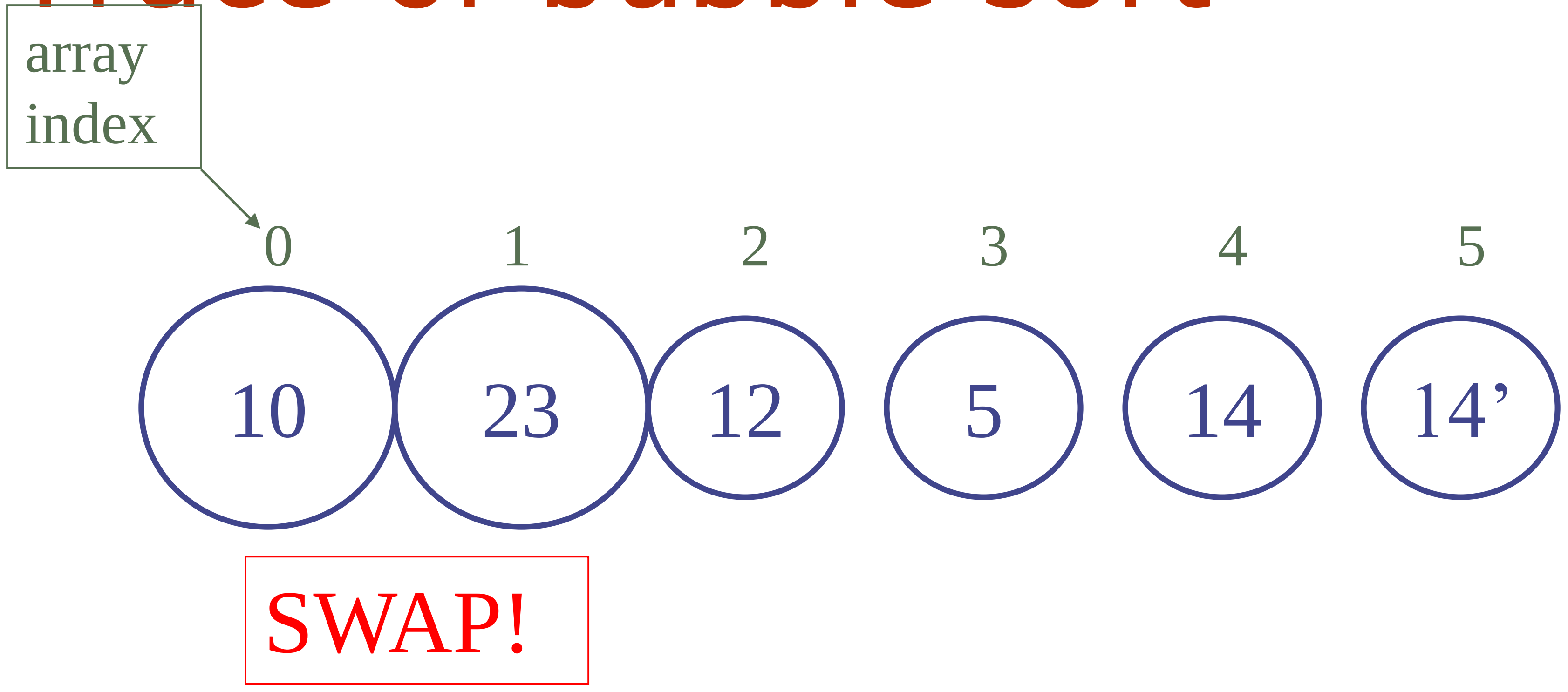
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 0, comparing arr[0] and arr[1]

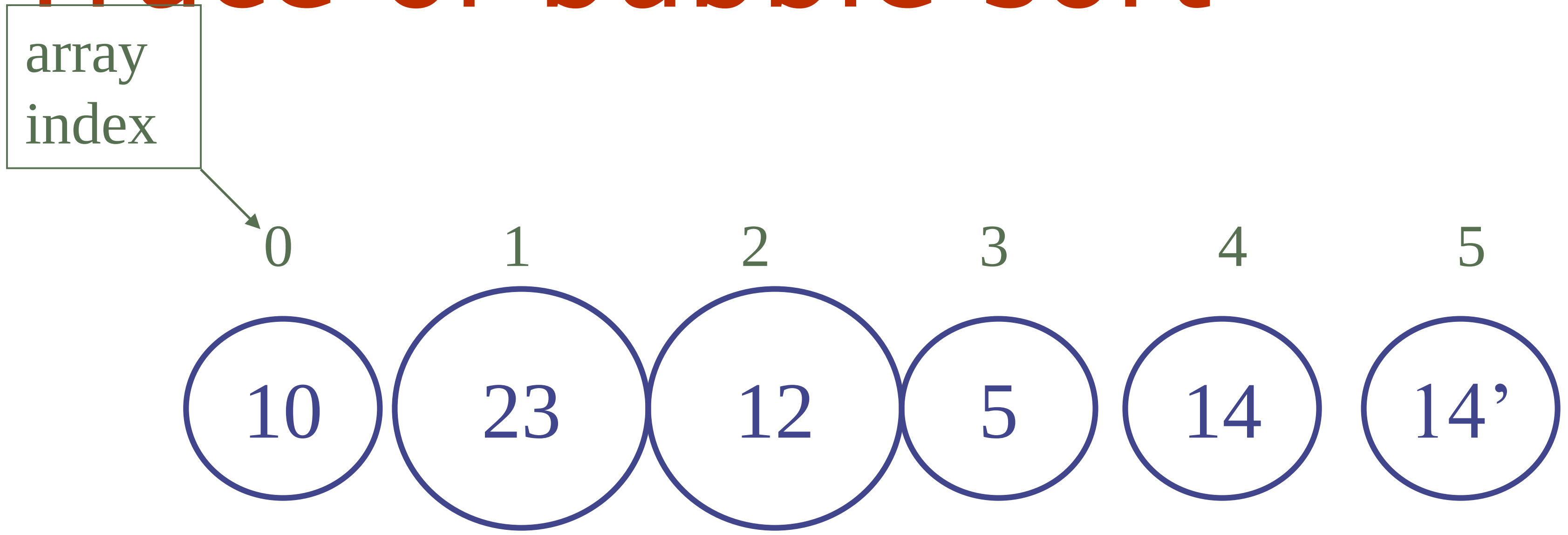
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 0, comparing arr[0] and arr[1]

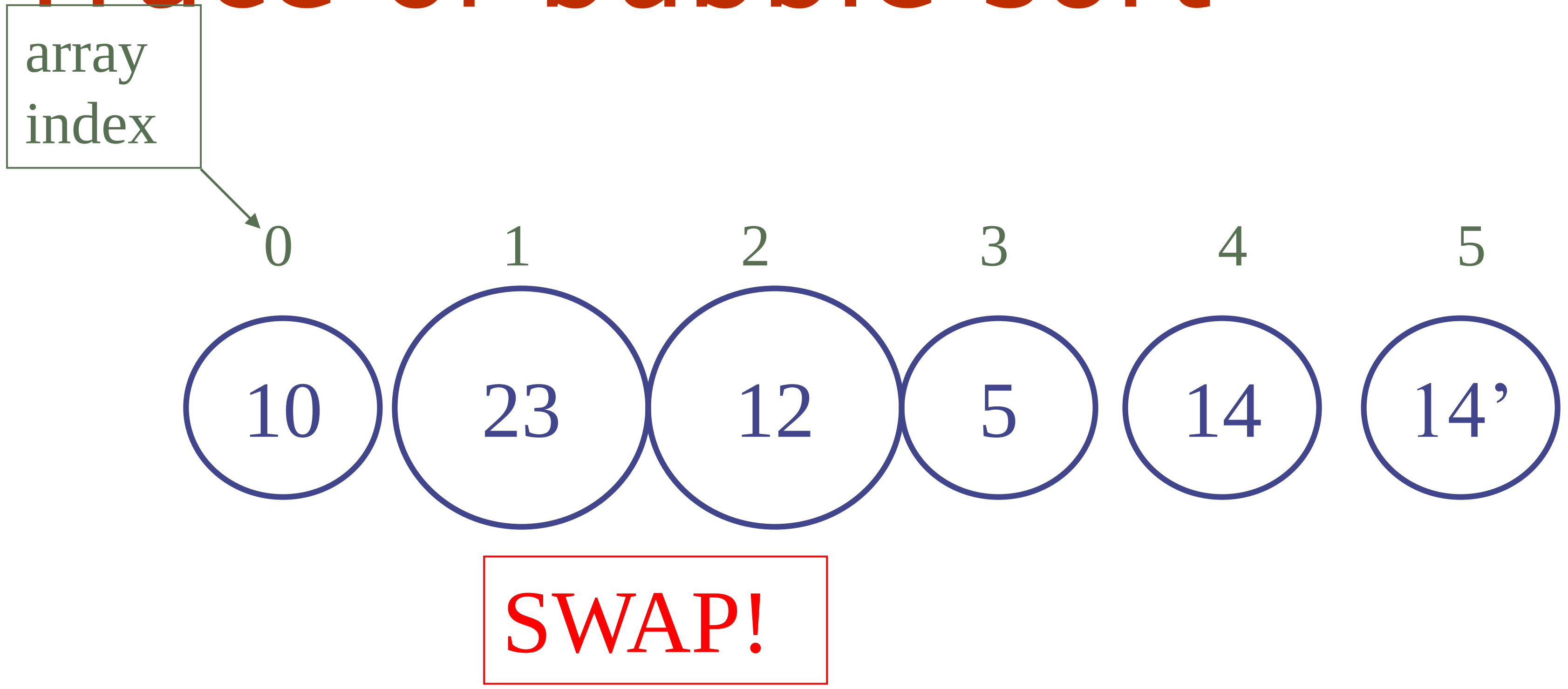
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 1, comparing arr[1] and arr[2]

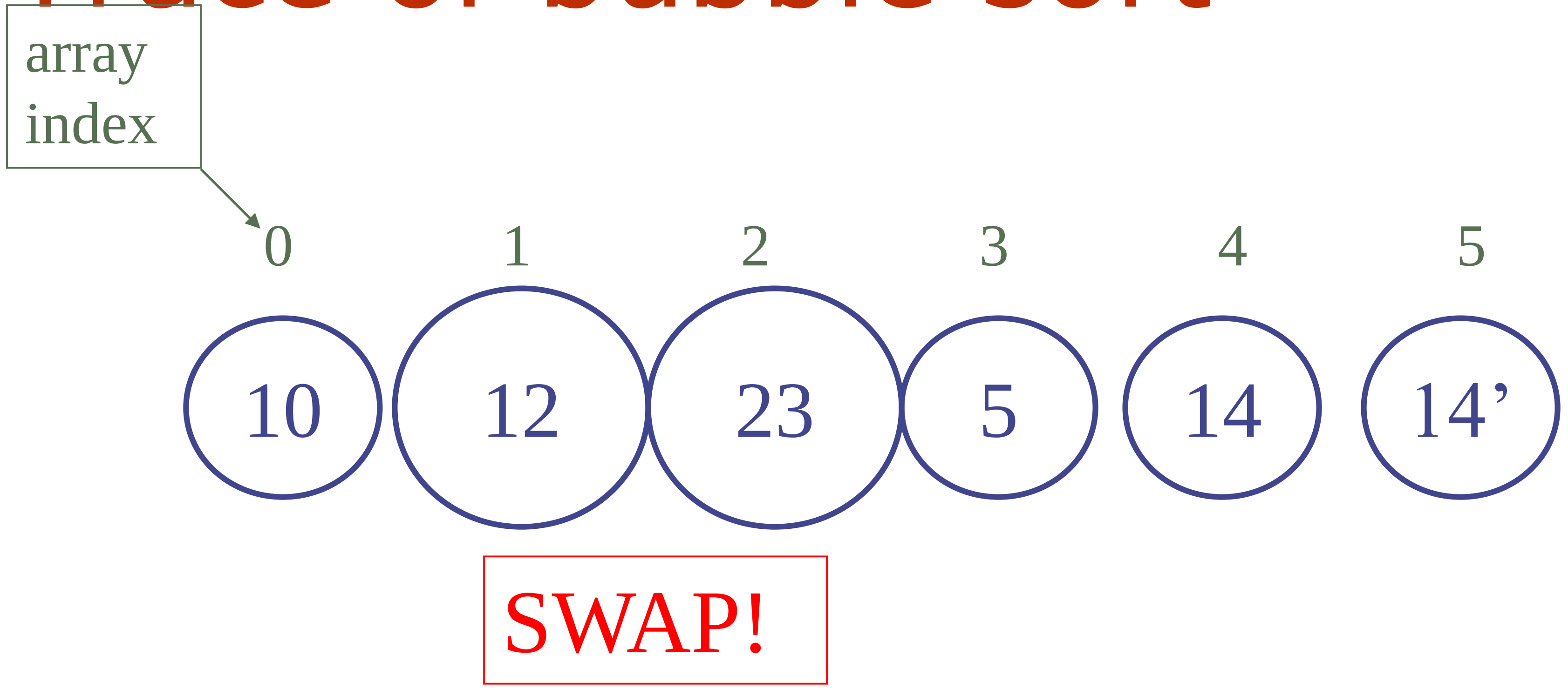
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 1, comparing arr[1] and arr[2]

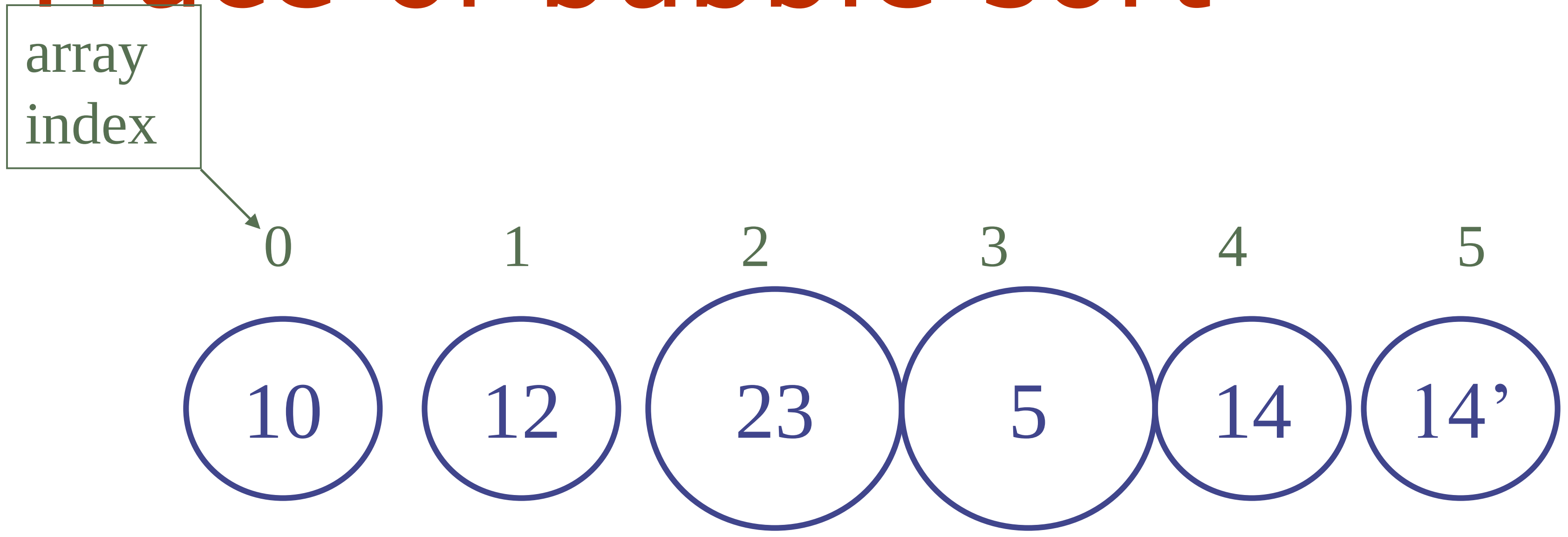
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 1, comparing arr[1] and arr[2]

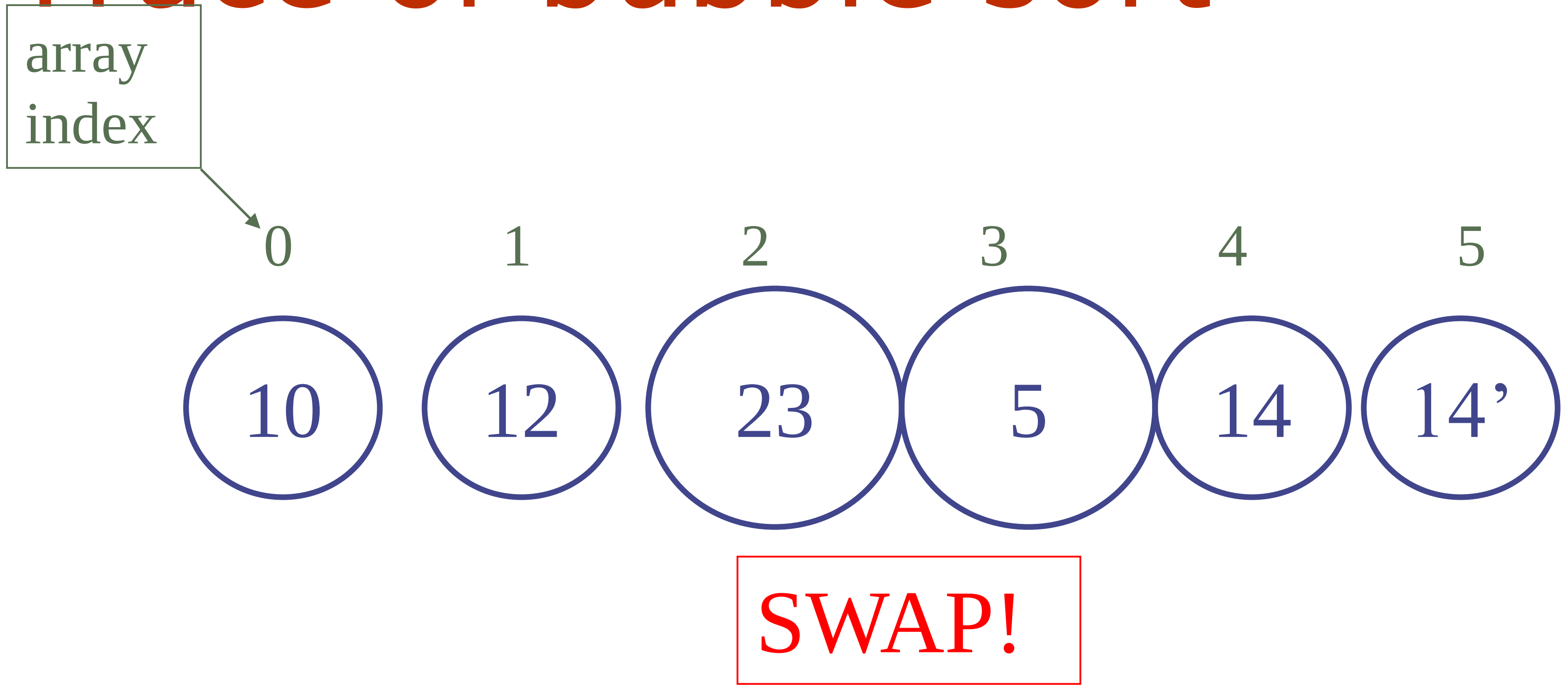
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 2, comparing arr[2] and arr[3]

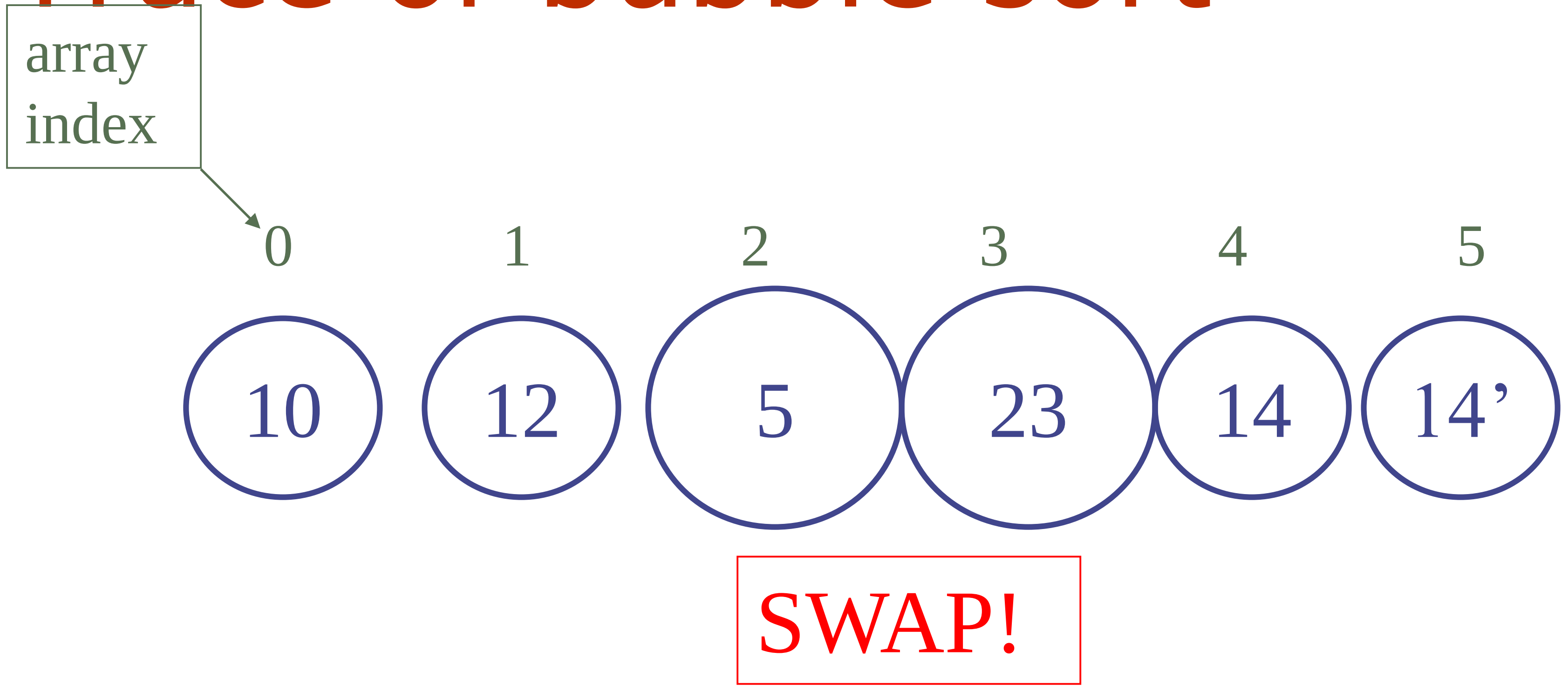
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 2, comparing arr[2] and arr[3]

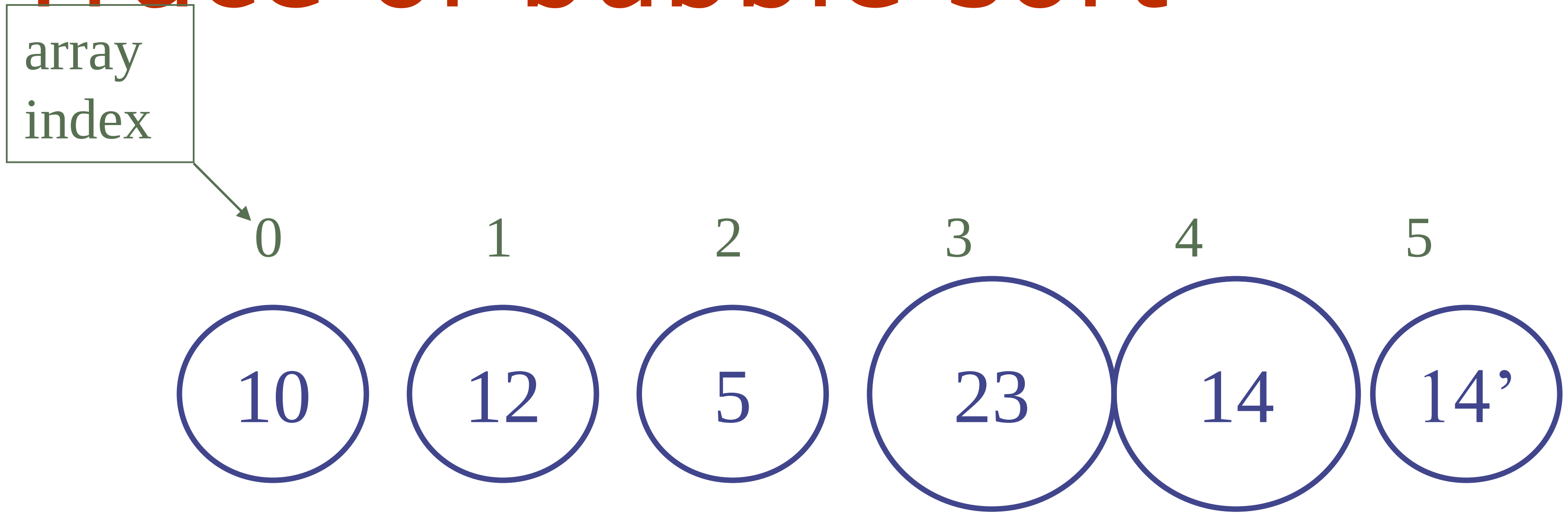
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 2, comparing arr[2] and arr[3]

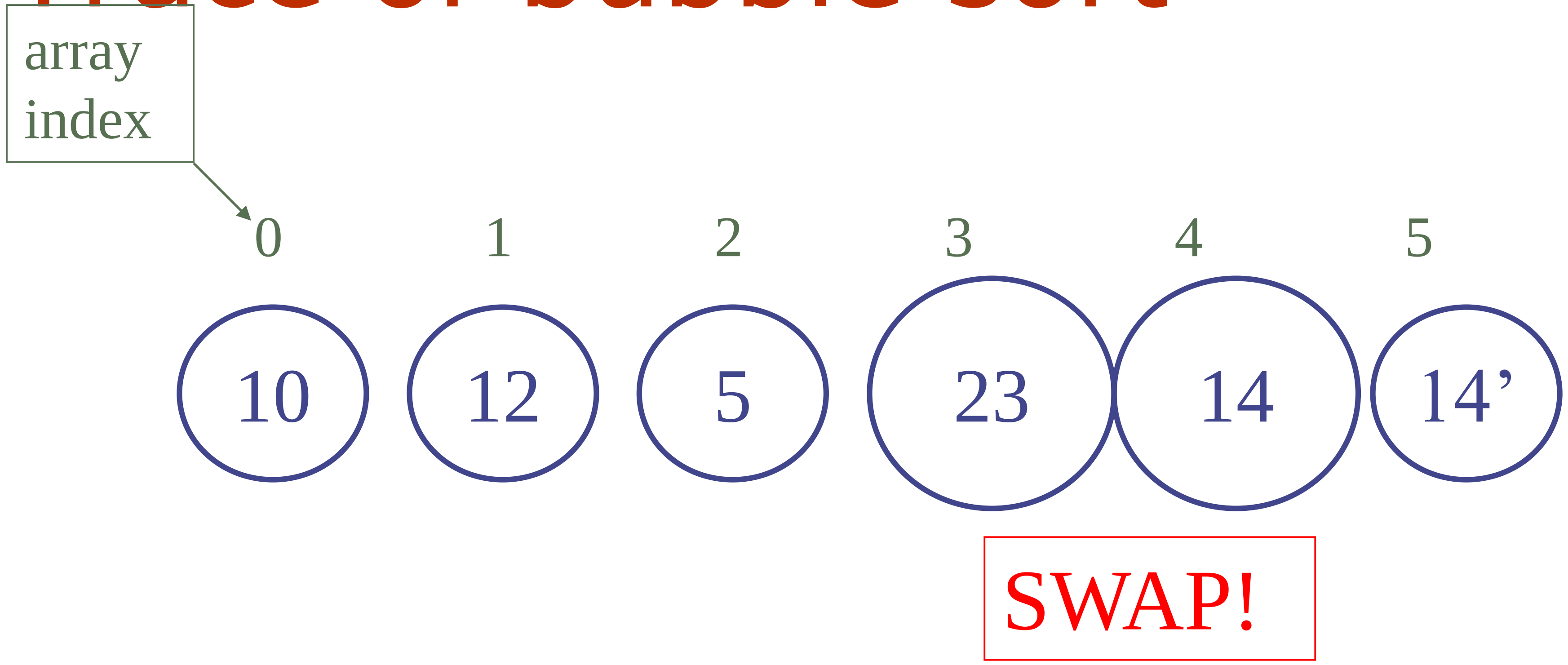
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 3, comparing arr[3] and arr[4]

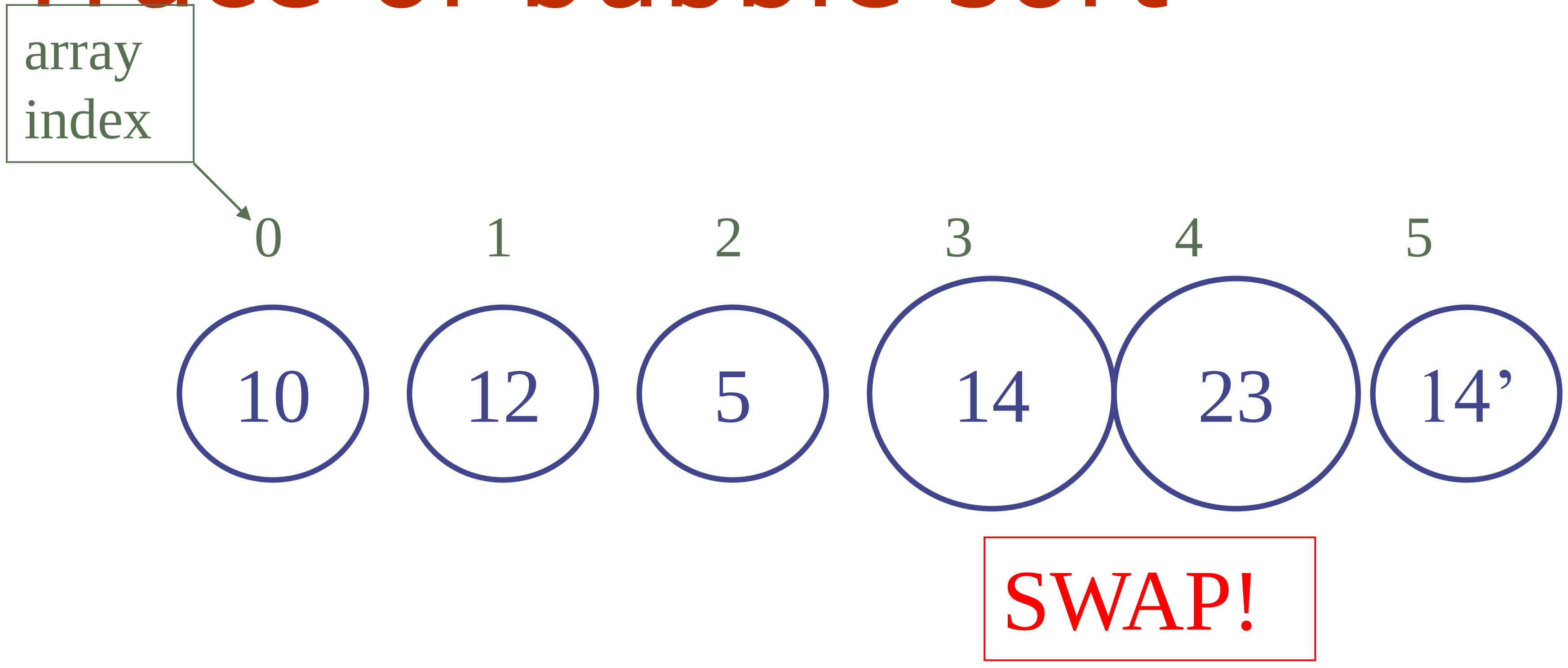
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 3, comparing arr[3] and arr[4]

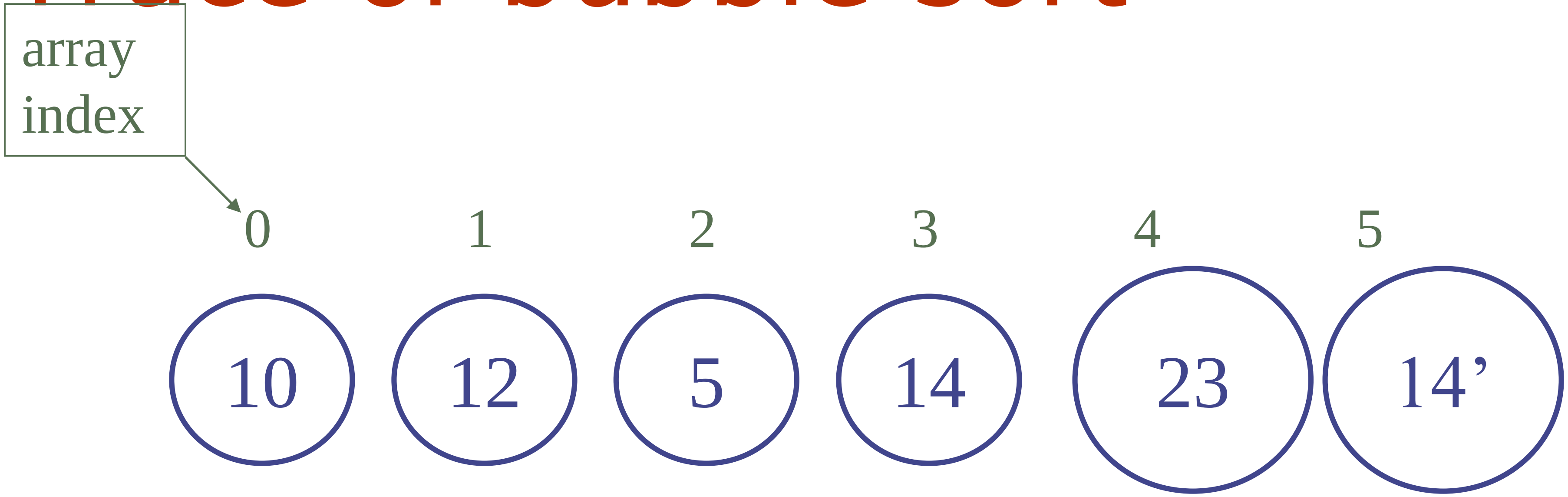
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 3, comparing arr[3] and arr[4]

