Name William Chen

Partner 1 ____________

BERMUDA TREASURE EXPLORATION

For this project, you will follow the below storyline in order to find lost pirate treasure! This project will take from various parts of our class, so you can start working on it rather early, though you likely will not be able to complete it until the end of the semester. You may work with up to one partner on this assignment, or you may work alone. Show ***ALL*** necessary work for each problem and be sure to box or circle your final answers.

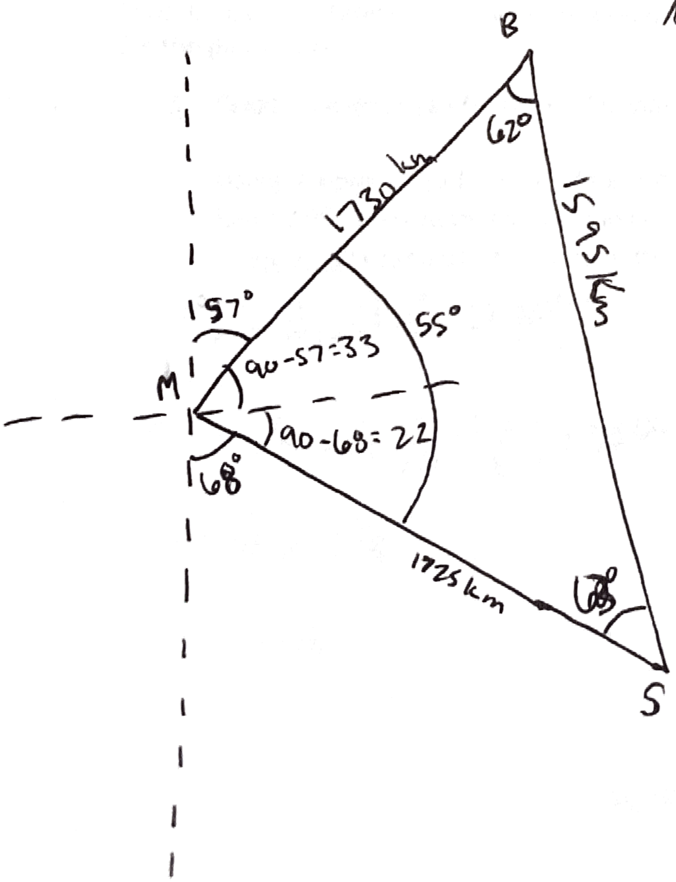
You are hired as the navigator aboard the *Blacktide,* a vessel piloted by some of the saltiest, scurvy scallywags you have ever met, but they pay well. Your mission is the sail into the Bermuda Triangle and recover the wreck of *The Arabella*, the lost pirate ship of the notorious Captain Blood. The ship was rumored to have been carrying a fortune in stolen gold from the Dutch province of St. Martin before sinking near the center of the mysterious Bermuda Triangle.

Here is what you need to know about the Bermuda Triangle before going into this adventure: The Triangle itself is formed between three points: the city of Miami, Florida (M), the island of Bermuda (B), and the capital of Puerto Rico, San Juan (S). The island of Bermuda is roughly 1,730 kilometers $N57°E$ of Miami and the city of San Juan is roughly 1,725 kilometers $S68°E$ of Miami

1. As the *Blacktide* departs from its port in San Juan heading toward the center of the triangle, you decide to do some quick calculations for the dimensions of the Bermuda Triangle. Draw a picture of the triangle given the dimensions and bearings above and determine the values of all three angles inside of the Bermuda Triangle and the distance from Bermuda to San Juan in kilometers. Round to the nearest whole number for all values.

$M^2 = 1730^2 + 1725^2 - 2(1730 \cdot 1725) \cdot \cos(55)$

$\approx \boxed{1595 \text{ km}}$



$\overline{BS} = 1595\text{km}$
$\angle B = 62°$
$\angle M = 55°$
$\angle S = 63°$

