

# Studying the Response of RLC Circuits to Sinusoidal Inputs Using Simulink

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## IMPORTANT

Provide here the hash value you calculated for your group denoted by  $\Theta$  (see lab instructions).

Hash value  $\Theta = \text{round}(((\text{mod}(72 * 113, 128)) + 1)/128 + 1, 2) = 1.57$

Include all other requested screenshots of the plots and answers as needed below.

## 1. Natural frequency of an RLC Circuit

### 1.1 Exercise 1.1

Calculate the natural frequency for the following systems (1 pt)

1.  $C = 0.01 \times \Theta, L = 0.01 \times \Theta$  10.137Hz

2.  $C = 0.02 \times \Theta, L = 0.01 \times \Theta$  7.168Hz

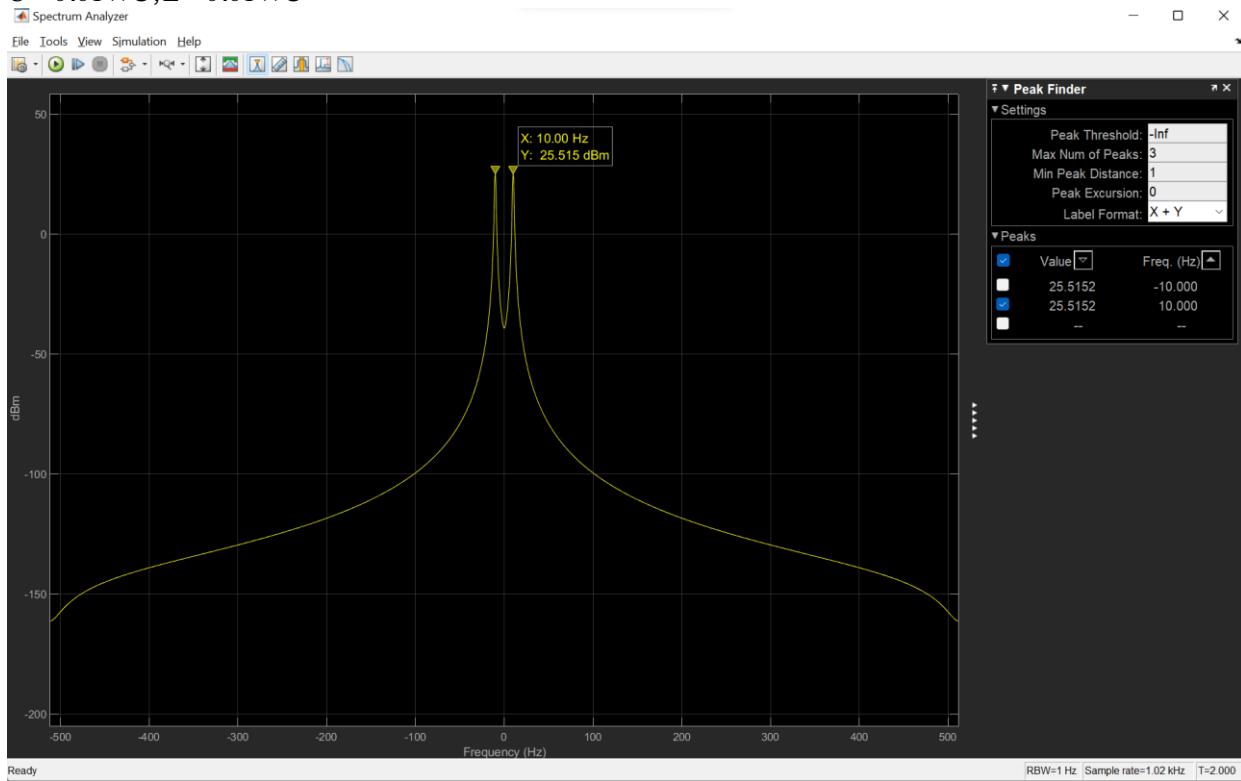
3.  $C = 0.04 \times \Theta, L = 0.01 \times \Theta$  5.068Hz

4.  $C = 0.01 \times \Theta, L = 0.02 \times \Theta$  7.168Hz

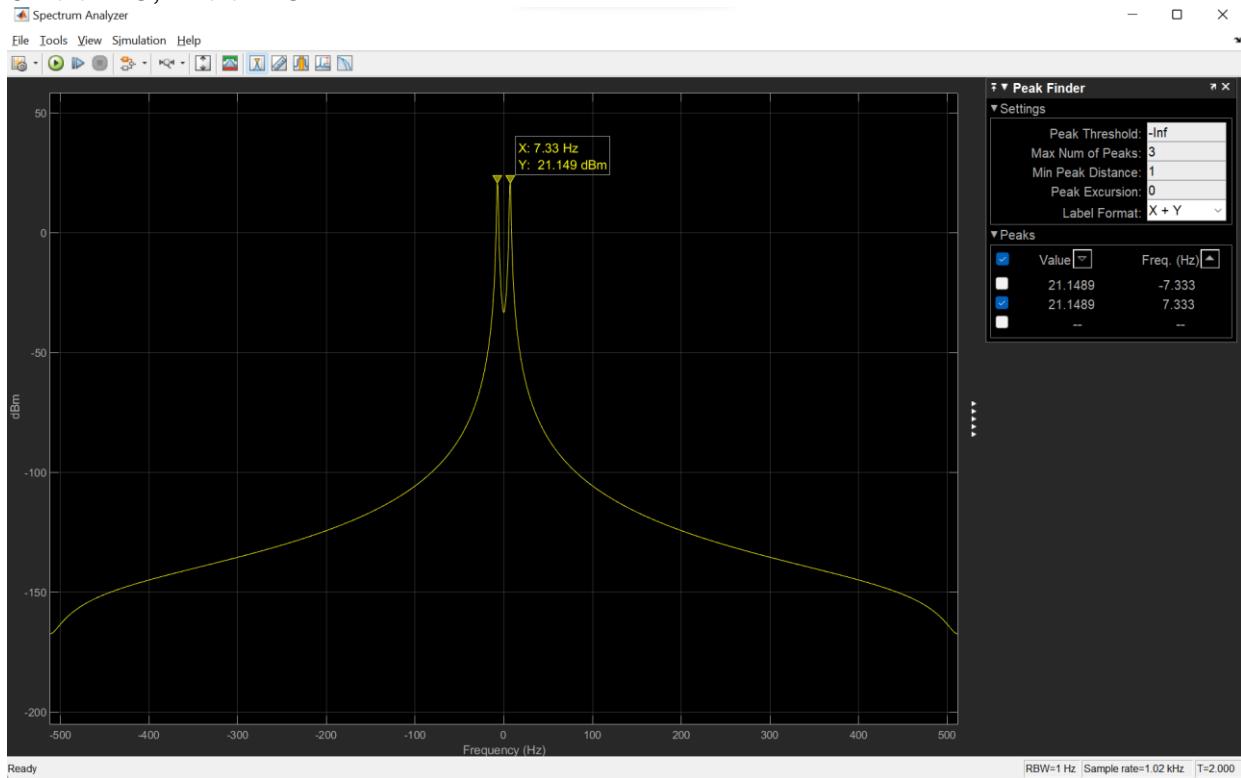
5.  $C = 0.01 \times \Theta, L = 0.04 \times \Theta$  5.068Hz

Compare them with the experimental results. Obtain a screenshot of the spectrum plot to show your results and include with your submission.

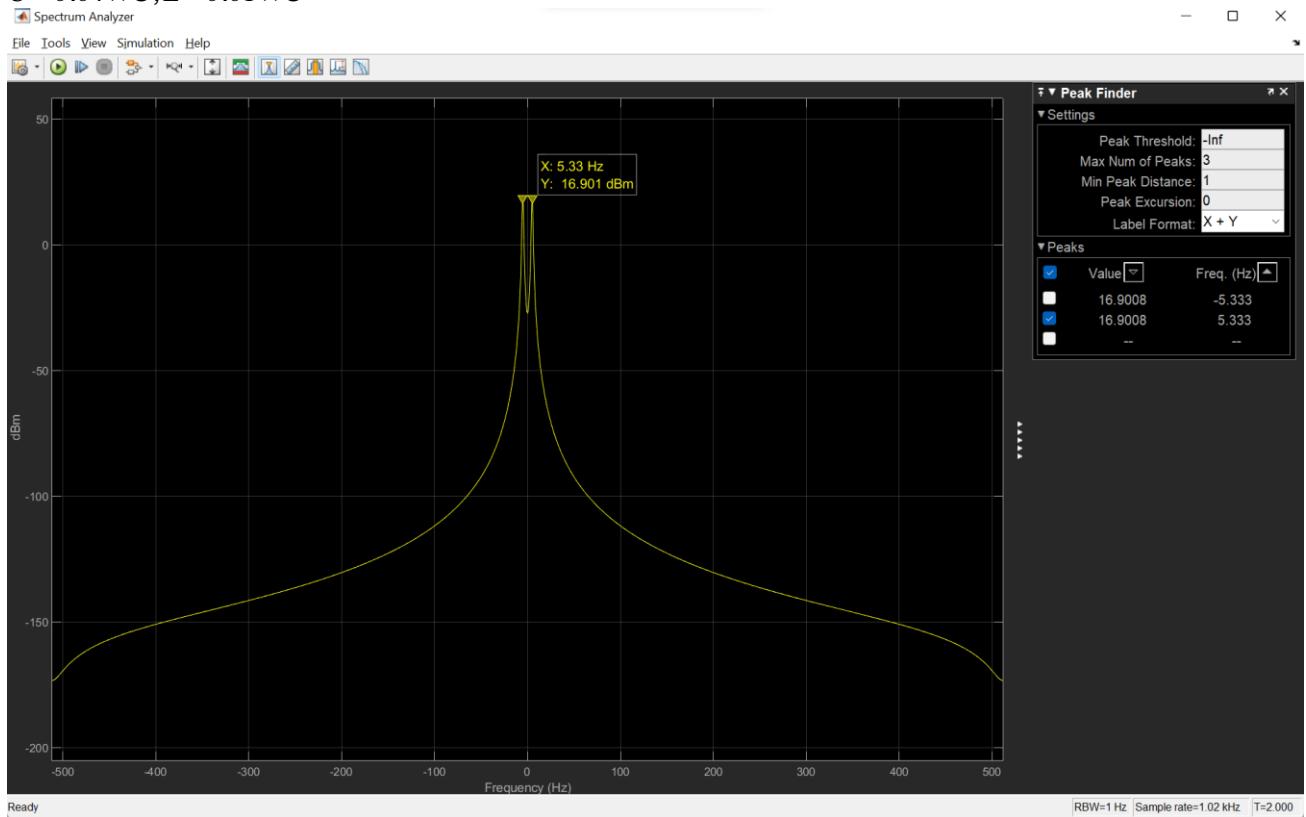
1)  $C = 0.01 \times \Theta, L = 0.01 \times \Theta$



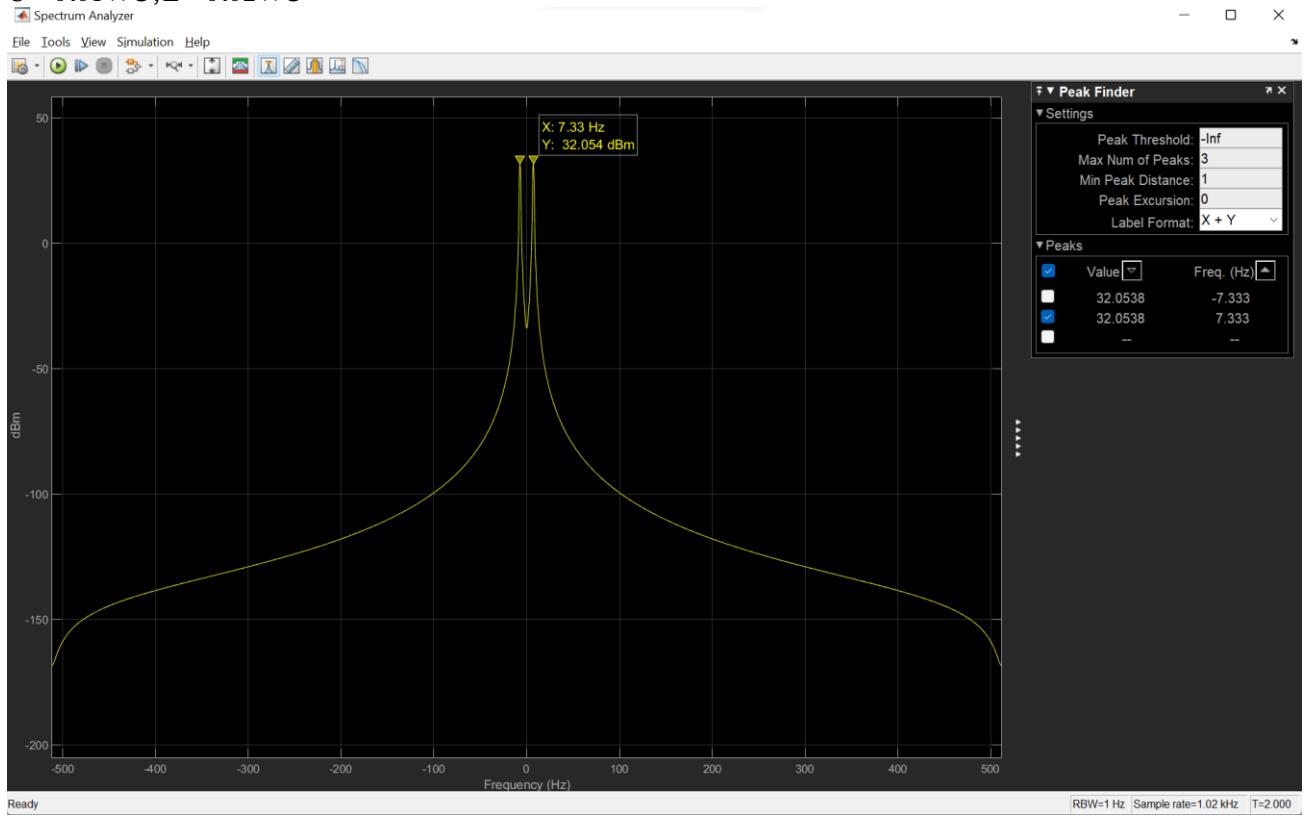
2)  $C = 0.02 \times \Theta, L = 0.01 \times \Theta$



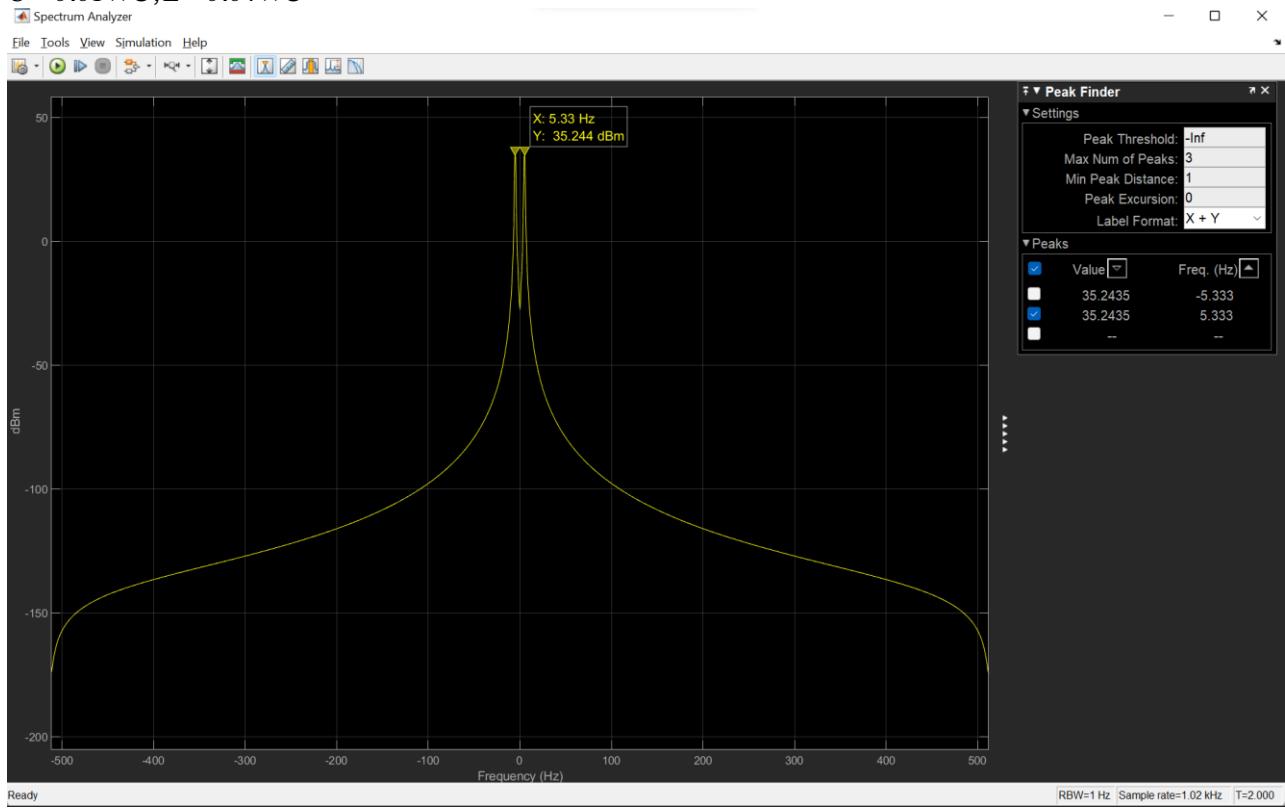
3)  $C=0.04 \times \Theta, L=0.01 \times \Theta$