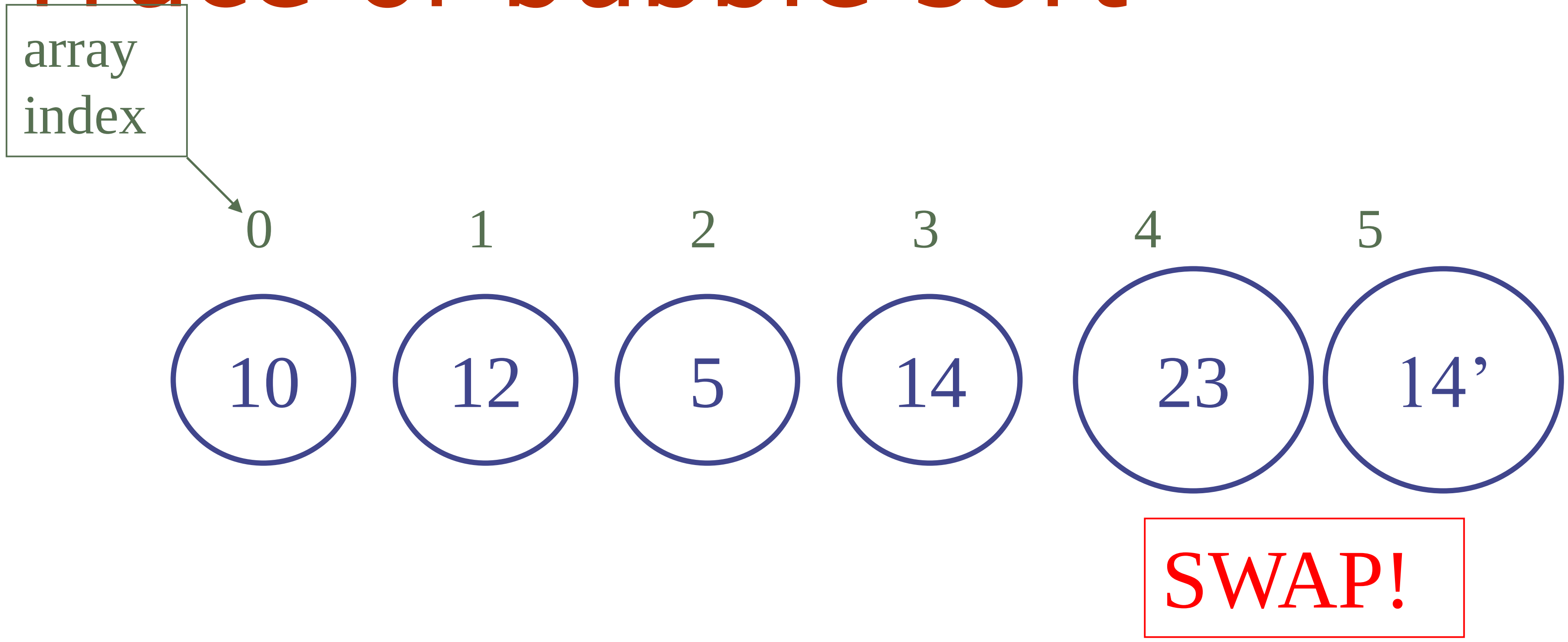
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 4, comparing arr[4] and arr[5]

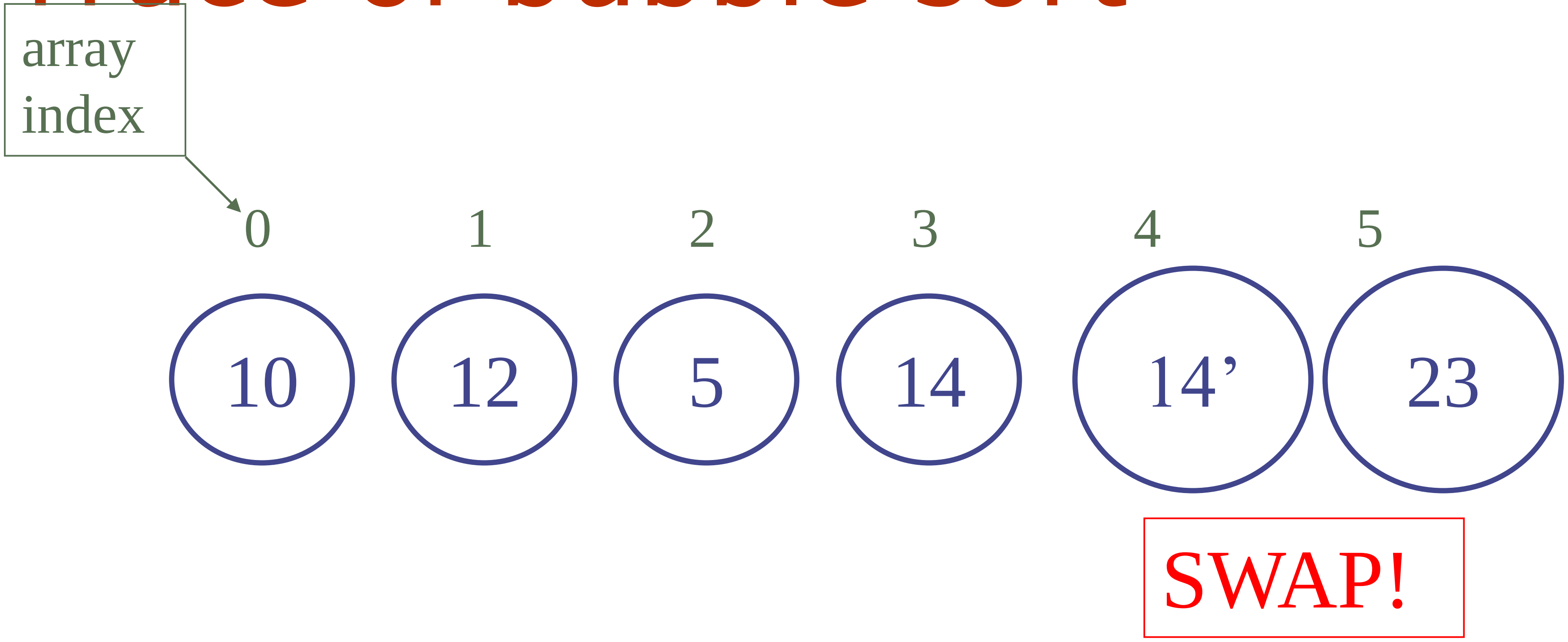
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 4, comparing arr[4] and arr[5]

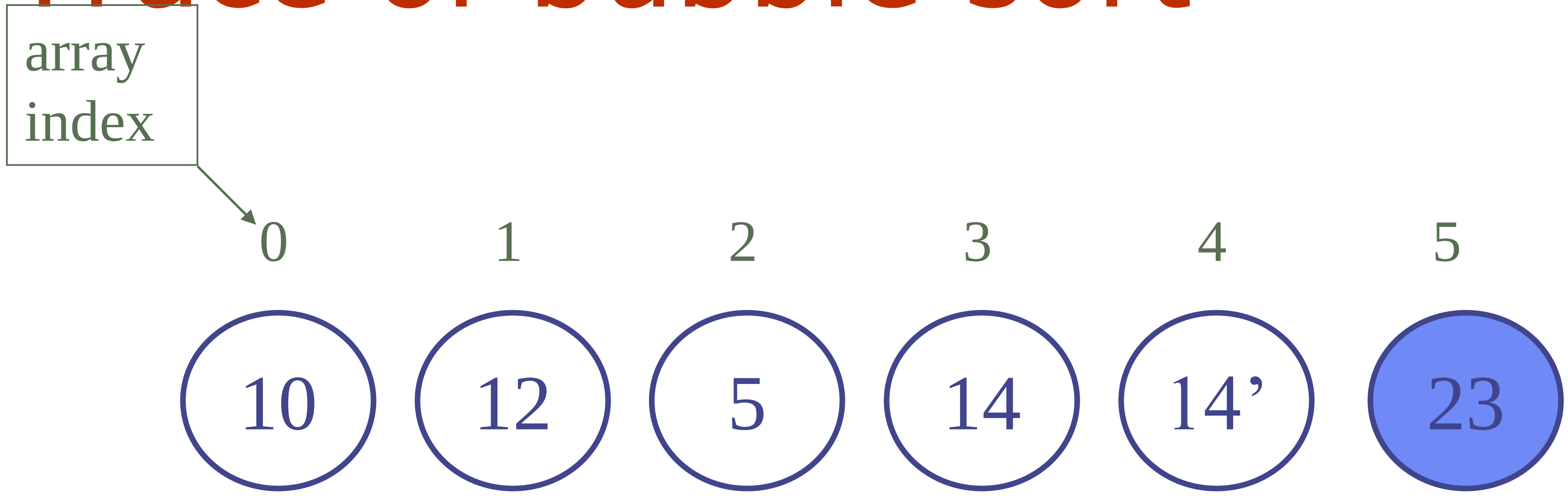
Trace of bubble sort



i = 5, first iteration of the outer loop

j = 4, comparing arr[4] and arr[5]

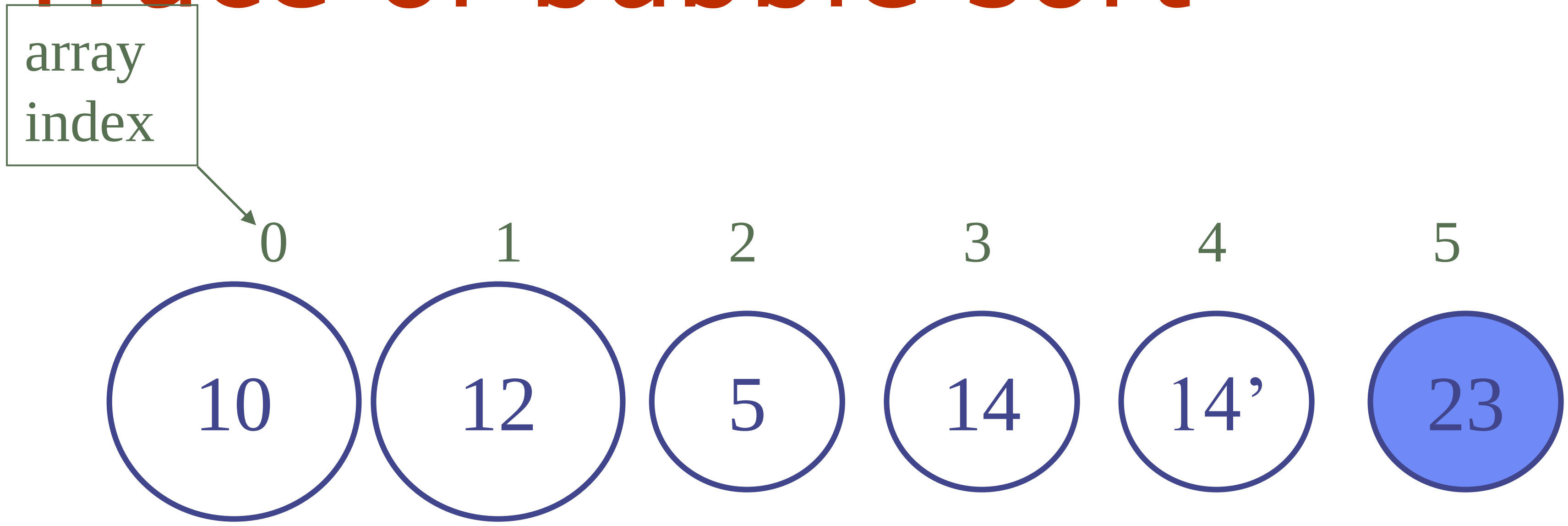
Trace of bubble sort



i = 5, first iteration of the outer loop

inner loop finished; largest element in position 5, positions 0-4 unsorted

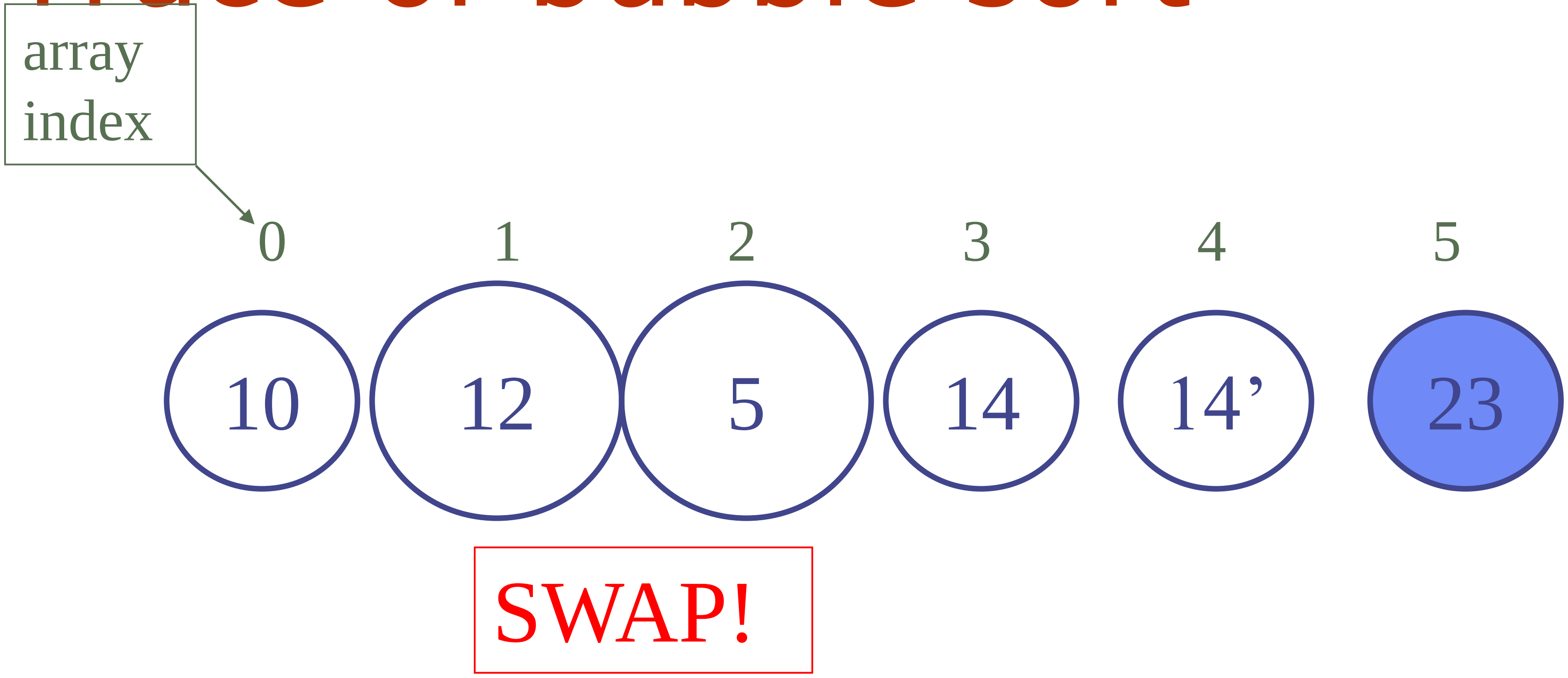
Trace of bubble sort



i = 4, second iteration of the outer loop

j = 0, comparing arr[0] with arr[1]

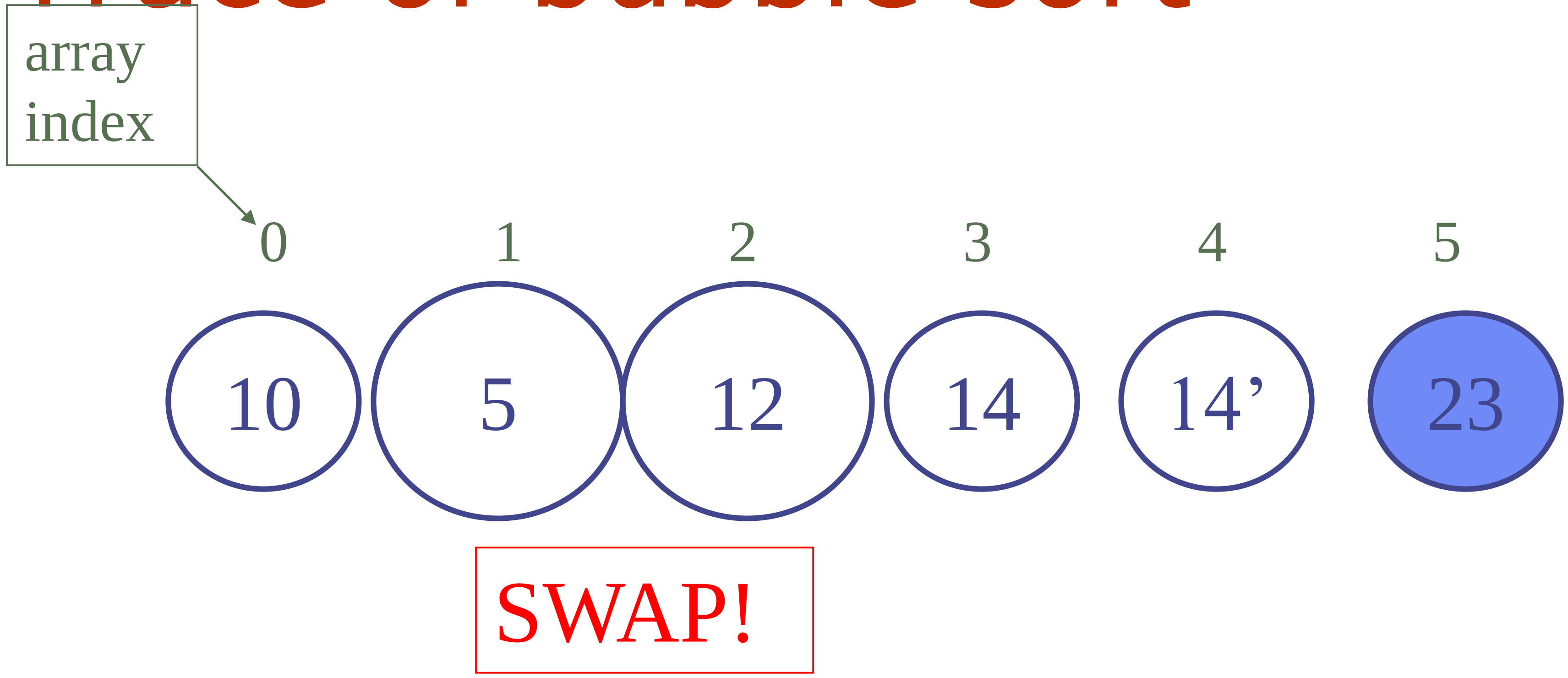
Trace of bubble sort



i = 4, second iteration of the outer loop

j = 1, comparing arr[1] with arr[2]

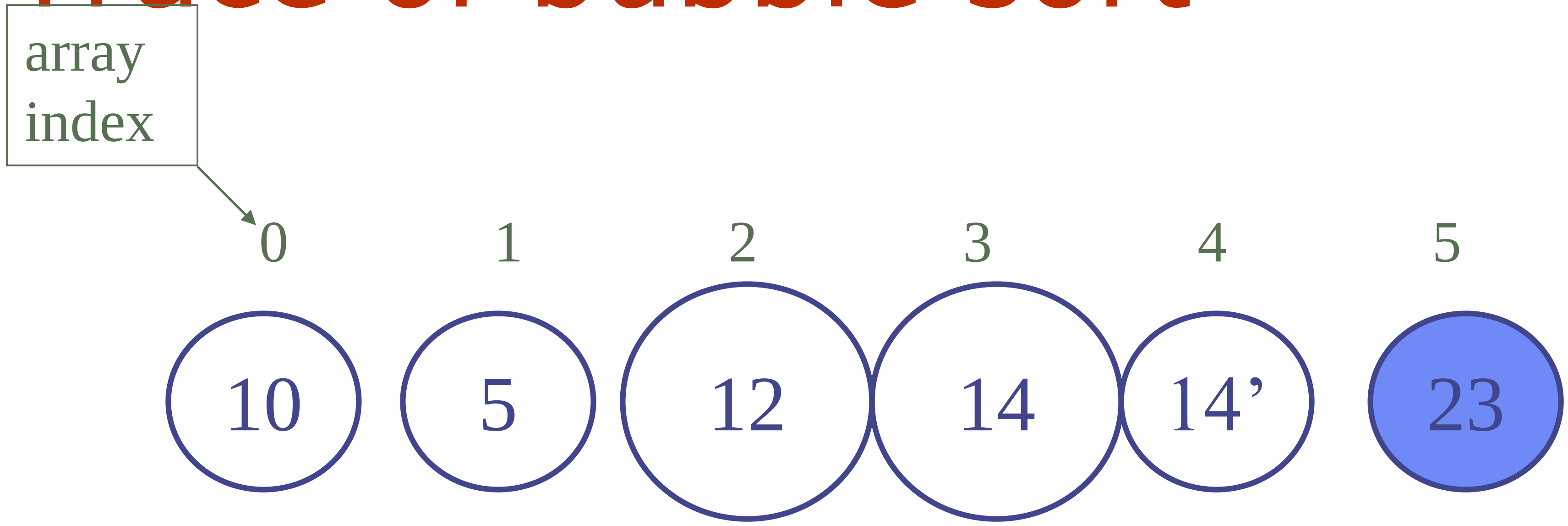
Trace of bubble sort



i = 4, second iteration of the outer loop

j = 1, comparing arr[1] with arr[2]

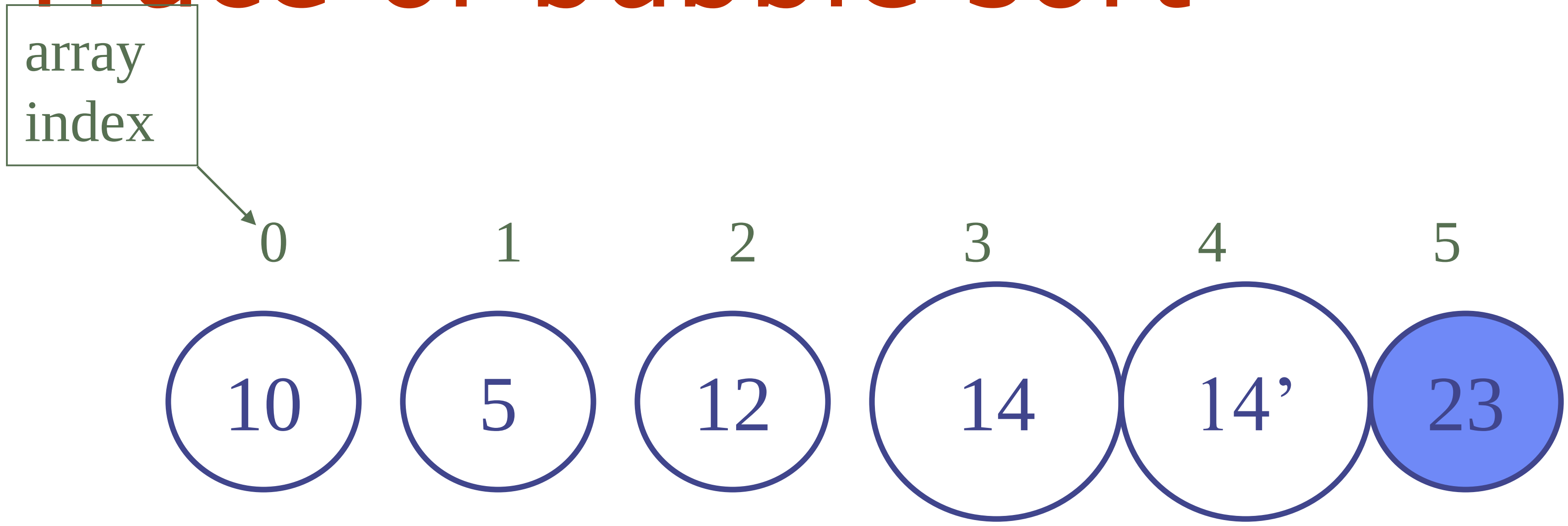
Trace of bubble sort



i = 4, second iteration of the outer loop

j = 2, comparing arr[2] with arr[3]

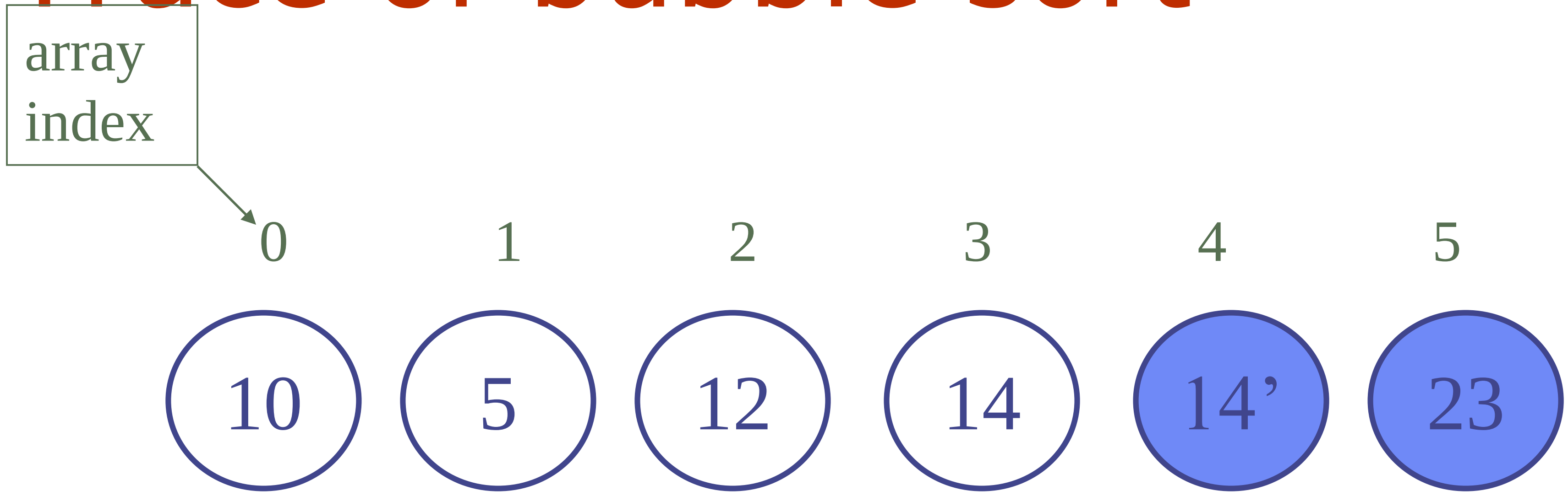
Trace of bubble sort



i = 4, second iteration of the outer loop

j = 3, comparing arr[3] with arr[4]

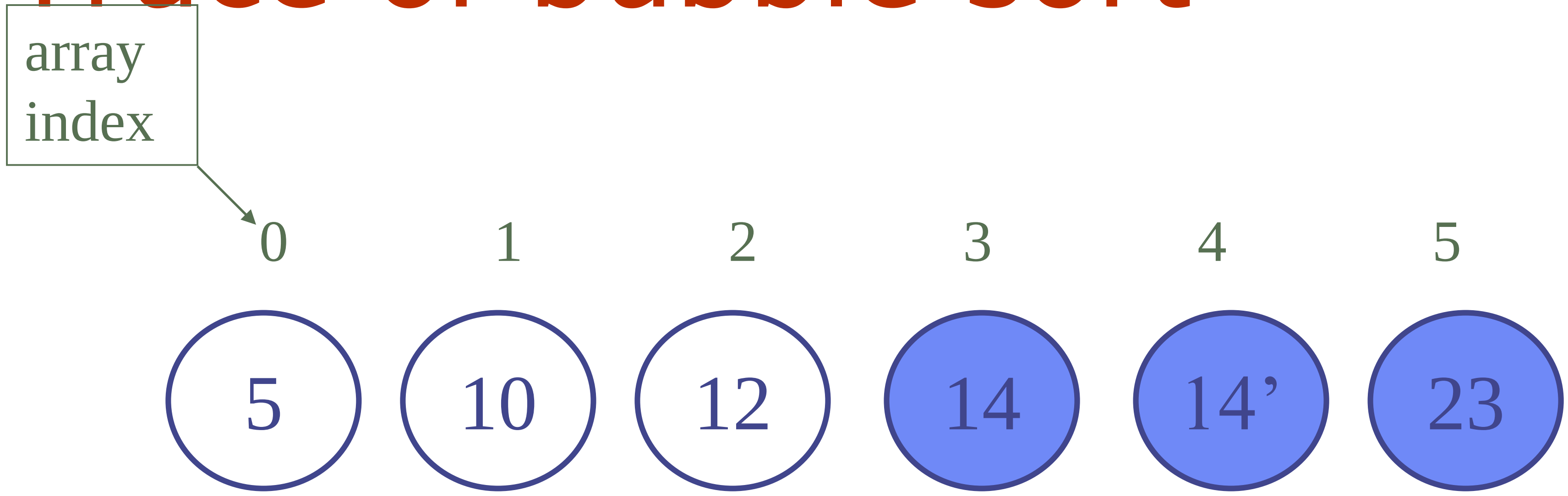
Trace of bubble sort



i = 4, second iteration of the outer loop

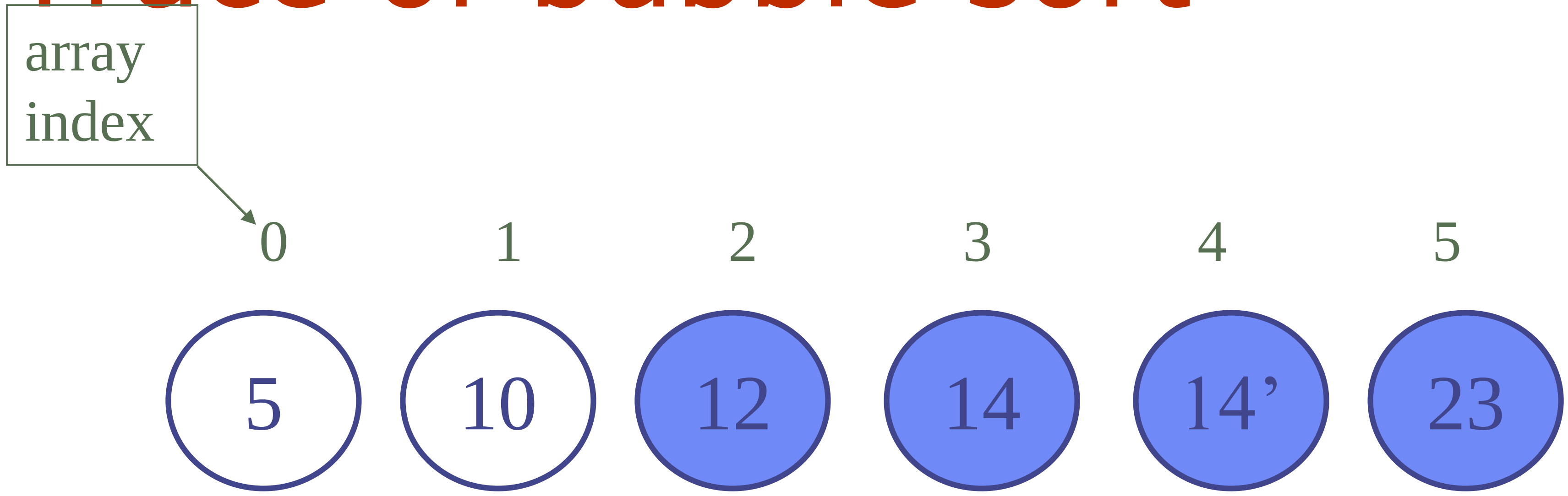
inner loop finished, second largest element in position 4, positions 0-3 unsorted

Trace of bubble sort



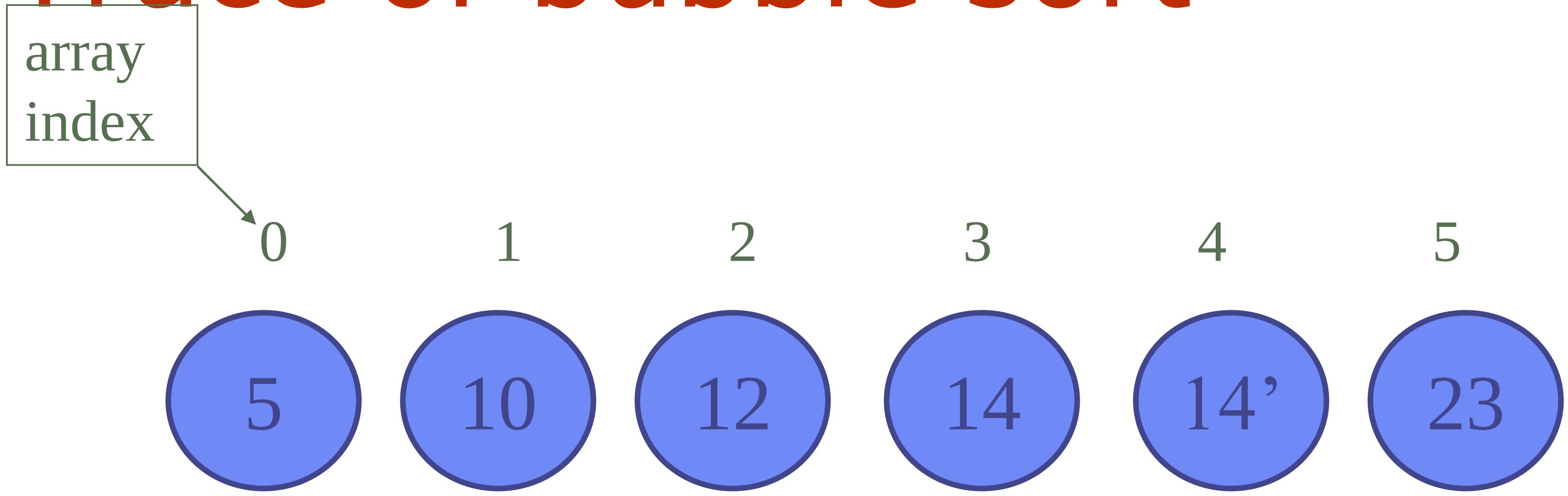
After third iteration…

Trace of bubble sort



After fourth iteration…

Trace of bubble sort



After fifth iteration…

Note: 14 and 14’ are in same relative order as they started

bubble sort is stable

Sorting “Stability”

- If sorting int[] then it does not really matter if the entries are swapped
- Exercise: so why care about stability at all?

