Difference-in-Difference: Pre and Post, Treatment and Control

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September 16, 2019

Overview

Origin

The *Broad Street cholera outbreak* was a severe outbreak of cholera that occurred in 1854 near Broad Street. Physicians and scientists held two competing theories on the causes of cholera in the human body: **miasma** theory (by air) and **germ** theory (by water).

- What is the true cause.

on 31 August 1854, It is the major outbreak of cholera that occurred. Before that, several outbreaks still happened. For more info: https://en.wikipedia.org/wiki/1854_Broad_Street_cholera_outbreak

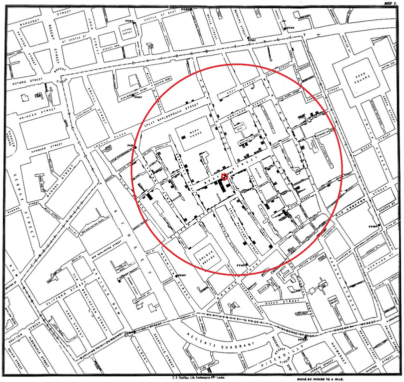
Origin

John Snow(1855), a physician, compared the changes of death rates of two resident regions where water were provided by two different waterworks companies: Southwark and Lambeth.

- In 1849, both companies were taking water from sewage-polluted sections.
- In 1852, Lambeth relocated to a less-polluted section.
- John Snow found that compared with the death rate in regions supplied water by Southwark, the death rate in regions supplied by Lambeth had a significant decrease.

Origin

Snow later used a dot map to illustrate the cluster of cholera cases around the pump.

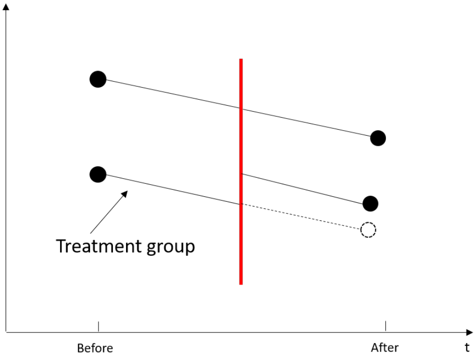


Intuition

- Difference in differences is a statistical technique used in econometrics and quantitative research in the social sciences that attempts to **mimic** an experimental research design.
- Use **observational** panel data to control for unobserved-but-fixed omitted variables
- To understand **the effect of a sharp change** in the economic environment or government policy.
- **Key conceptions**: pre, post, treatment, control.

Intuition

Key conceptions: pre, post, treatment, control.



Experiments vs Quasi Experiments

- **Experiments**: the experimenter *manipulates* some aspects of people's reality and checks for what effect this has on the targeted outcome, usually **with random assignment**.
  - Medical experiment
  - Lab experiment: Stanford prison experiment
- **Quasi Experiments**: a study that appears to be an experiment but is actually not. Control is bound to barely exist in such an experiment, usually **without random assignment**.
  - The relocation in Broad Street cholera outbreak case
  - Military mobilization & Female labor supply

Random Assignment

- It ensures that each participant or subject has an equal chance of being placed in treatment group or control group.
- It ensures that any differences between and within the groups are not systematic at the outset of the experiment.

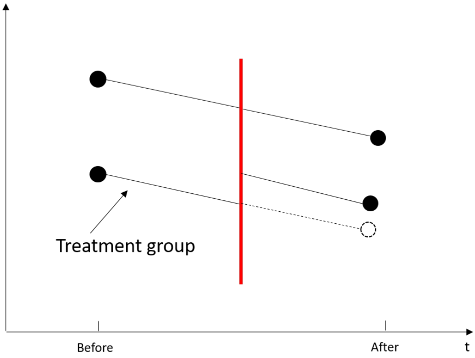
- Thus, any differences between groups recorded at the end of the experiment can be more confidently attributed to the experimental procedures or treatment.

Data

Observational Data: In fields such as epidemiology, social sciences, psychology and statistics, an observational study draws inferences from a sample to a population where the independent variable is not under the control of the researcher because of ethical concerns or logistical constraints.