4)  $C=0.01 \times \Theta, L=0.02 \times \Theta$



5)  $C = 0.01 \times \Theta, L = 0.04 \times \Theta$



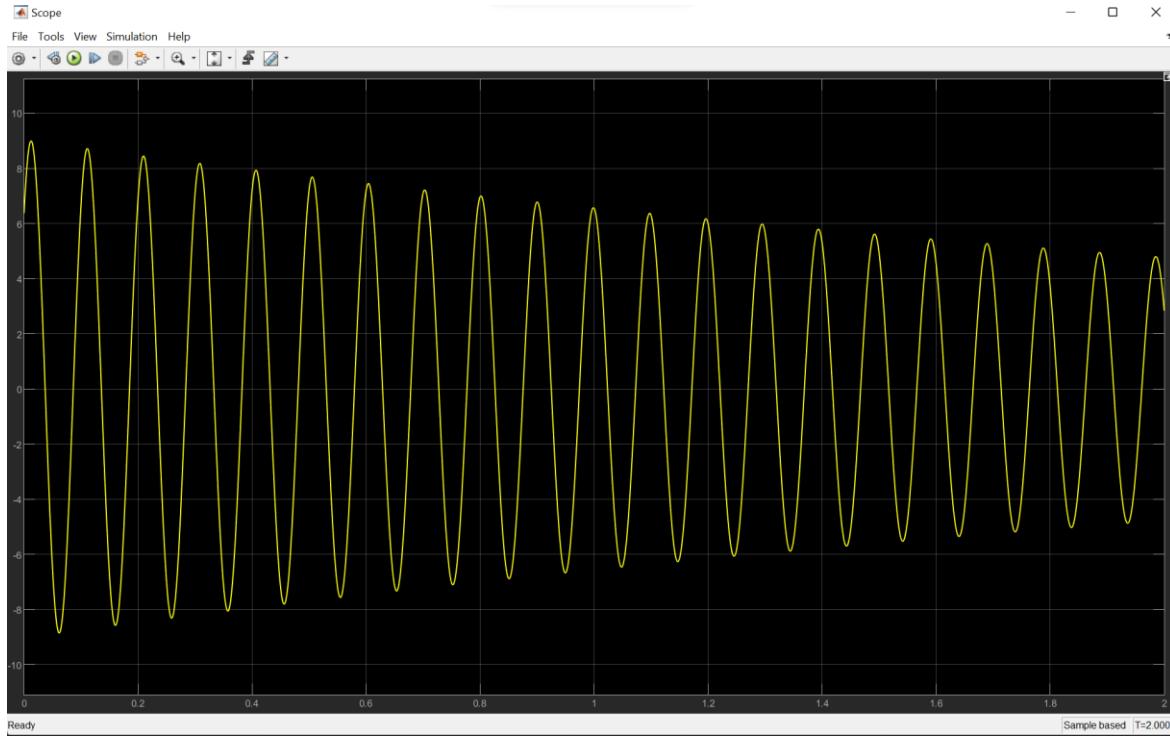
The Experimental results are approximately close to our calculations. There are some differences because the experimental results round off the values.

## 1.2 Exercise 1.2

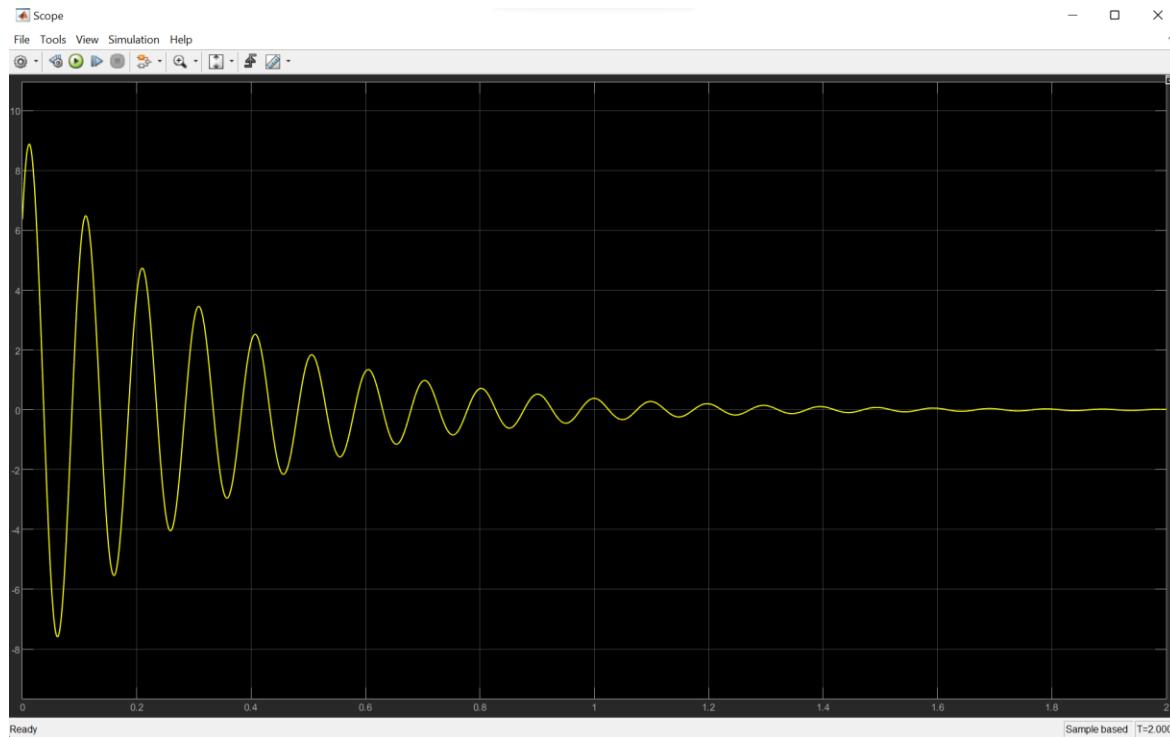
- a. Vary the resistance values to show that damping factor increases as you increase the resistance (set  $C = 0.01 \times \Theta$  and  $L = 0.01 \times \Theta$ ).

Save your screenshot of the capacitor voltage over time and attach to this document. (1 pt)

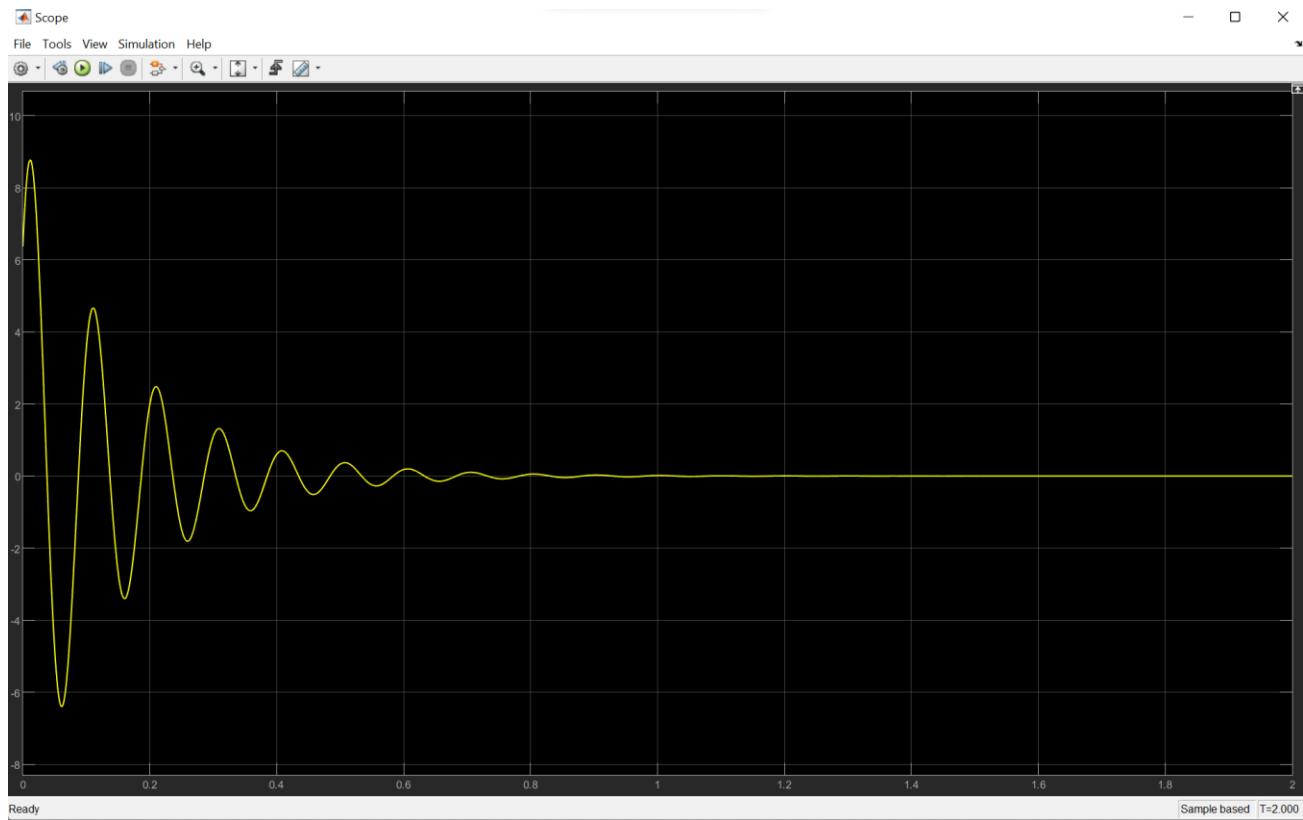
1)  $R = 0.01$  Ohms:



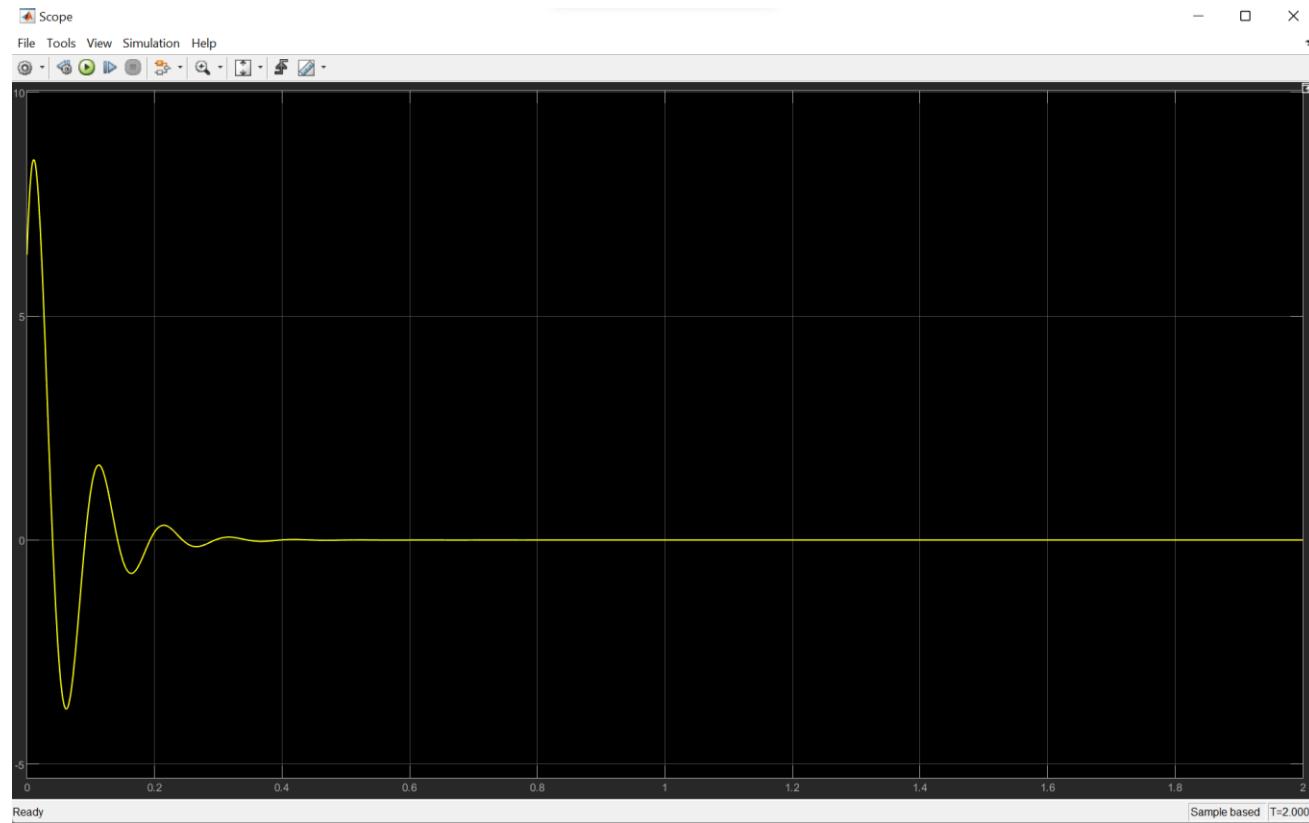
2)  $R = 0.1$  Ohms:



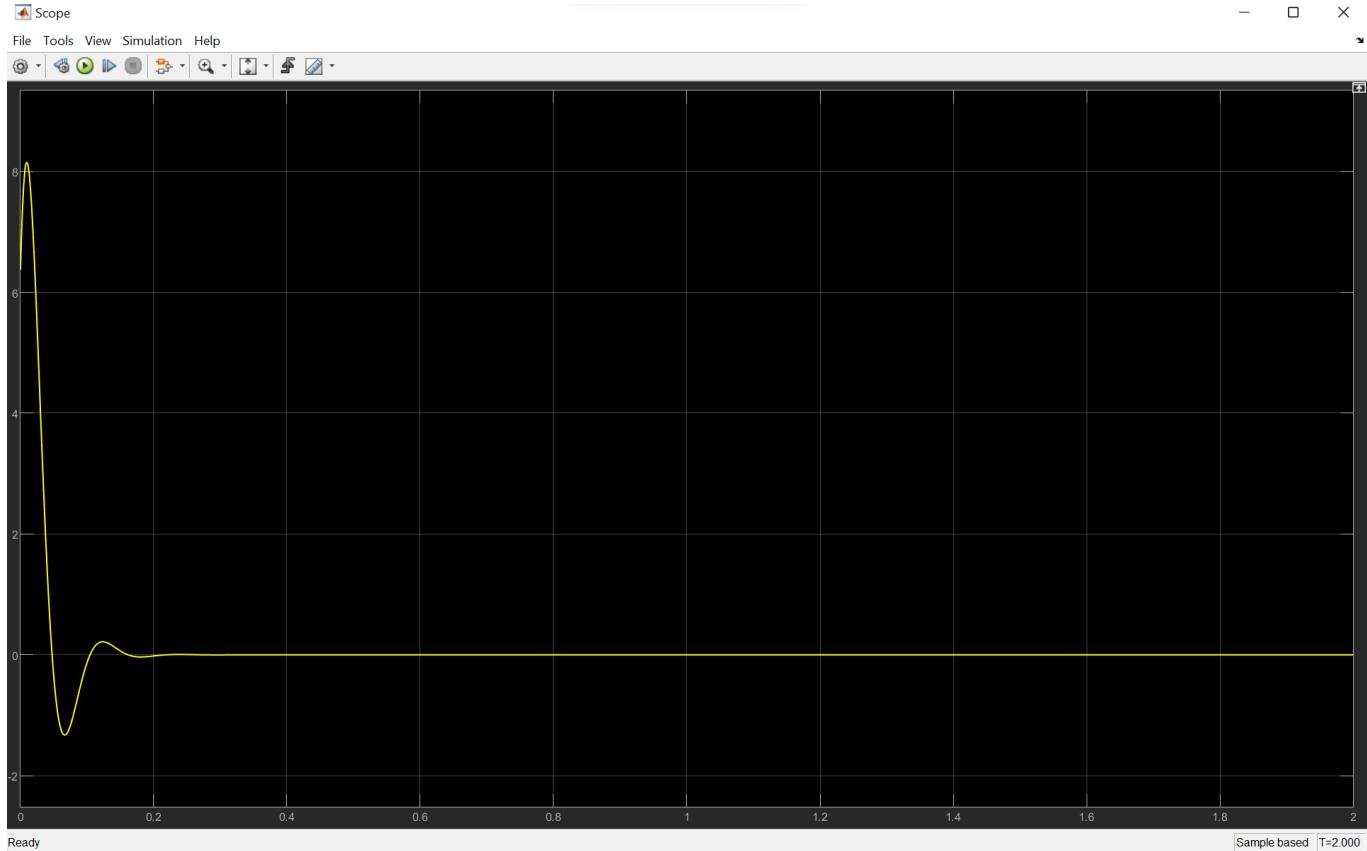
3)  $R = 0.2$  Ohms



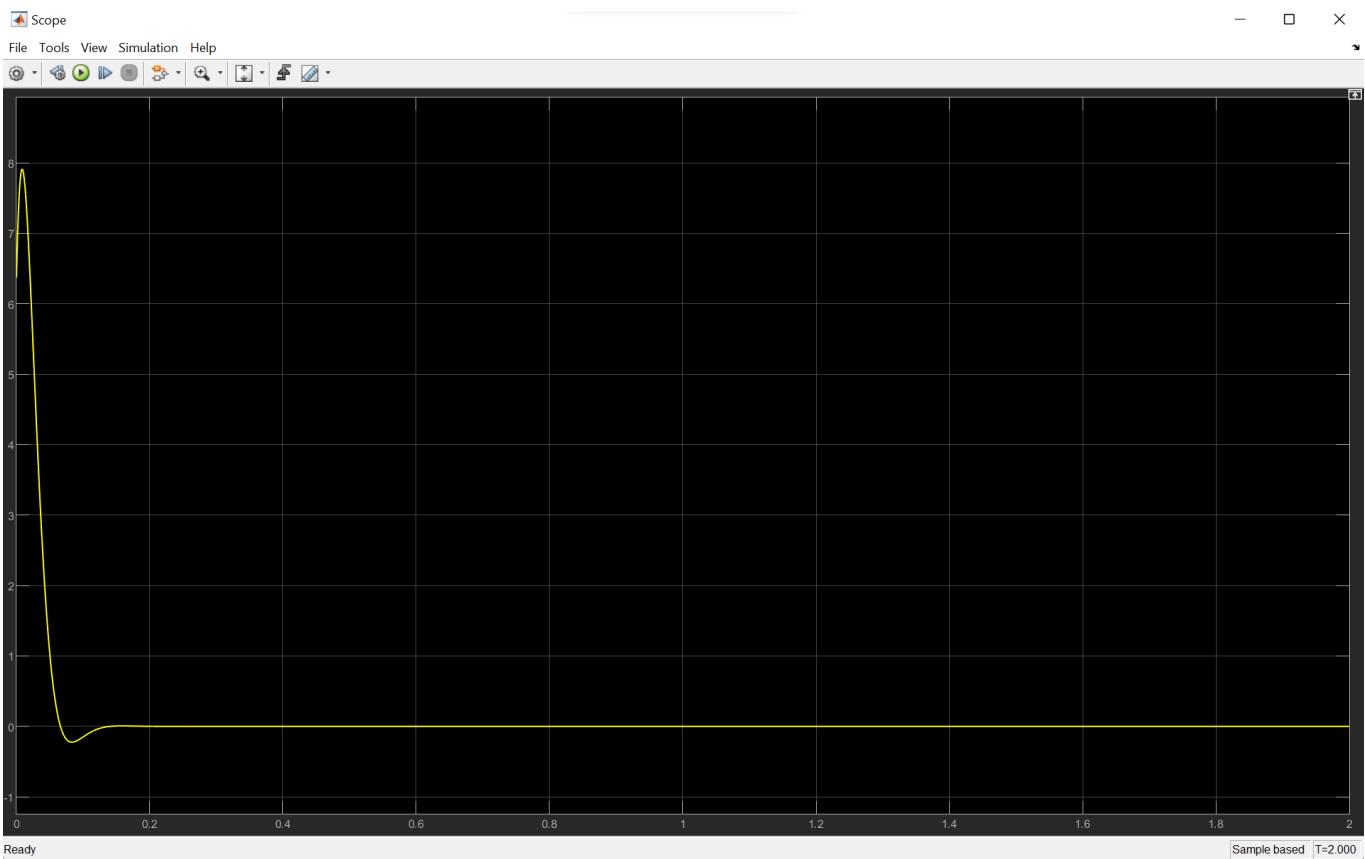
4)  $R = 0.5$  Ohms



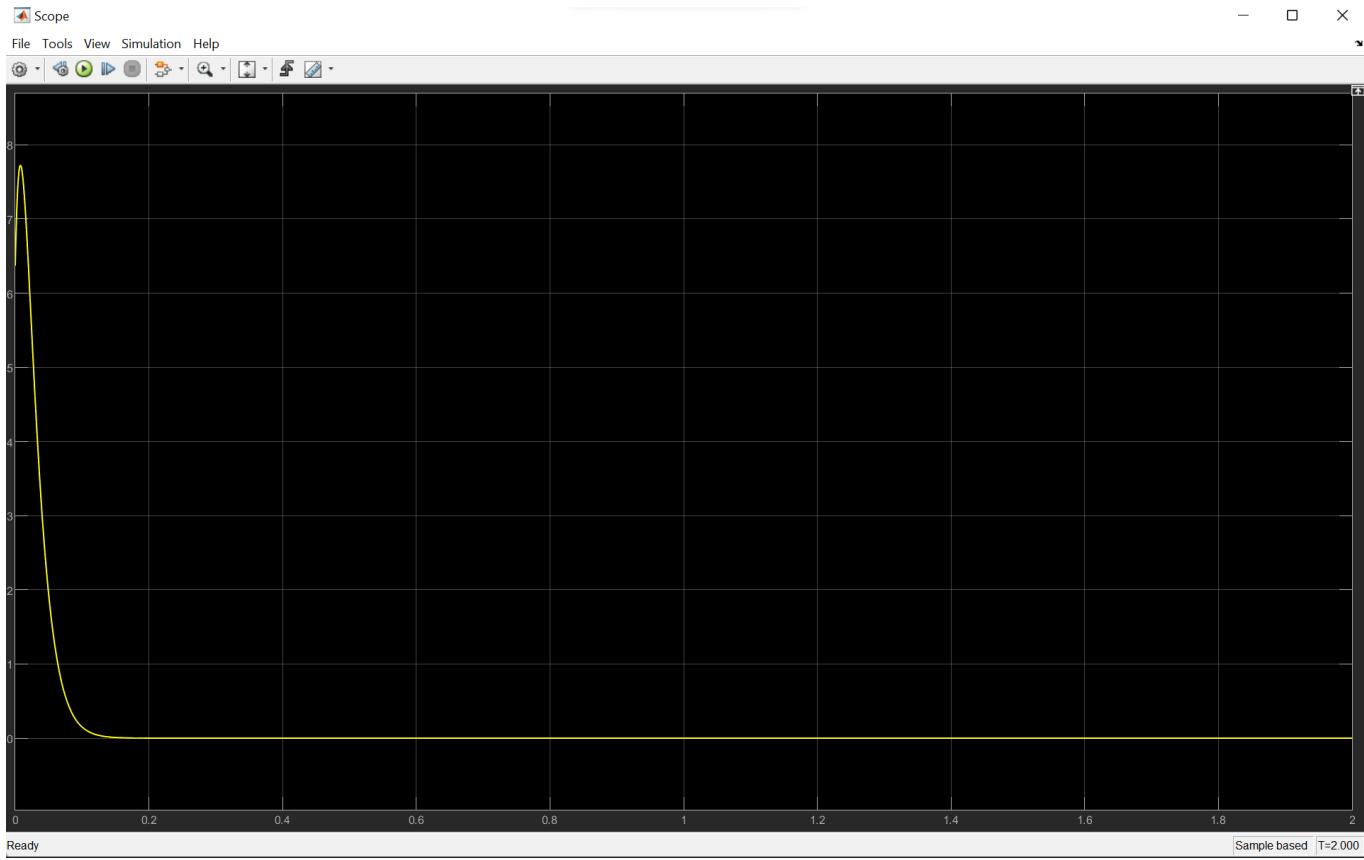
5)  $R = 1 \text{ Ohm}$



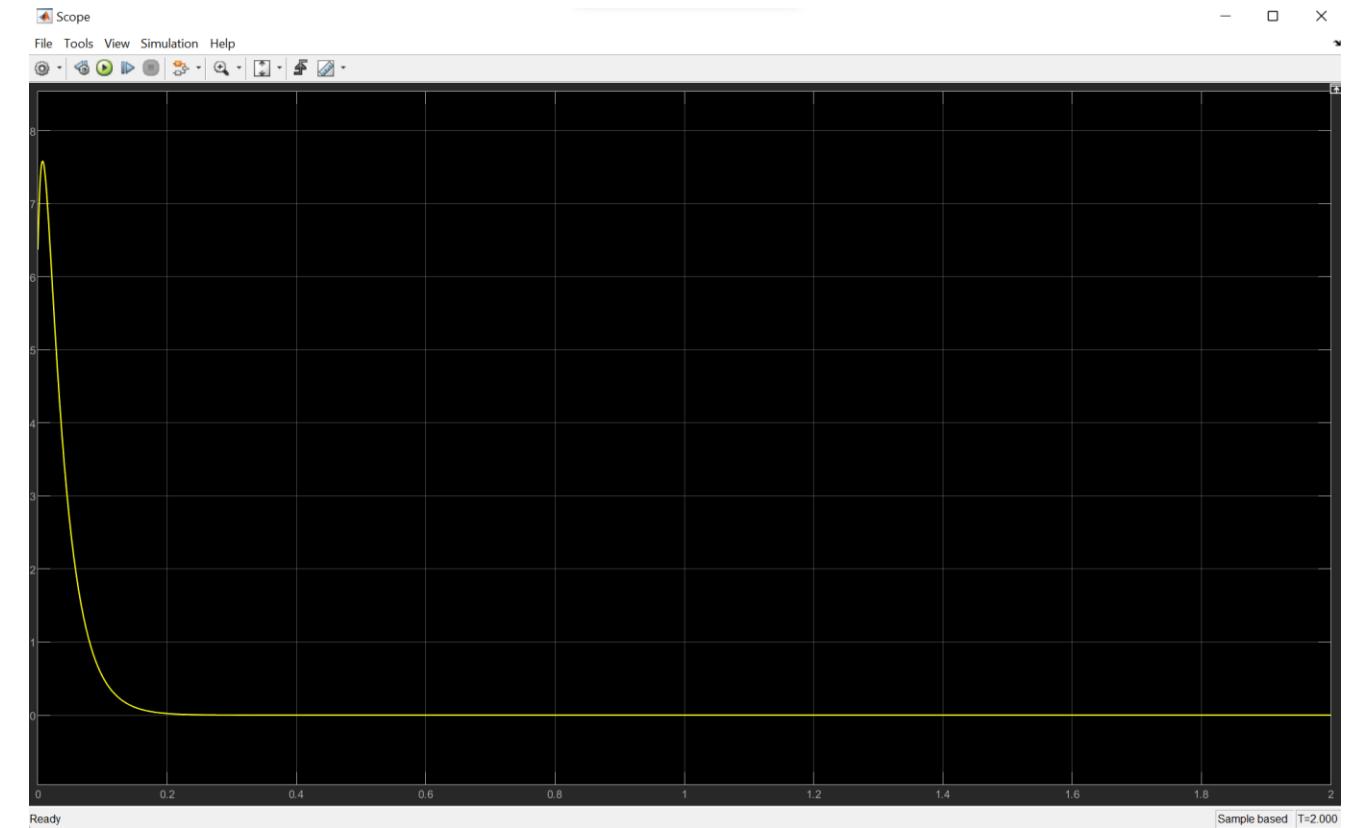
6)  $R = 1.5 \text{ Ohms}$



7)  $R = 2$  Ohms



8)  $R = 2.5$  Ohms



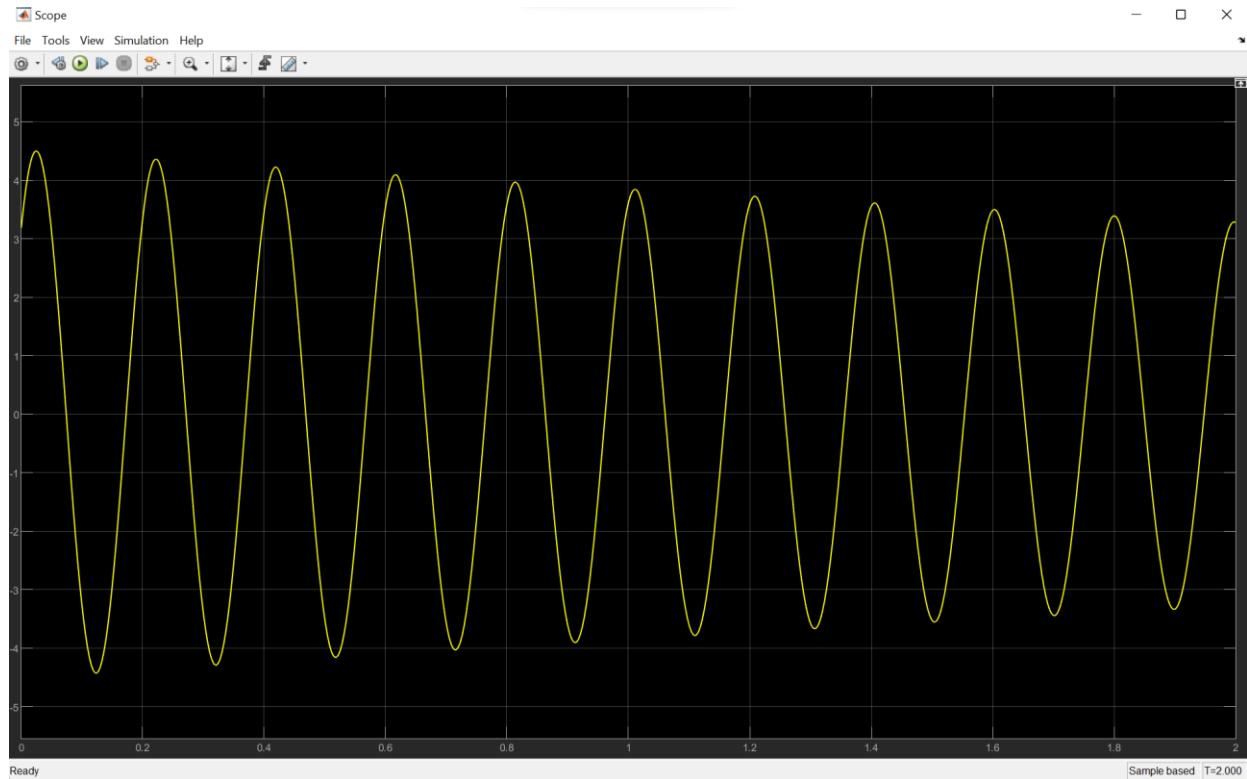
b. At what resistance does the system transition from underdamped to overdamped? (Keep  $L=0.01 \times \Theta$  and  $C = 0.01 \times \Theta$ ) (0.5 pt)