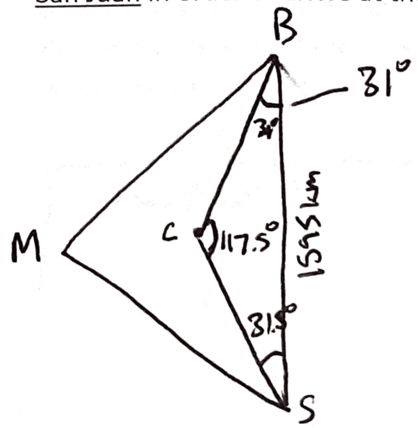
$\frac{\sin(55)}{1595} = \frac{\sin \angle B}{1725}$

$\angle B = 62°$

$180 - 62 - 55 = 63°$

About 105 kilometers into your journey, most of the navigation equipment goes haywire and becomes mostly useless, so the captain turns to you for advice. You know you have traveled 105 kilometers thus far along the correct path from San Juan toward the center of the triangle and the ship can keep the same bearing, so you just need to know how much further you need to travel in order to get to the center of the triangle

2. Draw the sketch of the Bermuda Triangle again, this time with the center (C) labeled. If you draw the triangle ΔBCS, you will notice that the line segment $\overline{CS}$ perfectly bisects angle S, and the line segment $\overline{BC}$ perfectly bisects angle B. With this in mind, determine how much further the *Blacktide* needs to travel along its path from San Juan in order to arrive at the center of the triangle. Round to the nearest hundredth of a kilometer.



$\angle C = 180 - 31 - 31.5 = 117.5$

$\dfrac{\sin(117.5)}{1595} = \dfrac{\sin(31)}{\overline{CS}}$

$\overline{CS} = 926.128\text{km}$

$926.128 - 105 = \boxed{821.128\text{km}}$

As you travel, you decide to watch the waves closely and do some calculations to see if there are any signs of a storm on the horizon. Wave height in the open ocean tend to average to about 2-6 meters high from the **crest** (or peak) of the wave to the **trough** of the wave. When a large hurricane or tropical storm is within 100 kilometers of your current location, the waves tend to increase to more than 6 meters high, oftentimes more than 9. You time from one peak of a wave to another peak to determine the period of the wave function, which you estimate to be about 6π. Further, you know that the average ocean depth in the current area is about 3,800 meters, which you use as the positive Vertical Shift. Lastly, you determine that cosine would be the best function to use, so that you do not have to calculate a value for the phase shift.

3. Create the equation for the cosine wave in the following format:

$$f(x) = A\cos(cx) + B$$

Using the point $P(53, 3799)$, determine the value of the amplitude A in your function rounded to the nearest hundredth. Be sure that you are in radian mode in your calculator. Should you be worried about a tropical storm in the area? Graph one full cycle of this function starting at $x = 0$ and list or label at least five points.

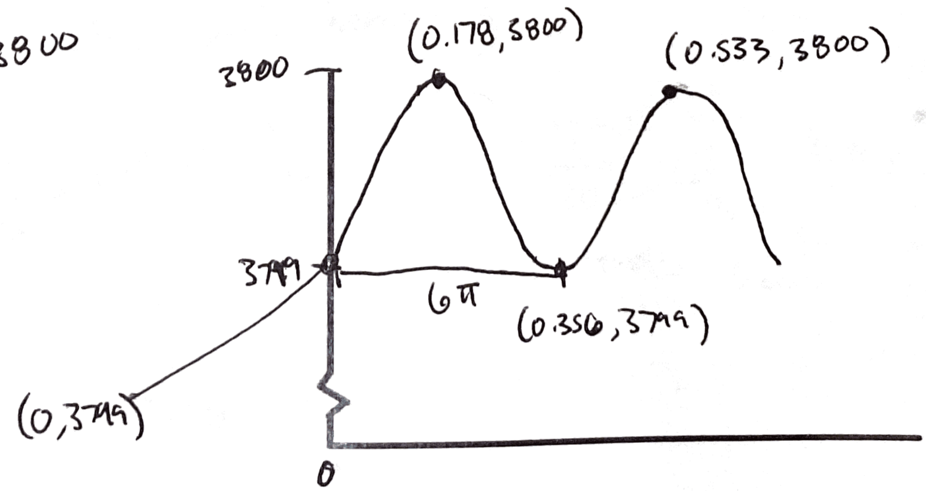
$f(x) = A\cos\left(\frac{2\pi}{P}x\right) + 3800$