- Time series data
- Cross-sectional data
- Panel data

General DID



Two-period Panel Data Analysis

- Say random assignment to treatment and control groups, like in a medical experiment
- One can then simply compare the change in outcomes across the treatment and control groups to estimate the treatment effect
- For time 1, 2, groups A, B
  - $(y_{2,B} - y_{2,A}) - (y_{1,B} - y_{1,A})$, or equivalently
  - $(y_{2,B} - y_{1,B}) - (y_{2,A} - y_{1,A})$, is the difference-in-differences

Difference-in-Difference in a Regression Framework

- A regression framework using *time* and *treatment* dummy variables can calculate this difference-in-difference as well.
- Consider the model:

$$y_{it} = \beta_0 + \beta_1 treatment_i + \beta_2 post_t + \beta_3 treatment_i * post_t + \mathbf{X}'_{it}\boldsymbol{\beta} + u_{it}$$

- $treatment_i = 1$, if individual i is in the treatment group.
- $post_t = 1$, if t period is after the treatment event.

Difference-in-Difference in a Regression Framework

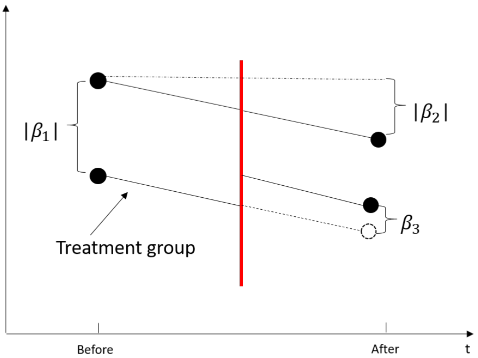
- Consider the model:

$$y_{it} = \beta_0 + \beta_1 treatment_i + \beta_2 post_t + \beta_3 treatment_i * post_t + \mathbf{X}'_{it}\boldsymbol{\beta} + u_{it}$$

- The estimated β_3 will be the difference-in-differences in the group means
- Additional covariates $\mathbf{X}_{it}$ can be added to the regression to control for differences across the treatment and control groups

General DID

$$y_{it} = \beta_0 + \beta_1 treatment_i + \beta_2 post_t + \beta_3 treatment_i * post_t + \mathbf{X}'_{it}\beta + u_{it}$$



Regression DID Advantages

- It's easy to add additional states or periods to the regression set-up.
- It facilitates empirical work with regressors other than switched-on/switched-off dummy variables
  - For example, we might look at all state minimum wages in the United States.
  - Some of these are a little higher than the federal minimum (which covers everyone regardless of where they live), some are a lot higher, and some are the same.
  - The minimum wage is therefore a variable with differing "treatment intensity" across states and over time.
- It is possible to incorporate additional covariates to control ommited trends.

Difference-in-Difference in a Regression Framework

There is another form of DID: **fixed effect**

- A regression framework using individual fixed effect and time fixed effect variables can calculate this difference-in-difference as well.
- Consider the model:

$$y_{it} = \beta_0 + f_i + T_t + \beta_1 * Treat\_Post_{it} + \mathbf{X}'_{it}\beta + u_{it}$$

- The estimated β_1 will be the difference-in-differences
- Additional covariates $\mathbf{X}_{it}$ can be added to the regression to control for differences across the treatment and control groups

Difference-in-Difference in a Regression Framework

What's the difference?

$$y_{it} = \beta_0 + \beta_1 treatment_i + \beta_2 post_t + \beta_3 treatment_i * post_t + \mathbf{X}'_{it}\boldsymbol{\beta} + u_{it}$$

$$y_{it} = \beta_0 + f_i + T_t + \beta_1 * Treat\_Post_{it} + \mathbf{X}'_{it}\boldsymbol{\beta} + u_{it}$$

Difference-in-Difference in a Regression Framework

What's the difference?

$$y_{it} = \beta_0 + \beta_1 treatment_i + \beta_2 post_t + \beta_3 treatment_i * post_t + \mathbf{X}'_{it}\beta + u_{it}$$

$$y_{it} = \beta_0 + f_i + T_t + \beta_1 * treat\_post_{it} + \mathbf{X}'_{it}\beta + u_{it}$$

- The individual fixed effect f_i can measure some unobserved time-invariant individual features.
- The second one would be better if there are possible omitted time-invariant variables.
- As a general rule, omitted time-invariant individual-specific effects are likely, time-specific individual-invariant effects somewhat less so.