$$R = 2 \text{ ohm}$$

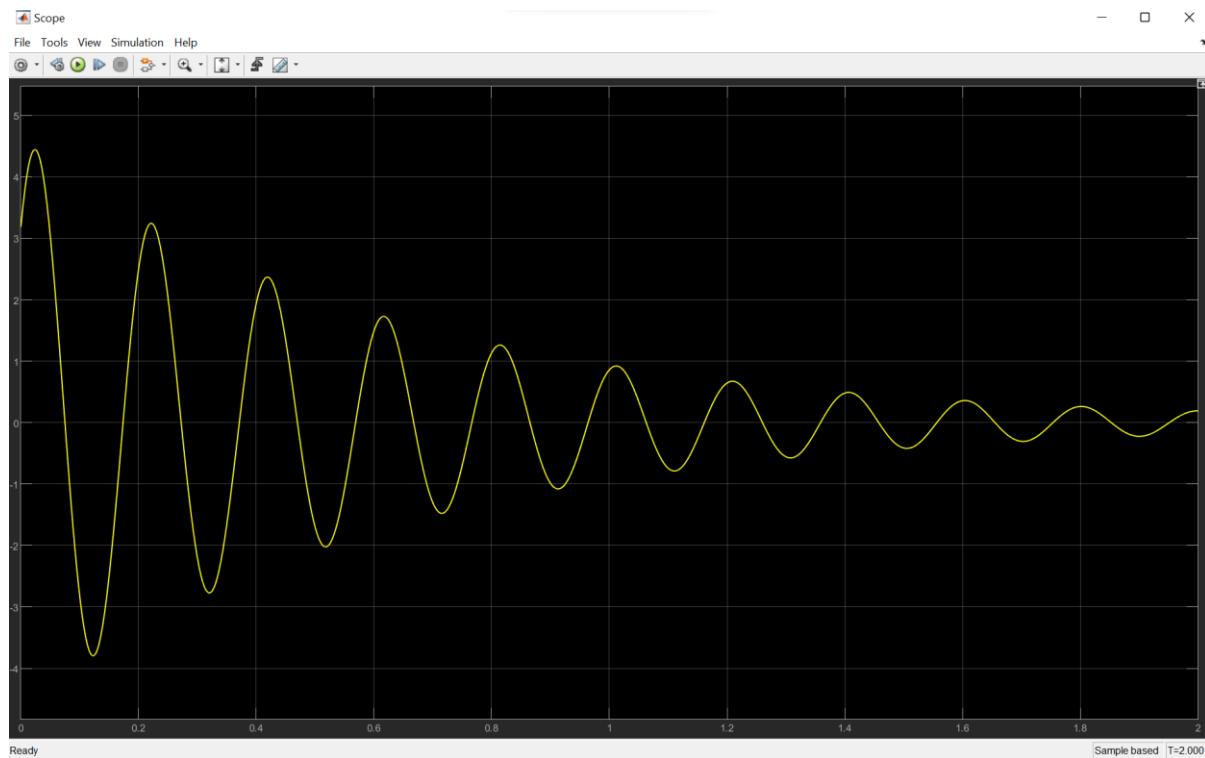
c. How would underdamped to overdamped transition change if you increase  $L$  to  $0.02 \times \Theta$  and  $C$  to  $0.02 \times \Theta$ ? (0.5 pt)

The value of R at which the system changes from critically damped to overdamped remains at 2 Ohms, as can be seen from the screenshots below

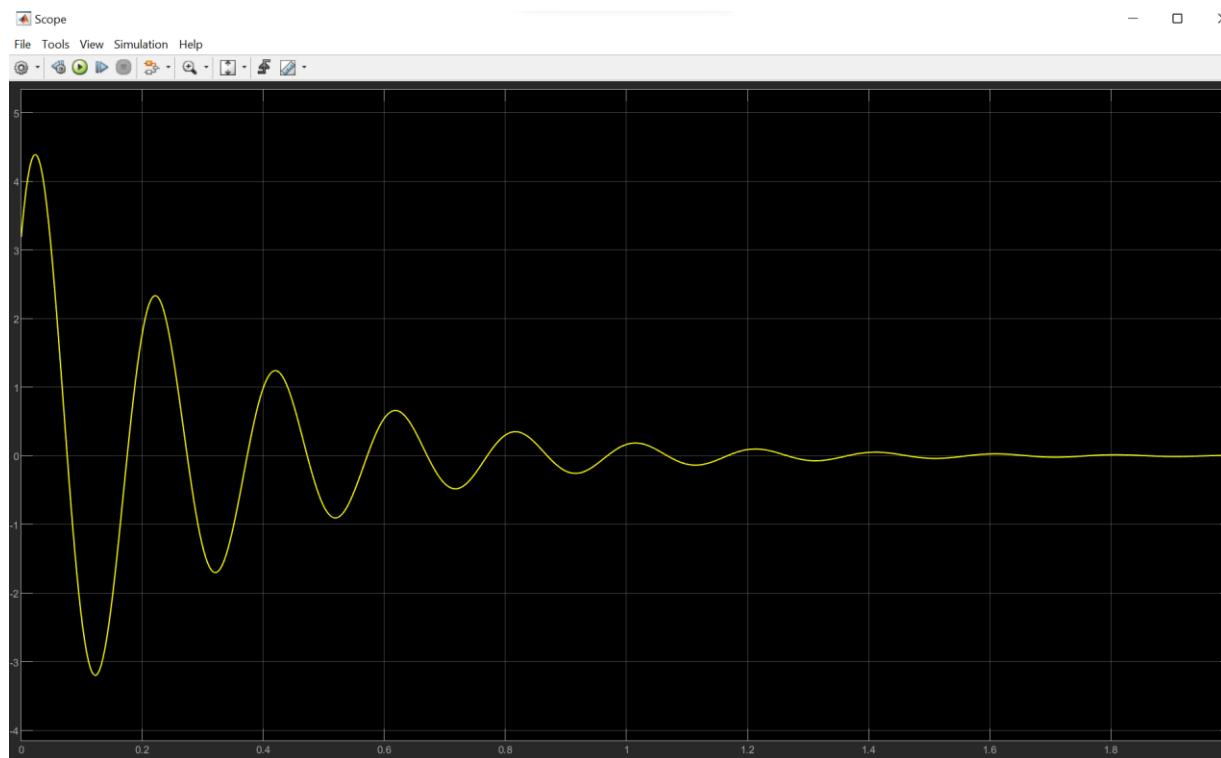
1)  $R = 0.01$

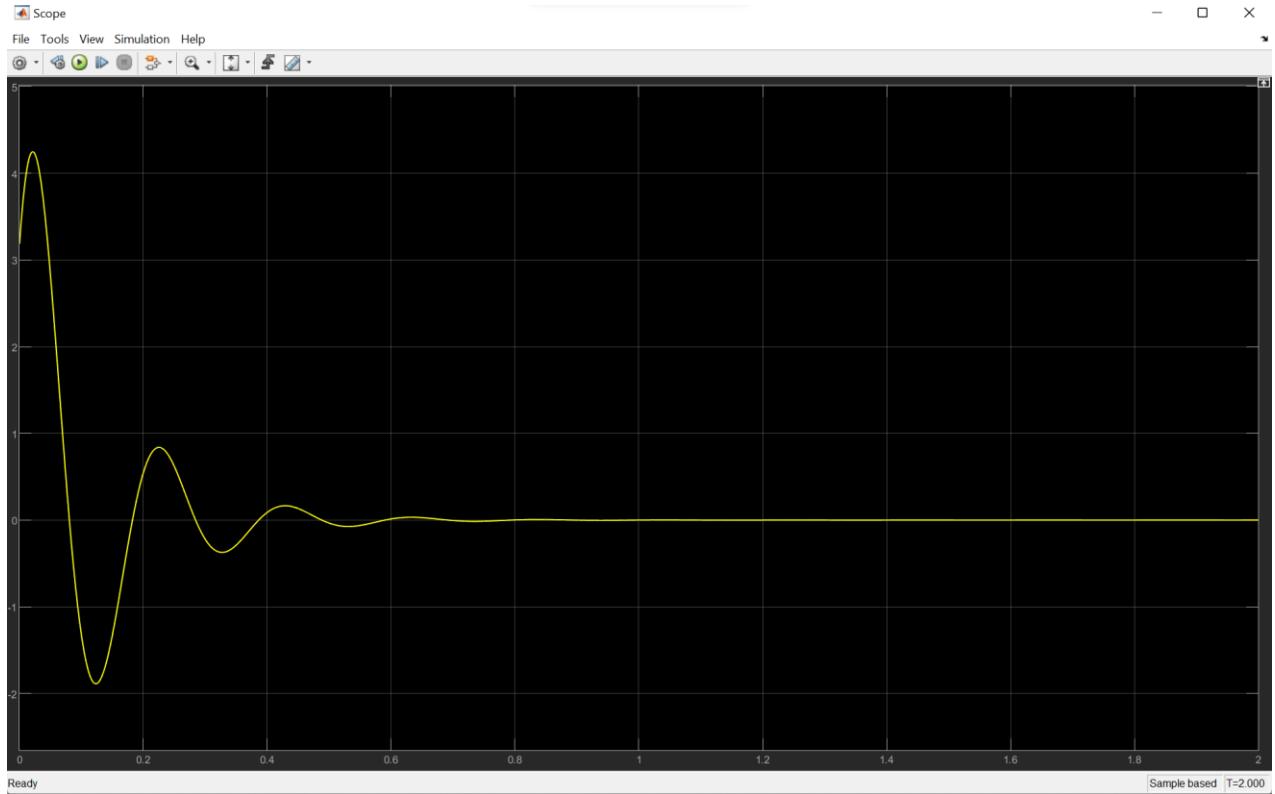
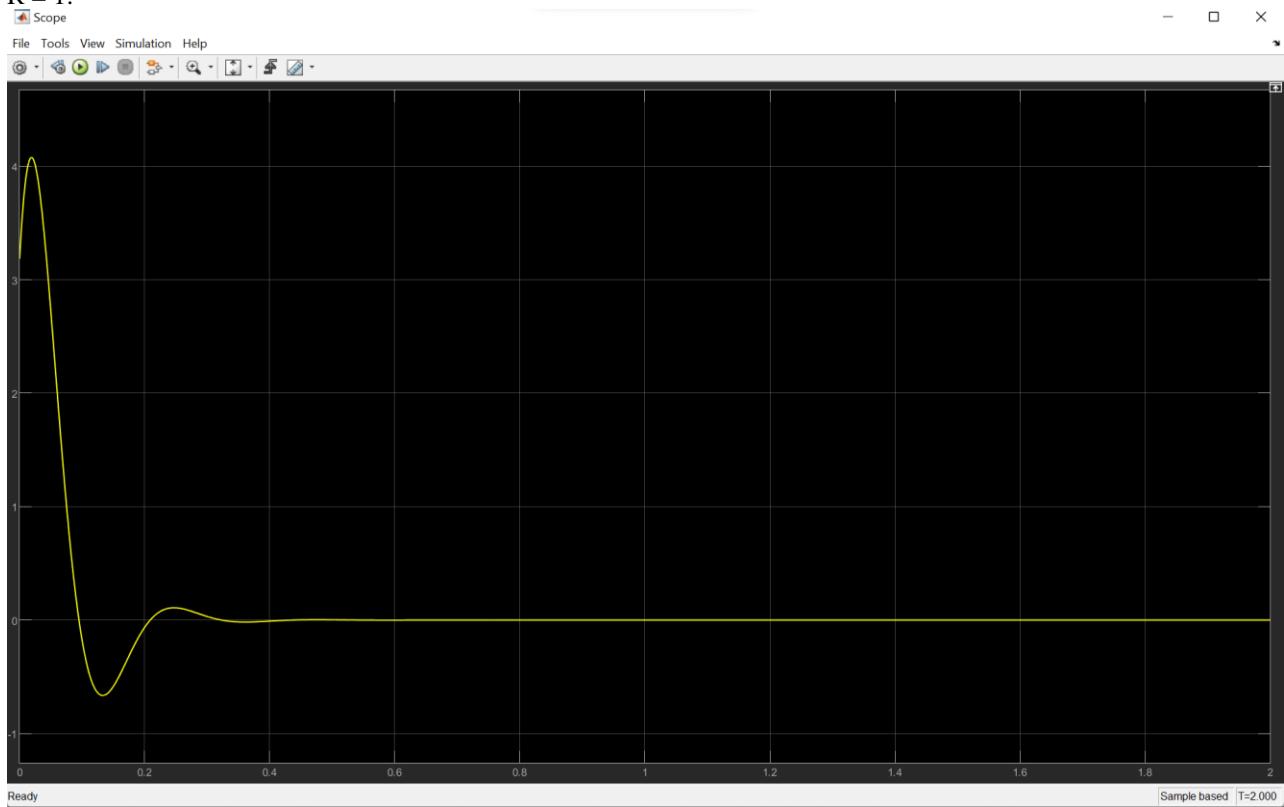


2)  $R = 0.1$

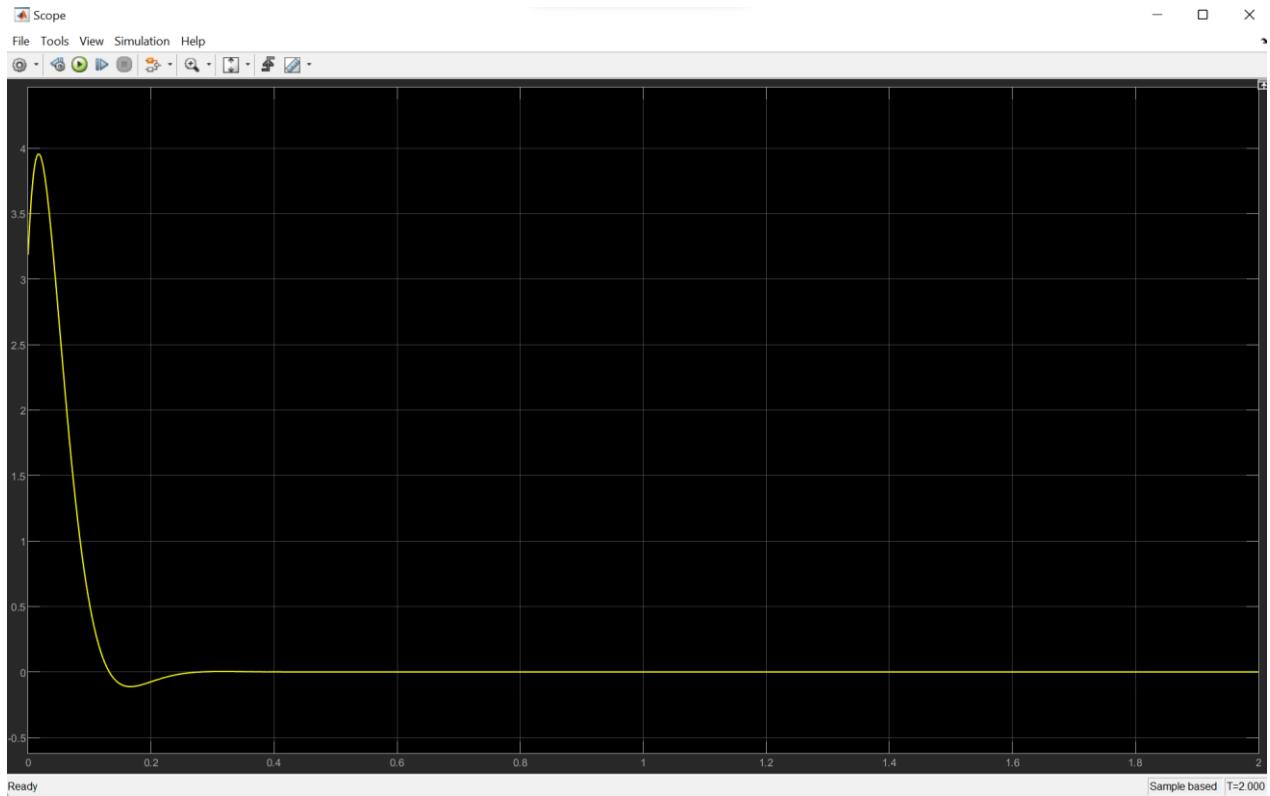


3)  $R = 0.2$ :