Sorting “Stability”

- Often we sorting objects according to some comparison function: compare(o1, o2) returns negative, zero, or positive for “less than”, “equal”, or “greater than”
- compare(o1,o2)=0 means objects o1 and o2 are
  - equal with respect to the desired ordering
  - but not necessarily that they have the same contents
  - “Some objects are more equal than others” (apologies to Orwell)
- E.g. object is a row of a spreadsheet and compare uses just one specified column - many different rows can be equal

Sorting “Stability”

- If sorting a spreadsheet, then might sort by one column then another.
- Do not want the sorting to unnecessarily change the order of the rows, as this can be annoying and confusing.
- “Sort by column A, followed by a stable sort on column B” means that still will have a secondary sort on column B

- **EXERCISE (offline): Experiment with Excel to see if it does stable sorts or not.**

Complexity of bubble sort

- For an array of size n, in the worst case: 1st passage through the inner loop: n-1 comparisons and n-1 swaps
- ...
- (n-1)st passage through the inner loop: one comparison and one swap.
- All together: $t((n-1) + (n-2) + \ldots + 1)$, where t is the time required to do one comparison, one swap, check the inner loop condition and increment j.
- We also spend constant time k declaring i,j,temp and initialising i. Outer loop is executed n-1 times, suppose the cost of checking the loop condition and decrementing i is t_1.

Complexity of bubble sort

$t\,((n-1) + (n-2) + \ldots + 1) + k + t_1(n-1)$

$(n-1) + (n-2) + \ldots + 1 = n(n-1)/2$
[see auxiliary material]
so our function equals

$$t\, n*(n-1)/2 + k + t_1(n-1) =$$
$$\tfrac{1}{2}\, t\,(n^2-n) + t_1\,(n-1) + k$$

complexity $O(n^2)$.

Proof: Complexity of bubble sort

Need to find n_0 and c, such that for all $n \geq n_0$, $\frac{1}{2} t (n^2-n) + t_1(n-1) + k \leq c * n^2$

$\frac{1}{2} t n^2 - \frac{1}{2} t n + t_1 n - t_1 + k \leq$

$\frac{1}{2} t n^2 + t_1 n + k \leq$

$t n^2 + t_1 n^2 + k n^2$ (if $n \geq 1$)

Take $c = t + t_1 + k$ and $n_0 = 1$.

Bubble sort of lists?

- Exercise (online): Is bubble sort also workable for linked lists?
- Bubble sort is just as efficient (or rather inefficient) on linked lists.
    - We can easily bubble sort even a singly linked list.

Bubble sort of singly-linked lists

- Assume we have a class Node with fields: element of type E and next of type Node.
  - (Strictly speaking getter/setter methods would be better, but this is just for the sake of brevity…)
- The List class just has head field.
- Which way are we going to traverse?
  - There is only one way we can traverse! From the head.
- What should we keep track of?
  - A “border” between the unsorted beginning part of the list, and the sorted end of the list

Bubble sort of a linked list

```
Node border = null; // first node in the sorted part
while (border != head) {
   Node current = head; // start from the first node
   while (current.next != border) {
      if (current.element > current.next.element) {
         E element v = current.element;
         current.element = current.next.element;
         current.next.element = v;
      }
      current = current.next;
   }
   border = current; // the sorted part increases by one
}
```

$\Big\}$ swap with the next node if elements out of order

Bubble sort of a linked list

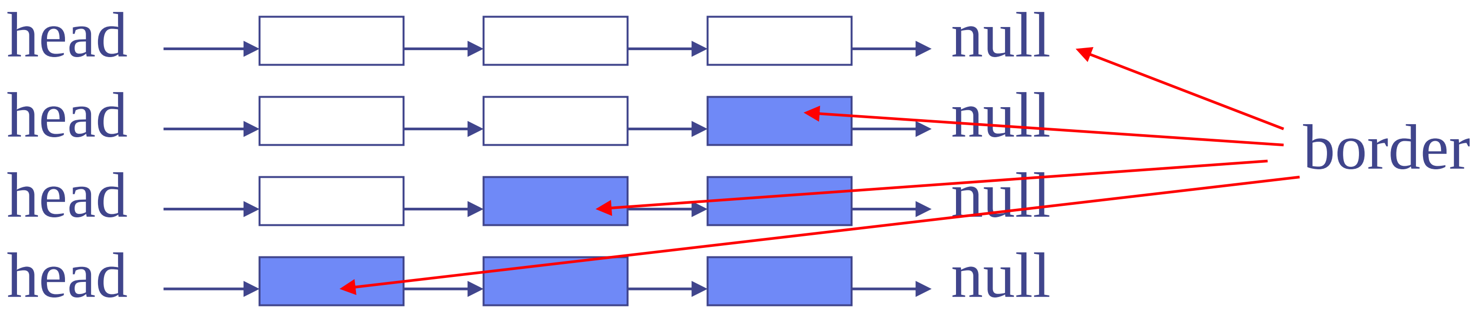
In the previous slide

```
    E element v = current.element;
    current.element = current.next.element;
    current.next.element = v;
```

- Does a direct swap of the contents. This is fine if the element is small.
- This is usually the case as it will be a primitive data type or an object reference.
- One very rarely swaps the actual contents of objects, but usually swap the references to them.
- If necessary then one could instead the references/pointers that give the structure of the list – that is, swap the way that the nodes are built into the list by rearranging the ‘next’ values.

Complexity of bubble sort on lists

- Same complexity as for arrays O(n^2):
  - First time we iterate until we see a null (swapping elements)
  - Second time we iterate until we see the last node;
  - … each time the border is one link closer to the head of the list
  - until border == head.



Selection Sort: Basic Idea

- Similar to bubble sort;
- On each scan:
  - instead of always try to move the “greatest element so far” immediately, we just remember its location and move it at end of scan

EXERCISE: Why one want to do this!?

Exercise: Why delay swaps?

Answer:

- Suppose that the entries are large then a swap operation might be quite expensive
- So might want to reduce the number of swaps by directly moving entries to “the right place”

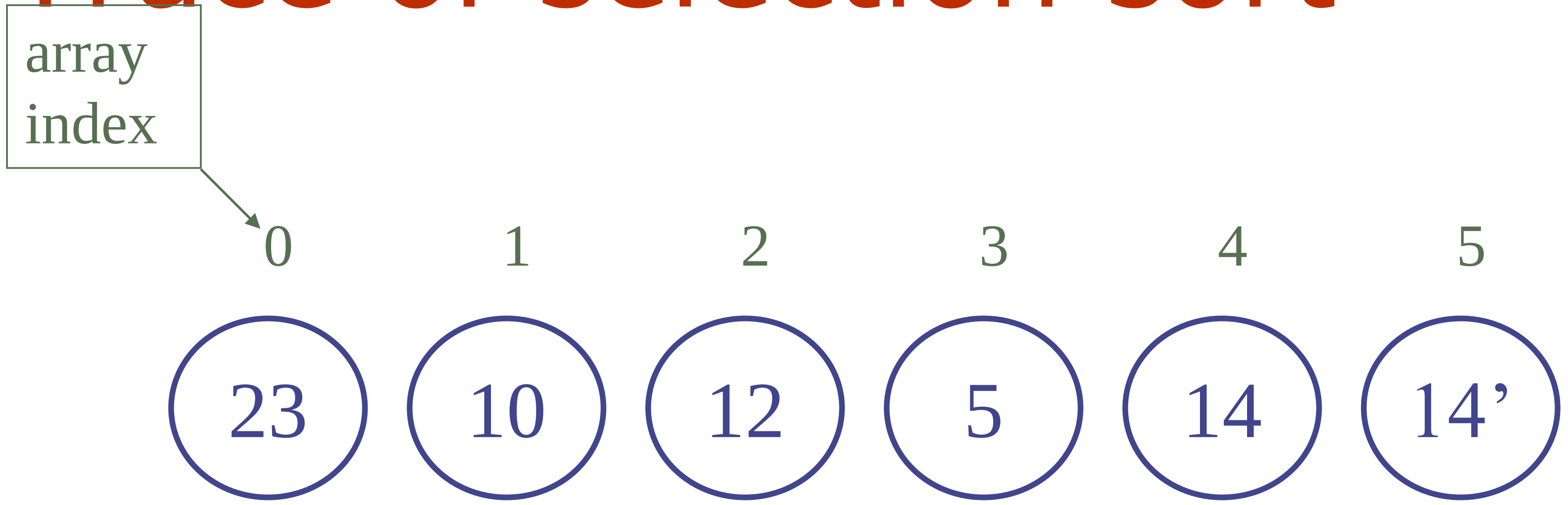
Selection sort

```
void  selectionSort(int arr[]){
  int i, j, temp, pos_greatest;
  for( i = arr.length-1; i > 0; i--){
     pos_greatest = 0;
     for(j = 0; j <= i; j++){
       if( arr[j] >= arr[pos_greatest])
          pos_greatest =  j;
     }//end inner for loop
     if ( i != pos_greatest ) {
       temp = arr[i];
       arr[i] = arr[pos_greatest];
        arr[pos_greatest] = temp; }
   }//end outer for loop
}//end selection sort
```

compare the current element to the largest seen so far; if it is larger, remember its index

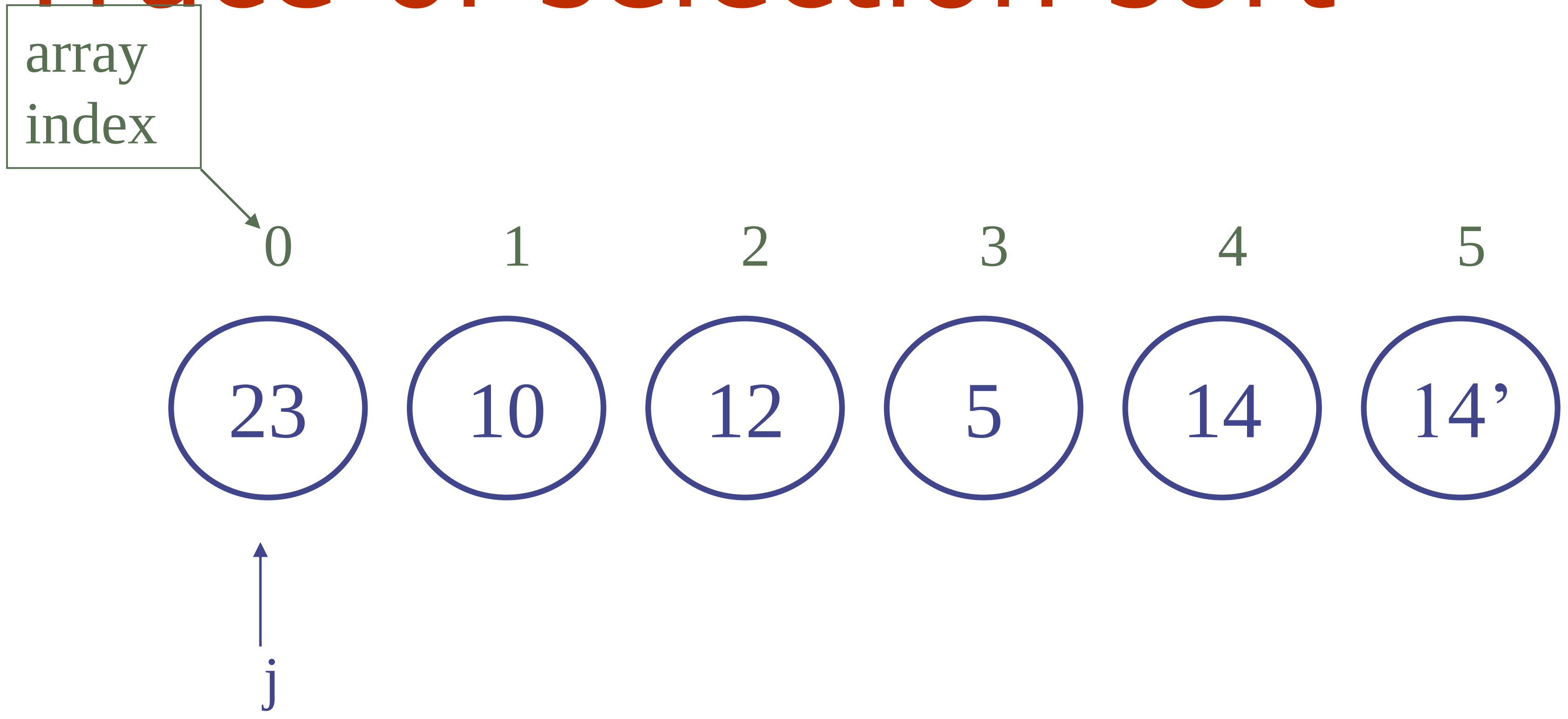
swap the largest element to the end of range, if not already there