$3799 = A\cos\left(\frac{2\pi}{6\pi} \cdot \frac{53}{1}\right) + 3800$

$-1 = A\cos\left(\frac{53}{3}\right)$

$\dfrac{-1}{\cos\left(\frac{53}{3}\right)} = A$

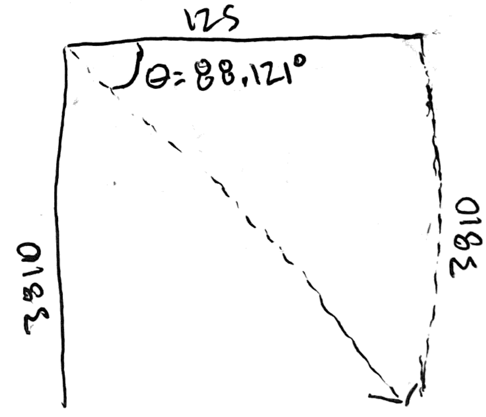
$\boxed{A = -2.644}$



No need to worry about a storm

Once you make it to the Center, you begin to search the ocean floor using acoustic location, i.e. sonar mapping, hoping that your instruments "hear" an acoustic reflection that differs from the rest of the ocean floor. Just as the sonar picks up a signal that seems different, the machine goes blank and becomes useless. However, the technician assigned to the machine recorded a couple numbers: the ocean depth at this location is about 3,810 meters and the machine was scanning the circular region around the ship with a radius of 125 meters. Your crew decides to send down the two piloted submersibles to explore the region themselves.

4. Using the dimensions given, first find the **angle of depression** (in degrees) rounded to the nearest hundredth that the submersibles should follow in order to reach the edge of the circular region from your ship. Then, determine the area of the circular region at the bottom of the ocean in meters squared rounded to the nearest whole square meter.



$\tan(\theta) = \frac{3810}{125}$

$\tan^{-1}\left(\frac{3810}{125}\right) = \theta$

$\boxed{\theta = 88.121°}$

Area $= \pi(125)^2$

$= \boxed{49087\ m^2}$

5. The submersibles together can search 850 square meters of area thoroughly every ten minutes, but they need to return to the surface to refill the air supply and fuel after every four hours of searching. When the submersibles need to return to the surface, it takes an additional 2 hours to surface, refuel, and submerge again. Determine the maximum amount of time it will take to search the entire region using these submersibles, including the time needed to refuel. Assume that both submersibles start with a full tank of fuel. Round your answer in hours to the nearest hundredth.

~~$\frac{49087}{850}$ = 57.749 = 577.492 minutes~~

$\frac{49087}{85} = 577.494$ minutes total searching

$\frac{850}{10\ min} \times \frac{60\ min}{1\ hour} \times \frac{2\ hours}{1\ trip} = 10200\ ft^2/trip$

$\frac{49087\ ft^2}{10200\ ft^2/trip} = 4.8$ trips for every hours

$\frac{577.494\ min}{60} = 9.625$ hours

$4.8 \times 2 = 9.6$ hours

$9.6 + 4.8 = \boxed{14.4\ hours}$

The captain did not like to hear the answer from problem 5 and instead suggests searching the region by splitting the region into six sectors, starting with the three sectors in the direction the *Blacktide* was facing when the sonar equipment stopped responding. Searching in this way will increase the speed of the submersibles due to the focused region provided to the submersibles' computer systems. With the sector method, you estimate that the two submersibles together can search 1,000 square meters every ten minutes but still need to return to the surface after every four hours of searching. Surfacing, refueling, and submerging still takes a total of 2 hours. However, once a sector is complete, the submersibles need to return and recalibrate equipment to keep the submersibles in the correct sectors, which takes about 1 hour each time. This will add an additional 5 hours to the entire process. Assume that the submersibles cannot refuel and recalibrate at the same time, for simplicity.