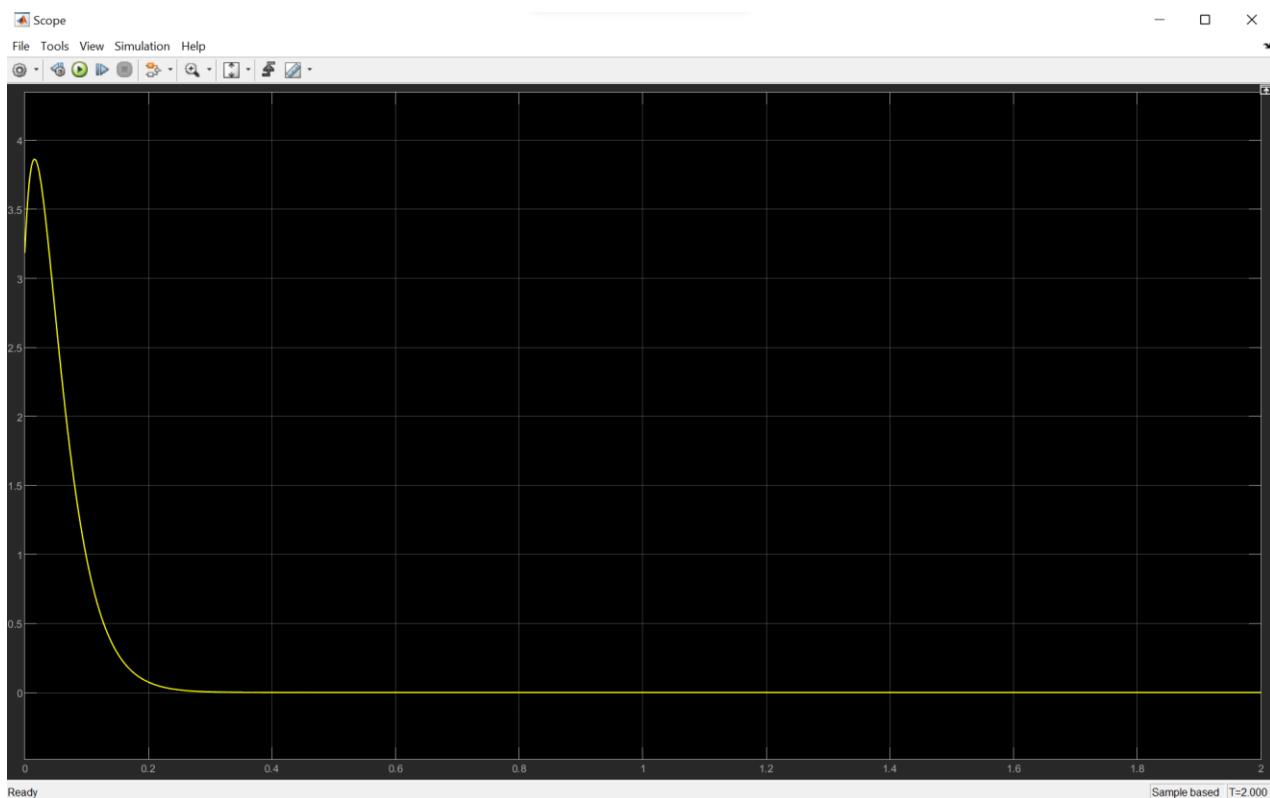


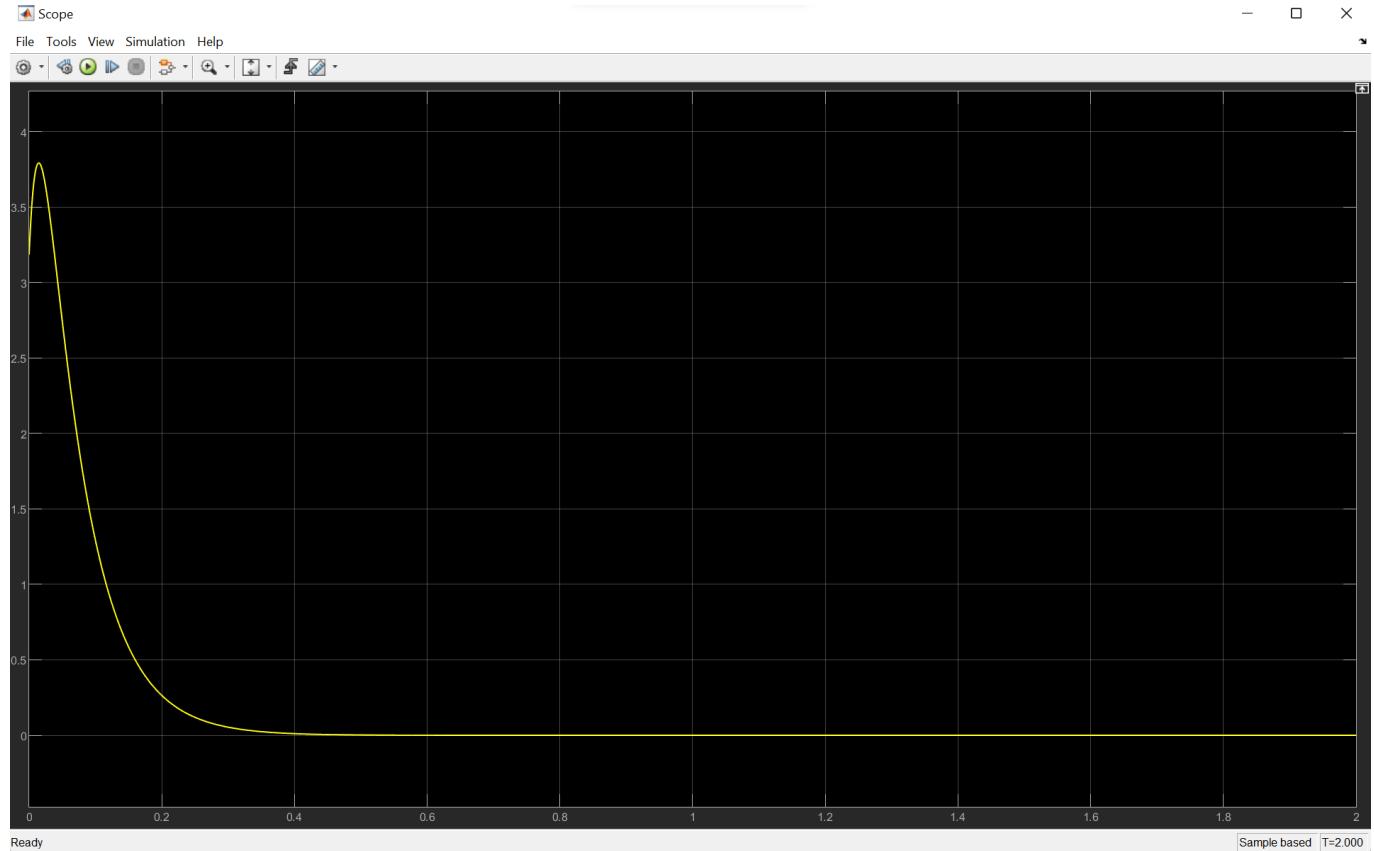
4)  $R = 0.5$ :5)  $R = 1$ :

6)  $R = 1.5$ :



7)  $R = 2$ :



8)  $R = 2.5$ :

## 2. RLC circuit response to an external voltage source

### 2.1 Exercise 2.1

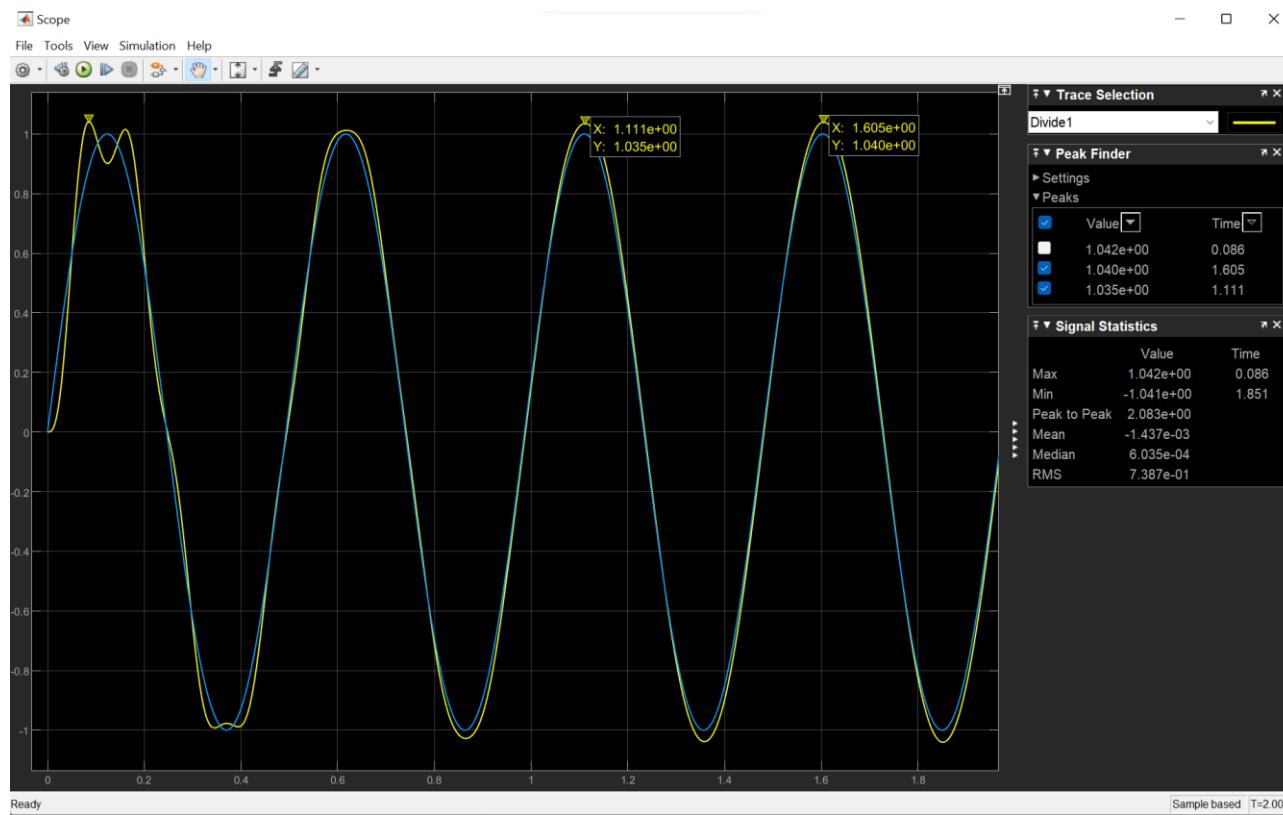
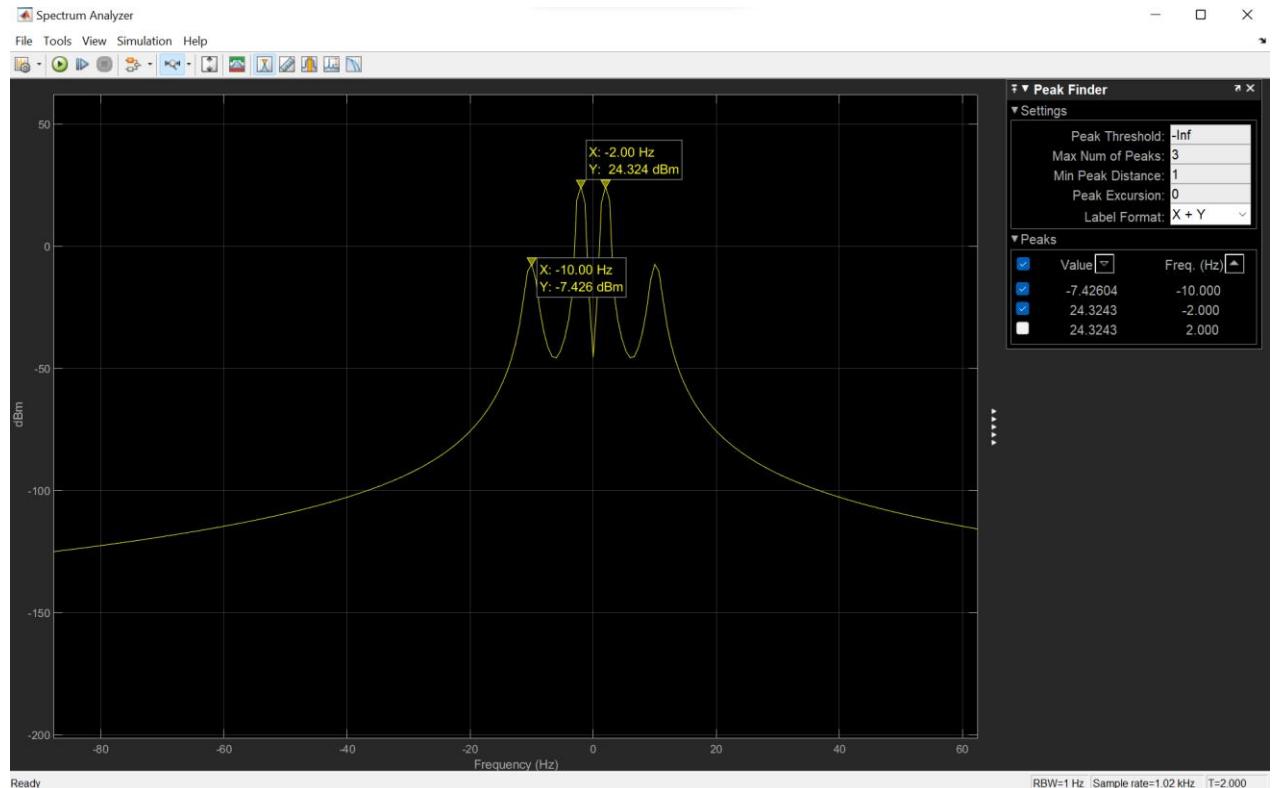
- a. Set the amplitude of your voltage source to 1 and measure the amplitude of the output response of the circuit for the case when the sine-wave input has the following frequencies. (1 pt)

1. Natural frequency / 5: 12.738 rad/sec
2. Natural frequency / 2: 31.845 rad/sec
3. Natural frequency: 63.690 rad/sec
4. Natural frequency \* 2: 127.38 rad/sec
5. Natural frequency \* 5: 318.45 rad/sec

Save your screenshots of the output signal and attach to this document, as well as your explanations.