Trace of selection sort



i = 5, first iteration of the outer loop

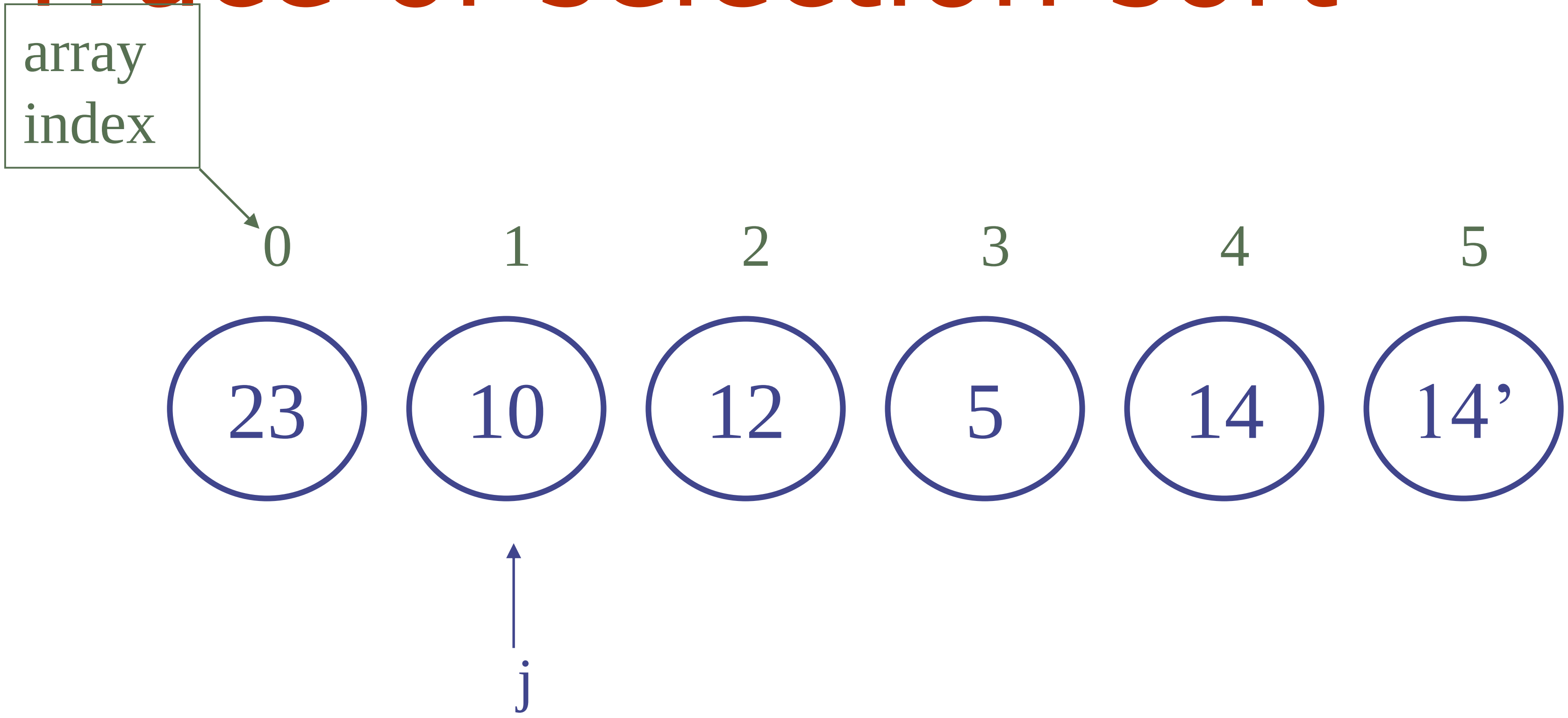
Trace of selection sort



i = 5, first iteration of the outer loop

j = 0, pos_greatest = 0

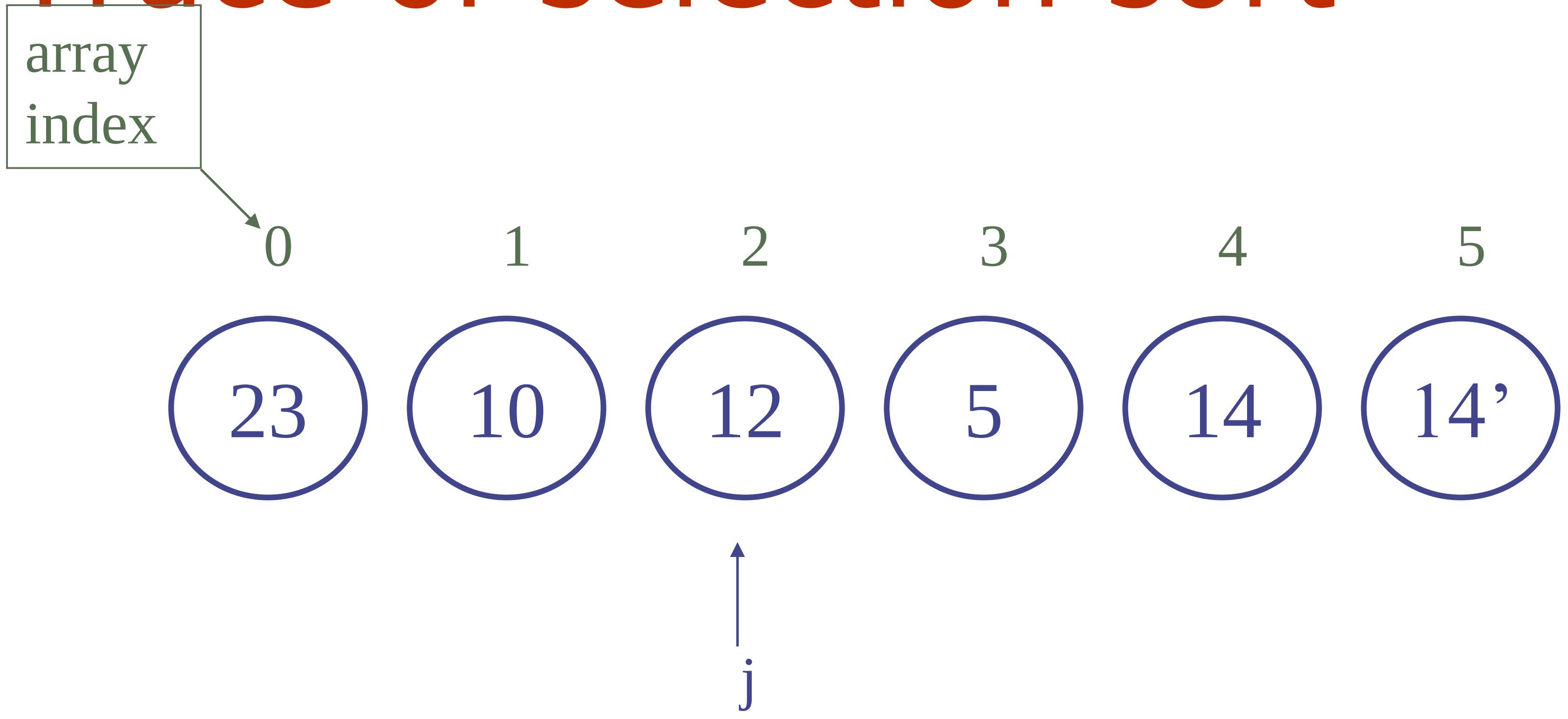
Trace of selection sort



i = 5, first iteration of the outer loop

j = 1, pos_greatest = 0

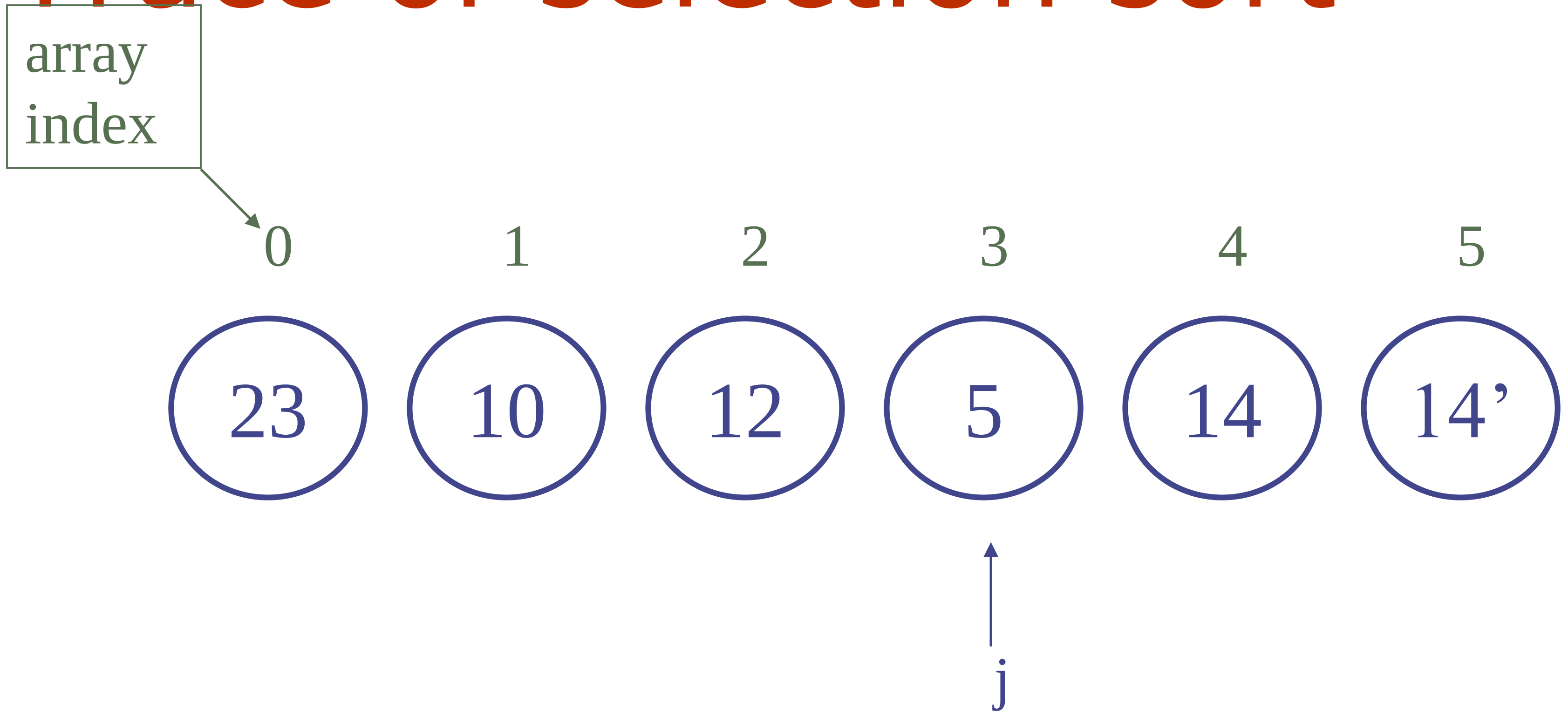
Trace of selection sort



i = 5, first iteration of the outer loop

j = 2, pos_greatest = 0

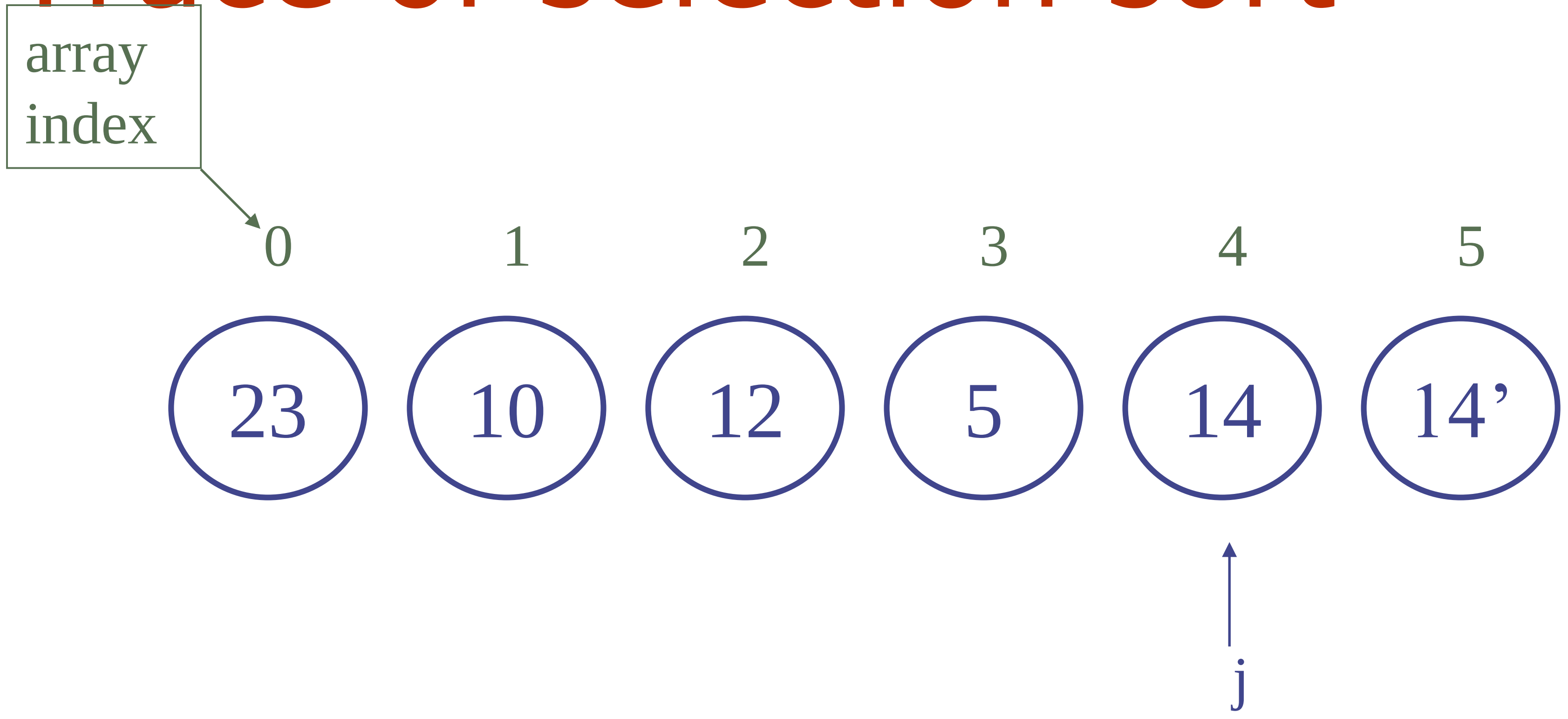
Trace of selection sort



i = 5, first iteration of the outer loop

j = 3, pos_greatest = 0

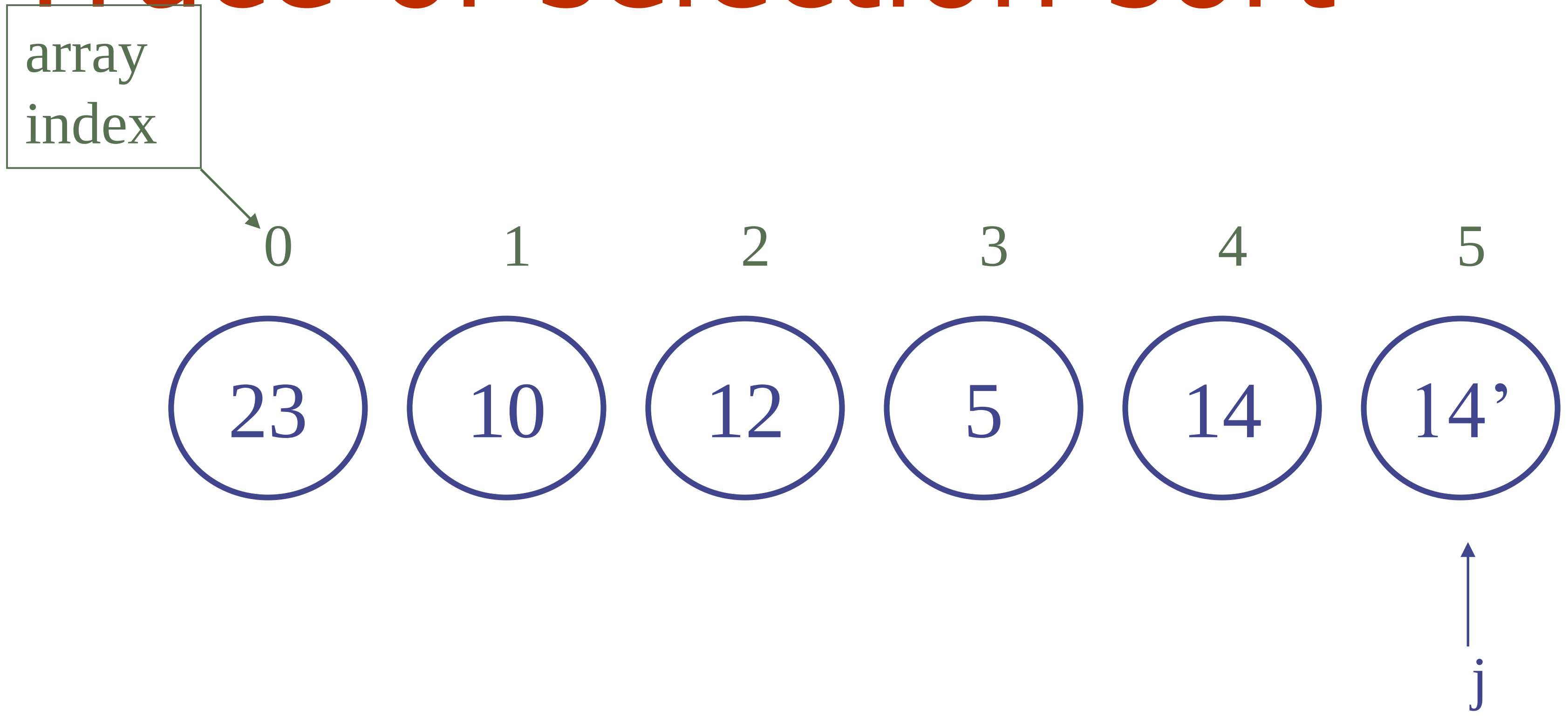
Trace of selection sort



i = 5, first iteration of the outer loop

j = 4, pos_greatest = 0

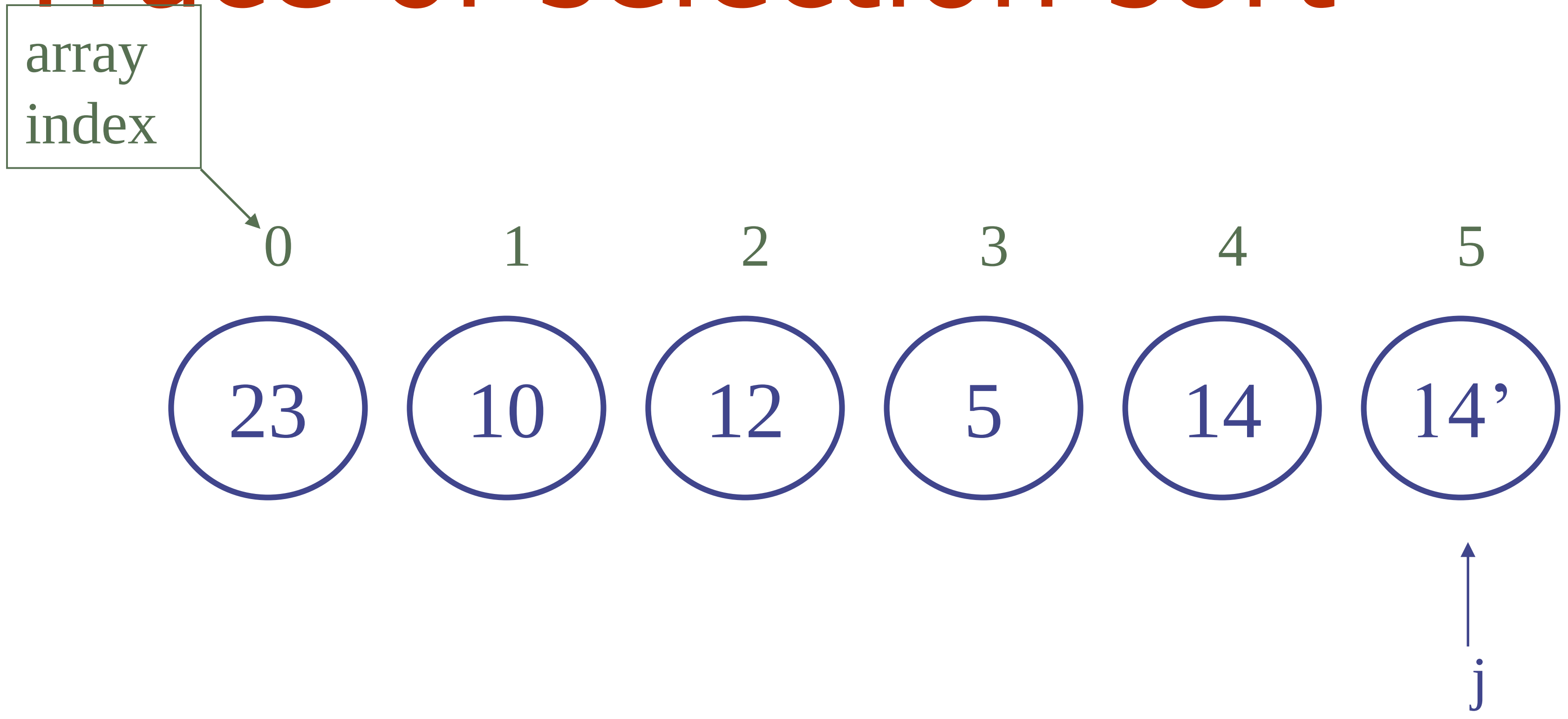
Trace of selection sort



i = 5, first iteration of the outer loop

j = 5, pos_greatest = 0

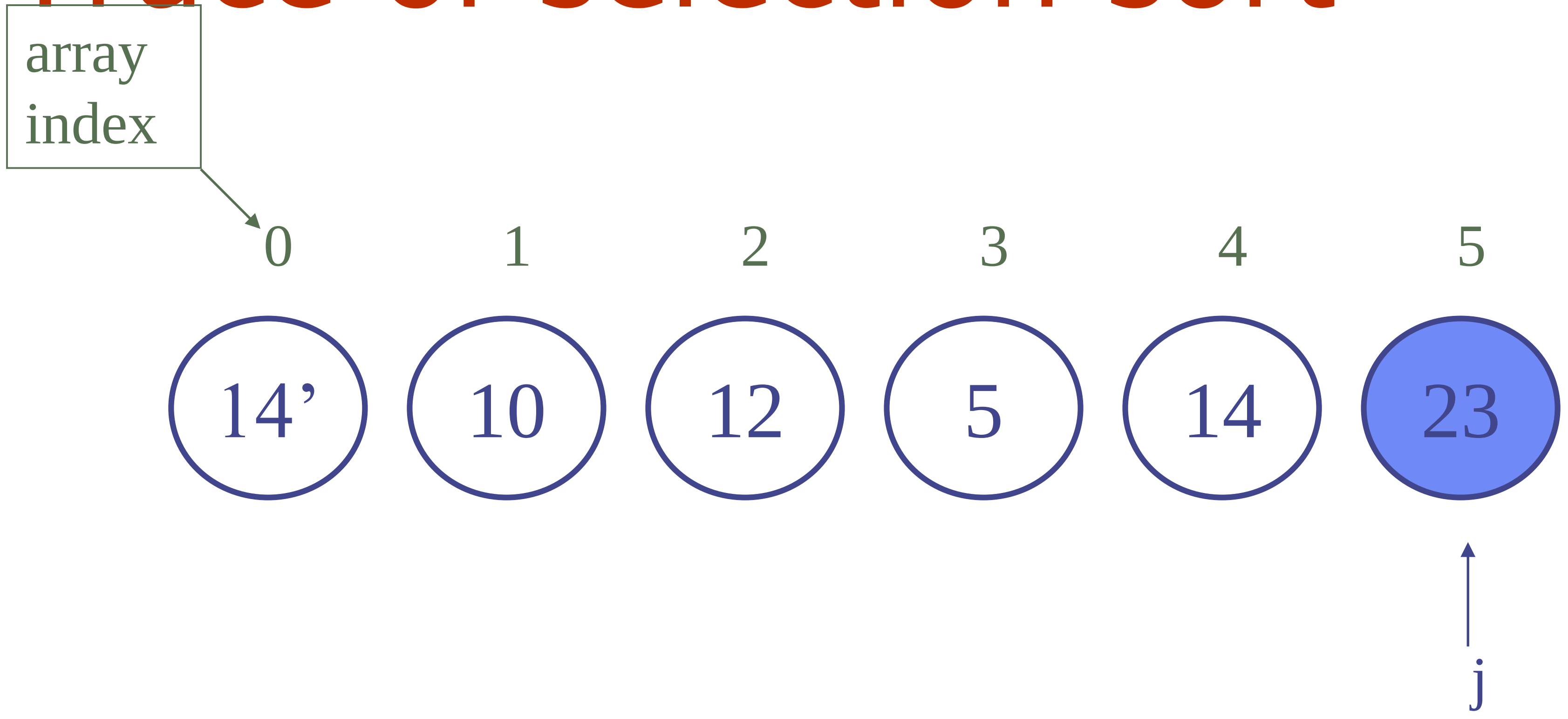
Trace of selection sort



i = 5, first iteration of the outer loop

swap element at pos_greatest=0 to 5

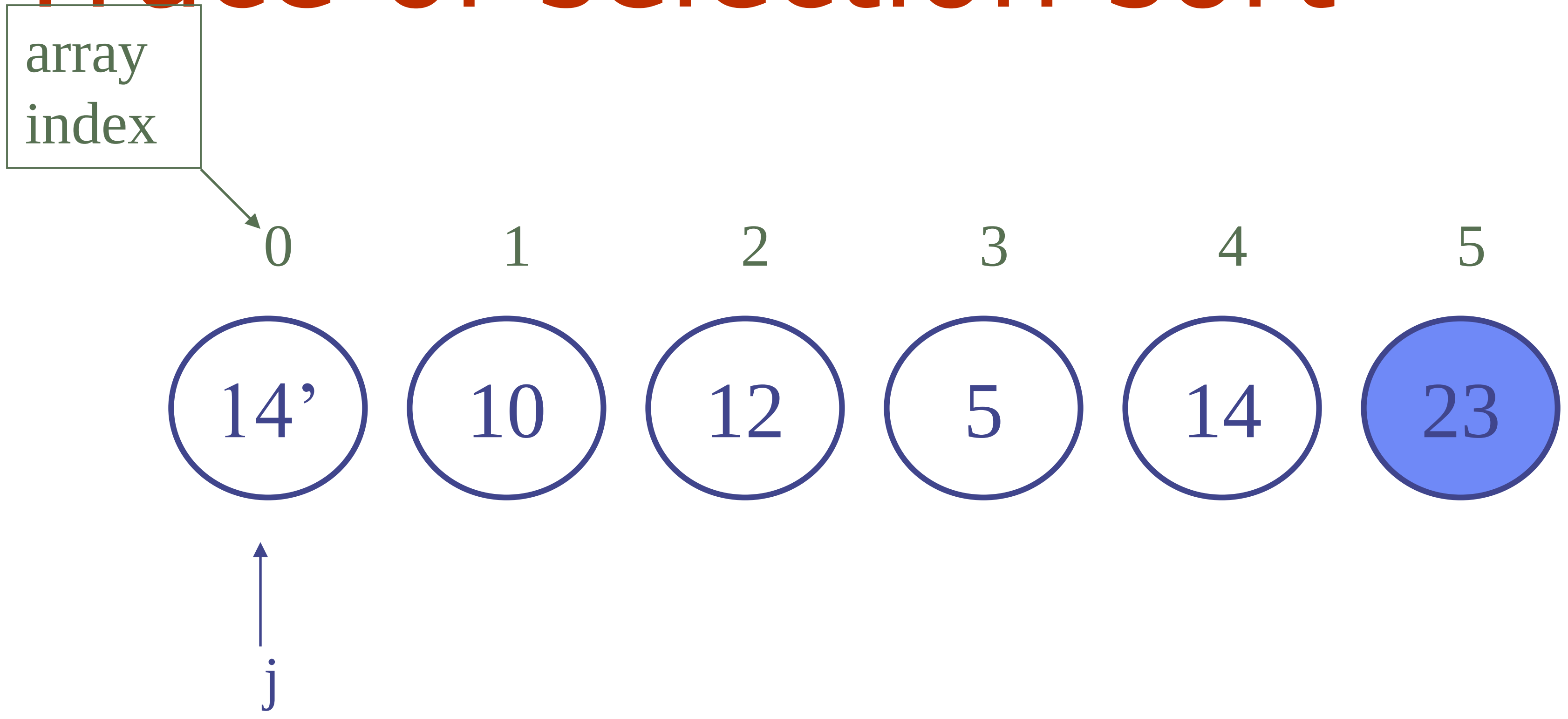
Trace of selection sort



i = 5, first iteration of the outer loop

swap element at pos_greatest=0 to 5

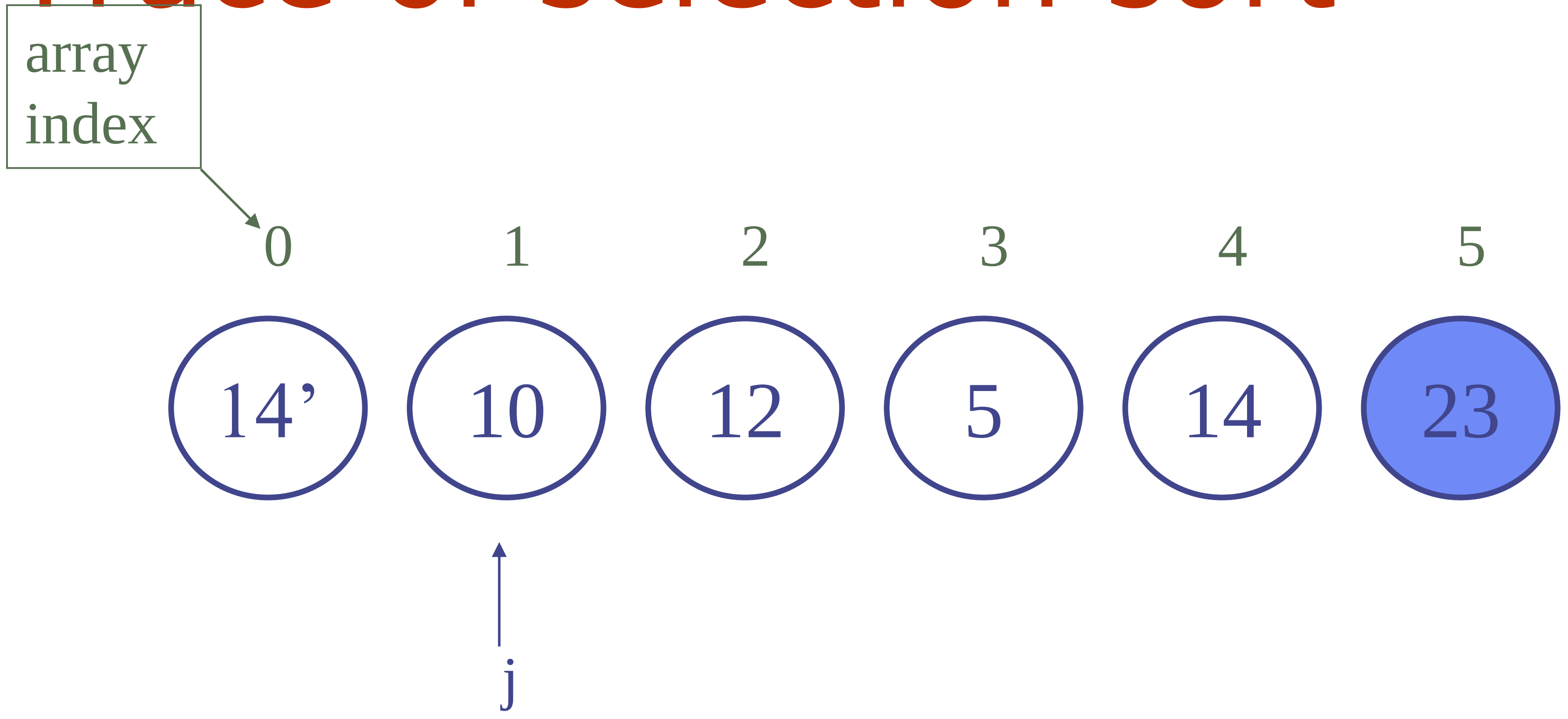
Trace of selection sort



i = 4, second iteration of the outer loop

j = 0, pos_greatest = 0

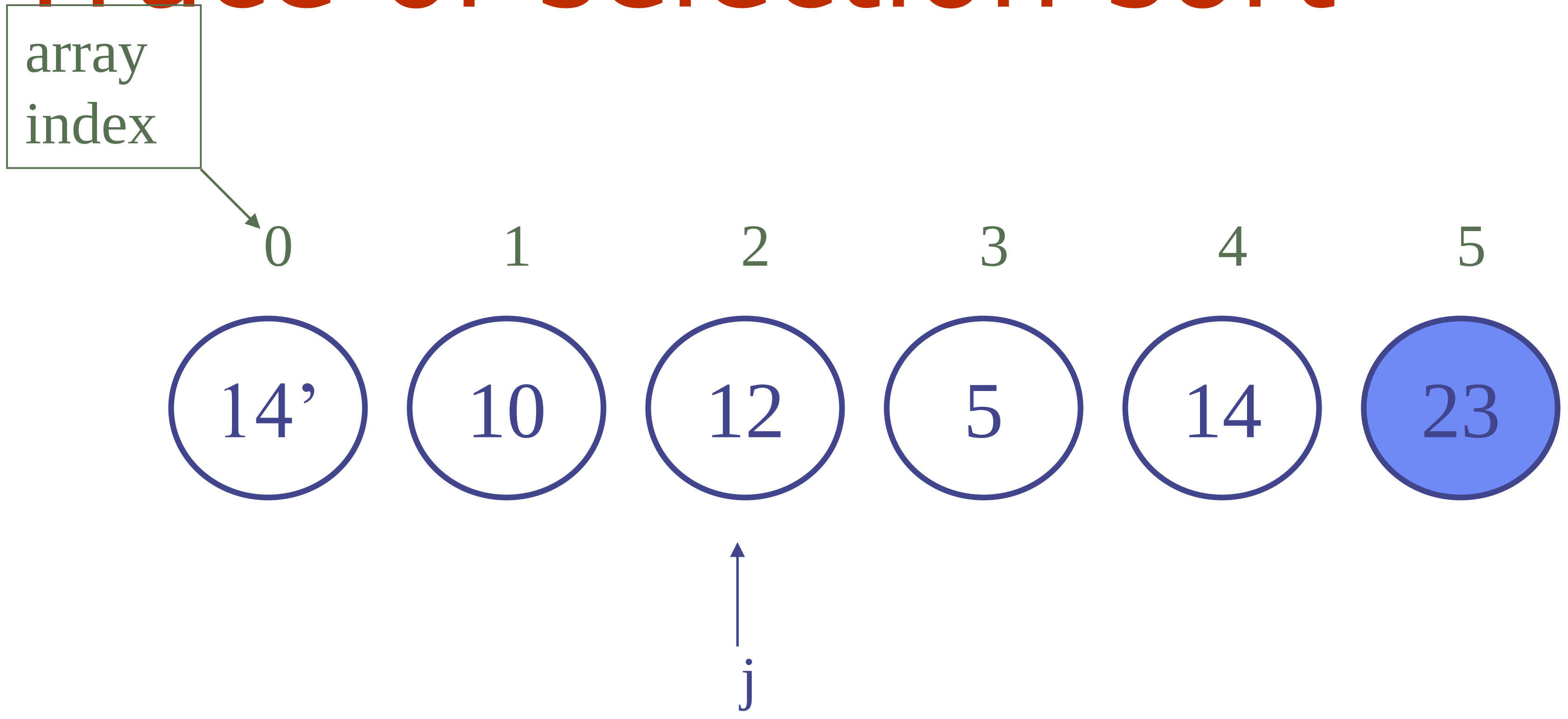
Trace of selection sort



i = 4, second iteration of the outer loop

j = 1, pos_greatest = 0

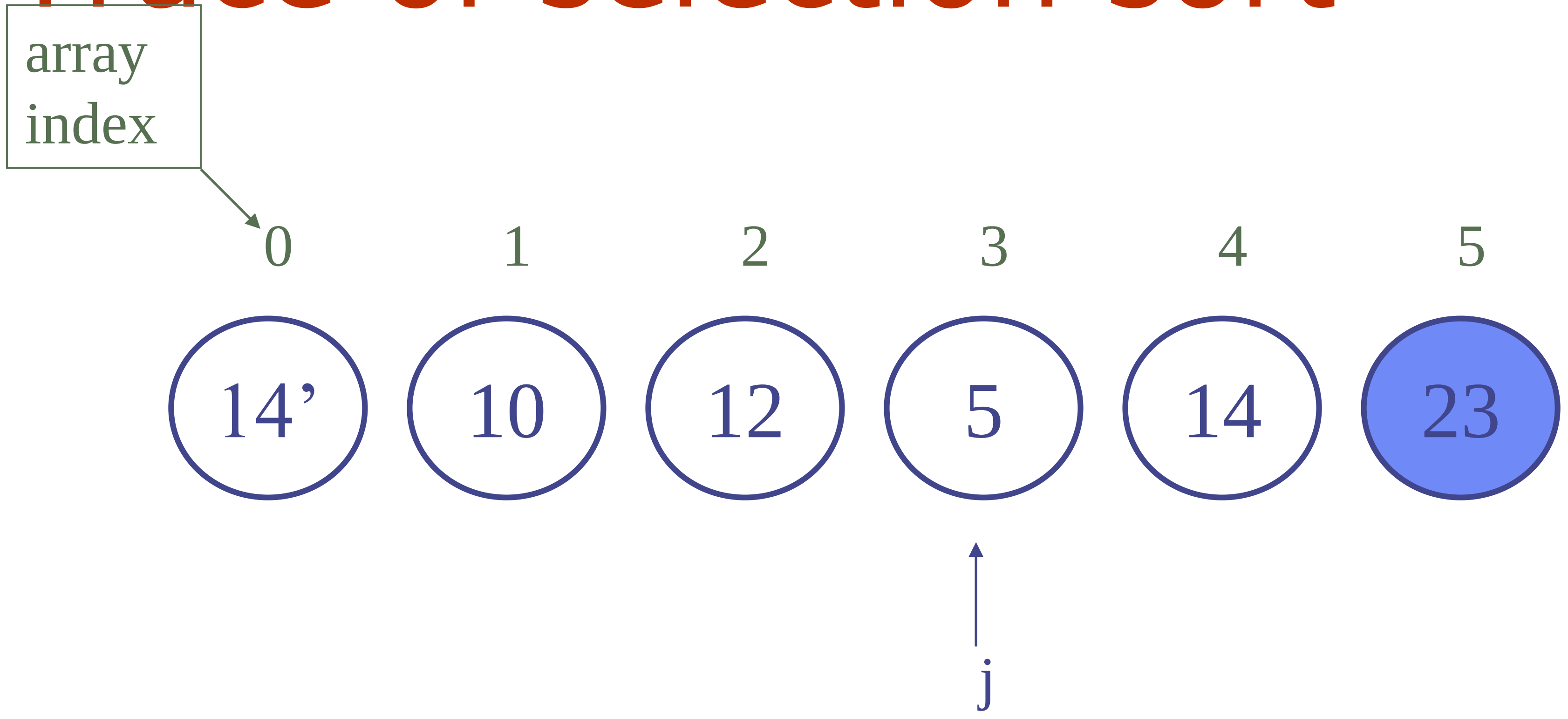
Trace of selection sort



i = 4, second iteration of the outer loop

j = 2, pos_greatest = 0

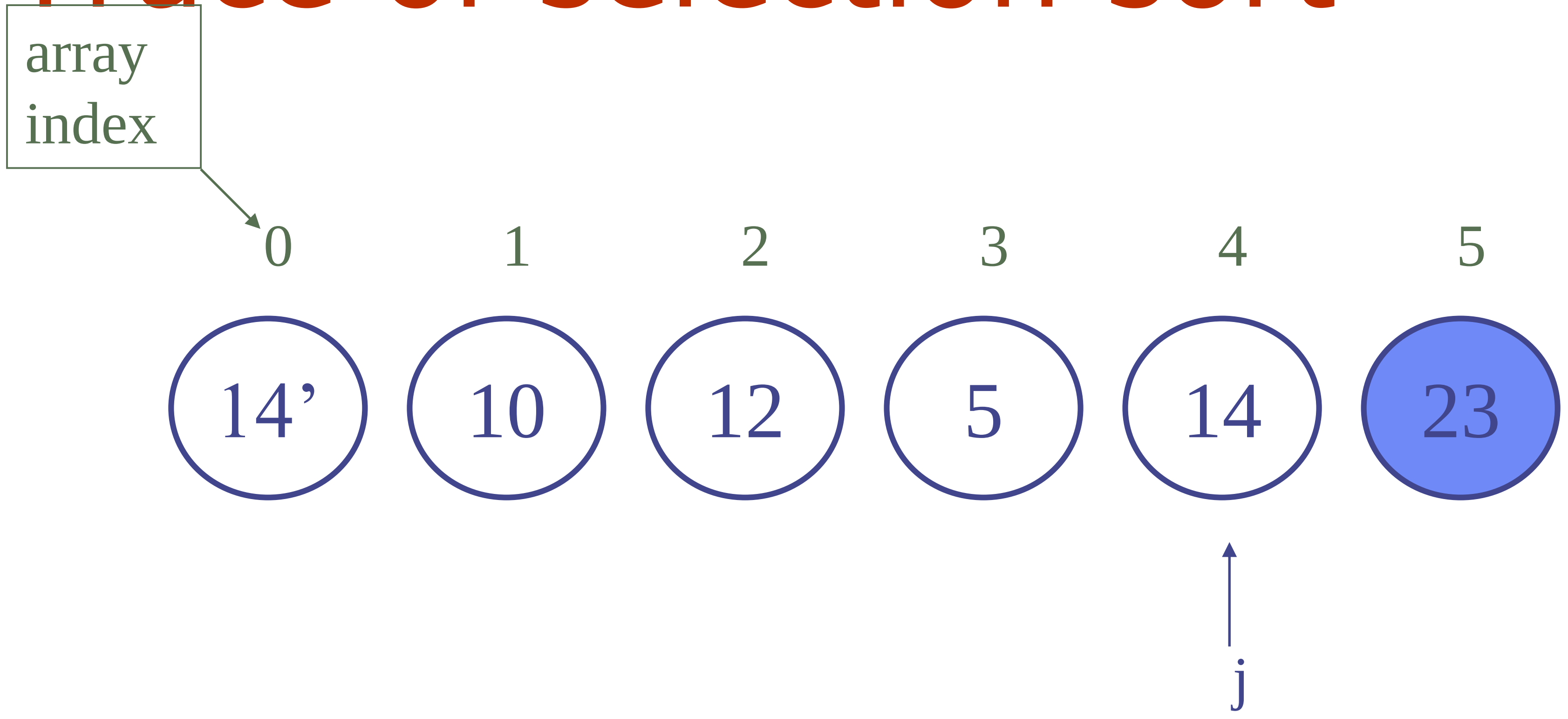
Trace of selection sort



i = 4, second iteration of the outer loop

j = 3, pos_greatest = 0

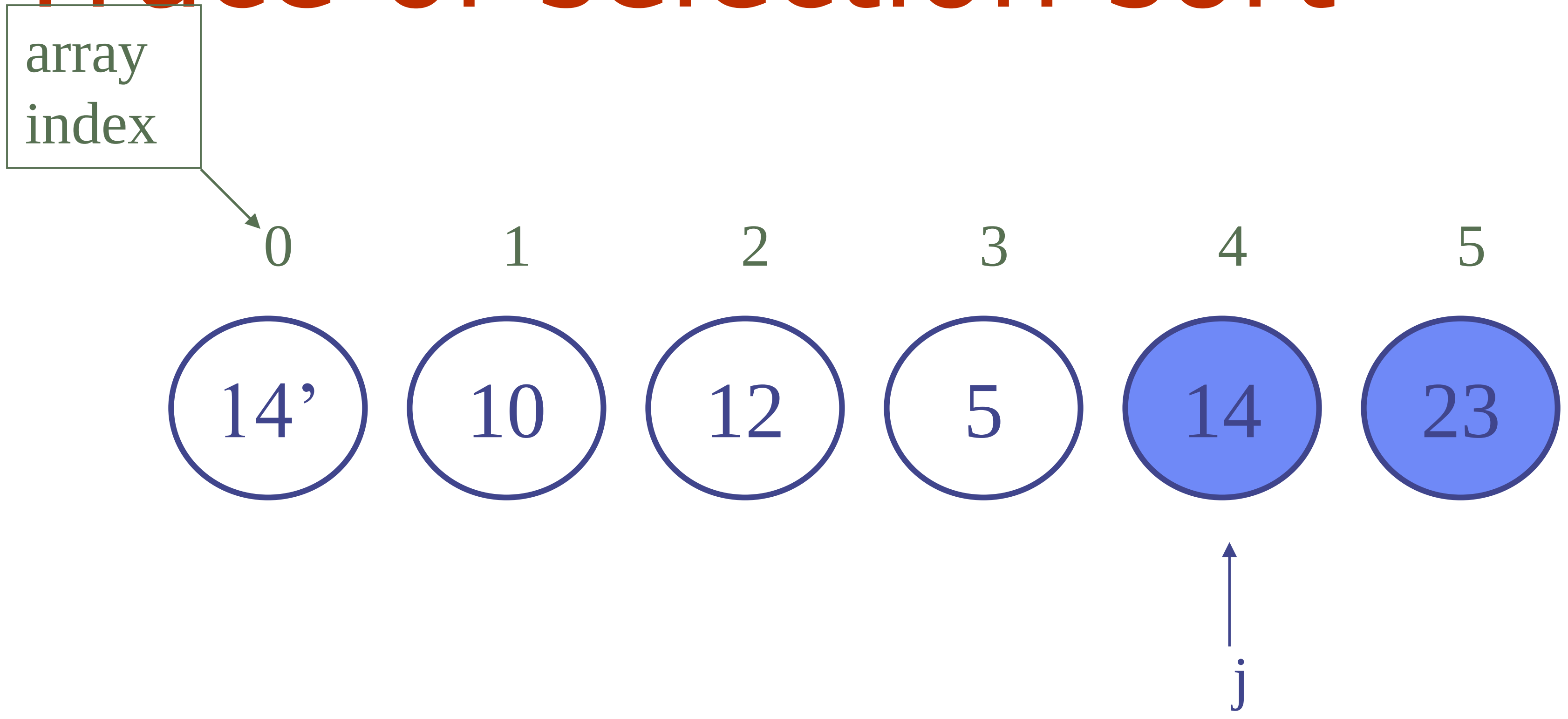
Trace of selection sort



i = 4, second iteration of the outer loop

j = 4, pos_greatest = 4 (because the ">=" in the comparison)