Difference-in-Difference in a Regression Framework

Is this regression formulation right?

$$y_{it} = \beta_0 + f_i + T_t + \beta_1 treat_i + \beta_2 post_t + \beta_3 treatment_i * post_t + \mathbf{X}'_{it}\beta + u_{it}$$

Difference-in-Difference in a Regression Framework

Is this right?

$$y_{it} = \beta_0 + f_i + T_t + \beta_1 treat_i + \beta_2 post_t + \beta_3 treat_i * post_t + \mathbf{X}'_{it}\beta + u_{it}$$

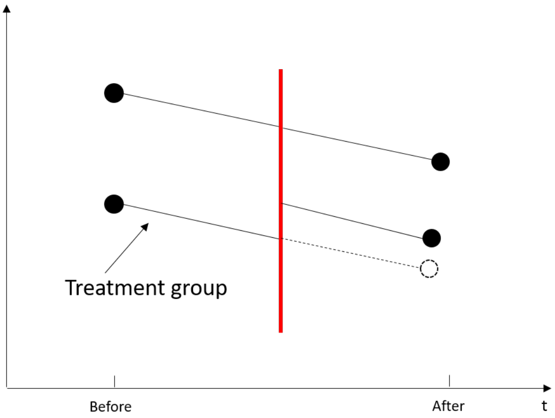
No, it's wrong

The *TREAT* variable will be colinear with the individual fixed effects f_i, and the *POST* variable will be colinear with the time fixed effects T_t.

Key Assumption Behind DID

- In the absence of treatment, the average change in the response variable would have been the same for both the treatment and control groups.
  - a.k.a. the **parallel trends** assumption since it requires that the trend in the outcome variable for both treatment and control groups during the pre-treatment era are similar.

Key Assumption Behind DID



Key Assumption Behind DID

- What this assumptions does *NOT* mean:
  - It does *NOT* mean that the there is no trend in the outcome variable during the pre-treatment era (just the same trend across groups).
  - It does *NOT* require that the level of the outcome variable for the two groups be same in the pre-treatment era.
    - This can create other issues.

Minimum Wages and Employment

In a competitive labor market, increases in the minimum wage move us up a downward-sloping demand curve. Higher minimums therefore reduce employment, perhaps hurting the very workers minimum-wage policies were designed to help.

Minimum Wages and Employment

- **Ideal but Impossible Experiment**:
  - Take a set of states, increase the minimum wage and measure employment.
  - “Rewind the clock,” take the same set of states and measure their employment.
  - Compare employment across two scenarios.
- **Desirable but Infeasible Experiment**:
  - Take a set of states and randomly select some fraction of states to be subject to the new minimum wage policy.
  - Compare employment across the two sets of states.

Minimum Wages and Employment

- The key to program evaluation is estimating the counterfactual: What would have happened had the treated not be treated?
- Therefore, quality of our evaluation is tied to how well we can estimate the counterfactual.
  - The Ideal but Impossible Experiment actually provides the counterfactual by "rewinding the clock."
  - The Desirable but Infeasible Experiment provides a good estimate of the counterfactual by the random assignment.

Minimum Wages and Employment

On April 1, 1992, New Jersey raised the state minimum from 4.25 to 5.05.

- Card and Krueger (1994) consider the impact of New Jersey's 1992 minimum wage increase from 4.25 to 5.05 per hour.
- Compare employment in 410 fast-food restaurants in New Jersey and eastern Pennsylvania before and after the rise.
- Survey data on wages and employment from two waves:
  - Wave 1: March 1992, one month before the minimum wage increase.
  - Wave 2: December 1992, eight month after increase.