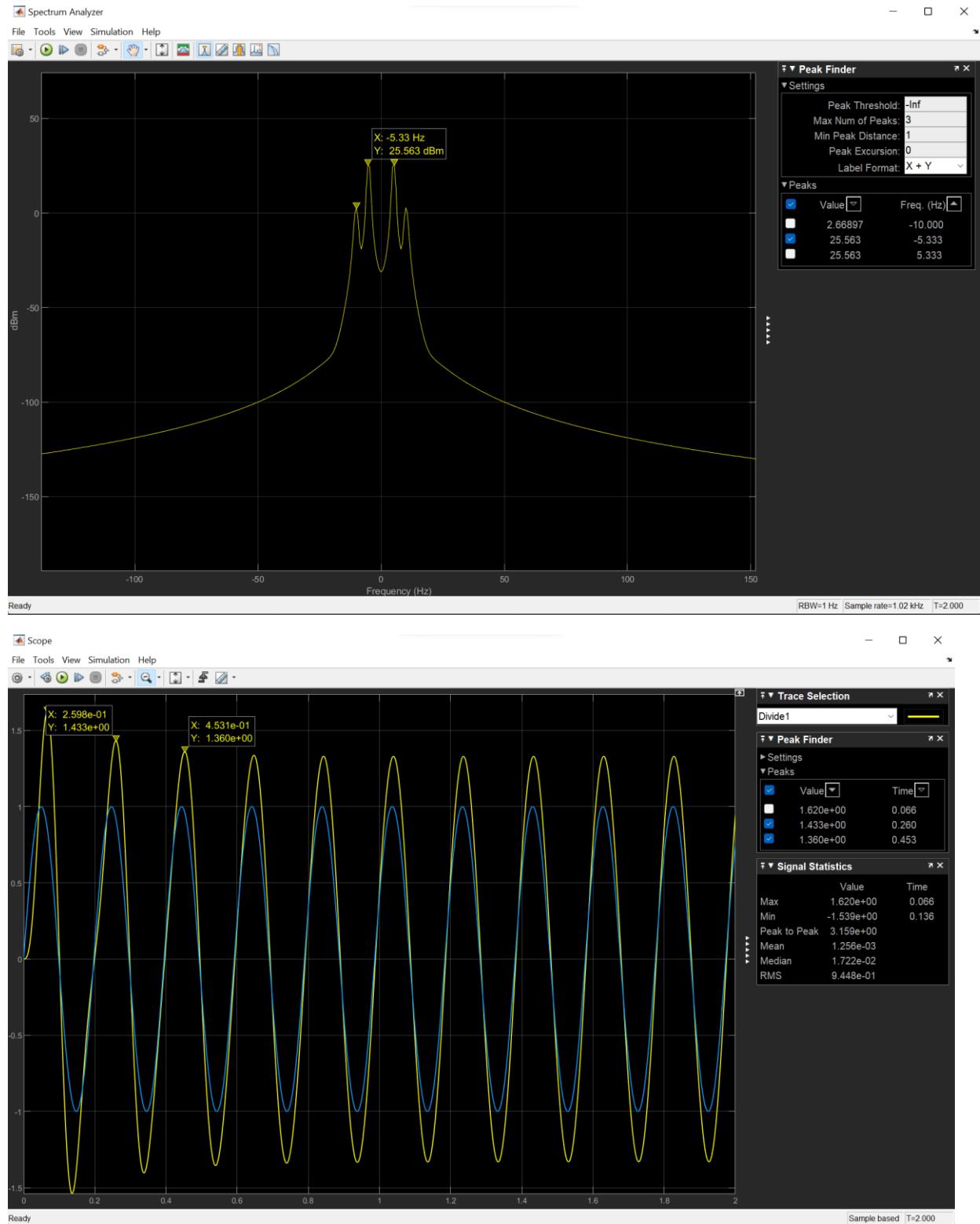
### 1) Natural Frequency /5:

Maximum and minimum amplitude was around 1.042V and -1.042V respectively. In the transient stage, the amplitude was around 1.035V



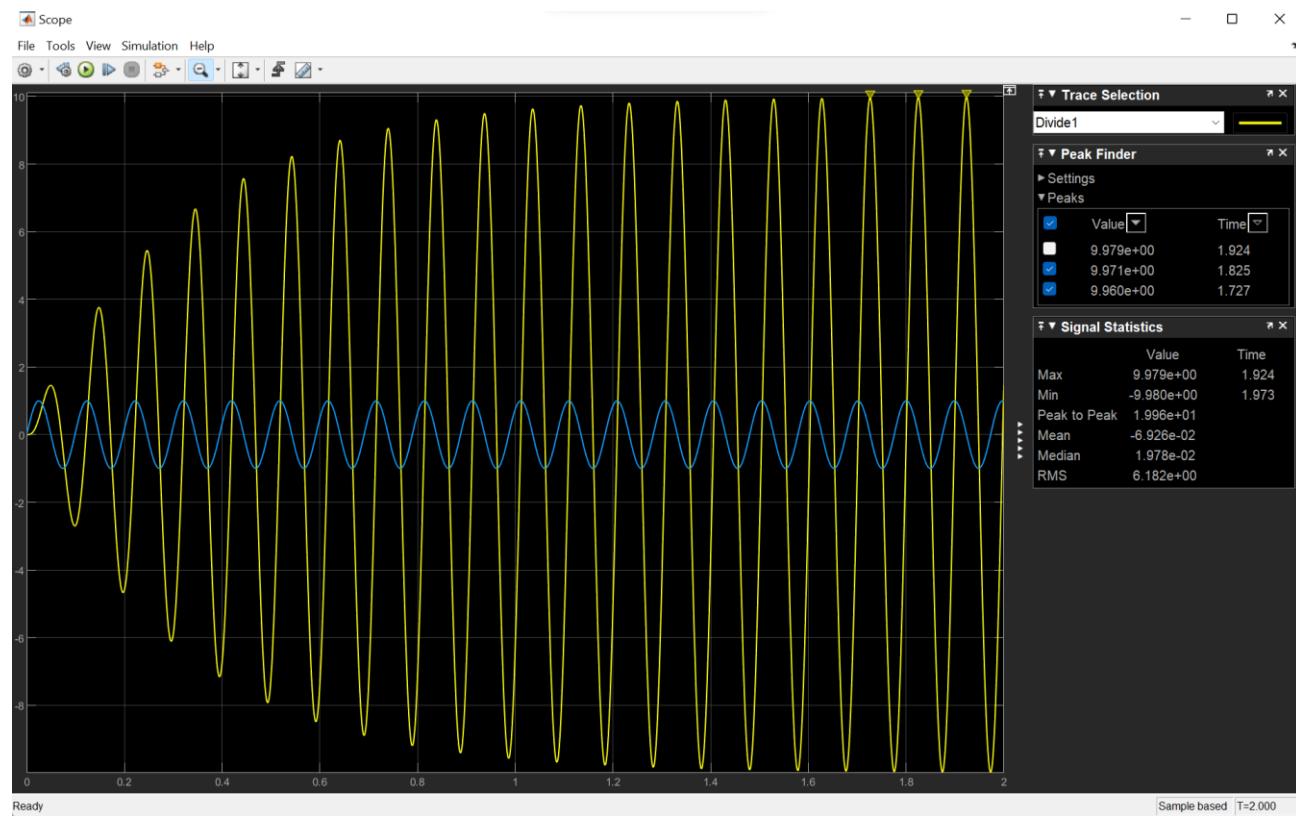
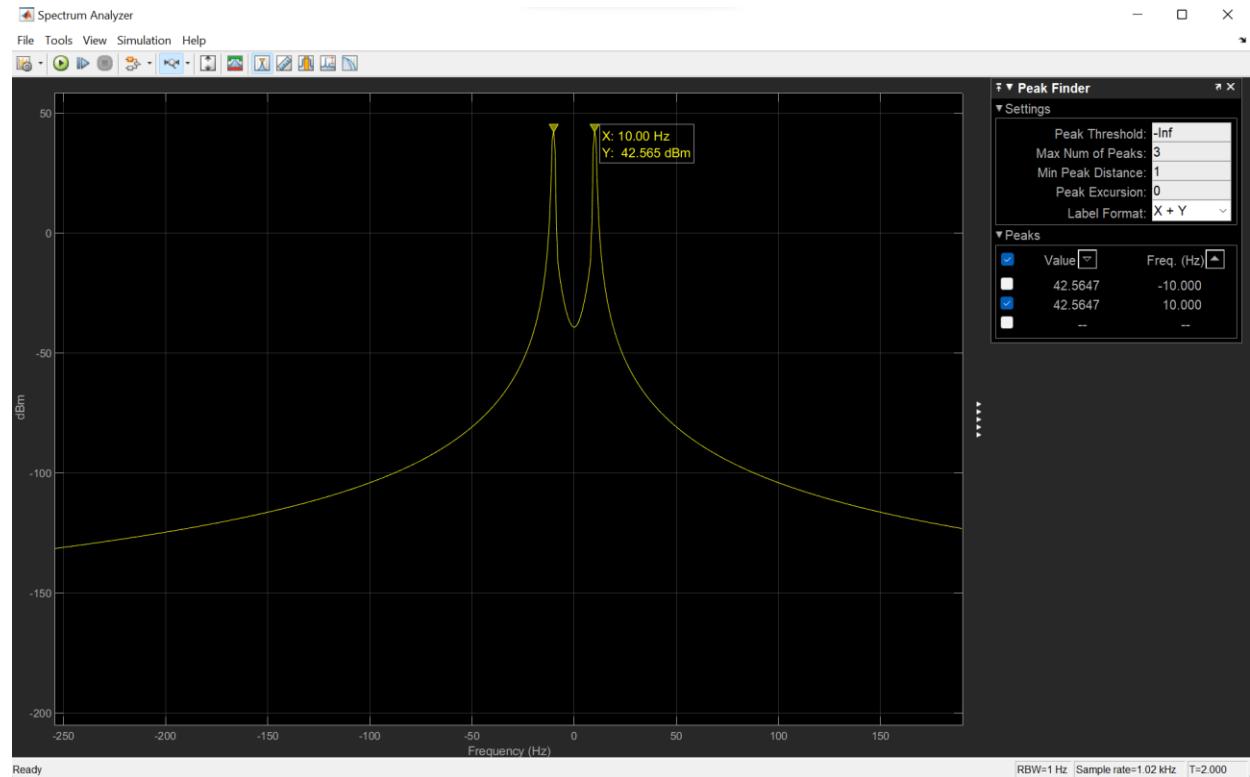
## 2) Natural Frequency /2

Maximum and minimum amplitude was around 1.62V and -1.539V respectively. In the transient stage, the amplitude was around 1.36V



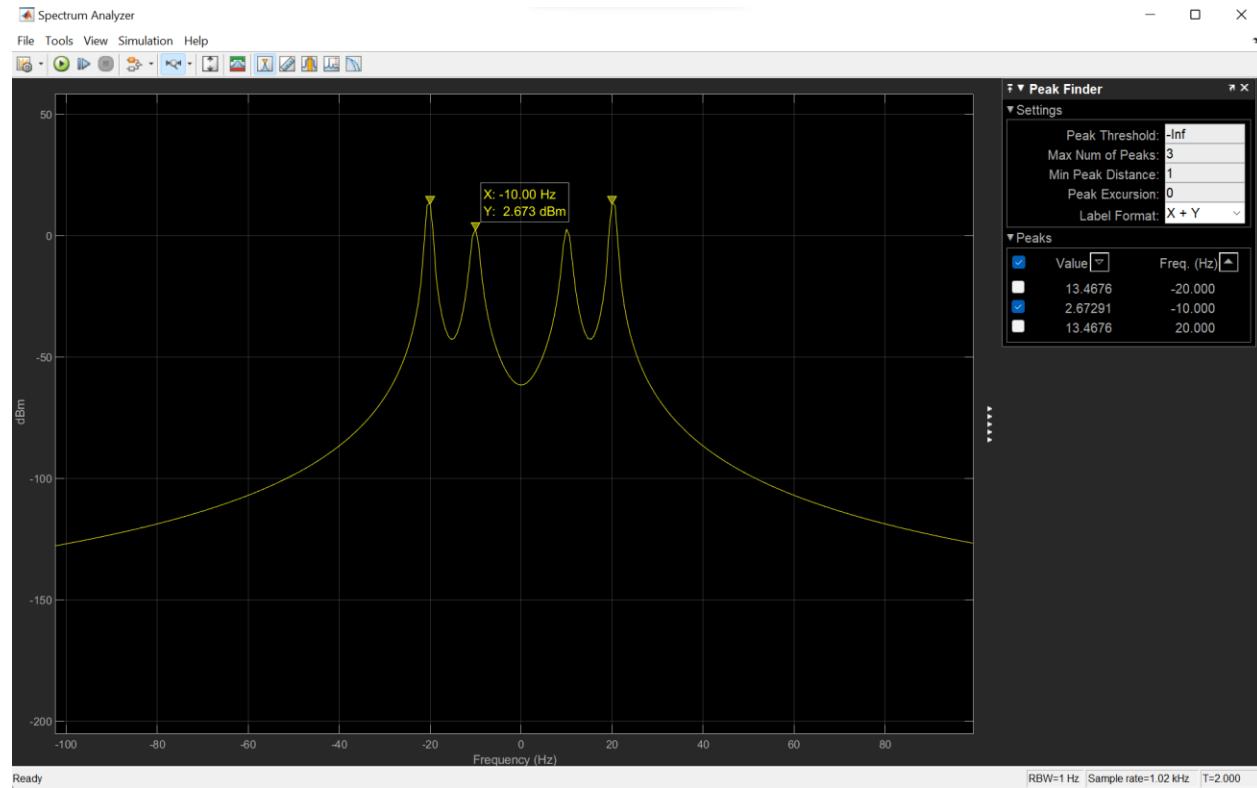
### 3) Natural Frequency

Maximum and minimum amplitude was around 9.979V and -9.980V respectively. In the transient stage, the amplitude was around 9.960V



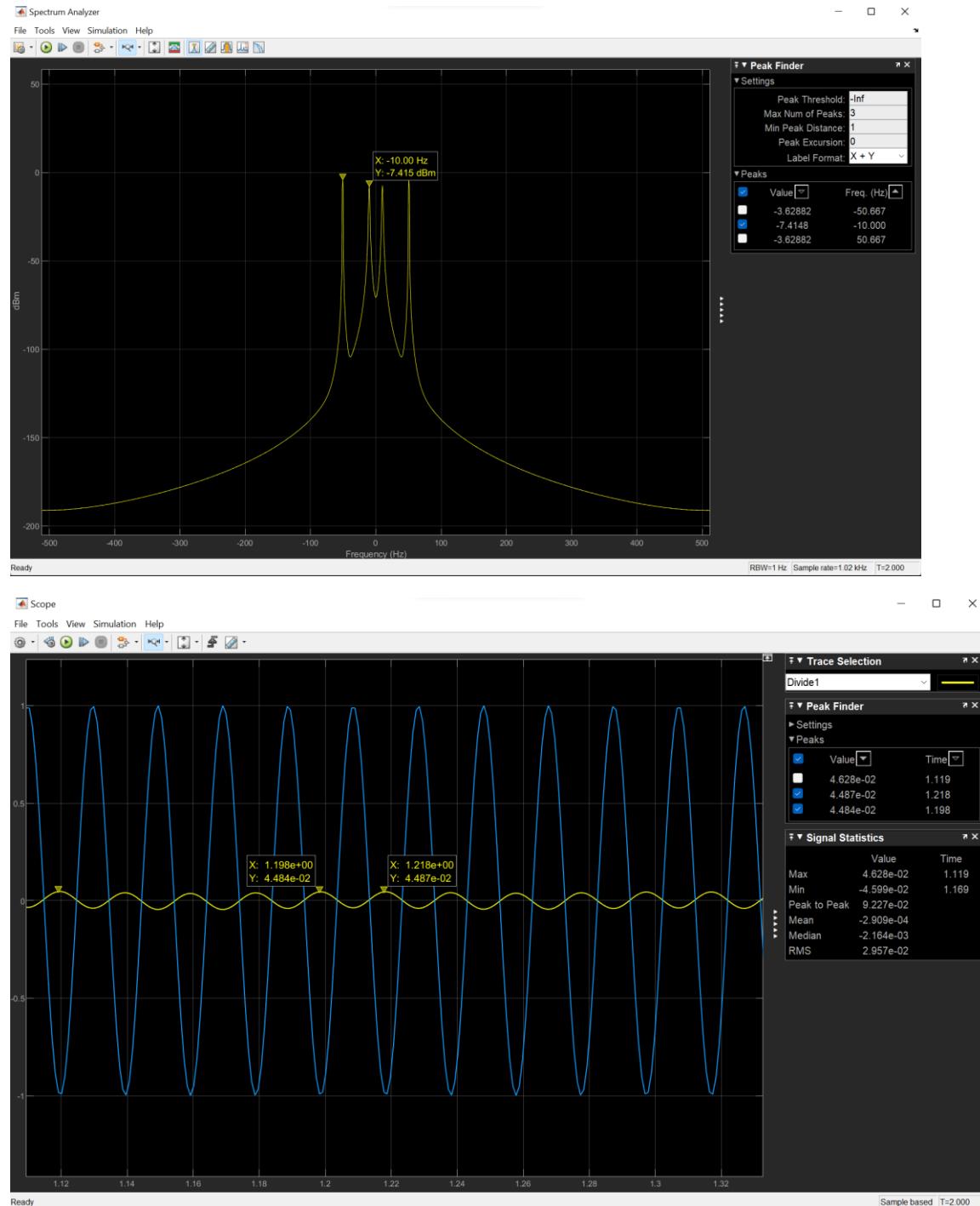
#### 4) Natural Frequency \*2

Maximum and minimum amplitude was around 0.3519V and -0.3481 respectively. In the transient stage, the amplitude was around 0.3426V



