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2019 Mathematical Contest in Modeling (MCM) Summary Sheet

Dragon for "real"

Summary

Although dragon doesn't exist in the real world, in this paper, our group creates a system with the presence of dragons. We build mathematical models to simulate the performance of the dragons. After setting parameters for dragons and build the system with some assumptions, we successfully find dragon's characteristics, behavior, habits, diet, and their interaction with the environment. Furthermore, by attaching the importance of climate conditions to our model, we understand the migration of dragons and manage to identify their needs under different environments. Our final goal is to apply this model for different subjects. We manage to explain some other real subjects using this model at the end of our study. By solving the problems above, we create a new mathematical way of studying new things that we knew little about before.

In this paper, we are faced with four related tasks. Our group took three major steps to accomplish them: building the model for dragons' growth, finding the pattern for dragons' migration, applying our model to other realistic subjects. In most cases, we regard the dragon as a black box and we formulate its growth path as the changing outcome in weight during the process of energy input and output where all the factors we study are linked together.

To understand this species' life pattern, our group establishes a block growth model with a steady state to describe the growth process of the dragon. We examined the mathematical effects of many factors and divided them into positive factors for growth and negative factors for growth. During the modeling process, we pay extreme attention to the unique characters of dragon, like its flying ability and etc. After that, we add climates factors into consideration and make some adjustment to our model to fit the migration situation. Based on the result of our model, we come up with a conclusion about dragon's migration theory. Furthermore, by analogy, we apply our model to the realm of reality, testing the universality of our model.

We establish a criterion of rational set for dragons. Because of the virtuality of dragon, and for simplifying the situations, we made several assumptions before analyzing the problem and set a serious of parameter. We mostly decide our settings for dragons according to historical data of other species from the database and sometimes we determined initial conditions reasonably according to reality.

Our suggested solution, which is easy to understand, shows the authenticity of the dragons in some degrees. Although our modeling subject is virtual, we are able to maintain the subject's realistic underpinning under the guidance of our work. On the other hand, since our model is based on many realistic features, it can be applied to many other domains.

Key words: dragon; growth block model; steady state; migration; energy transfer

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Note: 1. Our letter is in appendix one

I. Introduction

1.1 Problem Background

Dragons, creatures of western mythology and legend, are found in literature, art, and architecture. The earliest origin of the dragon in Western Europe represents a symbol of strength and sacred, while in the Christian culture is described as a synonym for evil.

In the modern society, the image of dragon is used in various situations, especially in the entertainment industry. For example, in the fictional television series Game of Thrones, there are three dragons, raised by a woman called “Mother of Dragons”. Nowadays, the western people's view of dragons has long been separated from absolute evil and fierce beasts, and they are more neutral creatures with power. Thus, such image is getting more and more popular among the people.

In today's society, no one has really seen the dragon, as to whether the dragon exists in this world, no one knows. However, we do regard the dragon as a kind of faith-like existence. Whether the dragon really exists, or whether it can fly or not, it is a kind of belief. In this paper, our group is able to apply a kind of analysis about it, telling the world what dragons will be like if it does exist in the real world.

On the one hand, describing and discussing a situation outside the realm of fictional dragons can provide us more insight of the unknown world and show us the way of studying new species. On the other hand, applying scientific factors to mythology beings can help us provide guidance about how to maintain the realistic ecological underpinning of the story when conducting movies or TV series.

1.2 Our works

In this paper our group uses mathematical models and some other methods to accomplish the following 4 tasks according to the problem we chose.

Task one:

Task one requires us to build and analyze a mathematical model to analyze the performance of a real dragon, including characteristics, behavior, habits, diet, and interaction with their environment.

Task two:

In task two, we consider the importance of climate conditions to our analysis. What will happen to the dragons and the resources if the dragon tends to move from regions to regions that have different climates?