Minimum Wages and Employment

- Individual i : 410 fast-food restaurants in New Jersey and eastern Pennsylvania
- States s : New Jerse (treatment group) and Pennsylvania (control group)
- Periods t : March 1992 and November 1992, Treatment time: April 1992. (The pre-event survey in the paper was conducted in late Feb and early March.)
- Treatment : raised the state minimum wage
- Potential Outcomes:
  - Y_{1ist} = fast food employment at restaurant i and period t if there is a high state minimum wage (treatment group)
  - Y_{0ist} = fast food employment at restaurant i and period t if there is a low state minimum wage.

Two-Period Panel Data Analysis

- Employment is determined by the sum of a time-invariant state effect and a year effect that is common across states in the absence of a minimum wage change.

$$E(Y_{0ist}|s,t) = \lambda_t + \gamma_s \tag{1}$$

- Let D_{st} be a dummy for high-minimum-wage states, where states are index by s and observed in period t.

$$D_{st} = \begin{cases} 1, & \text{if state } s \text{ is treated at period } t \\ 0, & o.w. \end{cases}$$

- Then, we have:

$$Y_{ist} = \lambda_t + \gamma_s + \beta D_{st} + \epsilon_{ist} \tag{2}$$

where $E(\epsilon_{ist}|s,t) = 0$.

Two-Period Panel Data Analysis

- From (2), we get:

  $$E(Y_{ist}|s=NJ,t=Nov)-E(Y_{ist}|s=NJ,t=Feb)=\lambda_{Nov}-\lambda_{Feb}+\beta$$

  and

  $$E(Y_{ist}|s=PA,t=Nov)-E(Y_{ist}|s=PA,t=Feb)=\lambda_{Nov}-\lambda_{Feb}$$

- The population DID:

  $$E(Y_{ist}|s=NJ,t=Nov)-E(Y_{ist}|s=NJ,t=Feb)$$

  $$-[E(Y_{ist}|s=PA,t=Nov)-E(Y_{ist}|s=PA,t=Feb)]=\beta$$

Two-Period Panel Data Analysis

	Nov (Post)	Feb (Pre)	Diff
NJ (Treat Group)	$\lambda_{NJ} + \gamma_{Nov} + \beta$	$\lambda_{NJ} + \gamma_{Feb}$	$\gamma_{Nov} - \gamma_{Feb} + \beta$
PA (Control Group)	$\lambda_{PA} + \gamma_{Nov}$	$\lambda_{PA} + \gamma_{Feb}$	$\lambda_{Nov} - \lambda_{Feb}$
Diff	$\lambda_{NJ} - \lambda_{PA} + \beta$	$\lambda_{NJ} - \lambda_{PA}$	β

- $\lambda_{NJ} - \lambda_{PA} + \beta$,$\lambda_{NJ} - \lambda_{PA}$: **Cross-Sectional** difference
- $\gamma_{Nov} - \gamma_{Feb} + \beta$,$\lambda_{Nov} - \lambda_{Feb}$: **Time-series** Difference

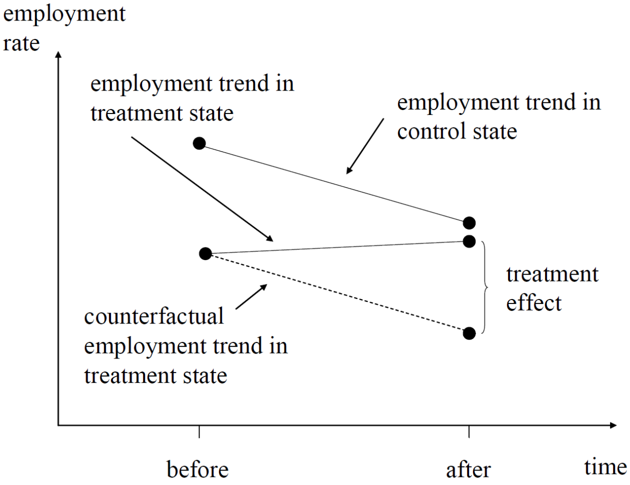
Results

Table 5.2.1: Average employment per store before and after the New Jersey minimum wage increase