### 5) Natural Frequency \*5

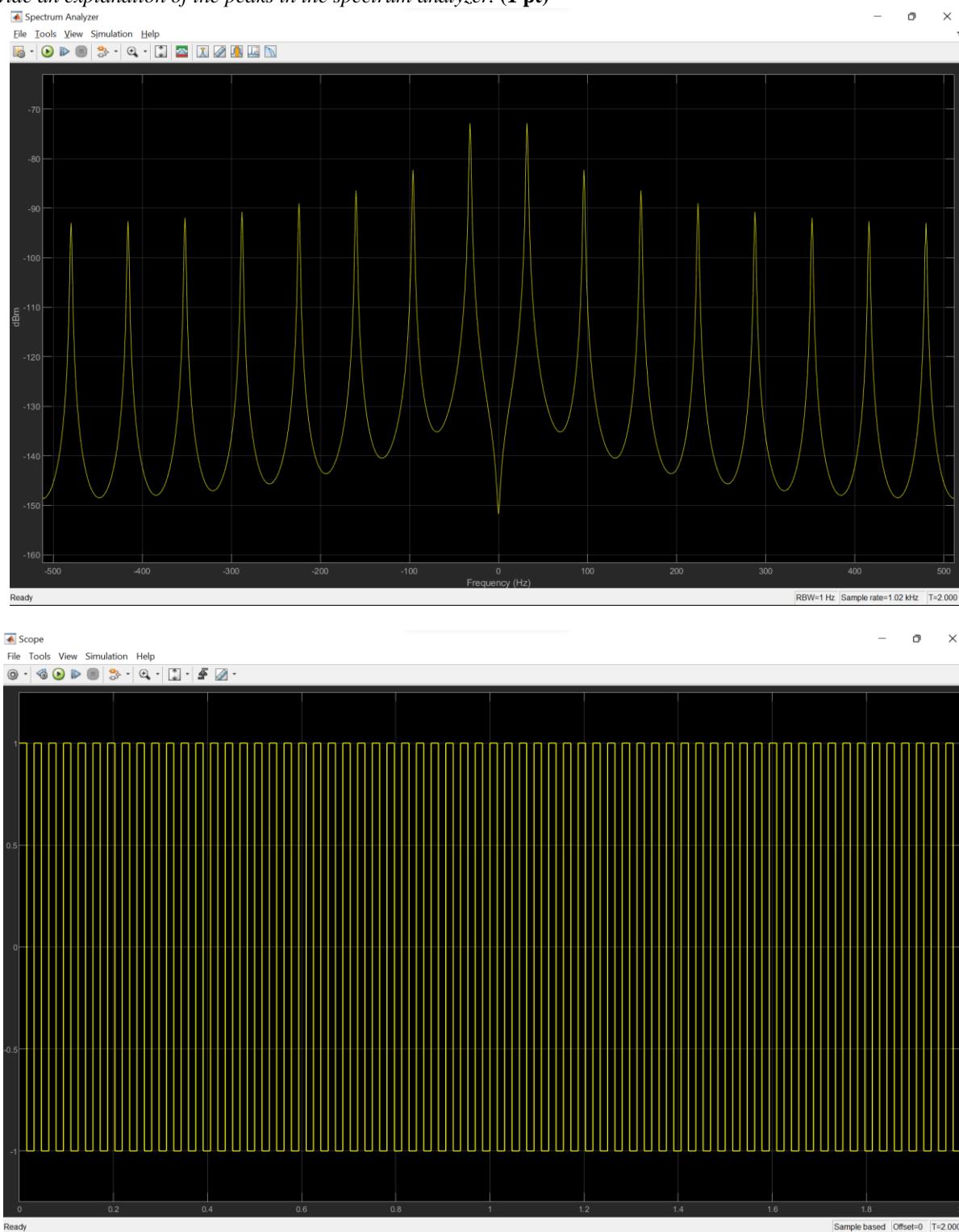
Maximum and minimum amplitude was around 0.0463V and -0.0460 respectively. In the transient stage, the amplitude was around 0.0449V

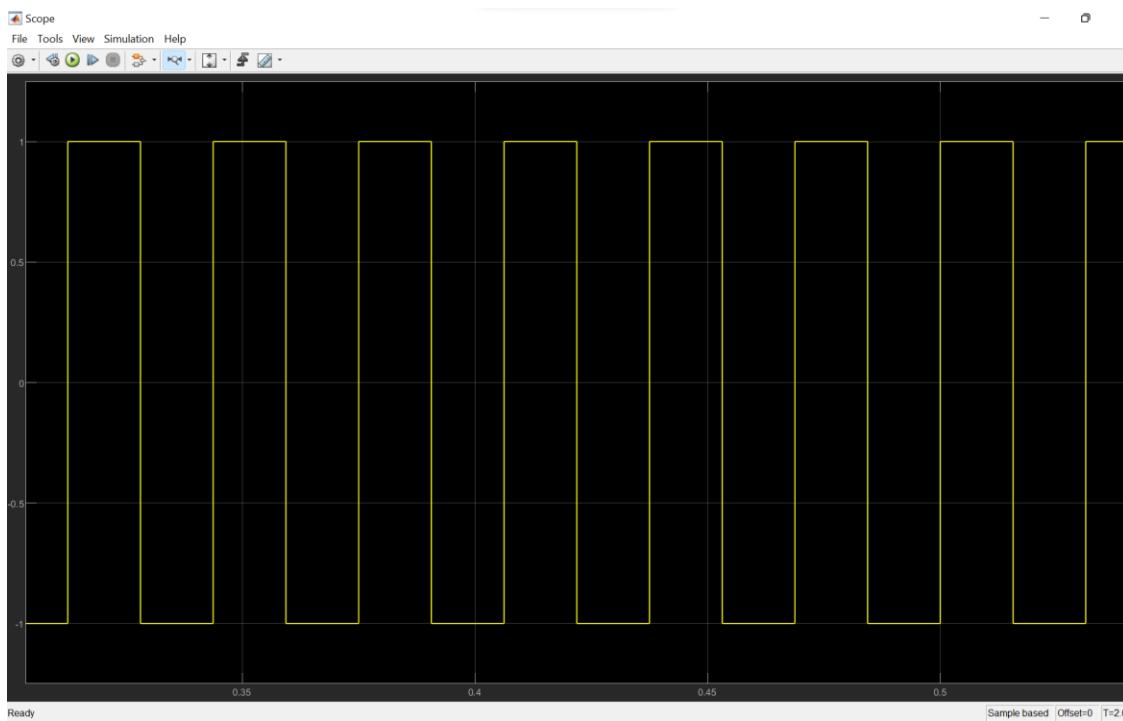


The amplitude is the highest at the natural frequency. Below the natural frequency, increasing the frequency increases the amplitude. Above the natural frequency, increasing the frequency decreases the amplitude.

## 2.2 Exercise 2.2

Include a screenshot demonstrating the square-wave Simulink model (the time-domain plot and its frequency-domain spectrum) and provide an explanation of the peaks in the spectrum analyzer. (1 pt)





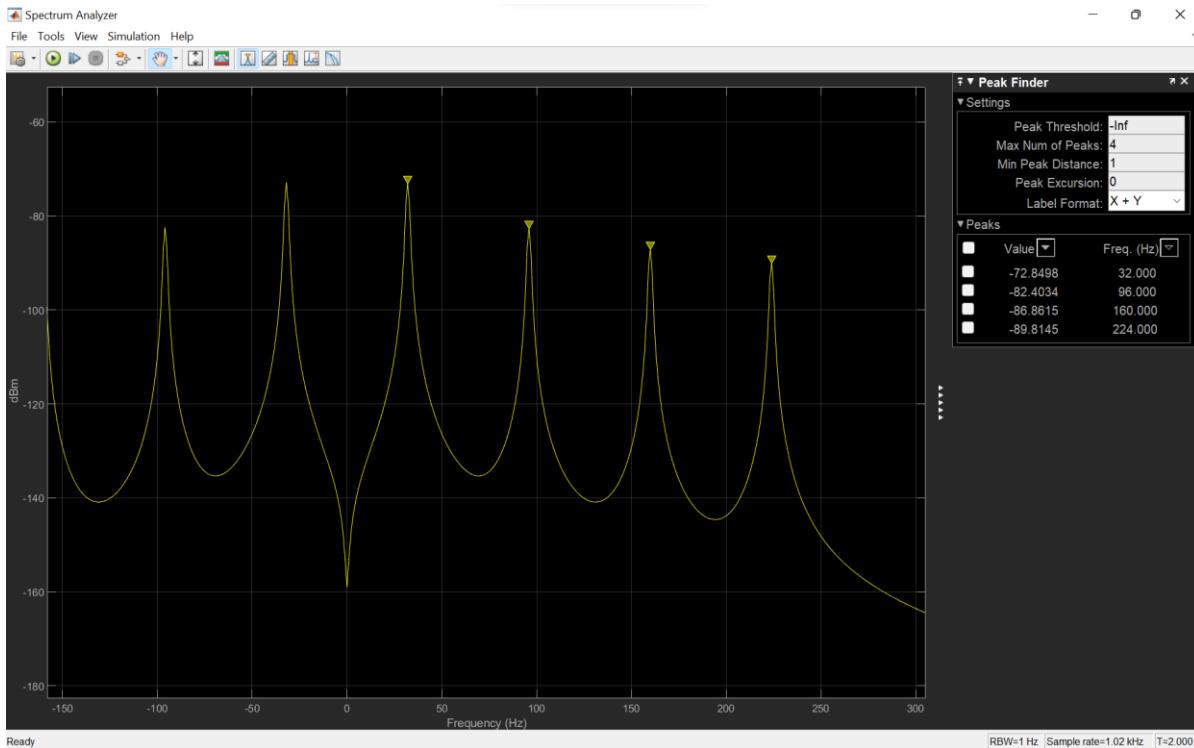
The high number of peaks is due to the higher frequency. At every multiple of 32Hz, there is a peak in the frequency domain plot.

### 3. Applying Fourier Series in circuit analysis

#### 3.1 Exercise 3.1

- a. Use a square wave with 1/32 sec period. Read the frequency of the first 4 peaks on the frequency-domain spectrum, and record the results below. Include a screenshot of your plots. (0.5 pt)

The first 4 peaks are at 32, 96, 160 and 224 Hz as shown in the screenshot below.



- b. Calculate the first 4 terms of the Fourier series for the square wave using the equation provided, and write down the frequency and amplitude of each term from the Fourier approximation below. (0.5 pt)

## Exercise 3.1 Calculations

The given formula is:  $\text{Square}(t) = \frac{4}{\pi} \sum_{n=1, \text{ odd}}^{\infty} \frac{1}{n} \sin\left(\frac{2\pi n t}{T}\right)$

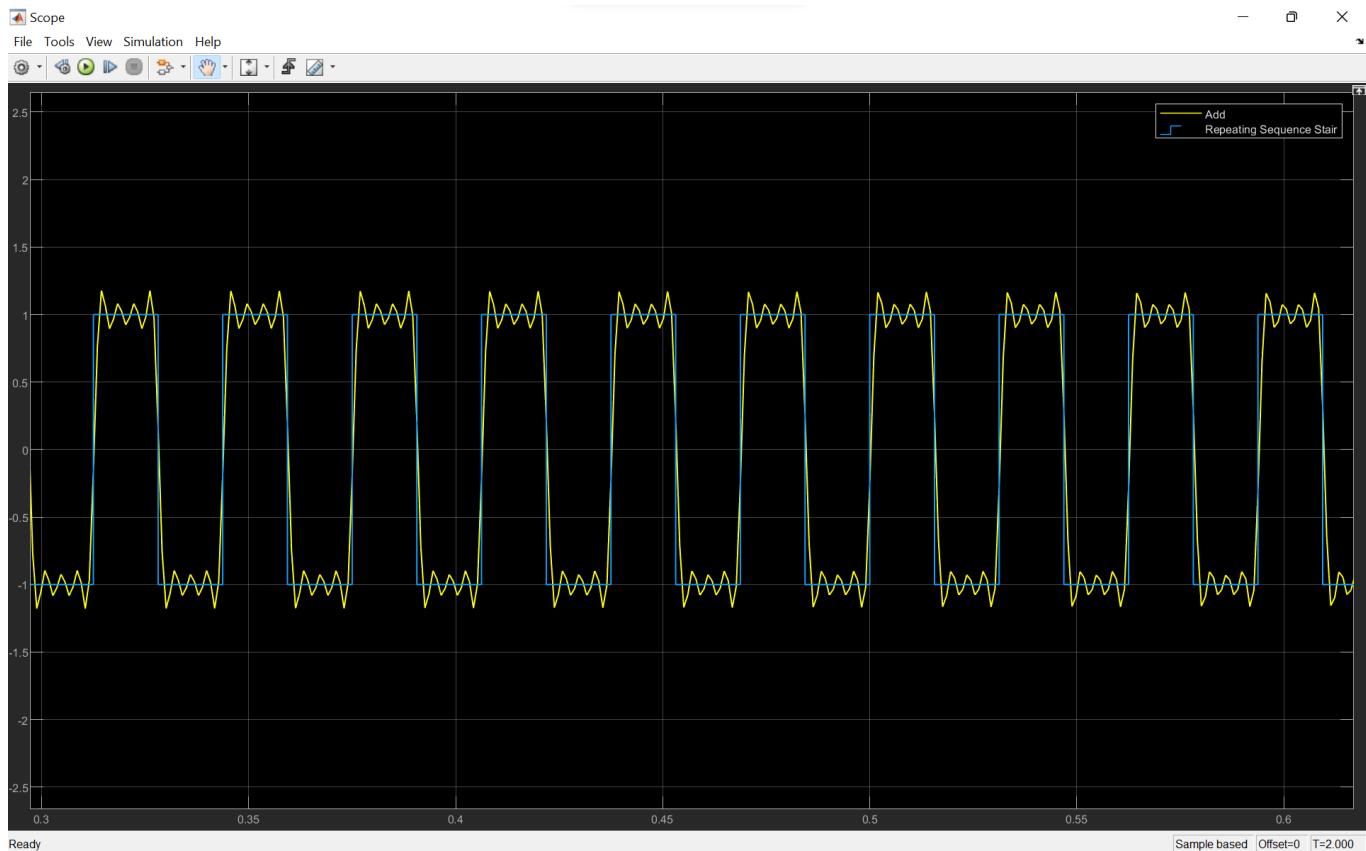
$$\text{At } n=1 \\ \Rightarrow \frac{4}{\pi} \times \frac{1}{1} \sin\left(\frac{2\pi(1)t}{1/32}\right) = \frac{4}{\pi} \sin(64\pi t) \quad \text{Amplitude} = \frac{4}{\pi} \\ \text{Frequency} = 64\pi \text{ rad/s}$$

$$\text{At } n=3 \\ \Rightarrow \frac{4}{\pi} \times \frac{1}{3} \sin\left(\frac{2\pi(3)t}{1/32}\right) = \frac{4}{3\pi} \sin(192\pi t) \quad \text{Amplitude} = \frac{4}{3\pi} \\ \text{Frequency} = 192\pi \text{ rad/s}$$

$$\text{At } n=5 \\ \Rightarrow \frac{4}{\pi} \times \frac{1}{5} \sin\left(\frac{2\pi(5)t}{1/32}\right) = \frac{4}{5\pi} \sin(320\pi t) \quad \text{Amplitude} = \frac{4}{5\pi} \\ \text{Frequency} = 320\pi \text{ rad/s}$$

$$\text{At } n=7 \\ \Rightarrow \frac{4}{\pi} \times \frac{1}{7} \sin\left(\frac{2\pi(7)t}{1/32}\right) = \frac{4}{7\pi} \sin(448\pi t) \quad \text{Amplitude} = \frac{4}{7\pi} \\ \text{Frequency} = 448\pi \text{ rad/s}$$