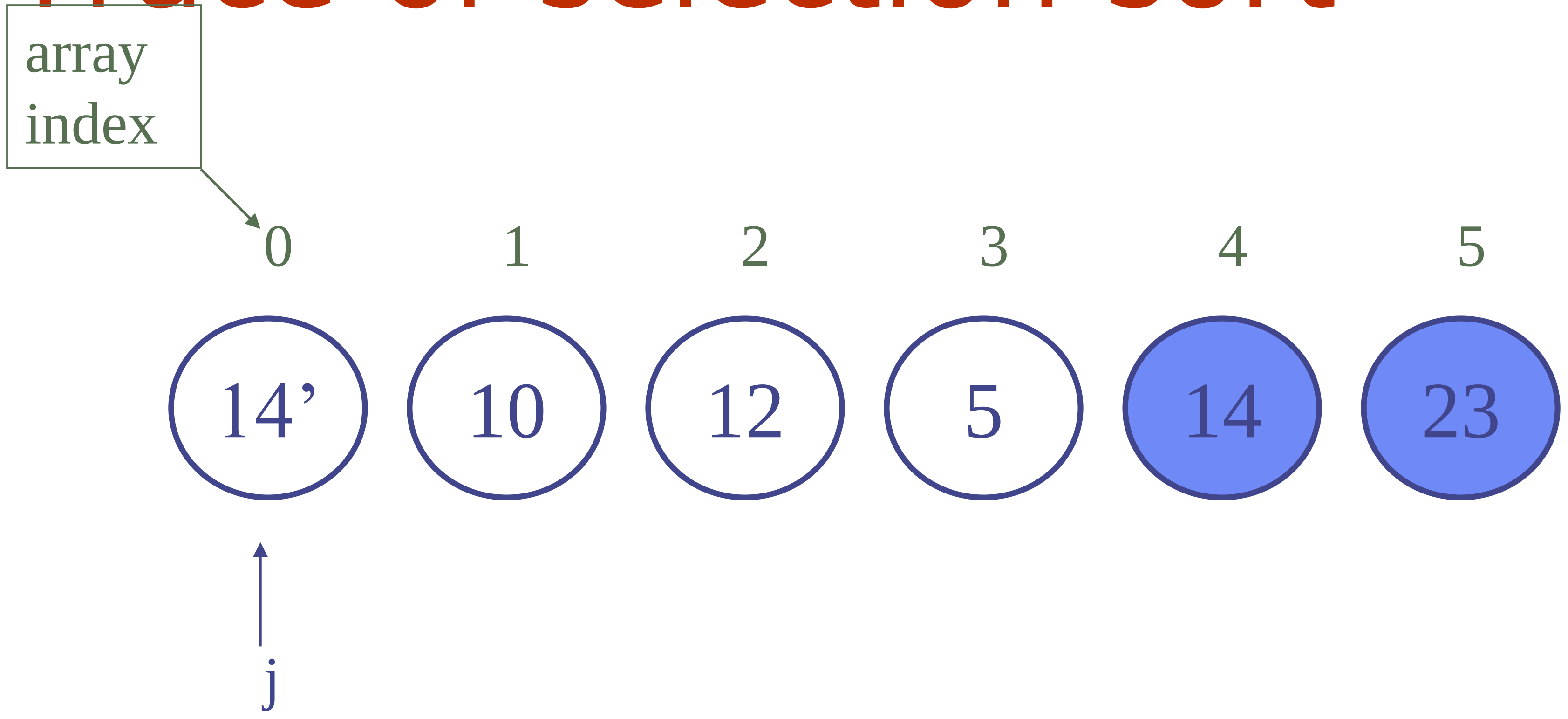
Trace of selection sort



i = 4, second iteration of the outer loop

No need to swap elements

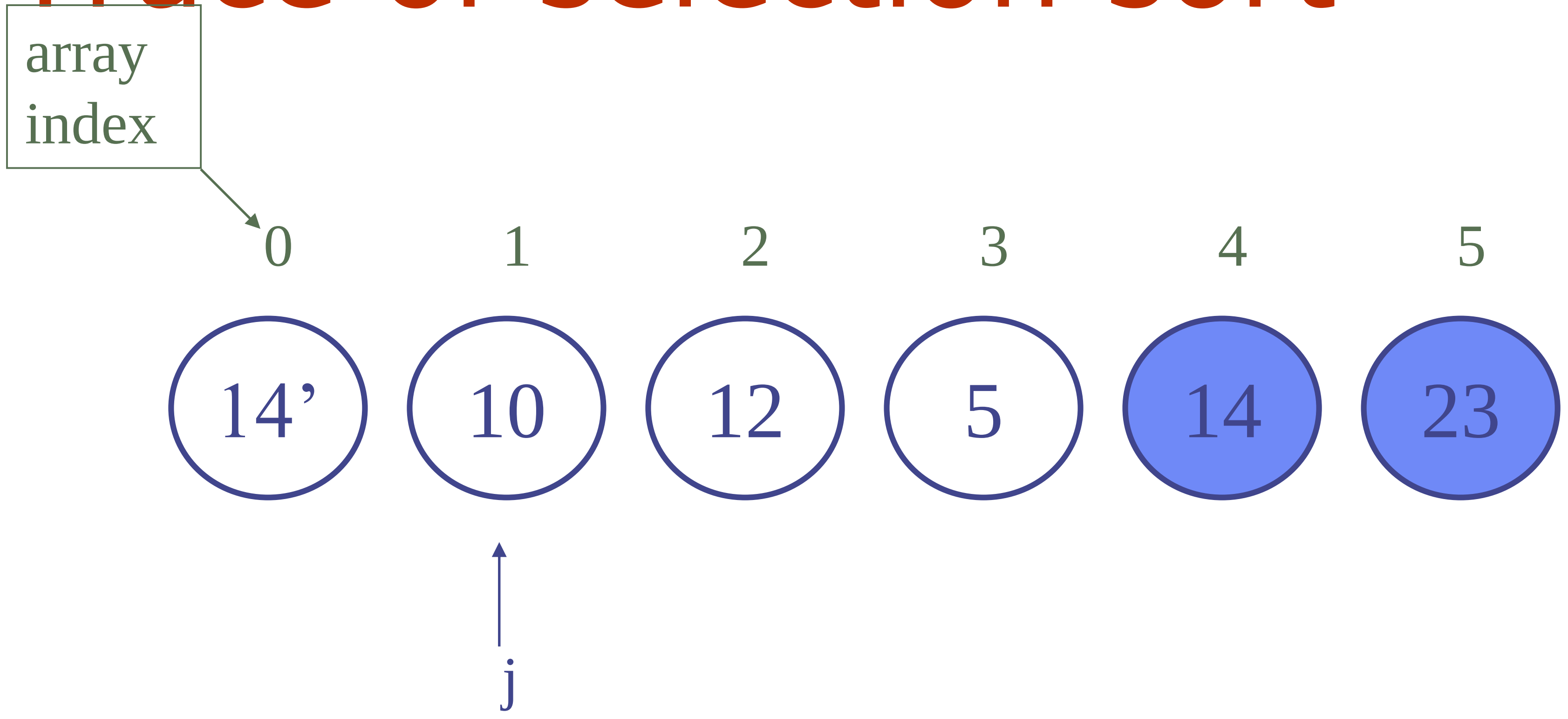
Trace of selection sort



i = 3, third iteration of the outer loop

j = 0, pos_greatest = 0

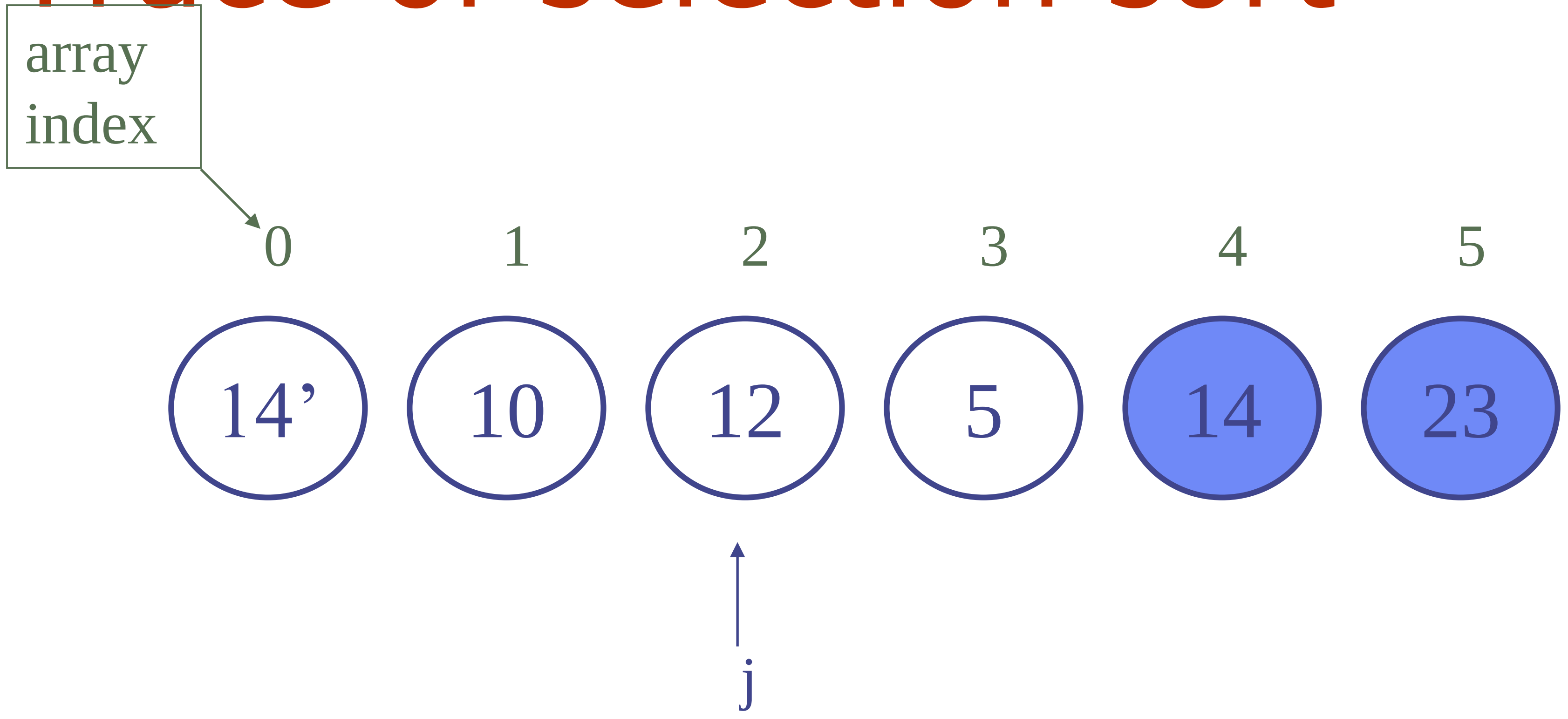
Trace of selection sort



i = 3, third iteration of the outer loop

j = 1, pos_greatest = 0

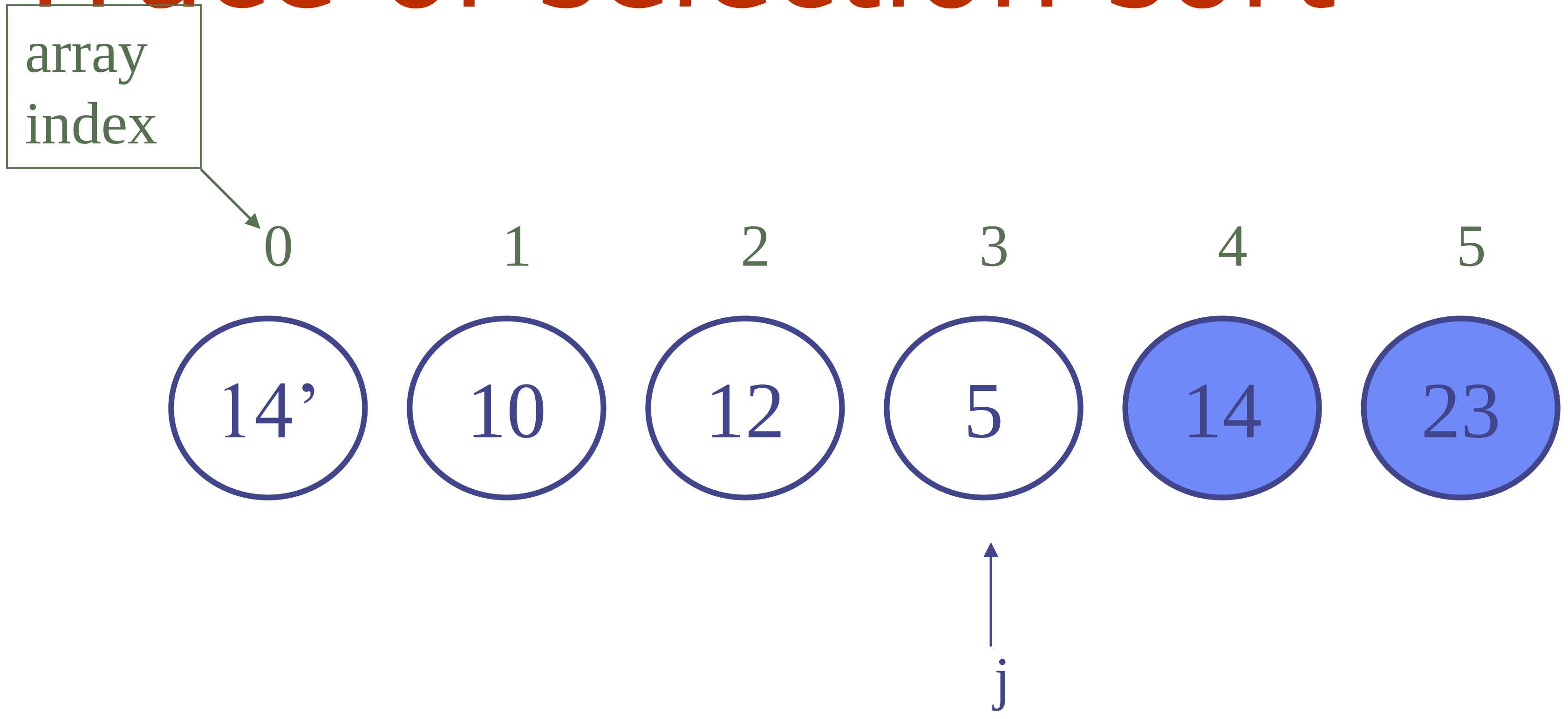
Trace of selection sort



i = 3, third iteration of the outer loop

j = 2, pos_greatest = 0

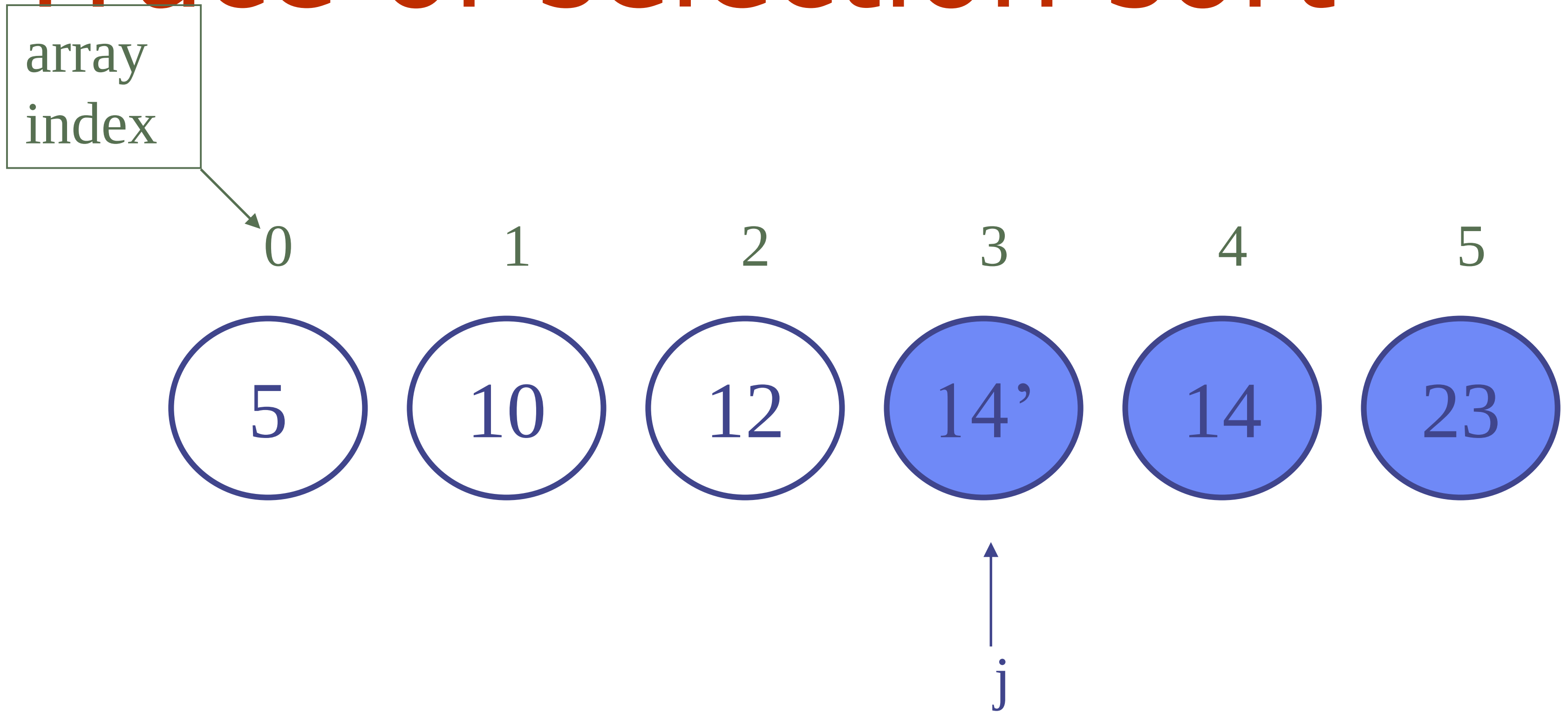
Trace of selection sort



i = 3, third iteration of the outer loop

j = 3, pos_greatest = 0

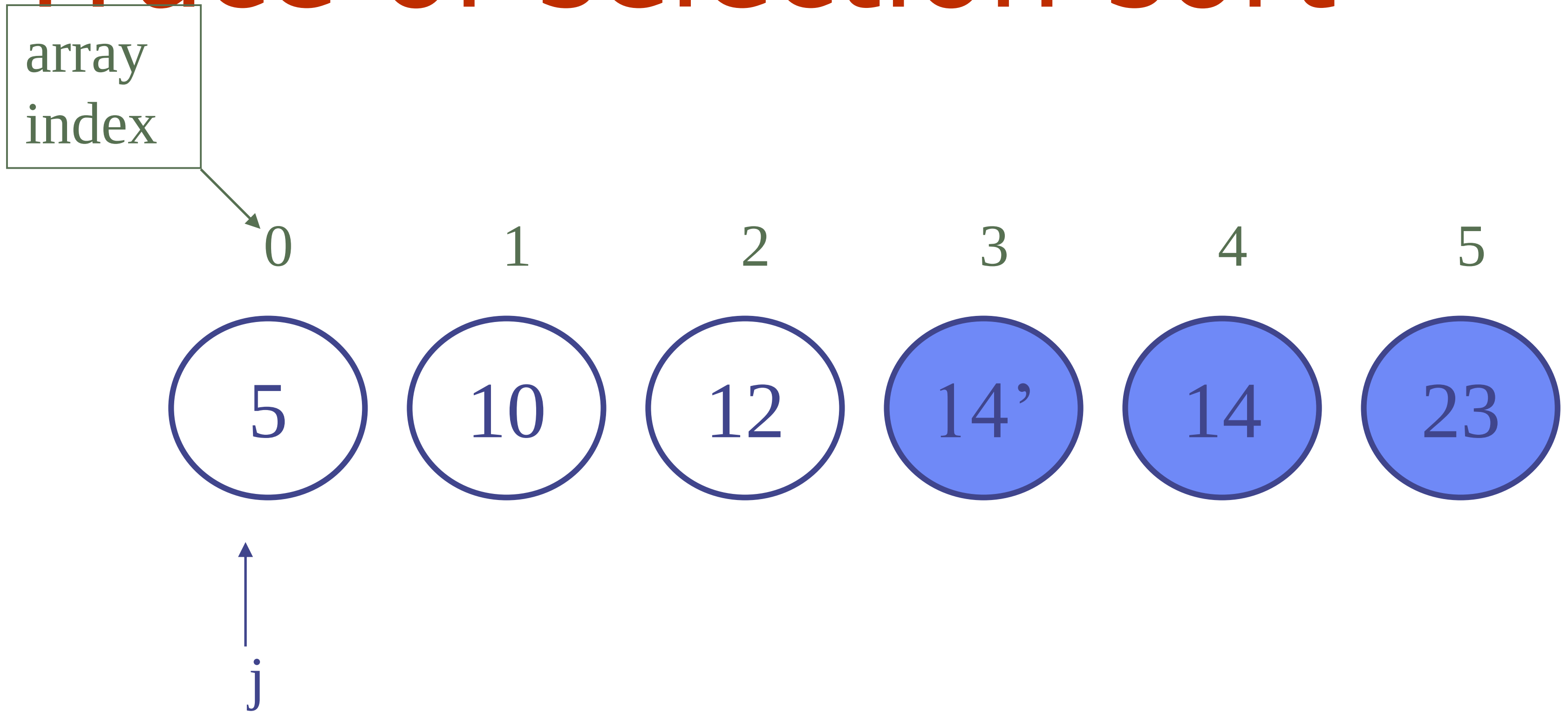
Trace of selection sort



i = 3, third iteration of the outer loop

swap elements at pos_greatest and 3

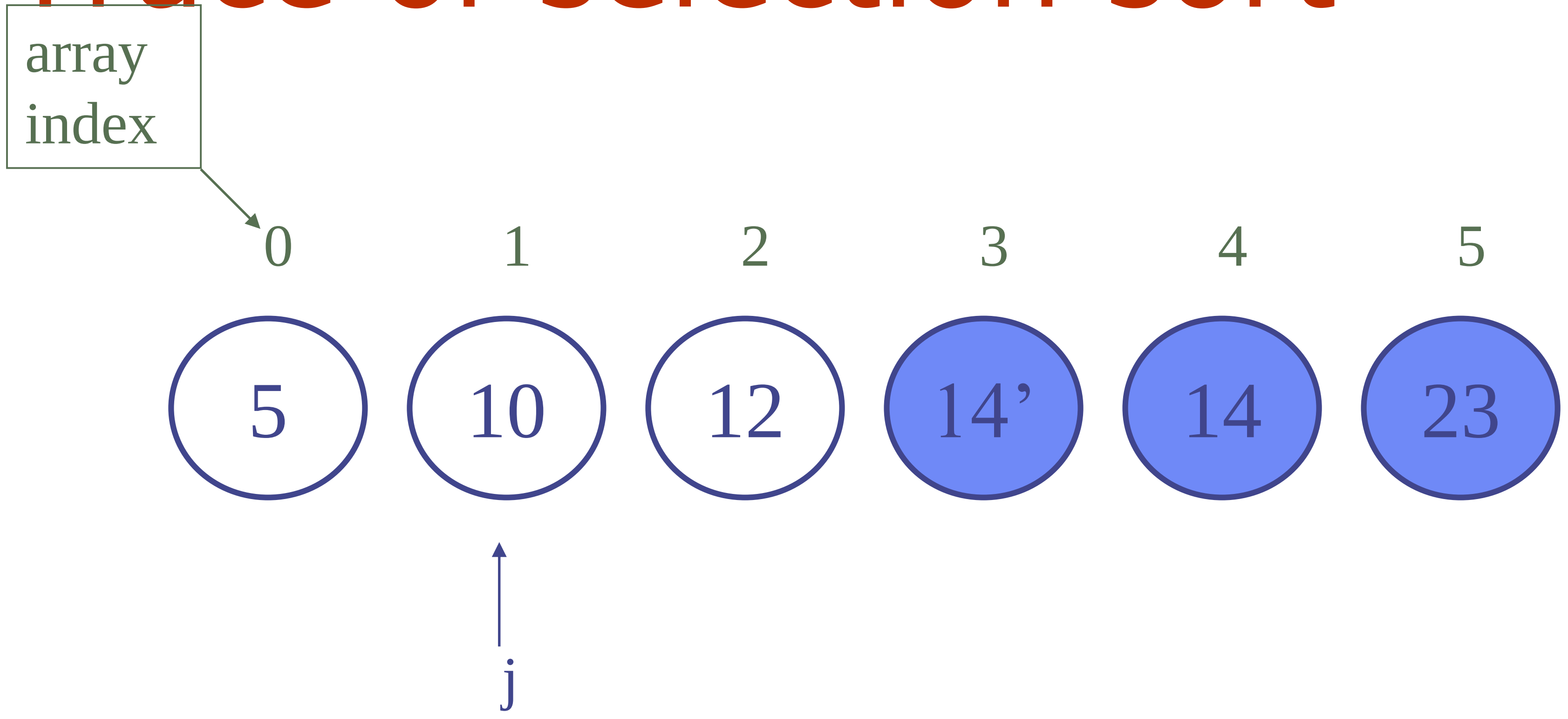
Trace of selection sort



i = 2, fourth iteration of the outer loop

j = 0, pos_greatest = 0

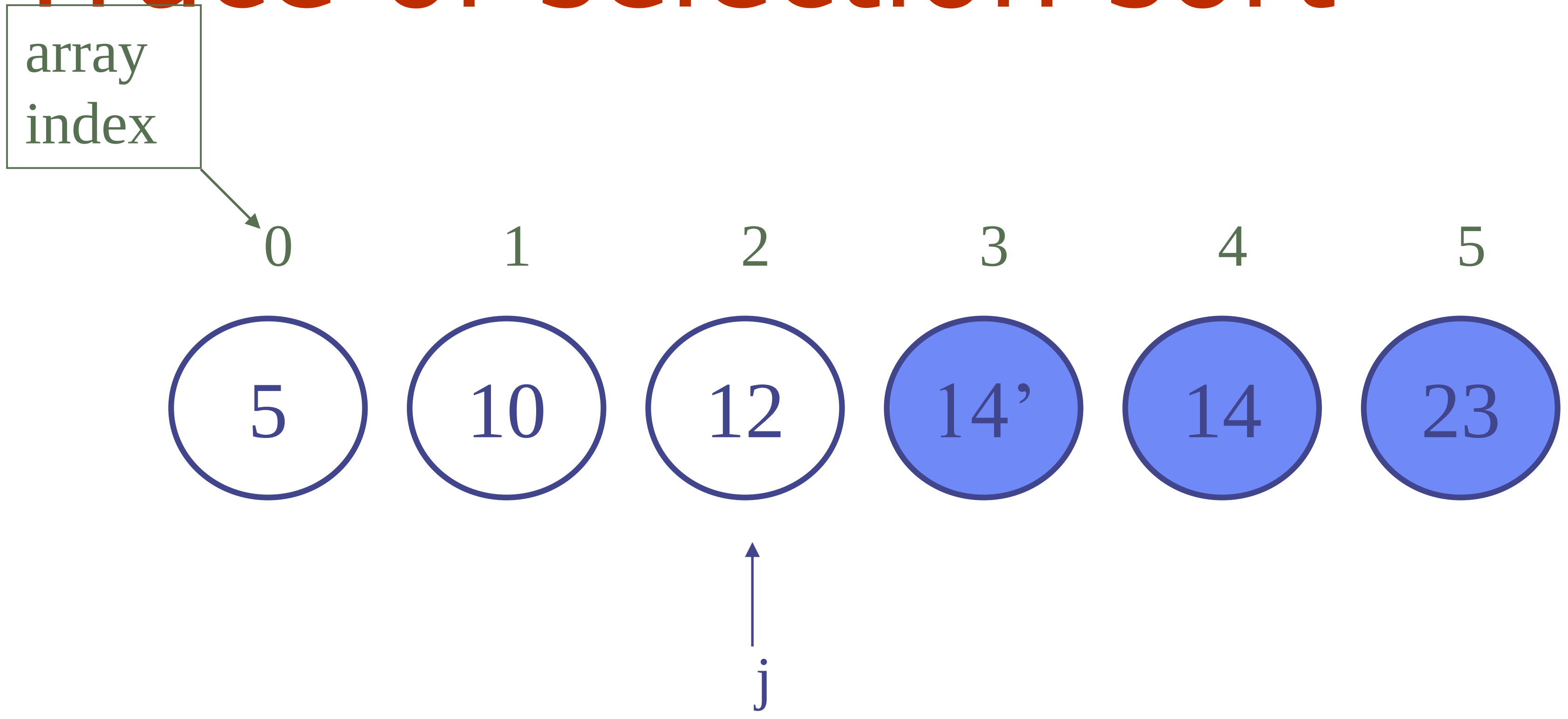
Trace of selection sort



i = 2, fourth iteration of the outer loop

j = 1, pos_greatest = 1 (changed!)

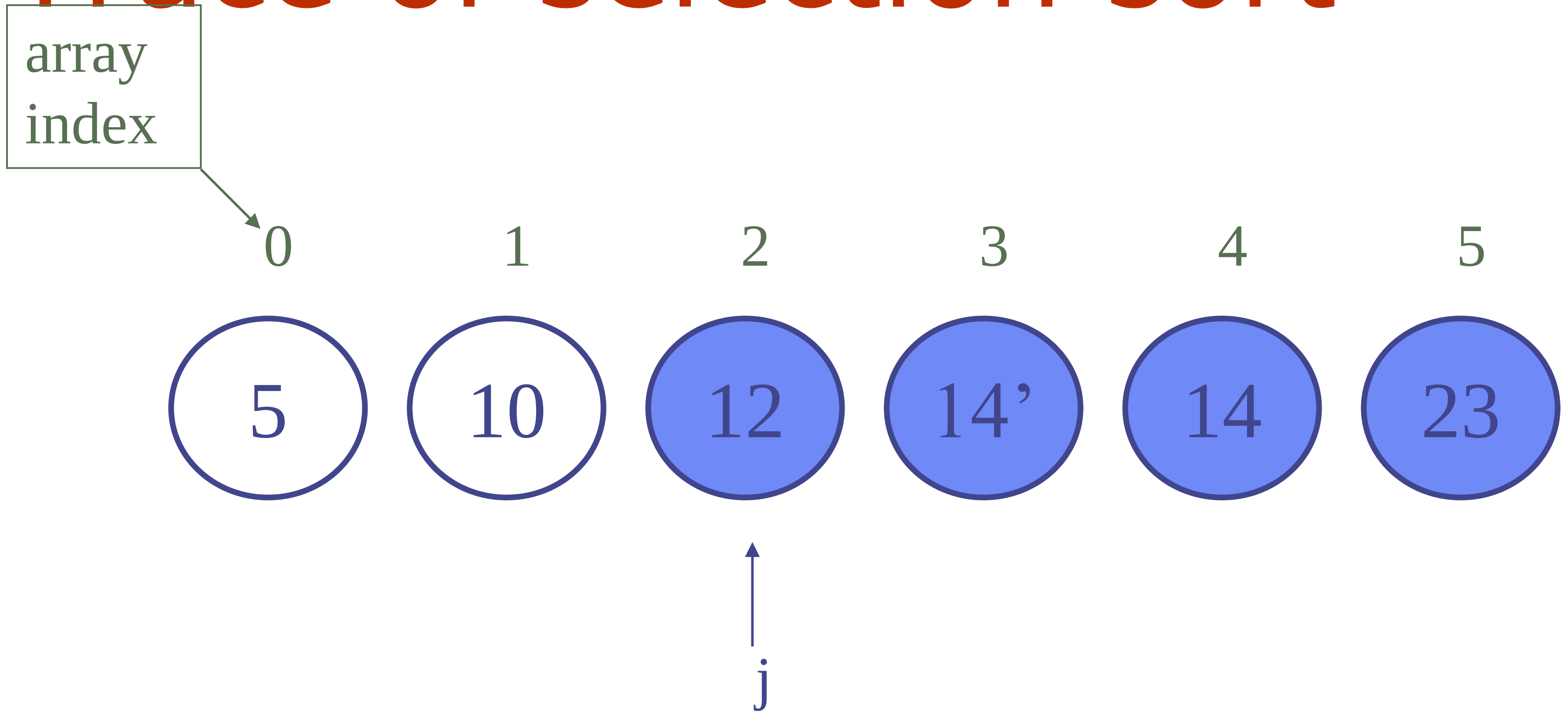
Trace of selection sort



i = 2, fourth iteration of the outer loop

j = 2, pos_greatest = 2 (changed again!)

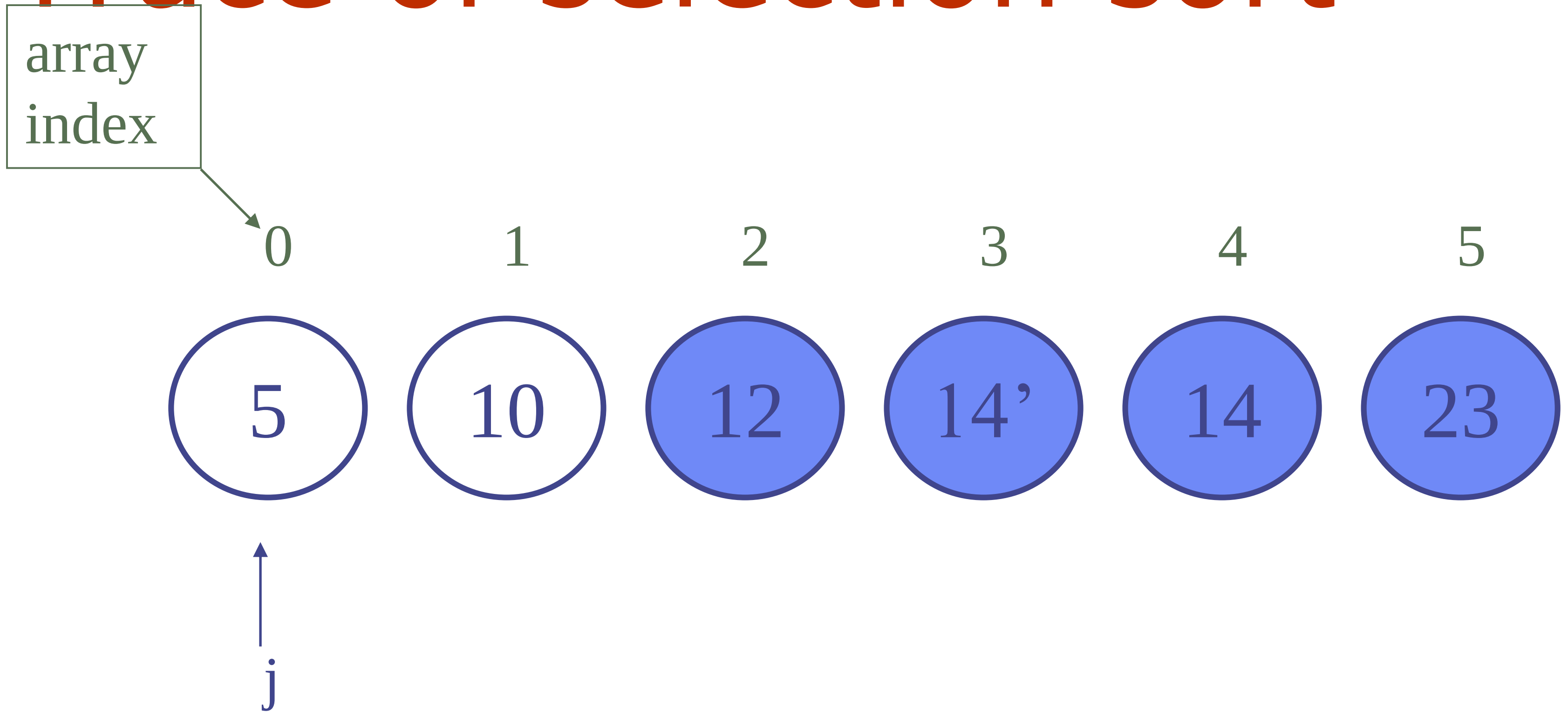
Trace of selection sort



i = 2, fourth iteration of the outer loop

swap elements at pos_greatest and 2 (element 12 with itself…)

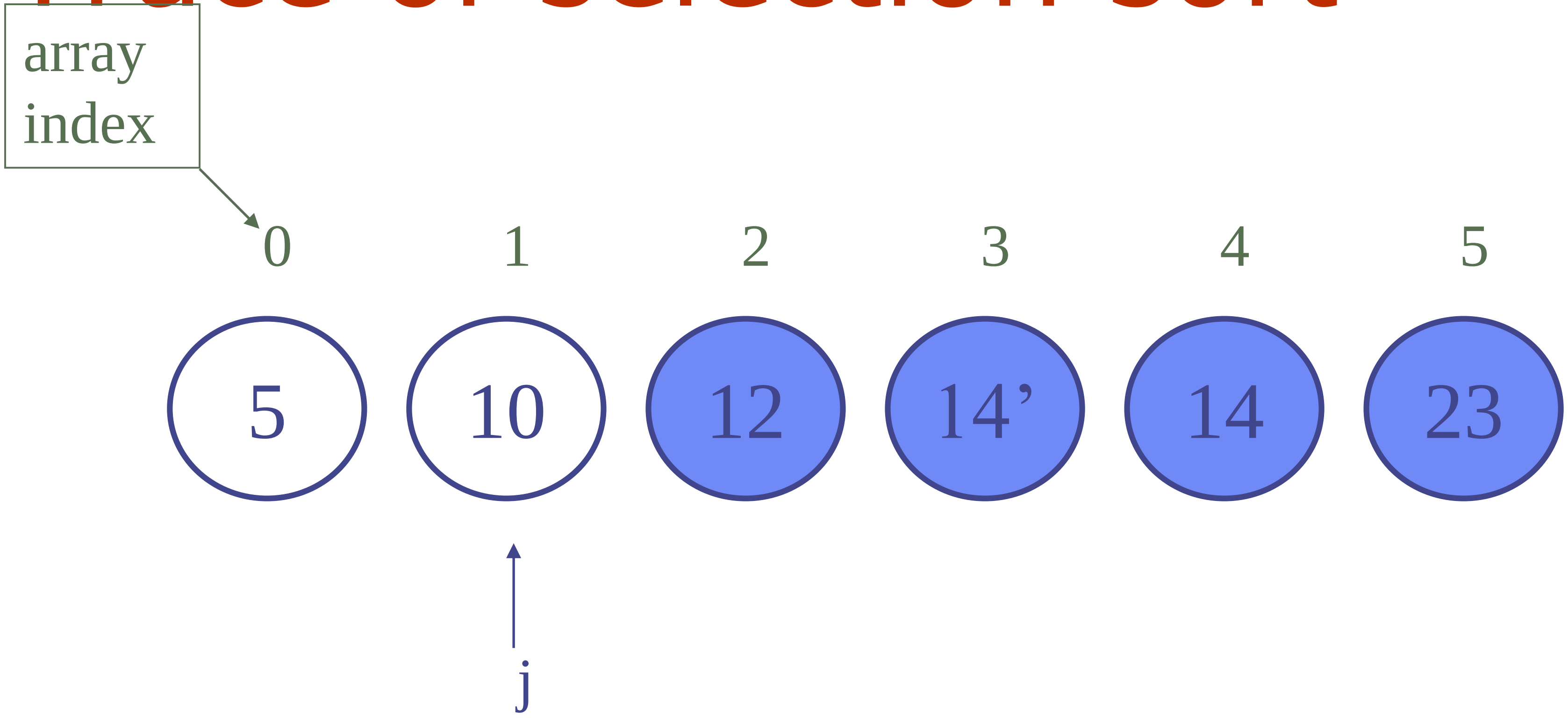
Trace of selection sort



i = 1, fifth iteration of the outer loop

j = 0, pos_greatest = 0

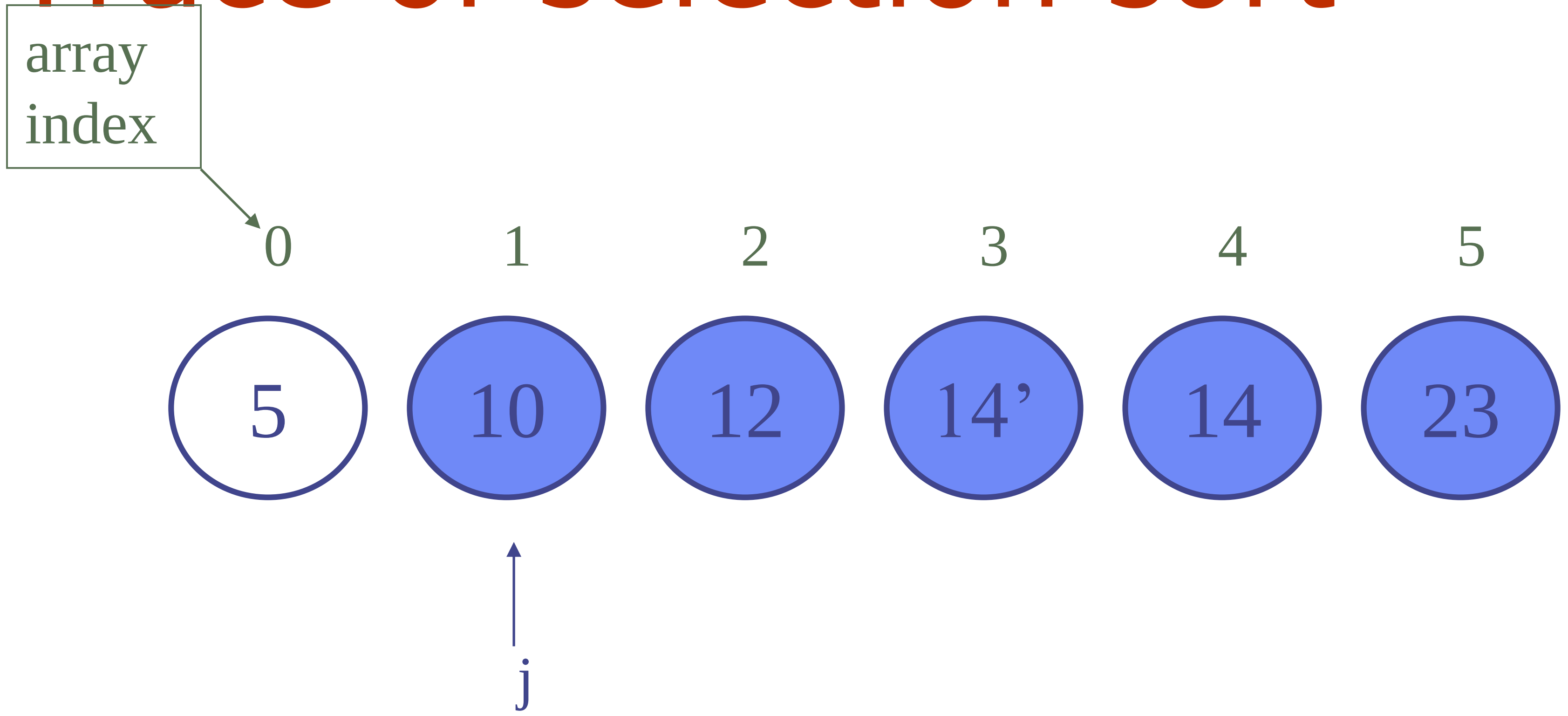
Trace of selection sort



i = 1, fifth iteration of the outer loop

j = 1, pos_greatest = 1 (changed)

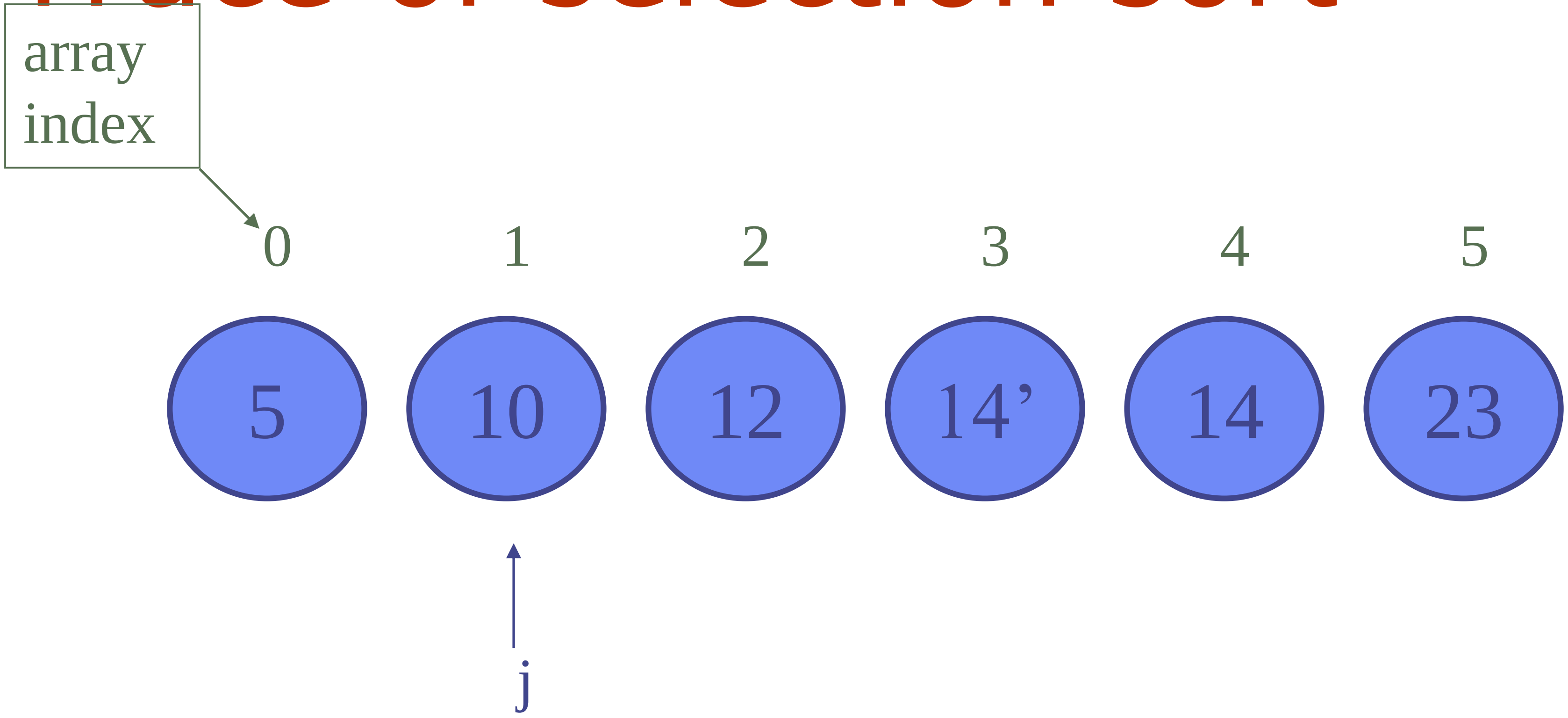
Trace of selection sort



i = 1, fifth iteration of the outer loop

swap element at pos_greatest with element at position 1 (10 with itself)

Trace of selection sort



i = 1, fifth iteration of the outer loop

NOTE: 14 and 14’ swapped ordering

This implementation of selection sort is not stable

<u>Exercise</u> (offline): can you make it stable or not?