Task three:

For this part, we will write a letter to the author of *A Song of Ice and Fire*, George R.R. Martin, to provide guidance about how to maintain the realistic ecological underpinning of the story, especially with respect to the movement of dragons from arid regions to temperate regions and to arctic regions.

Task four:

Although our model does not directly apply to a real physical situation, the mathematical modeling itself makes use of many realistic features used in modeling a situation. We will find what insight that this model can provide outside the realm of fictional dragons.

II. The Description of the Problem

2.1 Problem statement

In our discussion, we treat dragon like a black box. It receives material and energy from the environment or releases them to the outer environment through various activities, which results in the change of their weight. This process is the essence of growth for dragon.

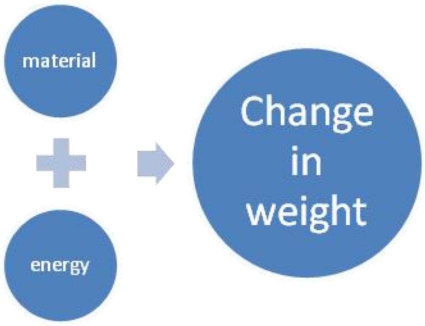

Fig1. The essence of growth for dragon

As we can see, the problem we are dealing with is closely related to the transfer of energy. In this way, we try to understand how dragons absorb energy, how they consume energy and how will it grow through this process. We will focus on the transfer of the energy to solve our task.

2.2 Analysis of our task

2.2.1 Analysis of problem in task one

The growing of the dragon is affected by many factors. We take the weight of the dragon as its index of growth. Through building the block growth model for dragon, we find the growth pattern of dragon. Using the model, we can explain how factors like behavior, diet and etc. is related to its weigh, which explain how the dragon maintain and grow. Especially, we pay extreme attention to the unique characters of dragon, like its flying ability and etc while building the model. To illustrate our idea for the performance of a real dragon, we will answer the following questions according to our result:

1. What is the ecological impact and requirements of the dragons?
2. What are the energy expenditures of the dragons, and what are their caloric intake requirements?
3. How much area is required to support the three dragons?
4. How large a community is necessary to support a dragon for varying levels of assistance that can be provided to the dragons?

2.2.2 Analysis of problem in task two

Dragons might travel to different regions of the world with very different climates, such activity is a need for dragon's life. In this section, we will add some adjustment to our model in task one to suit the change of climates to establish a model for migration situation. Such changes will definitely influence the resources required to maintain and grow a dragon.

2.2.3 Analysis of problem in task three

After understanding how the dragons perform under different circumstances, we will develop an understanding for dragon's migration. Using our conclusion above, we can tell what will happen if dragons move from arid regions to temperate regions and to arctic regions. With such information in the letter, the writer of A Song of Ice and Fire can manage to comprehend how to maintain the realistic ecological underpinning of the story.

2.2.4 Analysis of problem in task four

Although dragon actually doesn't exist, our solutions to the problem are based on many realistic features. Such method of studying new things can be applied to many other areas. The logic and insight of the model can be used when analyzing many other questions. In this part, we will bring another example which has similar characteristic with the dragon, and apply our model to study its performance to understand it.

III. Basic assumption

In order to build the model, we made the following assumptions:

1. Dragons are thermostatic animals.
2. Dragons have similar organs with other animals in reality.
3. Dragons are adapted to the environment on earth.
4. No matter how the environment changes, the assimilation efficiency of dragon stays the same.
5. The dragon's weight will eventually reach a steady state with time passing by.
6. Our growth model is divided into two periods, one before steady state and one after steady state.
7. The dragon will only have a routine daily life span (including basic flying and firing) before steady state.
8. The dragon will be only suffered from long distance migration, tremendous trauma or extra firing after steady state.[1]
9. Dragon's death is defined as having lost 45 percent of its body weight for any length of time.
10. The dragon can resist tremendous trauma by the loss of certain quantity of energy.

