

MATHEMATICS ANALYSIS AND
APPROACHES HL
Producing the IB Logo with the Fourier Series

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1 Rationale

I have shown interest in visual arts done through the means of software, with particular experience in 3D modelling and animation in Blender and Cinema 4D.

I never was experienced with drawing, therefore producing digital art on a 2D plane using artistic skill was not of interest to me. However, something that I came across online was the use of the Fourier Series in order to produce vector art, which instantly intrigued me.

While vector art files such as those with the file extension ".svg" relate to mathematics in the sense that it contains multiple graphed mathematical relationships in order to produce an image, the method of using the Fourier Series to produce similar art is more mathematically intriguing, as it proves use just one expression to produce the same result done by the numerous mathematical relationships.

2 Aim

The objective of this investigation is to link Fourier series with complex numbers to create a single series that is capable of reproducing the IB logo on the Argand plane.

3 Plan of Action

This exploration focuses on the following areas of math:

- Integral Calculus
- Series
- Trigonometry
- Complex Analysis
- Vectors

4 Background Information

4.1 Basics behind the Fourier Series

A periodic function is one where the output for a particular input equals to the output for the sum of the same input and the value of the function's period. This can be represented mathematically as:

$$f(x) = f(x + P)$$

where $P =$ the period of the function

The sine wave is widely known for being a periodic function for the ease of graphing a sinusoidal wave. However, there are periodic functions that are difficult to graph with an algebraic expression, such as one that alternates between 1 and -1 or one that is shaped as a zig-zag.

This is the motivation behind the Fourier Series, which is to be able to represent period functions that normally can't be represented by an algebraic function.

The idea behind the Fourier Series is to take an infinite sum of varying sinusoidal functions such that a desired periodic function is produced.

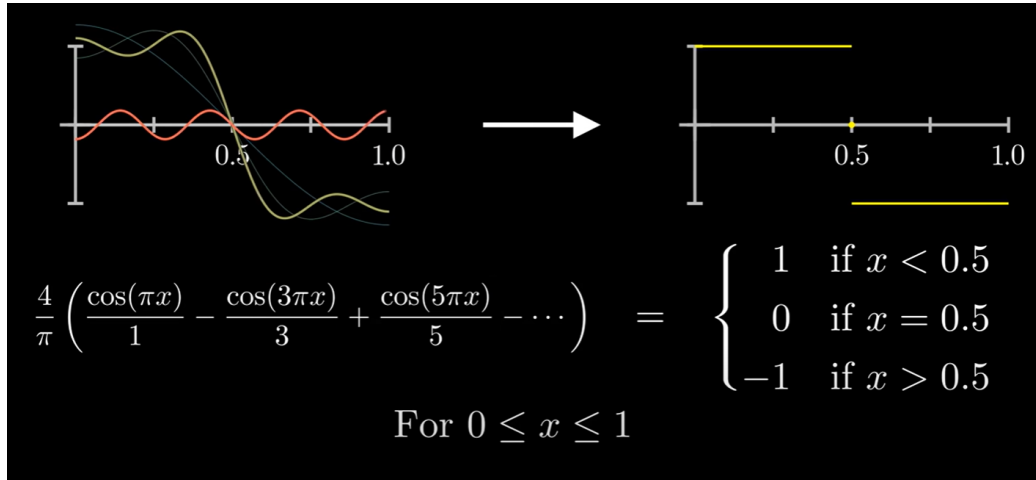


Figure 1: Visualization of the mechanism of the Fourier Series (Sanderson, 2019). The yellow line is the periodic function resulting from the previous iteration, the red line is the sinusoidal function to be added in the next iteration.

A general formula for representing a periodic function as a Fourier Series with a period of 2π is:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt$$

4.2 Fundamental application to draw on the Cartesian Plane

4.3 Rotating vectors on the Argand Plane

4.4 Enriched application to draw on the Argand Plane

References

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