Complexity of selection sort

Compared to bubble sort:

- Same number of iterations
- Same number of comparisons in the worst case
- fewer swaps (one for each outer loop = n-1)
- also O(n^2)

Selection sort on linked lists

- Implementation similar to bubble sort; also $O(n^2)$
- Instead of pos_greatest, have a variable Node largest which keeps the reference of the node with the largest element we have seen so far
- Swap elements once in every iteration through the unsorted part of the list:

```
E element v = current.element; current.element
=largest.element; largest.element= v;
```

Insertion Sort: Basic Idea

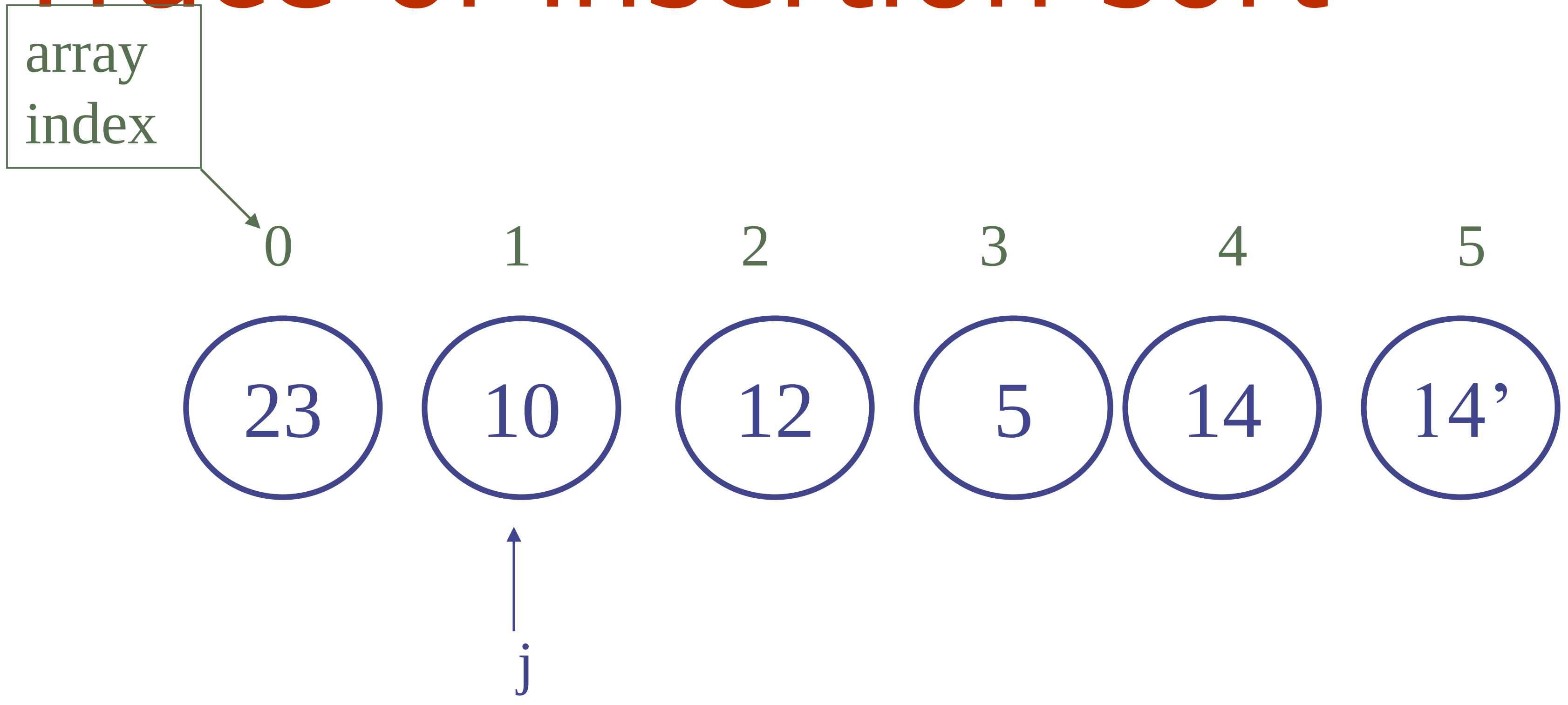
- **Keep the front of the list sorted**, and as we move through the back, elements we insert them into the correct place in the front

Insertion sort

```
void insertionSort(int arr[]){
  for(int j=1; j < arr.length; j++){
    int temp = arr[j];
    int i = j; // range 0 to j-1 is sorted
    while(i > 0 && arr[i-1] > temp){
      arr[i] = arr[i-1];
      i--;
    }
    arr[i] = temp;
  } // end outer for loop
}  // end insertion sort
```

Find a place to insert temp in the sorted range; as you are looking, shift elements in the sorted range to the right

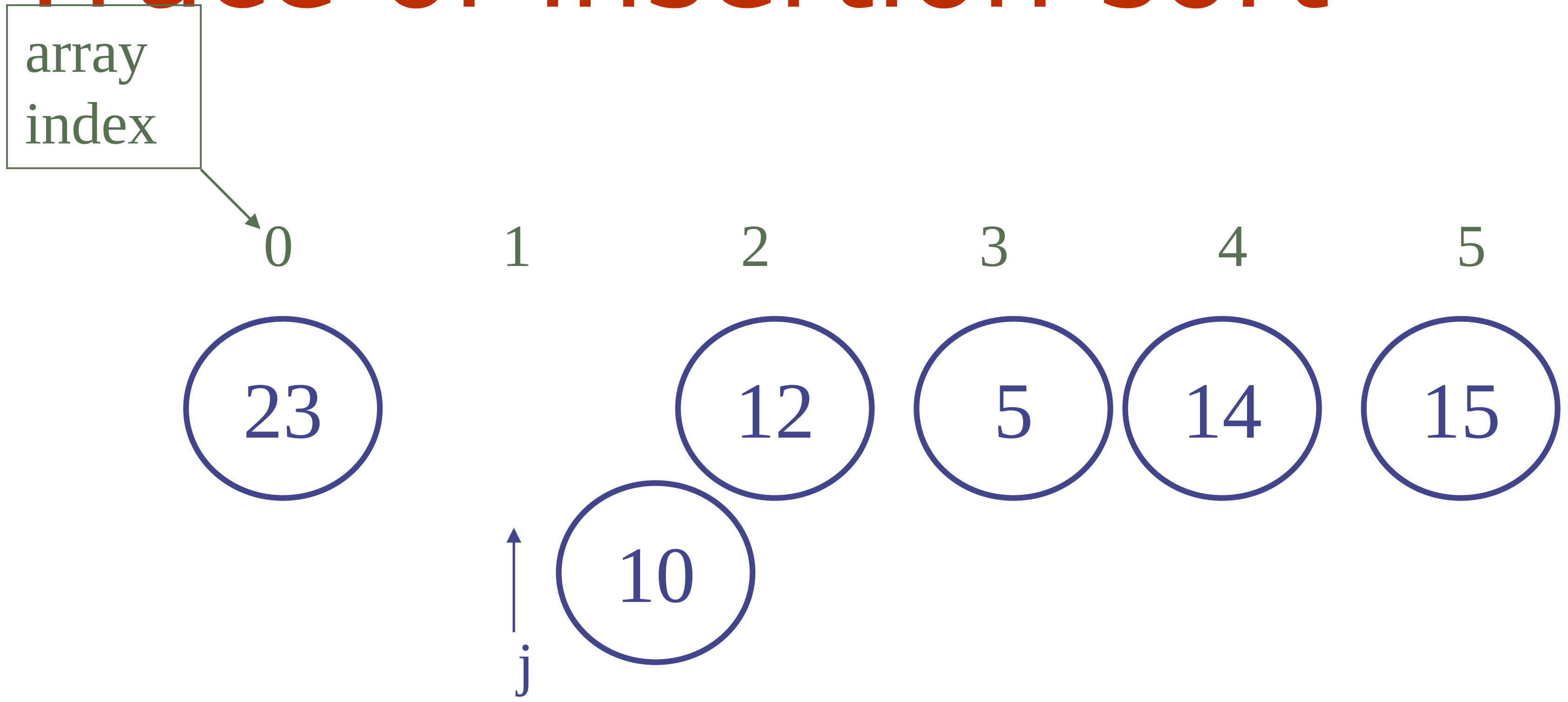
Trace of insertion sort



j = 1, first iteration of the outer loop

temp = 10; i = 1; arr[i-1] >= 10

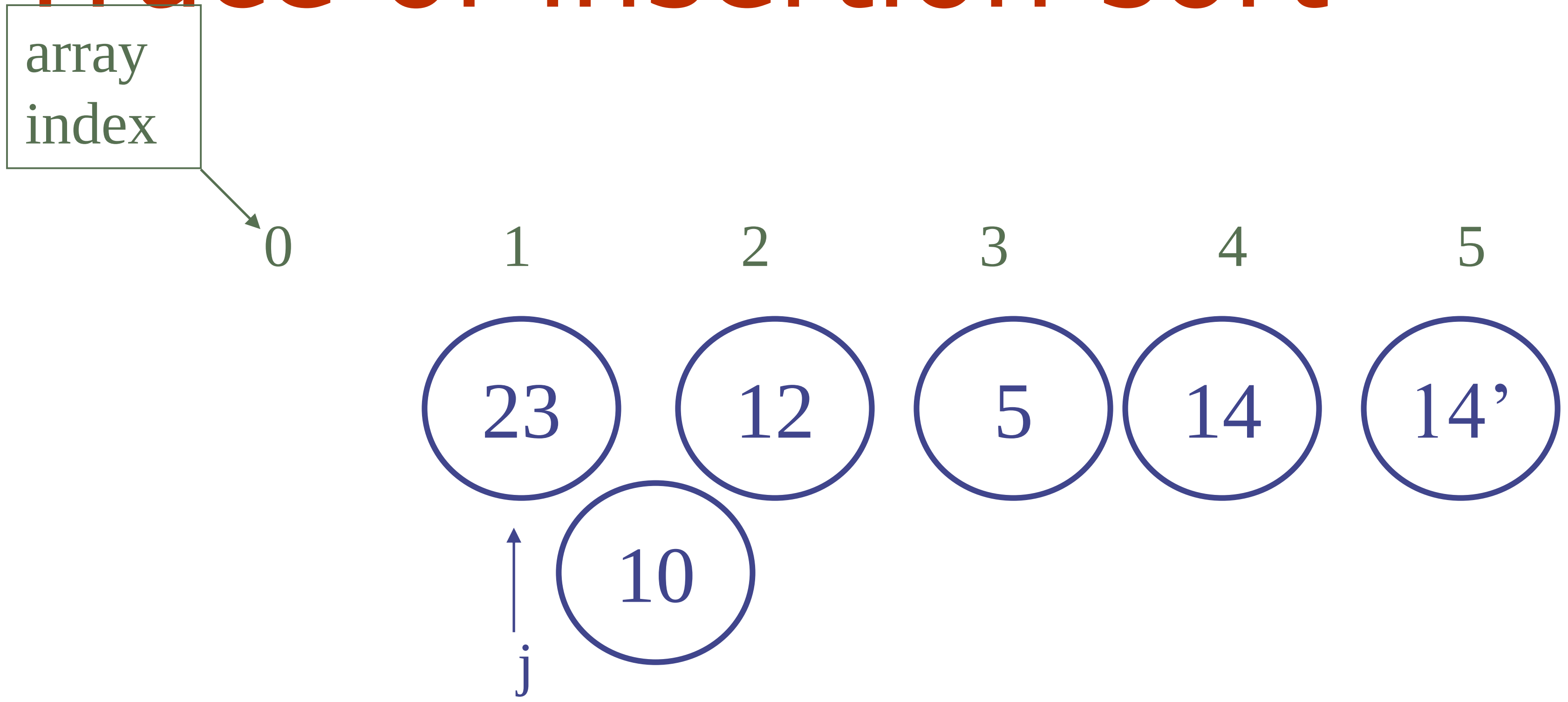
Trace of insertion sort



j = 1, first iteration of the outer loop

temp = 10; i = 1; arr[i-1] >= 10

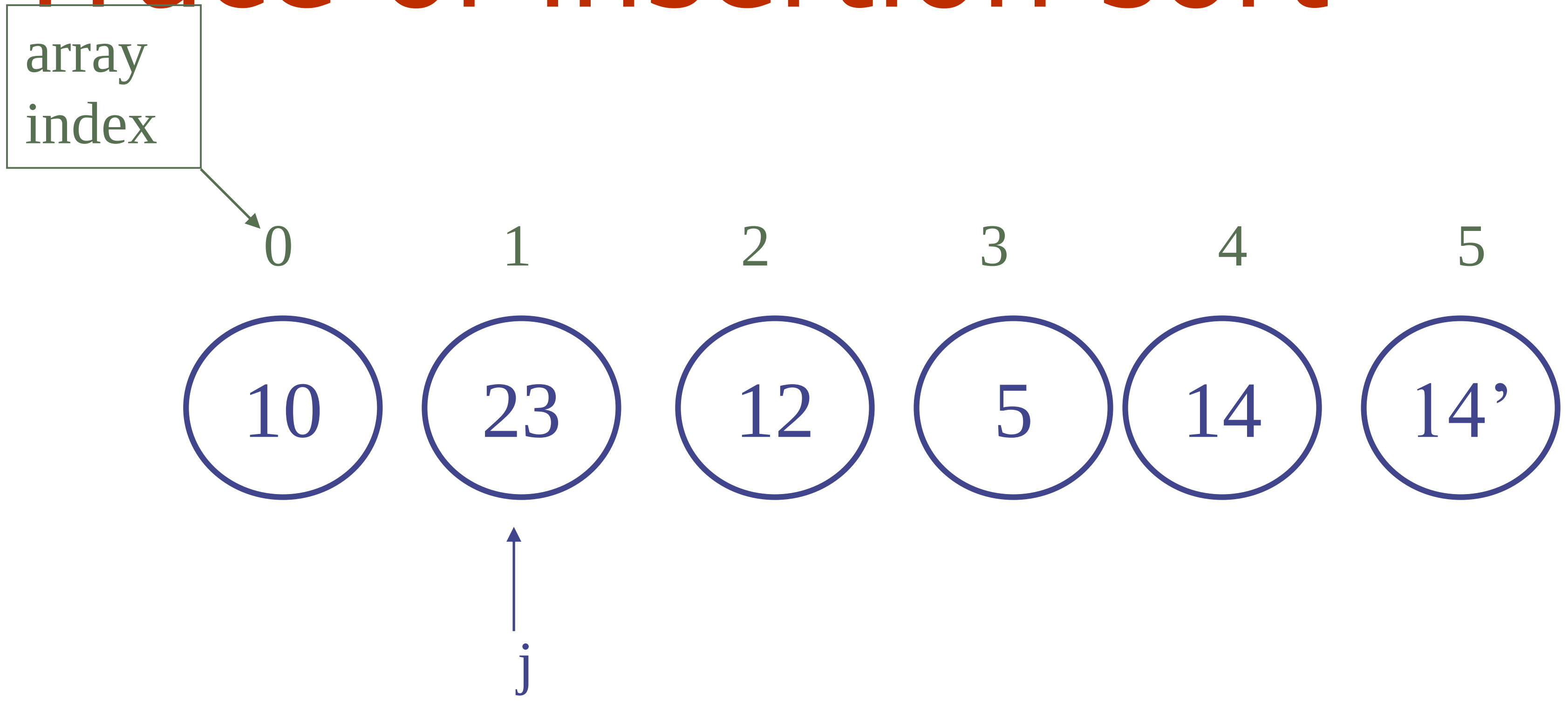
Trace of insertion sort



j = 1, first iteration of the outer loop

arr[i] = arr[i-1]

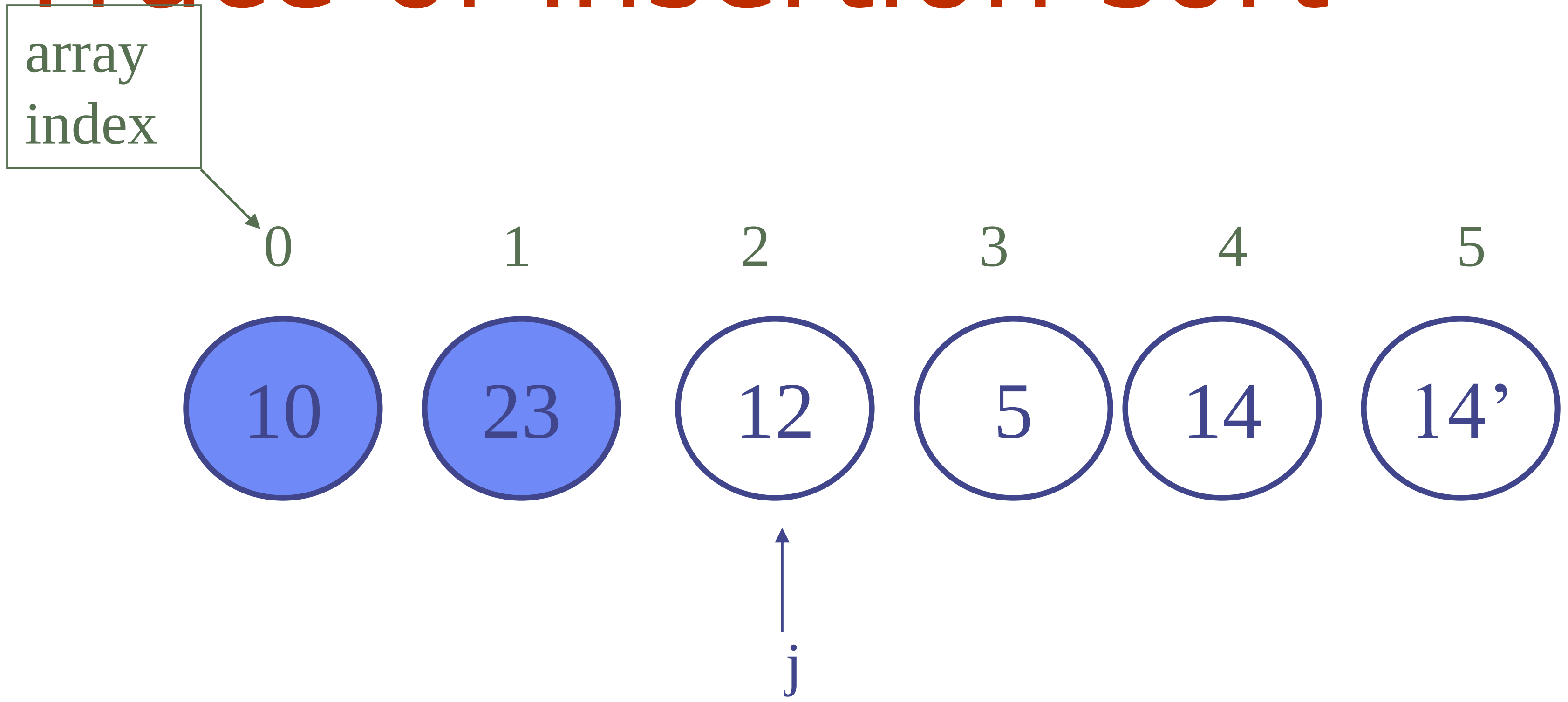
Trace of insertion sort



j = 1, first iteration of the outer loop

arr[i] = temp

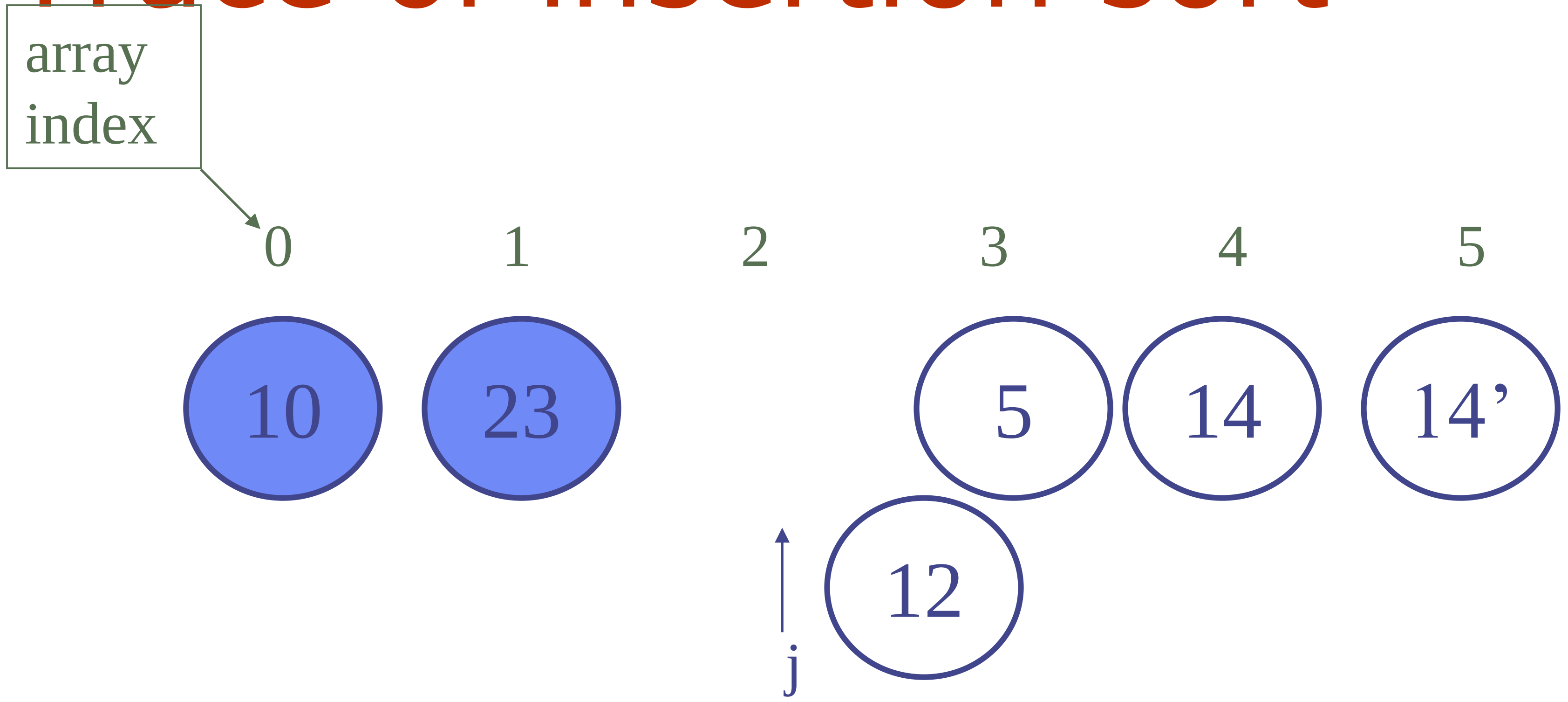
Trace of insertion sort



j = 2, second iteration of the outer loop

temp = 12; arr[i-1] >= temp

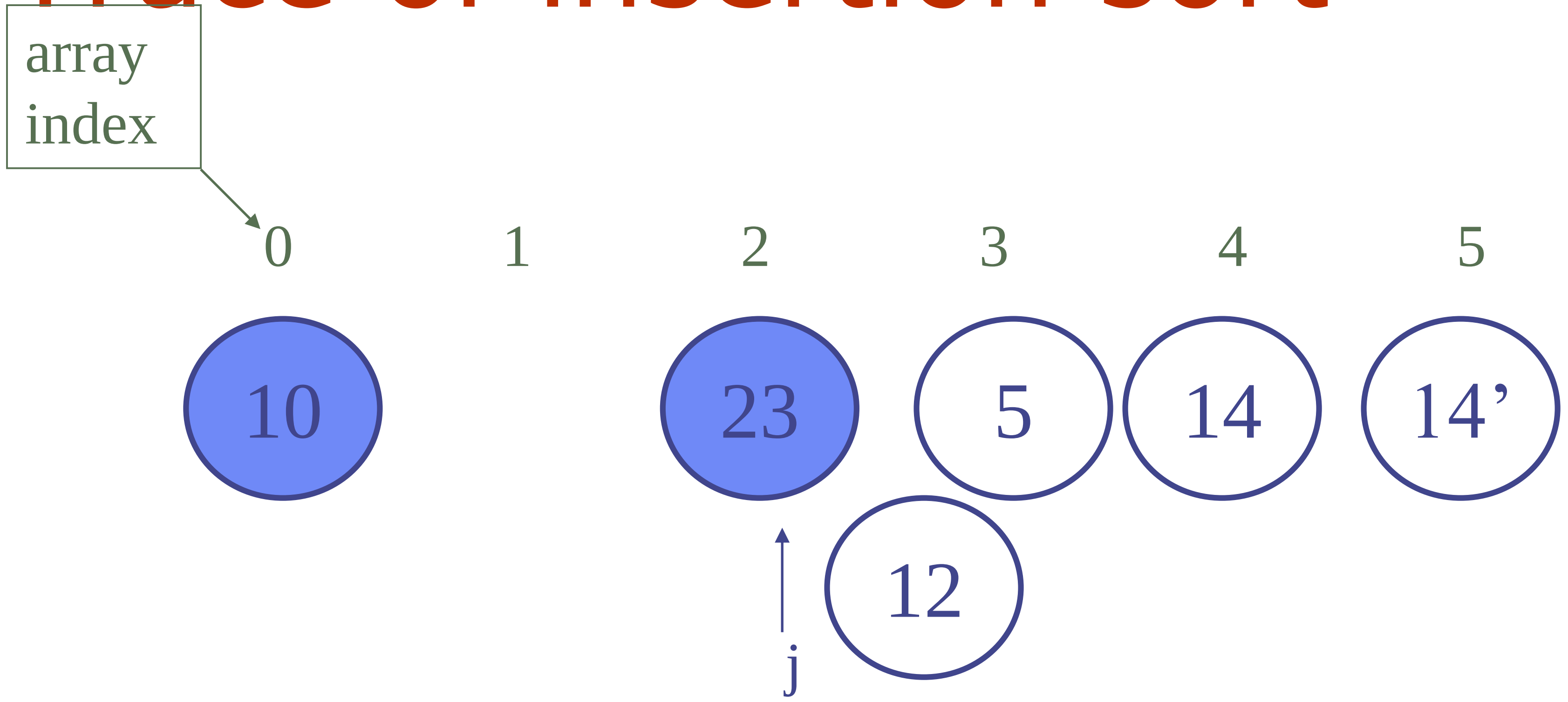
Trace of insertion sort



j = 2, second iteration of the outer loop

temp = 12; arr[i-1] >= temp

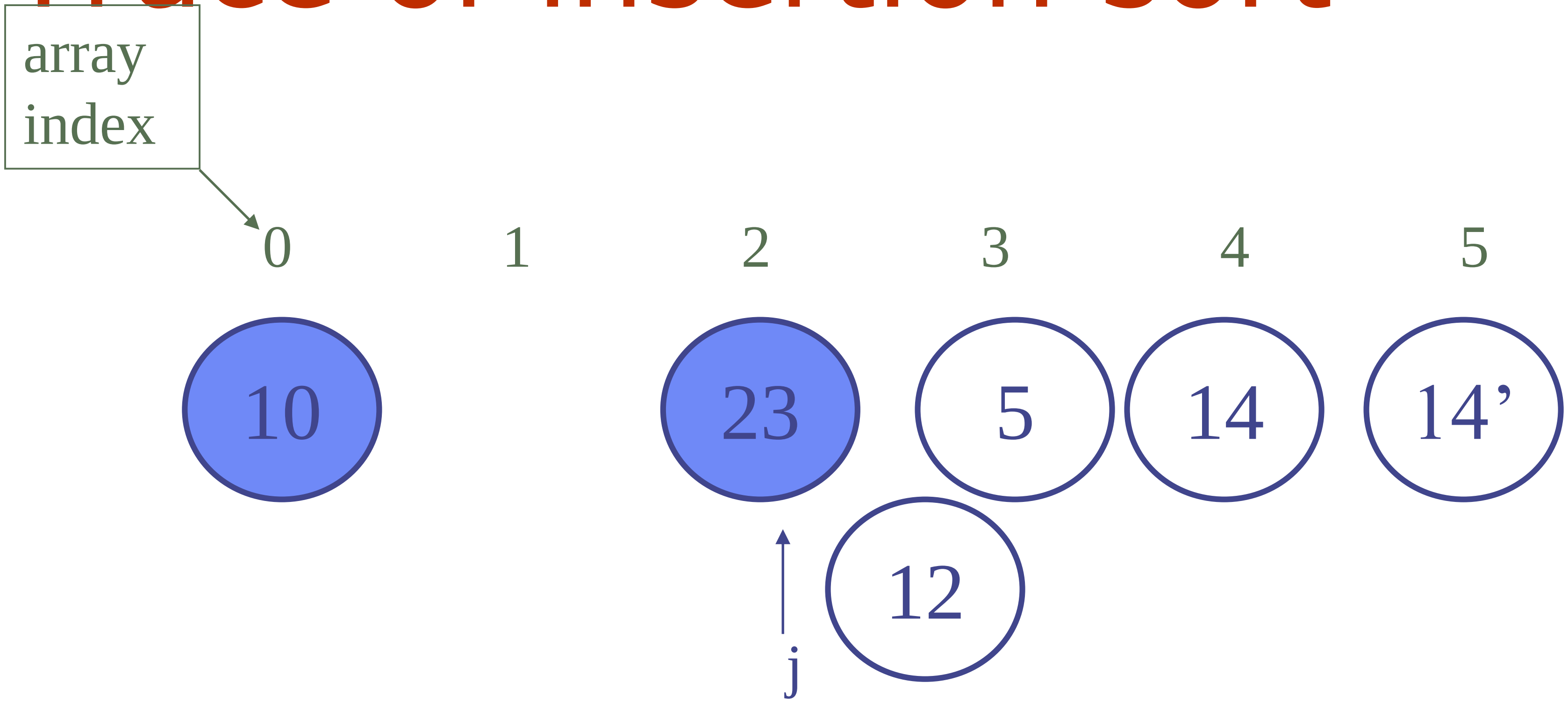
Trace of insertion sort



j = 2, second iteration of the outer loop

arr[i-1] = arr[i]