Variable	PA (i)	NJ (ii)	Difference, NJ-PA (iii)
1. FTE employment before, all available observations	23.33 (1.35)	20.44 (0.51)	-2.89 (1.44)
2. FTE employment after, all available observations	21.17 (0.94)	21.03 (0.52)	-0.14 (1.07)
3. Change in mean FTE employment	-2.16 (1.25)	0.59 (0.54)	2.76 (1.36)

Notes: Adapted from Card and Krueger (1994), Table 3. The table reports average full-time equivalent (FTE) employment at restaurants in Pennsylvania and New Jersey before and after a minimum wage increase in New Jersey. The sample consists of all stores with data on employment. Employment at six closed stores is set to zero. Employment at four temporarily closed stores is treated as missing. Standard errors are reported in parentheses

Figure


employment
rate
employment trend in
treatment state
employment trend in
control state
treatment
effect
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employment trend in
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before
after
time

Regression DID

- We can use regression to estimate equation like (2).
- NJ_s: dummy for restaurants in New Jersey
- d_t: a time-dummy that switches on for observations obtained in November,Then:

$$Y_{ist} = \alpha + \gamma NJ_s + \lambda d_t + \beta(NJ_s \cdot d_t) + \epsilon_{ist} \tag{3}$$

- If more additional covariates are introduced:

$$Y_{ist} = \alpha + \gamma NJ_s + \lambda d_t + \beta(NJ_s \cdot d_t) + \mathbf{X}'_{ist}\boldsymbol{\eta} + \epsilon_{ist} \tag{4}$$

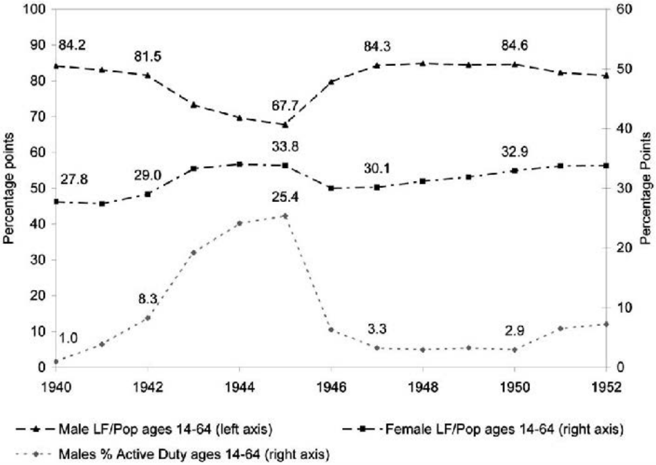
What if

- What if the treatment is not uniformly imposed on states?
- Then $Treatment_i$ cannot be a binary variable.
- Continuous variable?

Women, War, and Wages

- Acemoglu (2004) exploits the military mobilization for World War II (1939-1945) to investigate the effects of female labor supply on the wage structure. The mobilization drew many women into the workforce.
- They studied female labor force participation before and after World War II (WWII) as a source of plausibly exogenous variation in female labor supply.
- The war drew many women into the labor force as 16 million men mobilized to serve in the Armed Forces, with over 73 percent deploying overseas.

Women, War, and Wages



Women, War, and Wages

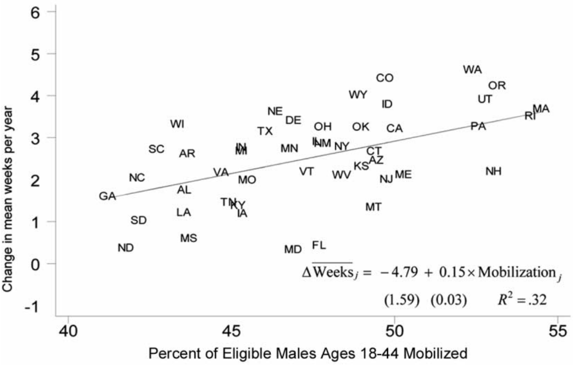
- But the impact was not uniform across states.
- The Selective Service's guidance for deferred exemption was based on marital status, fatherhood, essential skills for civilian war production, and temporary medical disabilities, but it left considerable discretion to the local boards.
- Because of the need to maintain an adequate food supply to support the war effort, states with a higher percentage of farmers had substantially lower mobilization rates.
- Most military units were still segregated in the 1940s, and there were relatively few black units, hence states with higher percentages of blacks also had lower shares.