IV. Glossary & Symbols

4.1 Glossary

1. ***Homothermal animal***: An organism that generates heat to maintain its body temperature, typically above the temperature of its surroundings.

[1] Our assumption 5-8 is based on the performance of dragon in the television series *Game of Thrones*, which is reasonable. It can help us build our model in two periods.

2. ***Assimilation efficiency***: The proportion of energy absorbed to total energy available from food.
3. ***Positive factors***: Factors which cause the variable we study to increase.
4. ***Negative factors***: Factors which cause the variable we study to decrease.
5. ***Steady state***: The state in the process of growing where the dragon's weight stay constant.

4.2 Symbols

Symbols	*Definition*	*Units*
M	The weight of the dragon	kg
L	The length of the dragon's wing(one side)	m
Q	The total energy available from the food that dragons take	J
m	The weight of food taken by the dragon	kg
η	Assimilation efficiency for one dragon	%
v	The growth rate of dragon's weight	kg/year
q	The energy absorbed by the dragon	J
q_0	The energy consumed through basic metabolism[2]	J
P_m	Basal metabolic power per unit of body weight	w
q_1	The basic energy used for daily flying[3]	J
q_2	The basic energy used for daily breathing fire	J
q^*	Extra energy consumption after steady state[4]	J
q_1^*	The extra energy used for long distance flying	J
q_2^*	The extra energy used for breathing fire	J
q_3^*	The extra energy used for resisting tremendous trauma	J
u	The flying speed of the dragon	m/s
t_1	The amount of time a dragon spend on flying during per year	s
t_2	The amount of time a dragon spend on breathing fire per year	s
T	The temperature in the region	℃
R	The precipitation in the region	mm

2 It doesn't include flying, firing and etc.
3 In the fraction, we don't consider the situation of long distance flying while migrating.
4 The energy that dragon needed for long distance migration, tremendous trauma or extra firing after it reaches steady state.

V. Task one: Model for the dragon's growth

5.1 Model Preparation

5.1.1 The Foundation of Model

The weight of the dragon is influenced by many factors. There are factors positive to it which cause the weight to increase while others are negative which cause the weight to decrease. In this model, we consider those factors as follow:

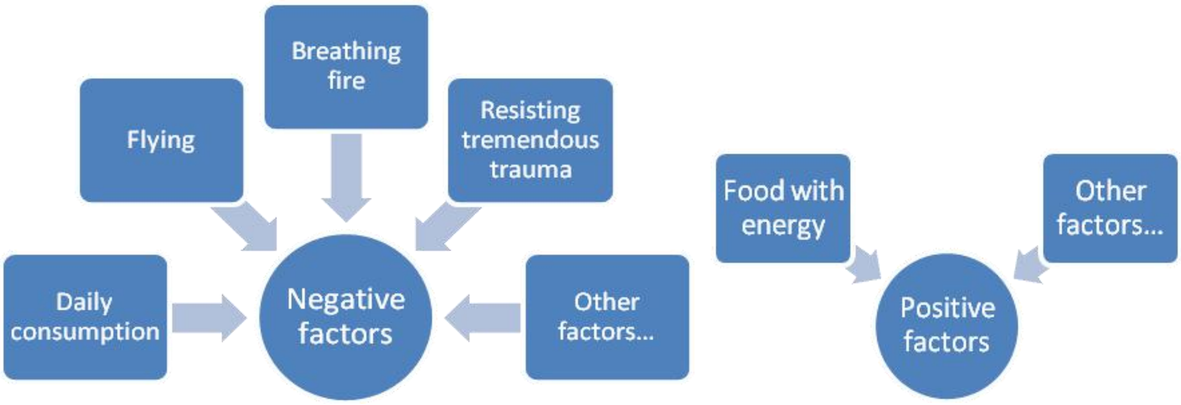

Fig2. Factors influencing the weight of dragons

5.1.2 Assumptions

In order to simplify the model, we assume that M and m are in linear relation, which means that $m = kM$. Also we assume that m and Q are in linear relation, which means that $Q = cm$.