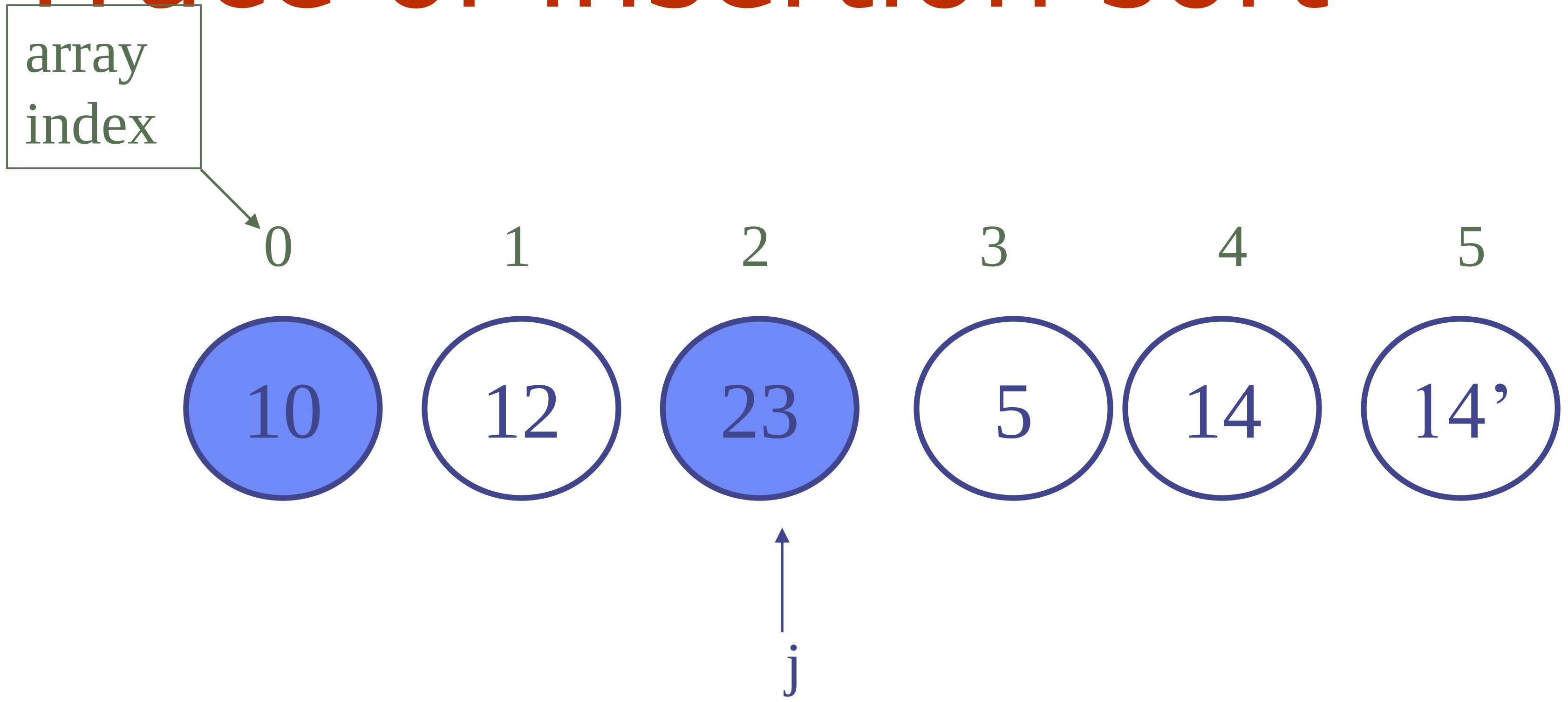
Trace of insertion sort



j = 2, second iteration of the outer loop

arr[i-1] < temp

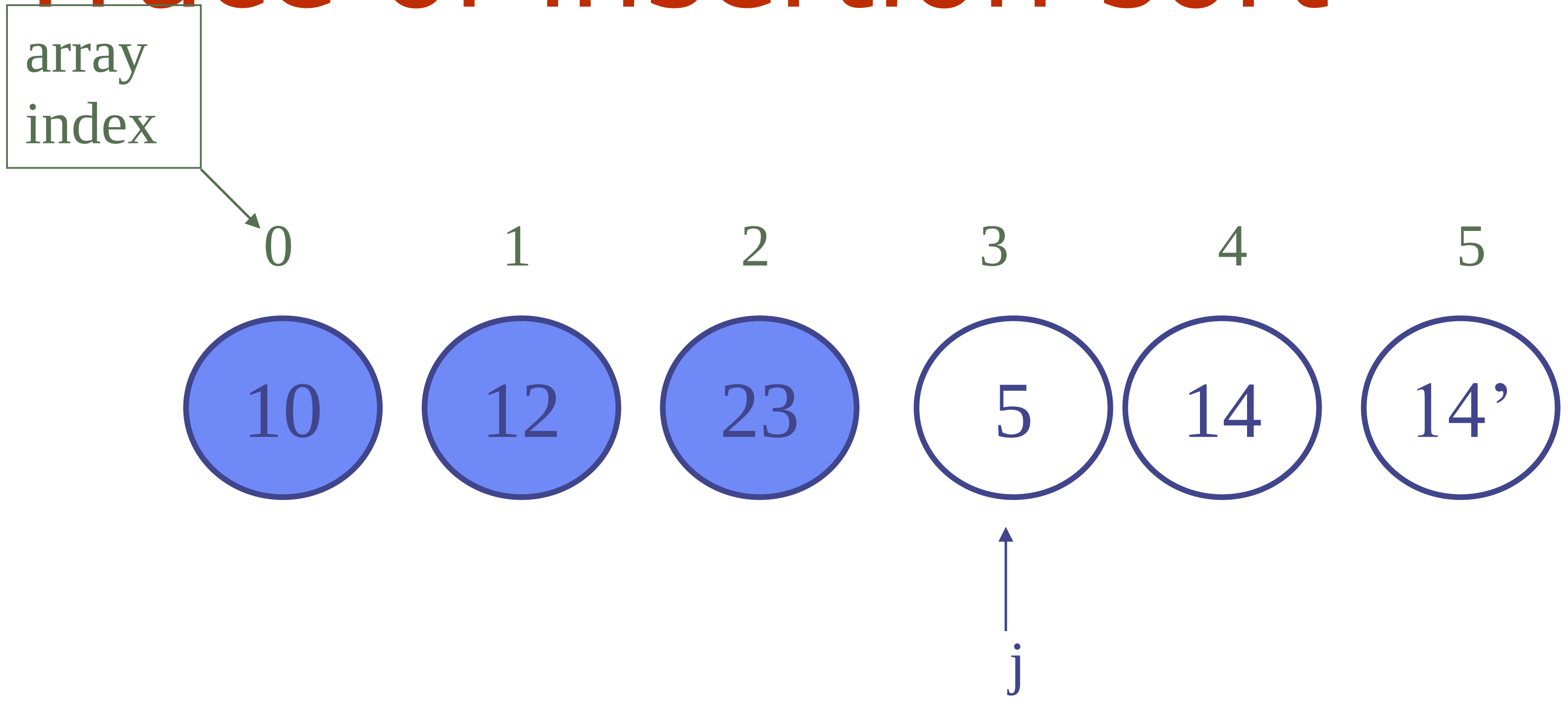
Trace of insertion sort



j = 2, second iteration of the outer loop

arr[i-1] = temp

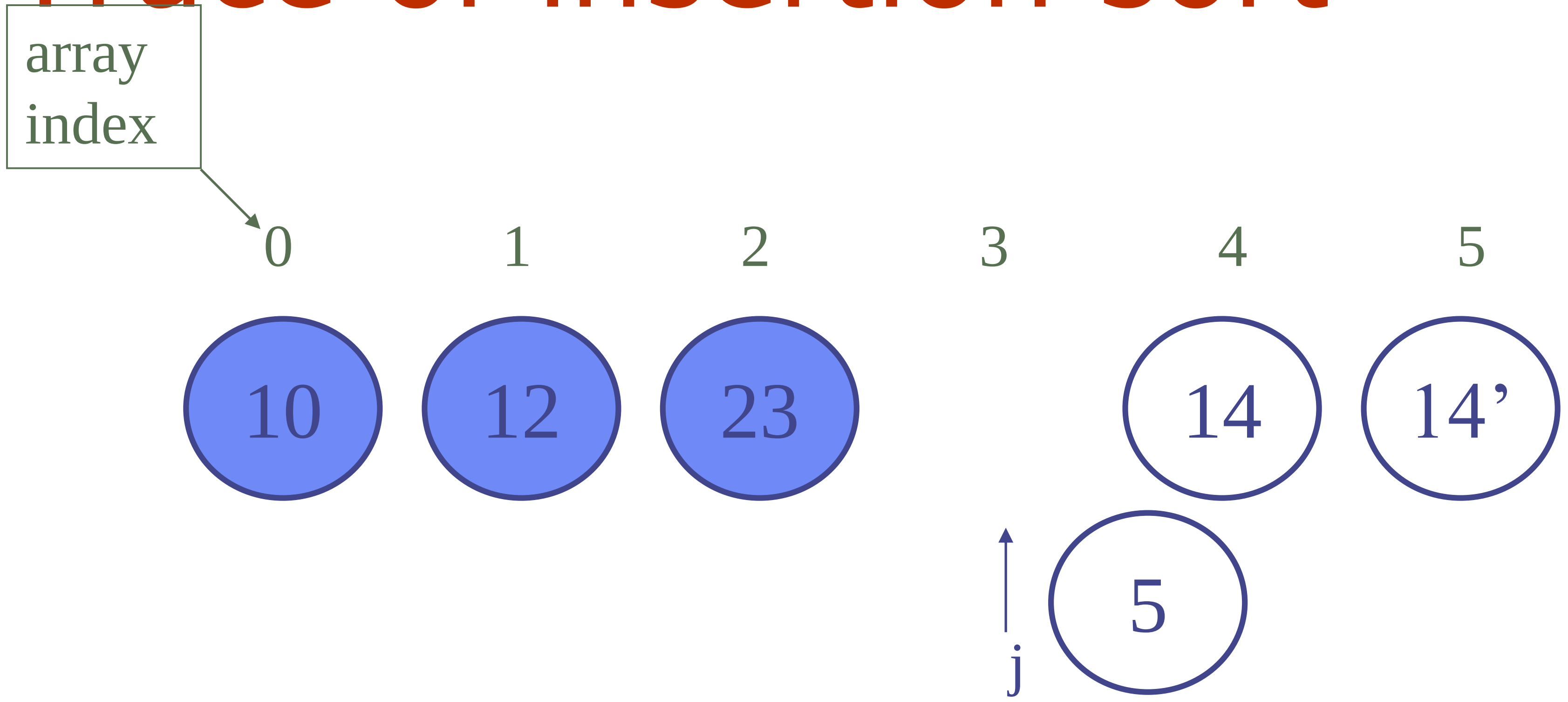
Trace of insertion sort



j = 3, third iteration of the outer loop

temp = 5

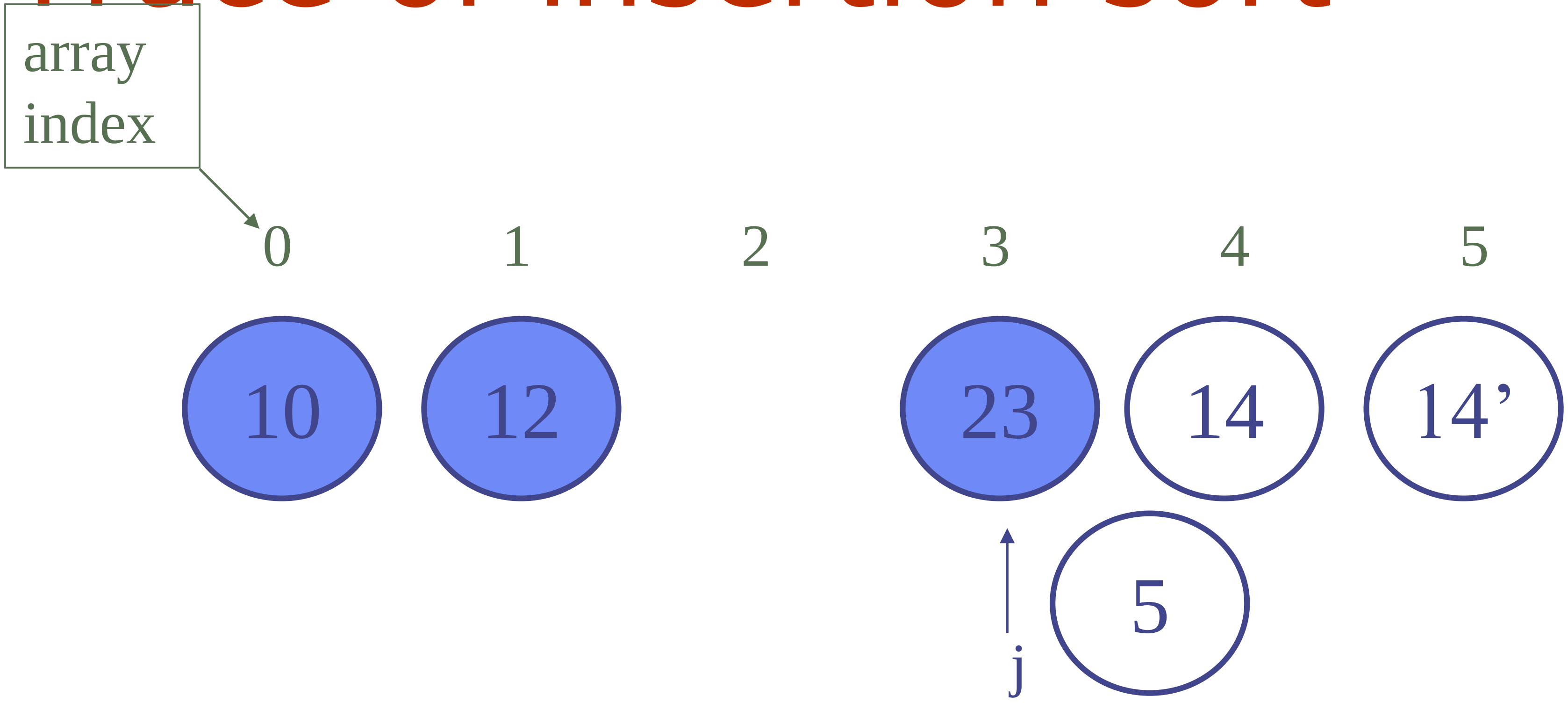
Trace of insertion sort



j = 3, third iteration of the outer loop

arr[i-1] >= temp

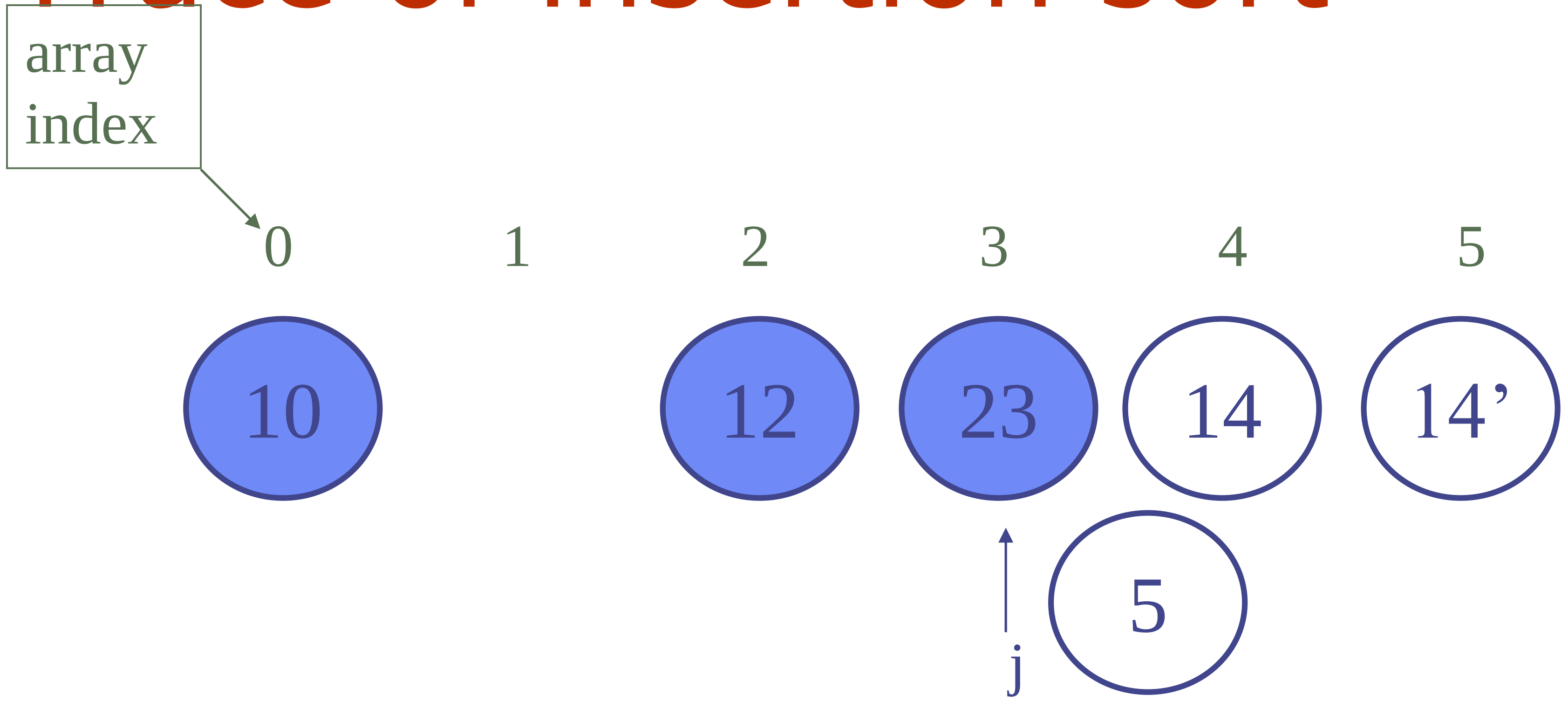
Trace of insertion sort



j = 3, third iteration of the outer loop

arr[i-1] >= temp

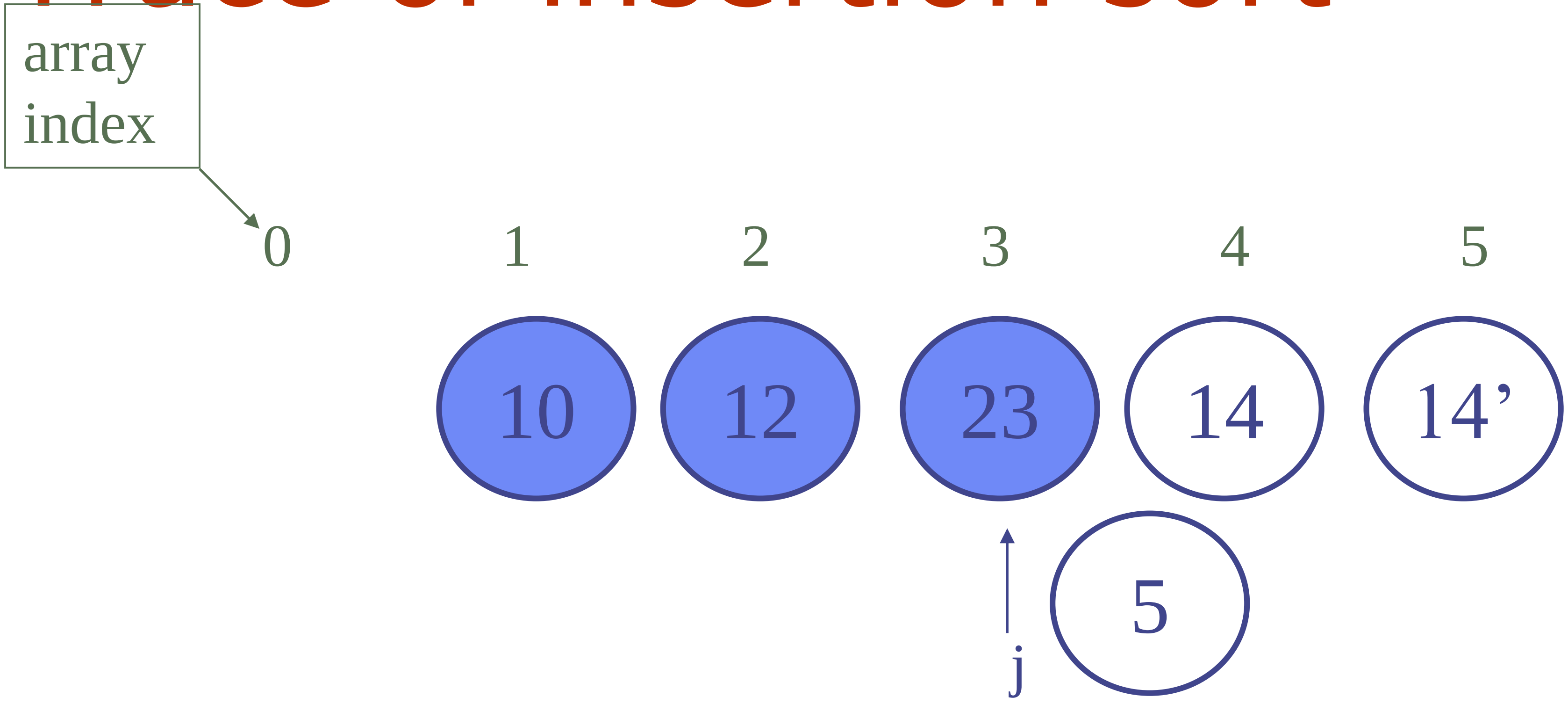
Trace of insertion sort



j = 3, third iteration of the outer loop

arr[i-1] >= temp

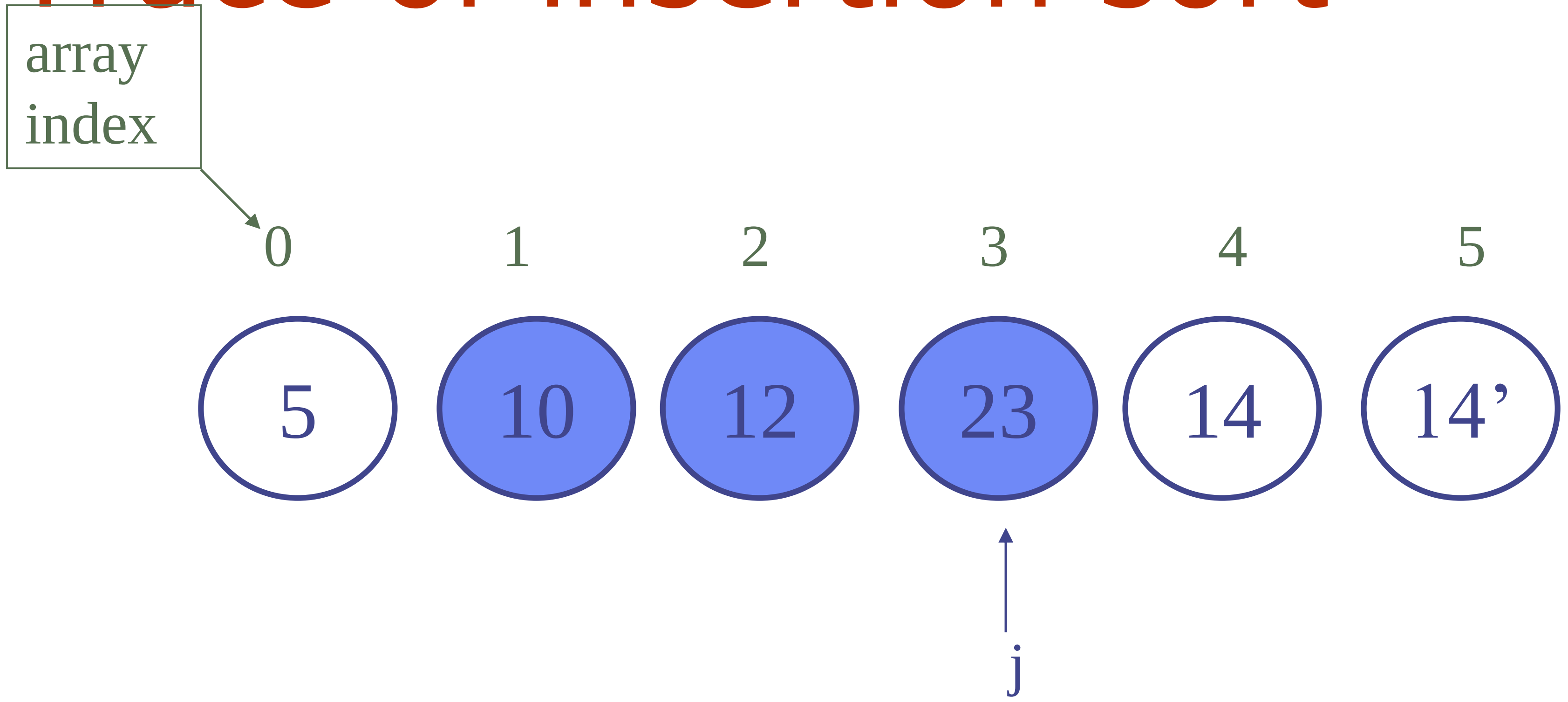
Trace of insertion sort



j = 3, third iteration of the outer loop

arr[i-1] >= temp

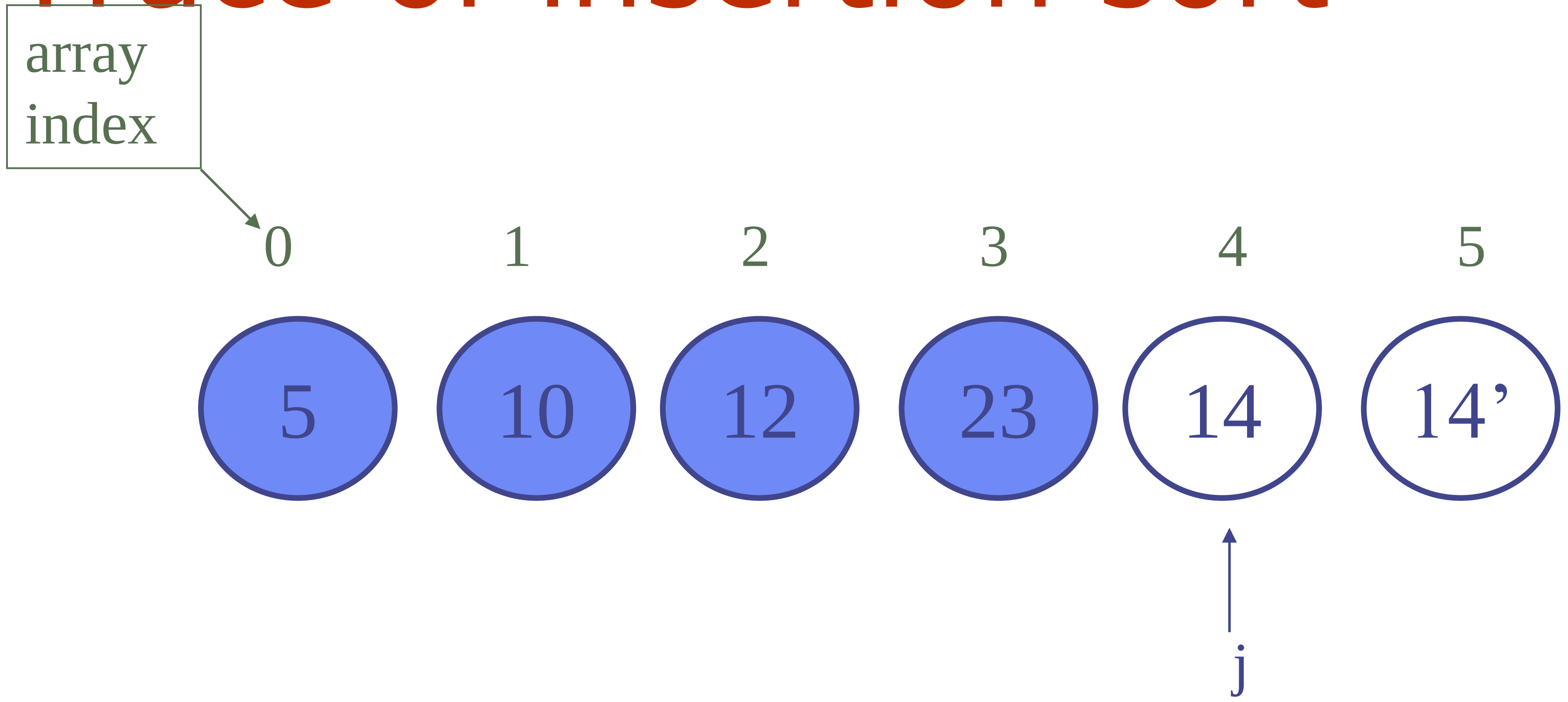
Trace of insertion sort



j = 3, third iteration of the outer loop

arr[i-1] = temp

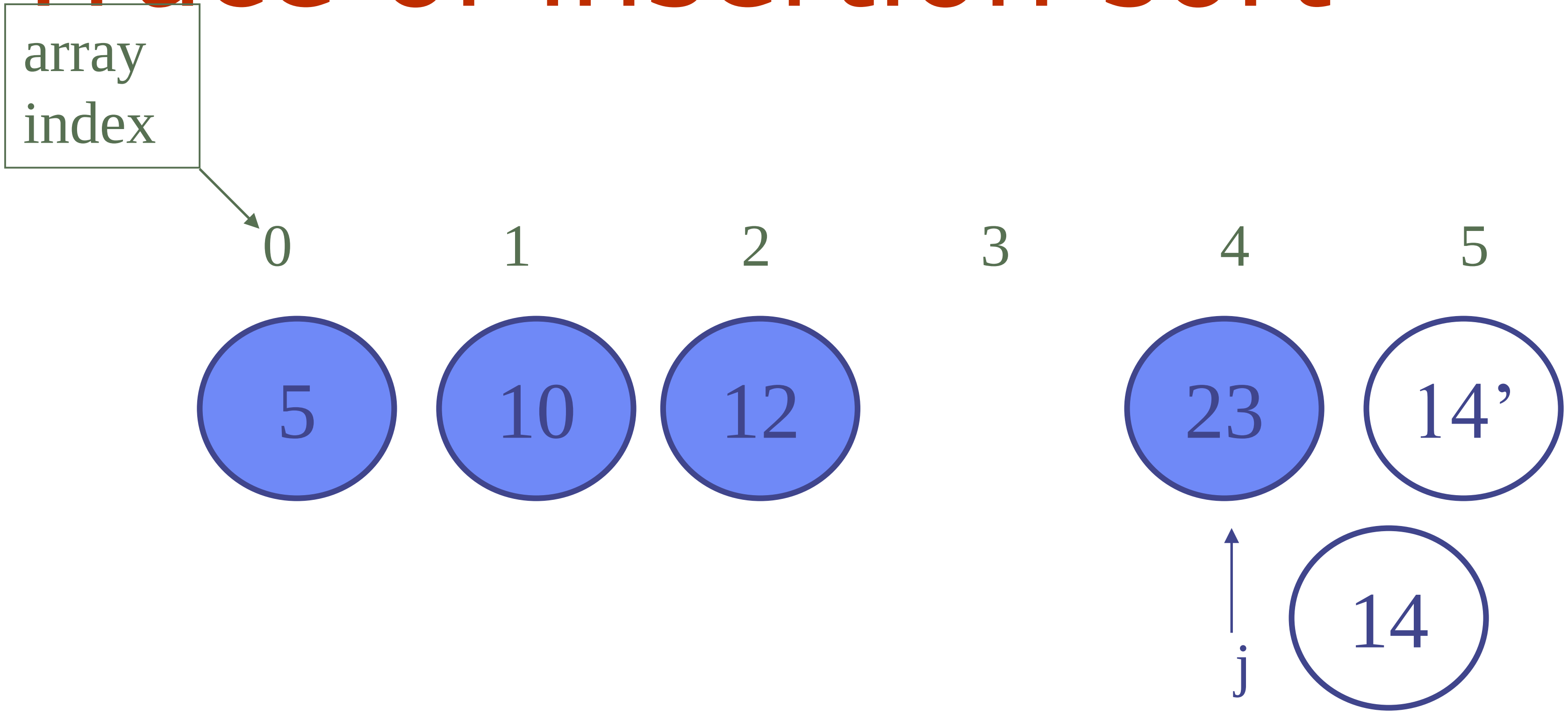
Trace of insertion sort



j = 4, fourth iteration of the outer loop

temp = 14

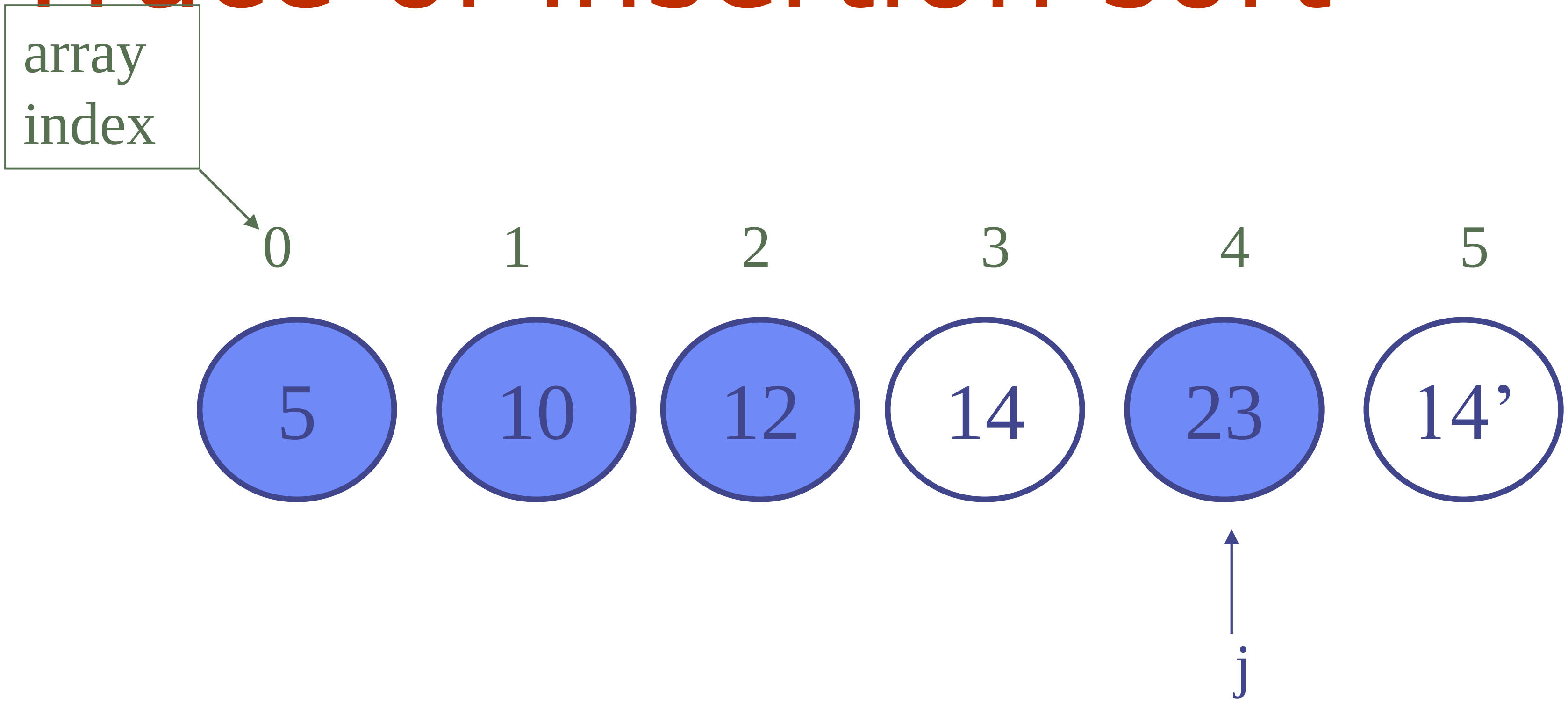
Trace of insertion sort



j = 4, fourth iteration of the outer loop

arr[i-1] >= temp

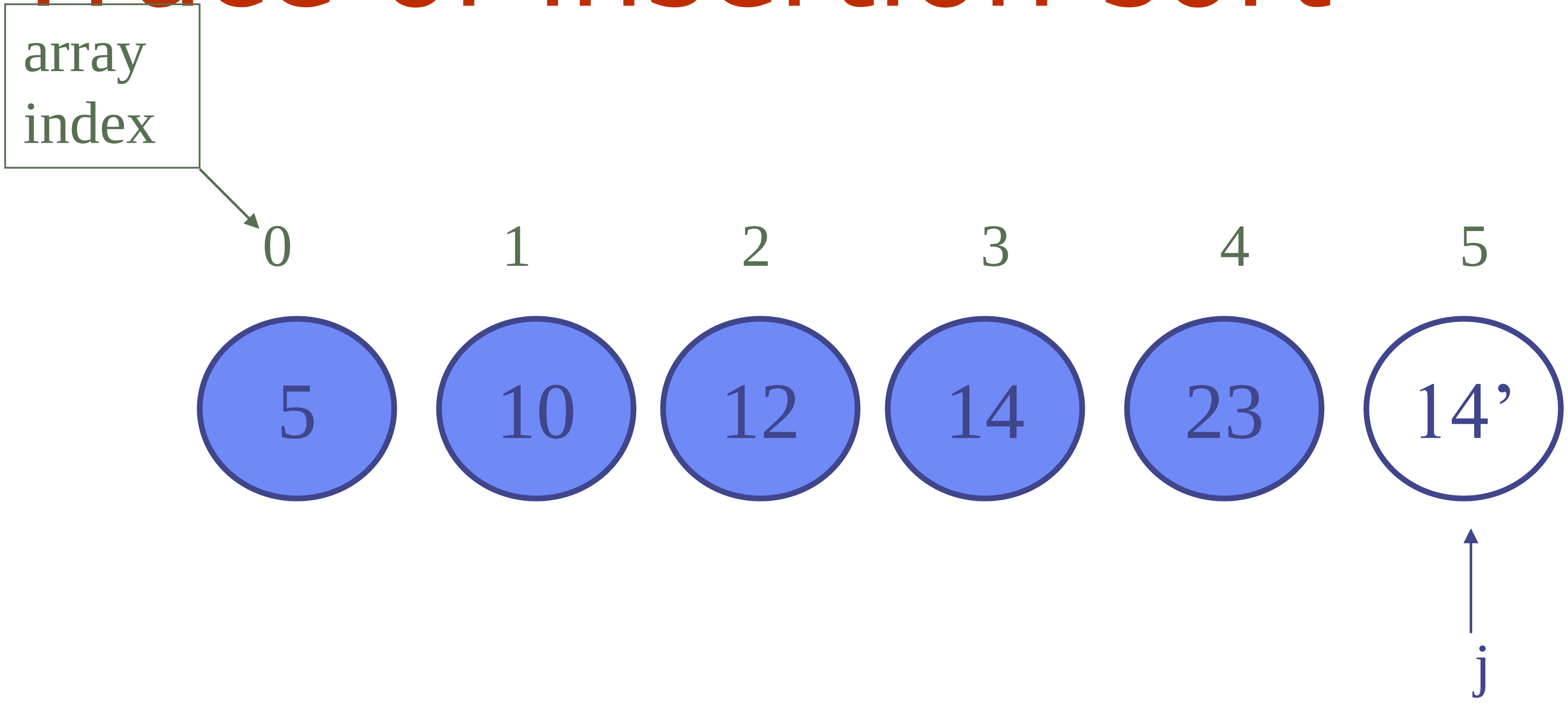
Trace of insertion sort



j = 4, fourth iteration of the outer loop

arr[i-1] = temp

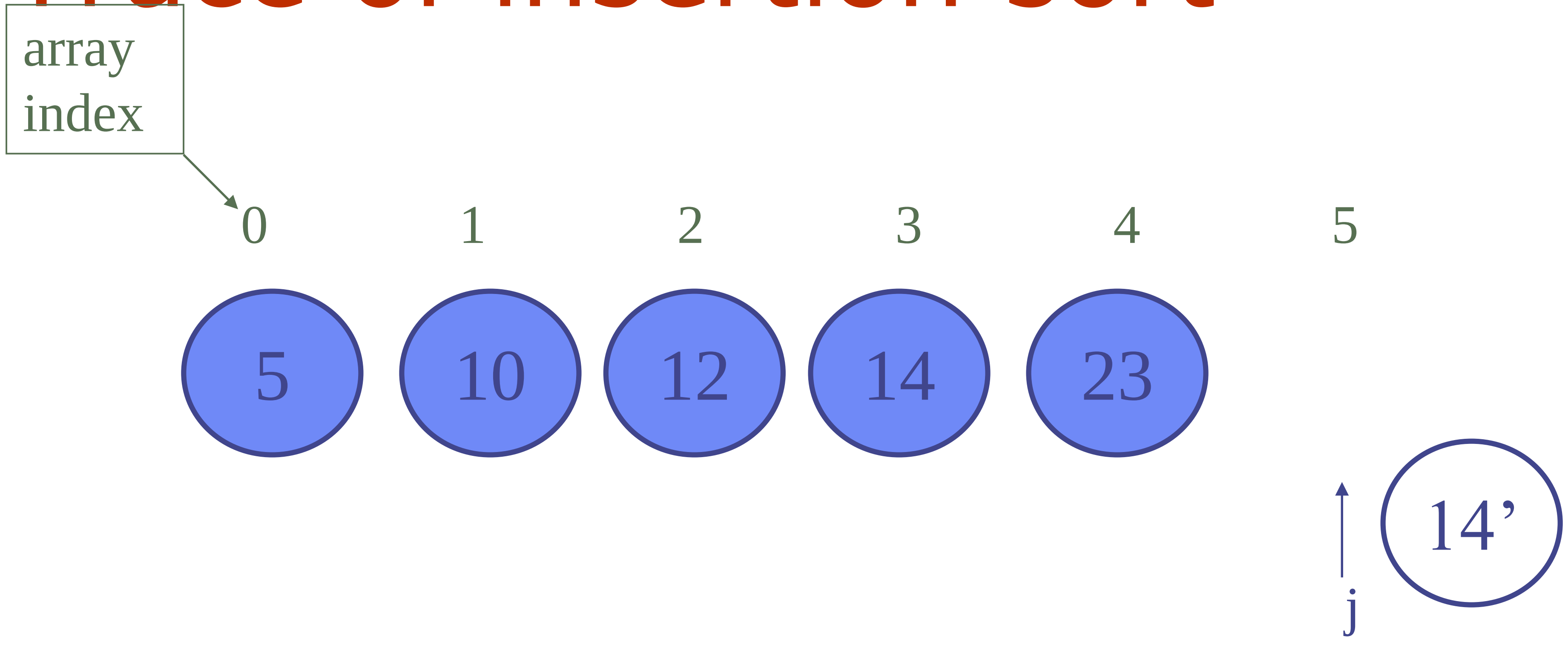
Trace of insertion sort



j = 5, fifth iteration of the outer loop

temp = 15

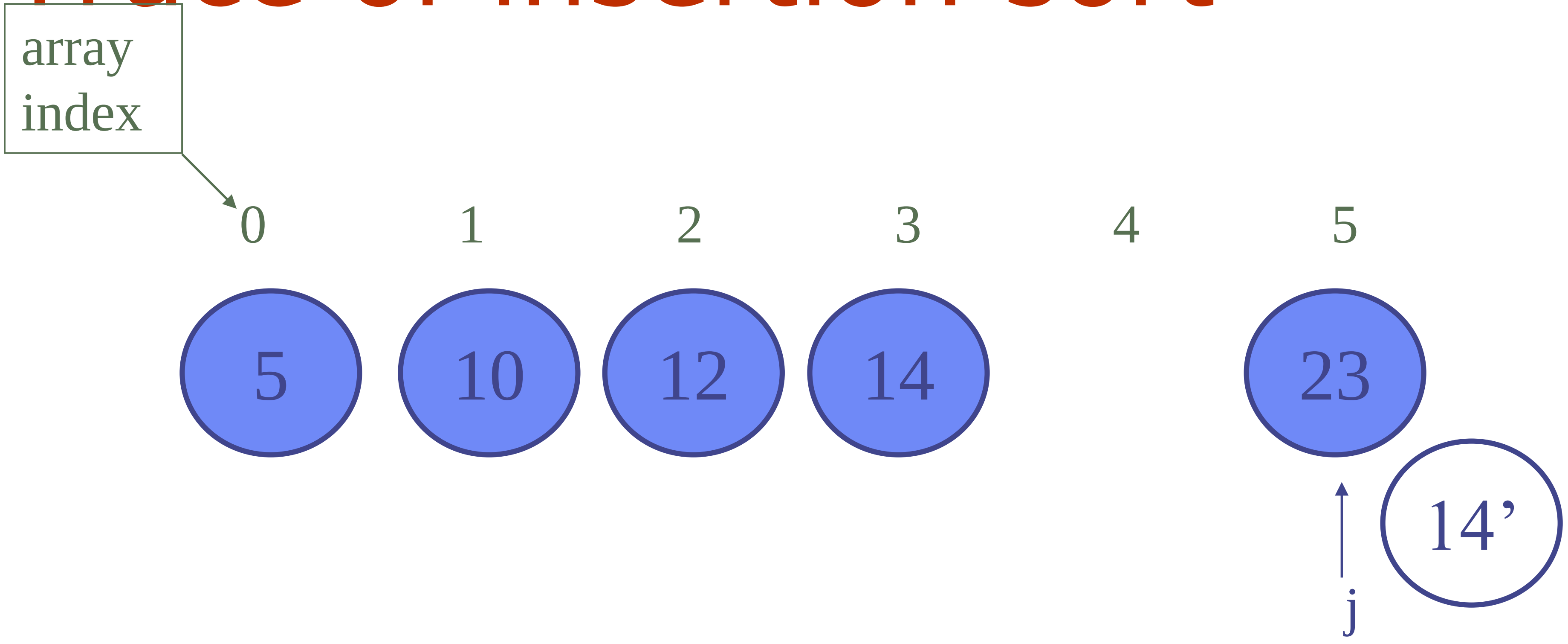
Trace of insertion sort



j = 5, fifth iteration of the outer loop

arr[i-1] >= temp

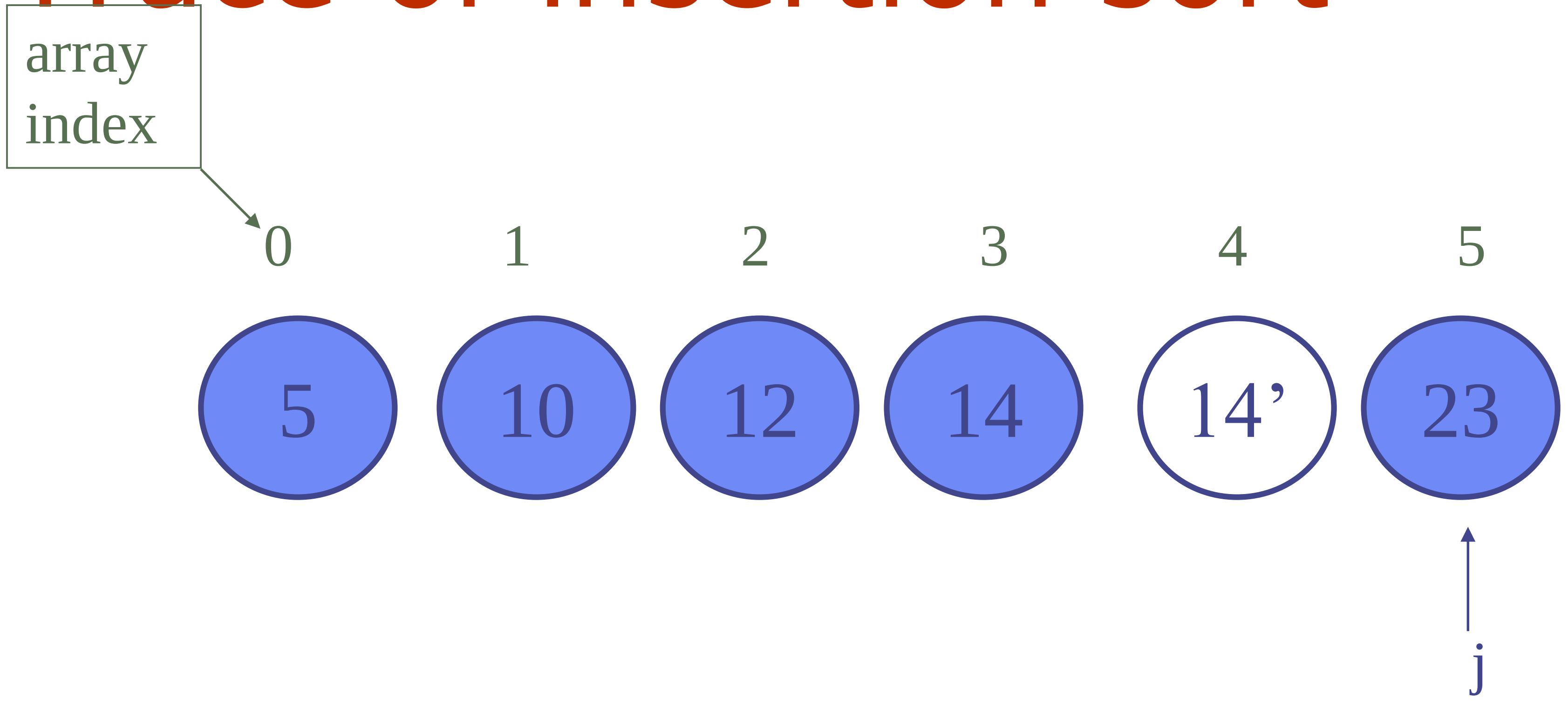
Trace of insertion sort



j = 5, fifth iteration of the outer loop

arr[i-1] >= temp

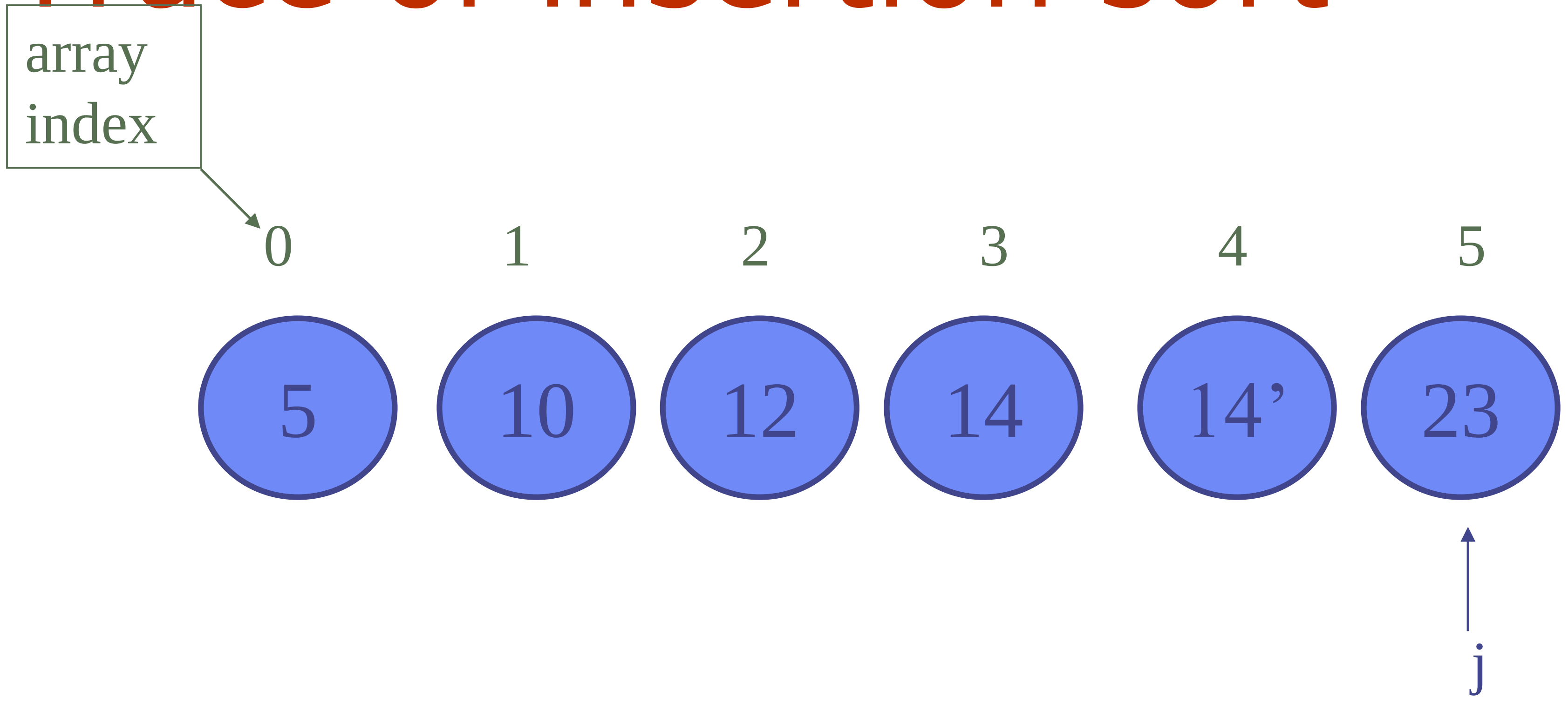
Trace of insertion sort



j = 5, fifth iteration of the outer loop

arr[i-1] = temp

Trace of insertion sort



j = 5, fifth iteration of the outer loop

arr[i-1] = temp

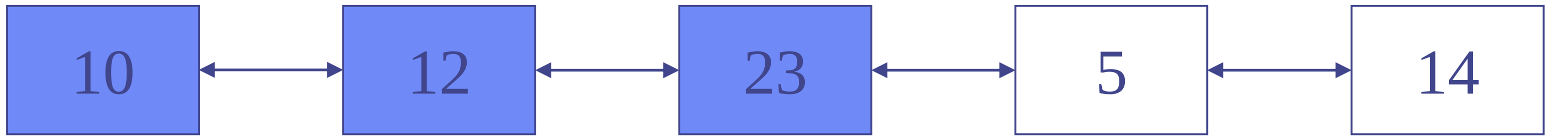
Note: the sort was stable

Complexity of insertion sort

- In the worst case, has to make n(n-1)/2 comparisons and shifts to the right
- also $O(n^2)$ worst case complexity
- best case: array already sorted, no shifts.

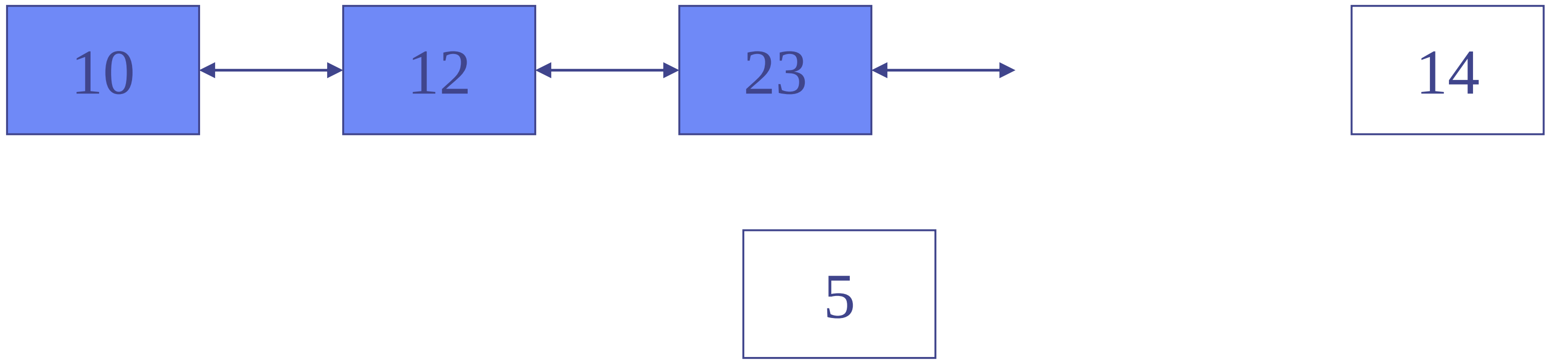
Insertion sort on linked lists

- This is a suitable sorting method for **doubly** linked lists
- We can just insert a node in a sorted portion of linked list in constant time, don’t need to shift other nodes to make space for it (but need to find the place…):



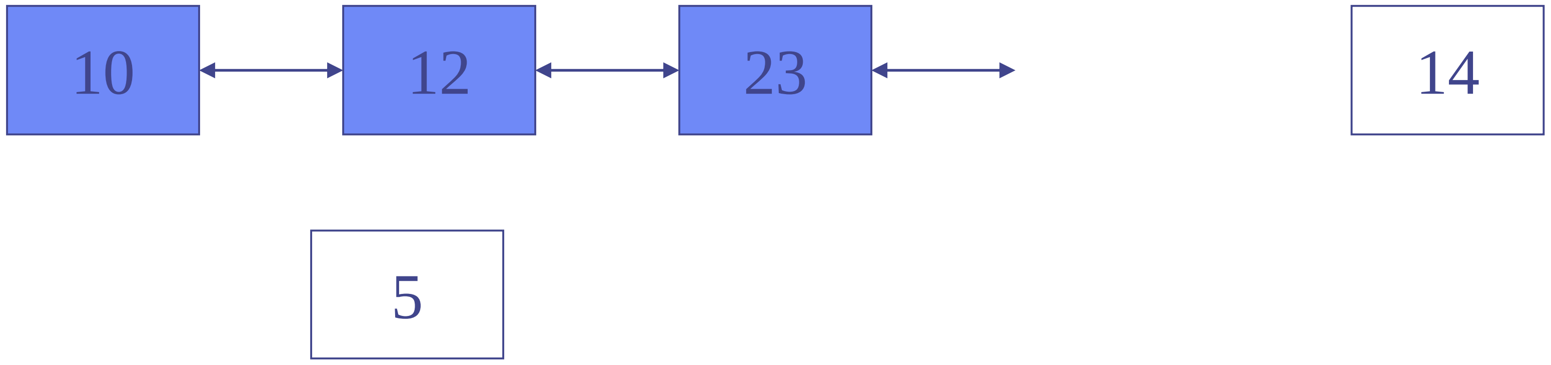
Insertion sort on linked lists

- This is a suitable sorting method for doubly linked lists
- We can just insert a node in a sorted portion of linked list in constant time, don’t need to shift other nodes to make space for it (but need to find the place…):



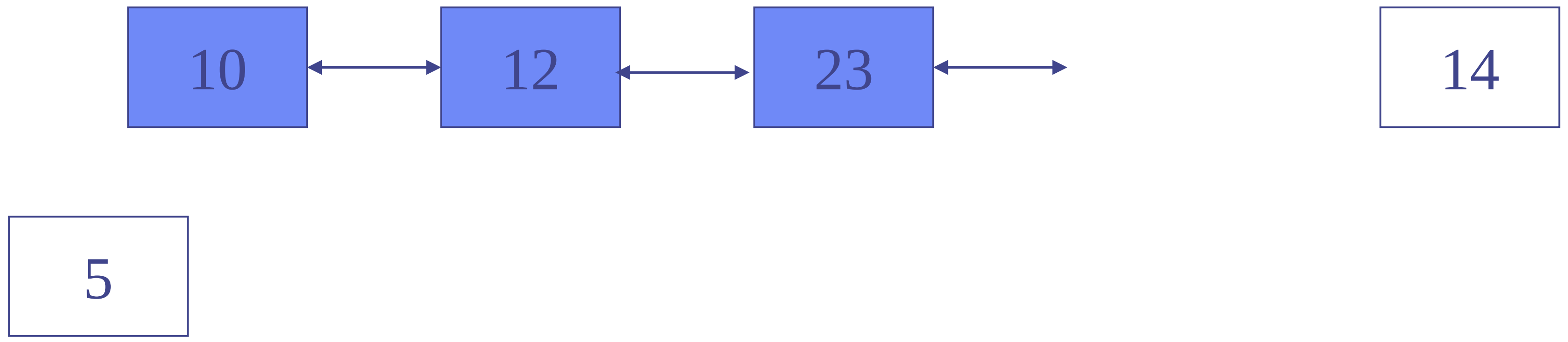
Insertion sort on linked lists

- This is a suitable sorting method for doubly linked lists
- We can just insert a node in a sorted portion of linked list in constant time, don’t need to shift other nodes to make space for it (but need to find the place…):



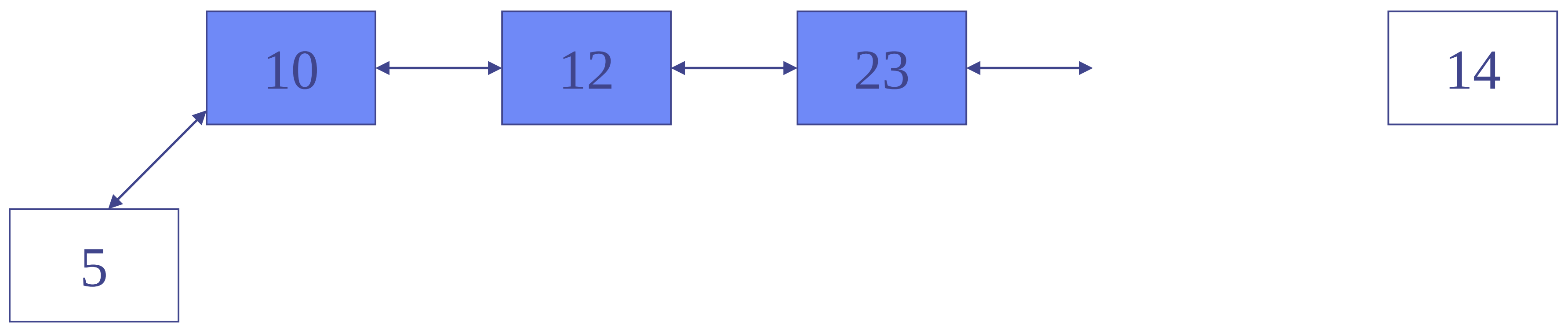
Insertion sort on linked lists