Women, War, and Wages

- **Mobilization intensity**: The mobilization variable is the number of men 18–44 who served divided by the number registered in each state.
- **Female labor supply**: female weeks worked per year.
- The only problem we care is how different mobilization intensities influent the female labor supply.
- **Pre**: 1940; **Post**: 1950

Women, War, and Wages

WWII mobilization rates and change in mean female weeks worked per year, 1940–50.



Regression DID

These models, which pool data from 1940 and 1950, have the following structure:

$$Y_{ist} = \delta_s + \gamma * d_{1950} + \mathbf{X}'_{ist}\beta_t + \varphi * d_{1950} * m_s + \epsilon_{ist} \qquad (5)$$

- y_{ist}: weeks worked by woman i residing in state s in year t (1940 or 1950).
- δ_s: states fixed effect.
- d_{1950}: a dummy for 1950.
- X_{ist}: denotes other covariates including state of birth or country of birth, age, race, share of farmers and nonwhites, and average schooling.
- m_s: the mobilization rate.
- φ: the coefficient of interest.

Results

TABLE 5
IMPACT OF WWII MOBILIZATION RATES ON LABOR SUPPLY, 1940–50
Dependent Variable: Annual Weeks Worked

	REGRESSION			
	(1)	(2)	(3)	(4)
	A. White Females (N=530,026)			
Mobilization rate × 1950	11.17	9.85	10.64	8.51
	(1.89)	(2.05)	(2.65)	(2.37)
1940 male share farmers × 1950			1.74	1.04
			(1.08)	(1.05)
1940 male share nonwhite × 1950			−1.96	−.72
			(1.15)	(1.37)
1940 male share average education × 1950				.52
				(.16)
R^2	.01	.18	.18	.18
Includes marital status, age, state of birth	no	yes	yes	yes

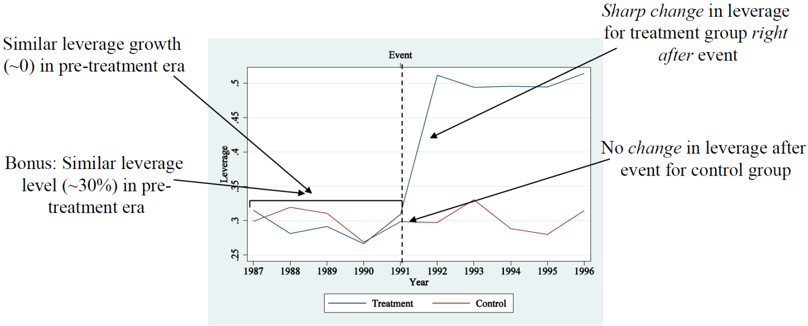
The Parallel Trends Assumption

In the absence of treatment, the average change in the response variable would have been the same for both the treatment and control groups.

- Strictly speaking, untestable
- But, we can inspect pre-treatment era outcomes.
  - By definition, this requires more than two periods of data.

Graphical Analysis

A "NICE" DID picture of bankruptcy law experiment



Statistical Analysis

- We could also perform a t-test of trend in average leverage growth rates across the treatment and control groups during the pre-treatment era.
  - This should turn out *insignificant* (statistically and economically) if the parallel trends assumption is valid
- You should be skeptical of any DD that doesn't show you a picture or an explicit test of this assumption.

Statistical Analysis

- Let's consider the regression DID framework:

$$y_{it} = \beta_0 + \beta_1 * Treat_i + \beta_2 * t * Treat_i + \beta_3 * t + T_t + \epsilon_{it} \quad (6)$$

- We want to verfity β_2 is statistically insignificant.