6. With all of the above information in mind, determine the area in square meters (rounded to the nearest whole number) of one sector, then determine how long it would take the submersibles to search a single sector, but not to refuel or recalibrate equipment. Round your answer in hours to the nearest hundredth.

$$\frac{49087\text{ ft}^2}{6\text{ sectors}} = 8181.17\text{ ft}^2/\text{sector}$$

$$\frac{1000\text{ m}}{10\text{ min}} \times \frac{60\text{ min}}{1\text{ hour}} \times \frac{5}{\text{hours}} = 3000$$

$$\frac{8181.17\text{ ft}^2/\text{sector}}{3000\text{ ft}^2} = 2.73\text{ dives/sector} \times 5 = \boxed{13.64\text{ hours}}$$

7. If the captain is wrong and this sector method does not work because the initial signal was delayed, how long will it take to search the entire 125 meter-radius region from problem 4 using this method, including the additional 5 hours of recalibration? Round your answer in hours to the nearest hundredth. The captain argues that this is better due to the increased speed of searching and the possibility of finding the treasure faster. Would you recommend to use the sector method suggested by the captain or stick to the original method from problem 5? Briefly explain your reasoning.

13.64hours / sector × 6 = 81.8hours total

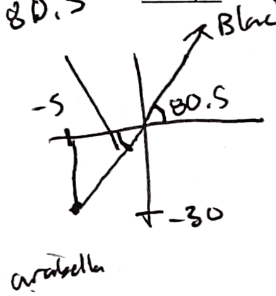
The method from problem 5 ~~was~~ would be better because it would take 14.4 hours instead of 81.

After a few days of searching, the submersibles have finally found what they believe to be the remains of *The Arabella*, though the ship is now in tatters and spread wide across the ocean floor due to the pressure. After a few more hours of searching, the team finds what they believe to be the chest, and the treasure, of Captain Blood, returning it promptly to the surface, which has now become coated in a deep and thick fog.

Before your captain can open the tightly-sealed chest, the *Blacktide*'s radar picks up something in the distance, again before immediately turning off and becoming worthless. Strangely, instead of giving a bearing or any seemingly useful information, the radar read "$-5-30i$", a complex number. While trying to fix the radar and wondering why there would be an imaginary coordinate in the first place, a crewman points out a ship off of the *Blacktide*'s starboard (right-hand) side. This is a resurrected *Arabella* with Captain Blood himself at the helm, here to reclaim his treasure!

8. You need to do a quick calculation to tell which direction the *Blacktide* needs to follow to escape the angry ghost pirate captain. You figure that going in the exact opposite direction from the ghost ship's position would suffice in order to escape, trusting in your more advanced ship's speed to outrun a decrepit wooden ship that shouldn't even be floating. Using the complex number as the position of the *Arabella,* determine the angle of the ghostly ship in reference to your ship (assume your ship is facing East along the Real Axis, so you're finding the **standard position angle** for the position of the *Arabella*) and give a **bearing** for the helmsman to follow in order to escape! Round both answers to the nearest positive whole degree.

80.5



arabella

$180+80.5=$

$\approx \boxed{261°}$

or $90-80.5=9.5$

$\approx 10°$

$\boxed{S\,10°\,W}$

Blactide

$\boxed{81°}$

or

$\boxed{N\,10°\,E}$

Giving the helmsman the correct bearing seemed to work and you're almost done with your adventure. The Captain's original plan was to make port in Miami once done exploring, but instead you decide to travel to Bermuda since it is much closer. Your current heading, or direction of travel, is defined by the vector $T = <16, 99>$ and the vector from your current position to Bermuda is defined by the vector $B = <330, 608>$. All values are in kilometers.

9. Using the above information, determine the angle between your current heading and Bermuda, as well as the distance (in kilometers) from your current position to Bermuda. Round both to the nearest whole number.

$$\cos\theta = \frac{\vec{T}\cdot\vec{B}}{\|T\|\cdot\|B\|}$$

$$d^2 = 10057 + 478564^2 - 2(10057)(478564)\cos 19$$

$$d = \boxed{469066}\text{ km}$$

$$\cos\theta = \frac{65472}{\sqrt{10057}\cdot\sqrt{478564}} = \cos^{-1}(0.9437) = \theta$$

$$= 19.31°$$

$$\boxed{19°}$$

Congrats! You are safely in the Bermuda port and survived your brief encounter with Captain Blood!

This is the end of the calculative part of the project. Next, each group member will write a personal reflection paper based on their experience with the project. This shouldn't be more than a page (250 word min.) but should be typed. Reflect on what you've learned from the project and what it shows you in regards to applications of Trigonometry. Upload a scanned copy of this project and your reflection into Canvas (do not need to be the same document)