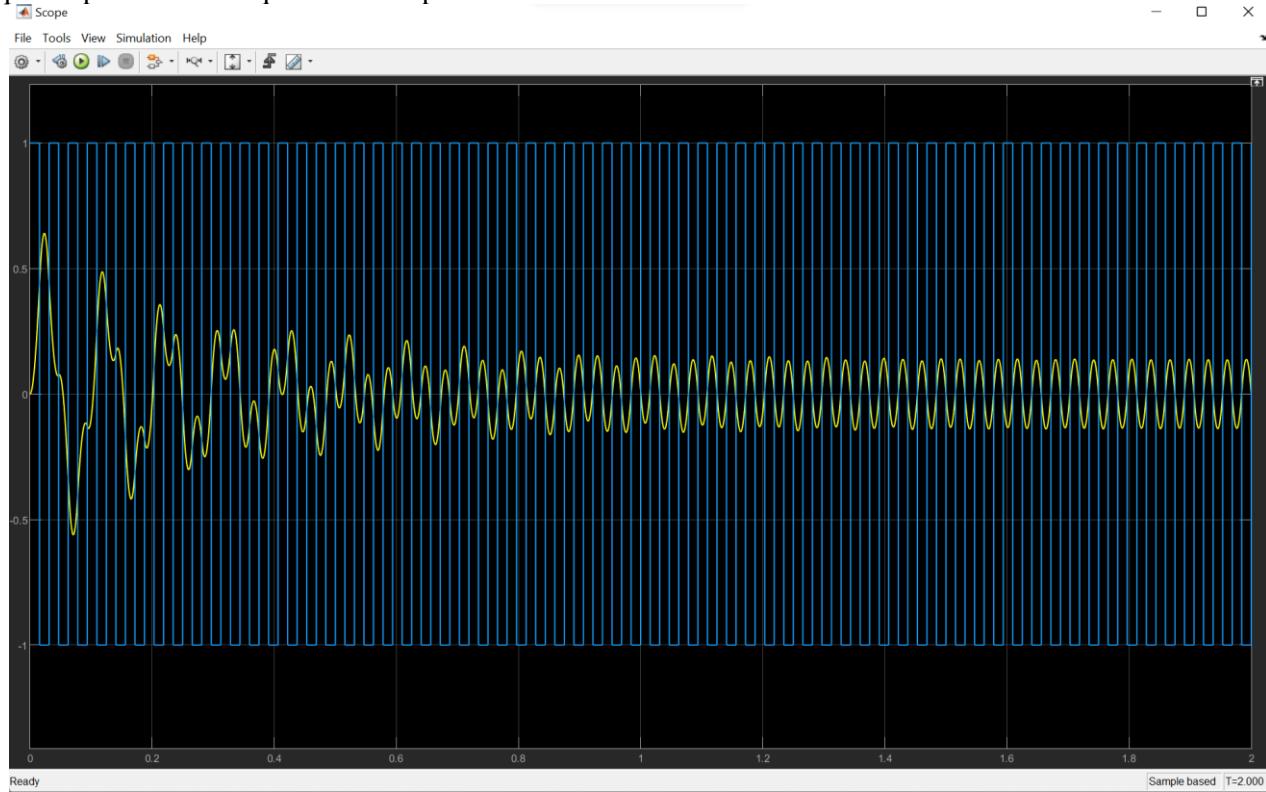
- c. Include a screenshot demonstrating how closely the 4-term Fourier series approximation matches the square wave. (0.5 pt)



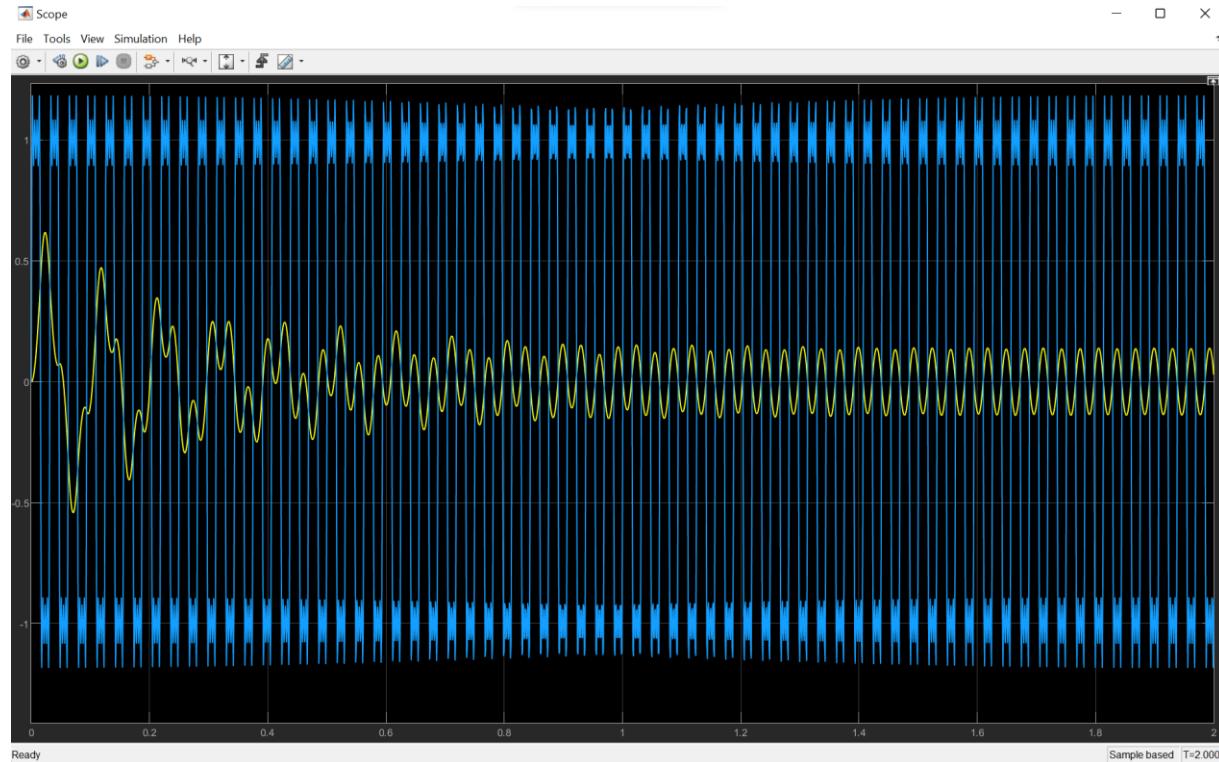
### 3.2 Exercise 3.2

Compare the output response of the RLC circuit to the 4-term Fourier series approximation input and the response to the square-wave input, respectively. Include a screenshot of the output response in the two cases, respectively. (1 pt)

Output response for the square-wave input:

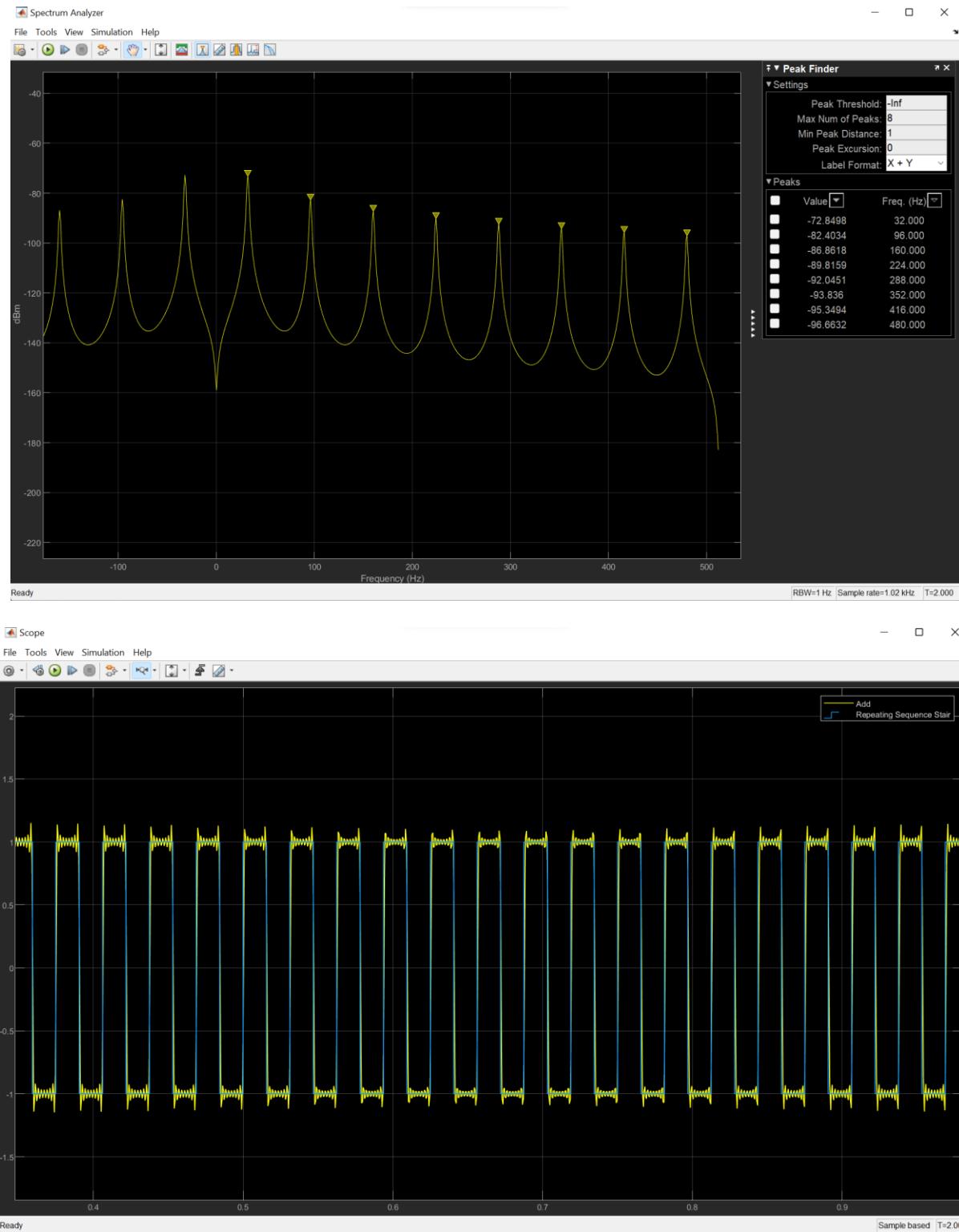


Output response for the 4-term Fourier series approximation:



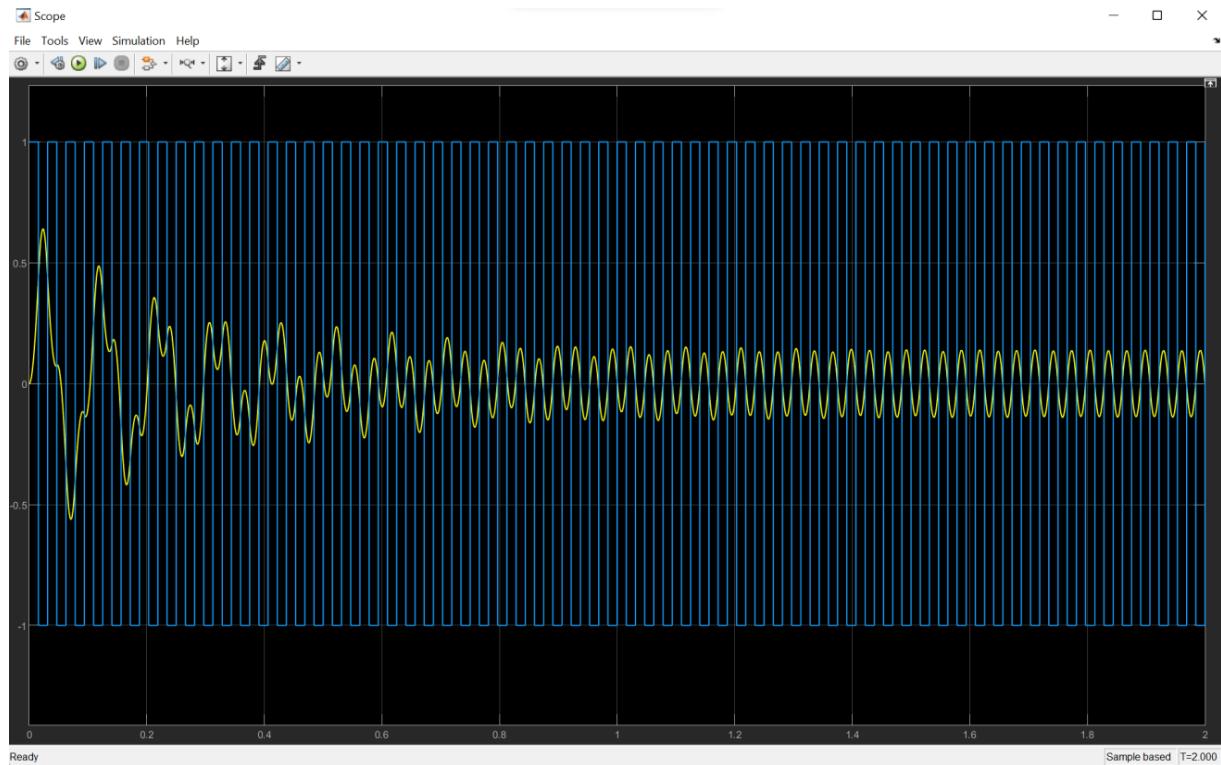
### 3.3 Exercise 3.3

a. Include a screenshot demonstrating how closely the 8-term Fourier series approximates the square wave. (0.5pt)

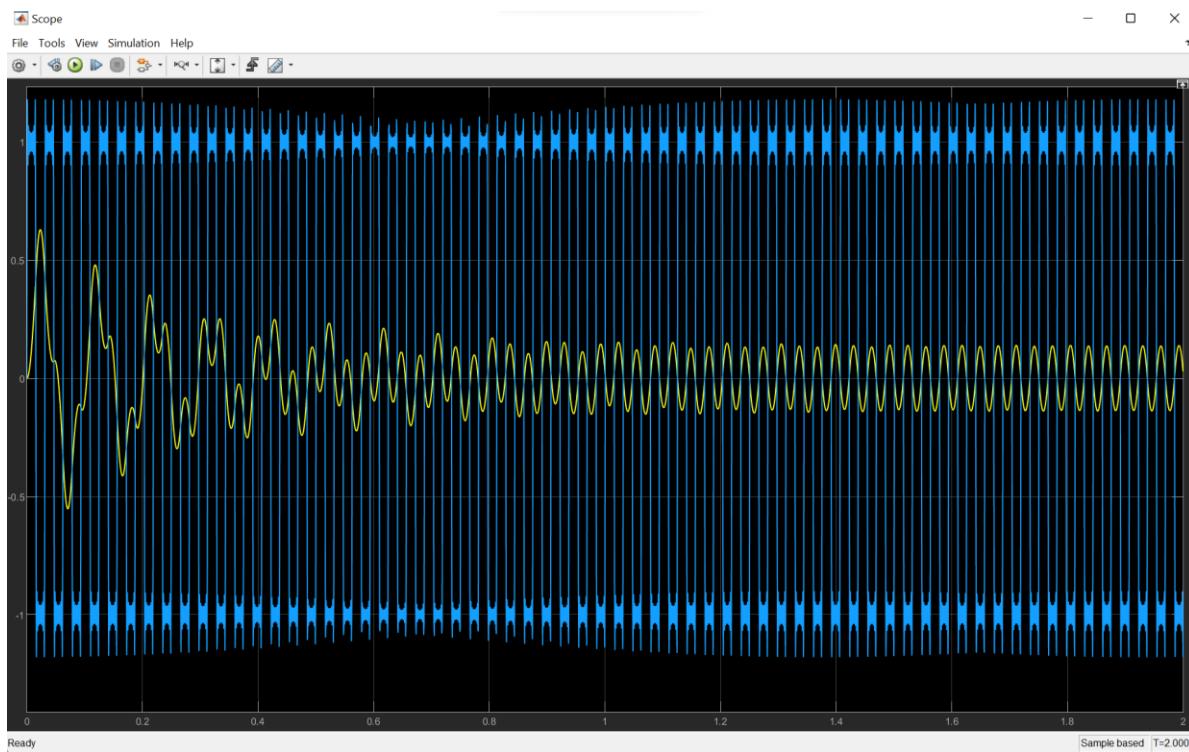


- b. Compare the output response of the RLC circuit to the 8-term Fourier series approximation input and the response to the square-wave input, respectively. Include a screenshot of the output response in the two cases, respectively. (1pt)

Output response for the square-wave input:



Output response for the 4-term Fourier series approximation:



- c. Does the 8-term Fourier series approximate the square wave better than the 4-term Fourier series? Does the output response of the RLC circuit to the 8-term Fourier series input approximate the output response to the square-wave input

*better than when the 4-term Fourier series was used as input? Include an answer and justify your answer. (1pt)*

Yes, the 8-term Fourier Series approximates the square wave better than the 4-term Fourier Series, and the output response of the RLC circuit for the 8-term Fourier series has a better approximation to the output response for the square wave input, as compared to the 4-term Fourier series. However, as can be seen in the graphs, there are still some distinctions at the discontinuities, due to Gibbs Phenomenon. The 8-term Fourier series is better at approximating because the higher number of terms results in more accurate values. However, when we visually compare the two graphs, we cannot see much difference since the difference in the number of terms are less, hence resulting in graphs that are approximated the same. However, it is expected that a higher number of terms will make the approximations better.