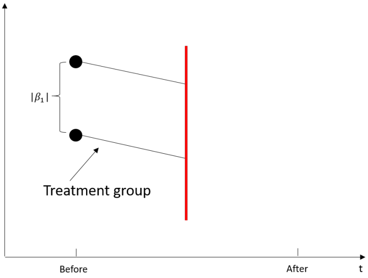


Figure: parallel

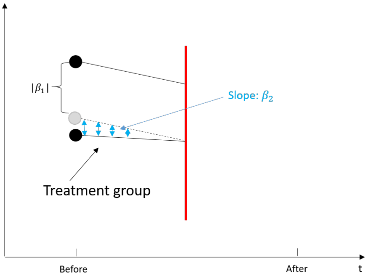


Figure: non parallel

Summary of DID

- Pre, post, treatment, control.
- Two general DID models:
  - $y_{it} = \beta_0 + \beta_1 treatment_i + \beta_2 post_t + \beta_3 treatment_i * post_t + \mathbf{X}'_{it}\beta + u_{it}$
  - $y_{it} = \beta_0 + f_i + T_t + \beta_1 * Treat\_Post_{it} + \mathbf{X}'_{it}\beta + u_{it}$
  - continuous variables.
- Key assumption: parallel
  - Graphical analysis
  - Statistical analysis

Potential Concerns

- Heterogeneous responses
- Time-variant fixed effects
- Extrapolation & counterfactual prediction

Heterogeneous Responses (1)

- Ideally control group shows no response to the treatment.
- Otherwise we need different responses by the treatment and control groups.
  - But, if intensity of responses is different, estimated effect may be meaningless. E.g., Imagine the following model:

    $$y_{it} = \mu_i + \alpha_t + \beta_i X_{it}$$

    where xit increases from 0 to X_T for treatment subjects and 0 to X_C for control subjects. The DID estimate is:

    $$\beta = [y_1^T - y_0^T] - [y_1^C - y_0^C] = \beta_T X^T - \beta_C X^C$$

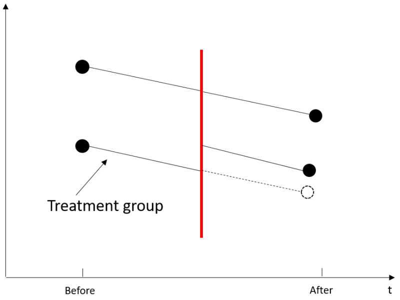
- If

  $$\beta_C = 2\beta_T,\ X^T = 2X^C$$

  then the DID estimator $\beta = 0$, despite a potentially large effect. (See Feldstein (1995))

Heterogeneous Responses (2)

- Although differences on treatment subjects and the control subjects during pre-treatment periods are allowed, potential heterogeneous responses due to exaggerative differences should be in the mind of researches.



Heterogeneous Responses (2)

Solution:

- Find comparable individuals.
- Matching algorithm

Time-variant fixed effects

Fixed effects and DID make sense only if unobservable features are time-invariant.

- Recall the case: minimum wage effect: $E(Y_{0ist}|s,t) = \lambda_t + \gamma_s$, where γ_s is the time-invariant state fixed effect.
  - What if the labor migrations from other states change the labor supply in NJ?

Time-variant fixed effects

Solution: lagged dependent variables

- Training programs & Earnings: Ashenfelter (1978) and Ashenfelter and Card (1985) find that training participants typically have earnings histories that exhibit a pre-program dip. What makes trainees special is their earnings *h* periods ago. We can then use panel data to estimate:

$$Y_{it} = \alpha + \theta Y_{i,t-h} + \lambda_t + \delta D_{it} + \mathbf{X}'_{it}\beta + \epsilon_{it}$$

- Time-series-related concerns (beyond the scope of DID)

Extrapolation & counterfactual prediction

As with all natural experiments and quasi-experimental methods, extrapolation is tricky.

- Take care in extrapolating the results.
- Internal validity is often inversely related to external validity.