- This is a suitable sorting method for doubly linked lists
- We can just insert a node in a sorted portion of linked list in constant time, don’t need to shift other nodes to make space for it (but need to find the place…):



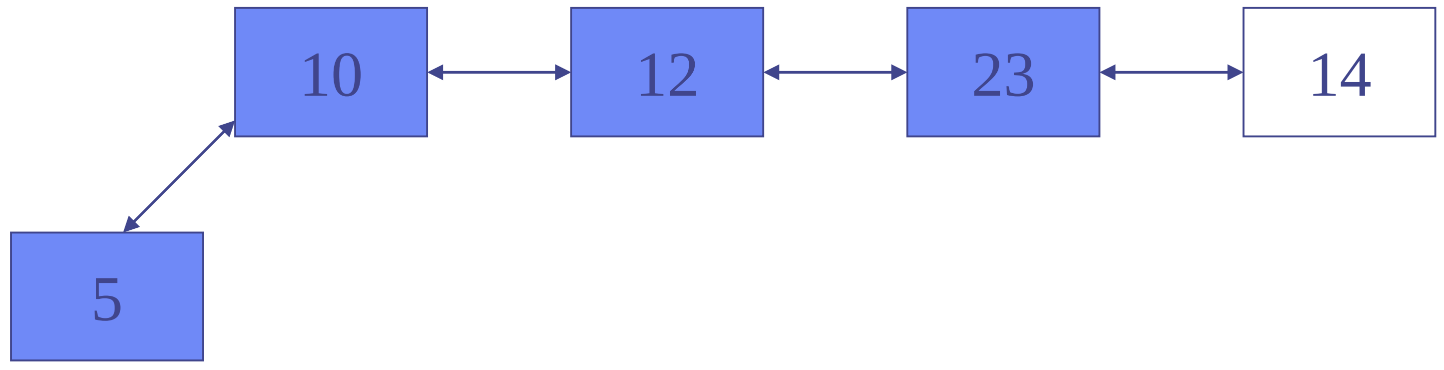
Insertion sort on linked lists

- This is a suitable sorting method for doubly linked lists
- We can just insert a node in a sorted portion of linked list in constant time, don't need to shift other nodes to make space for it (but need to find the place…):



Insertion sort on linked lists

- This is a suitable sorting method for doubly linked lists
- We can just insert a node in a sorted portion of linked list in constant time, don't need to shift other nodes to make space for it (but need to find the place…):



Implementation of insertion sort on linked lists

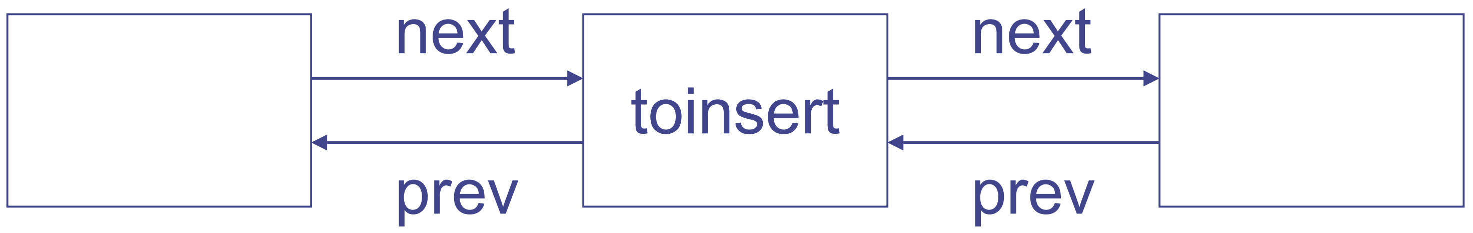
- So this time assume we have a doubly-linked list
- We will move nodes rather than swapping elements
- Assume we have DNode class which has fields element, next and prev
- DList class has head field

Implementation of insertion sort on linked lists

```
border = head; // last node in the sorted part
while (border.next ! = null) {
    DNode toinsert = border.next;
  // do we need to move it?
  if (toinsert.element >= border.element) {
     border = border.next;
  }
```

Implementation of insertion sort on linked lists

```
else { // toinsert.element < border.element
  // remove toinsert from the list
  toinsert.prev.next = toinsert.next;
  toinsert.next.prev = toinsert.prev;
```



Implementation of insertion sort on linked lists

```
// find a place between head and border to insert toinsert, in order
if (head.element > toinsert.element) {
   toinsert.next = head;
   toinsert.prev = null;
   head.prev = toinsert;
   head = toinsert;
}
```

Implementation of insertion sort on linked lists

```
// find a place between head and border to insert toinsert, in order
DNode current = head.next;
while (current.element < toinsert.element) {
   current = current.next; // this is O(n) in worst case
} // now current.element >= toinsert.element
toinsert.next = current;
toinsert.prev = current.prev;
current.prev.next = toinsert;
current.prev = toinsert;   // and close all brackets
```

Implementation of insertion sort on linked lists

- This is $O(n^2)$: in each iteration through the outer loop, we move border one step to the right
- In the inner loop, we iterate through the sorted portion, looking for a place to insert
- In the worst case (list sorted in reverse order) will have to do 1+2+…+n-1 comparisons in the sorted part.
- It would have been better (if we planned to use insertion sort for sorting almost sorted lists) to search for the insertion point not from the head of the list going towards the border, but from the border going towards the head; if the list is almost sorted, this would result in a shorter iteration.

Expectations

- Know these simple sorting algorithms
  - be able to implement them
  - recognise them
  - be able to analyse their complexity
  - (from future lectures) be able to give appropriate loop invariants
- Understand the meaning of “stable” in the context of sorting