In our model, we assume that a born dragon will finally reach a steady state of growth where its weigh stay constant. After reaching steady state, it weigh will flow due to some accidental energy consumption but will finally reach steady state again. We assume the dragon experience a war in the steady state.

Some other detailed assumptions are made and illustrated during the establishing of the model for calculation purpose.

5.1.3 The values of parameters in our assumptions in 5.1.2

Some other parameters are only used for detail calculation so they are illustrated during the establishing of the model.

①the value of η

We assume that $\eta = 80\%$. Our assumption is rational because we are just considering one dragon, in which case, the assimilation efficiency should stay relatively high.

②the value of k

We assume that $k = 4\%$. We collect the value of k for human, cow, eagle and tiger from the internet. We study their pattern and estimate k for the dragon is about 4%. [1]

③the value of c

We assume that $c = 3\times10^{7}\,\mathrm{J}\cdot kg^{-1}$. The calorific value of fat is about 38.9kJ/g, and that of protein is about 23kJ/g. Considering the diet that dragons have while also simplify the situation, we assume that the calorific value of its food is 30kJ/g, the average value of protein and fat, denoted by c. [2]

5.2 Model Establishment

5.2.1 Growth model before steady state

Basic equation

Because the weight of the dragon is changing through time, it's important to us to understand how the dragon will grow through time. According to the assumption, and in order to know the relations between them, we have:

$$\frac{dM}{dt} = rM\left(1-\frac{\sum q_i}{q}\right) \tag{1}$$

Where:

1. r is a parameter whose value will be determined later.

2. In the equation, q denotes positive factors and $\sum q_i$ denotes negative factors.

3. The positive factors and negative factors are both in relation with the dragon's weight, which we will study later.

To calculate the value of M, we need a basic equation and some initial data. According to the definition of v, we have:

$$\mathrm{v} = \frac{dM}{dt} \tag{2}$$

And we have the initial value listed below:

$$M_0 = 10kg\,,\ \ \mathrm{v}_0 = 25kg\cdot \mathrm{year}^{-1} \tag{3}$$

The determination of factors

To identify the positive factors, we have the following calculations:

According to the definition of η, we have:

$$q = 365 \times Q\eta \tag{4}$$

Therefore, from (4) and our assumption in 5.1.1(1), we have:

$$q = 365 \times k\eta cM \tag{5}$$

Where:
1. $k = 4\%$ according to 5.1.1(2)
2. $\eta = 80\%$ according to 5.1.1(2)
3. $c = 3\times10^7\,\mathrm{J}\cdot k\mathrm{g}^{-1}$ according to 5.1.1(2)

To identify the negative factors, we have the following calculations:
In our case, the negative factors can be approximately divided into three parts, we have:

$$\sum q_i = q_0 + q_1 + q_2 \tag{6}$$

① The study of q_0

To determine the value of q_0, we assume that q_0 and M are in linear relation. According to statistics, an adult's body weight was taken as 65kg denoted by m_0 and basal metabolic power as 78W denoted by P_0 [3], so we have:

$$P_m = \frac{P_0}{m_0}\ , q_0 = P_m \times M \times t \tag{7}$$

Where t denotes the amount of time of a year, so $t = 365\times24\times60\times60s$.

② The study of q_1

First, we regarded dragon as a cuboid. Let the dragon be x in both width and height. So it's body length $l = 4x$ approximately, its density denoted by ρ . According to these assumptions, we have

$$M = 4x^3\rho \tag{8}$$

Rearranging (8) produces:

$$l = 2\times(\frac{2M}{\rho})^{\frac{1}{3}} \tag{9}$$

To determine the value of q_1, we assume that L and M are in linear relation, so we have:

$$L = aM^{\frac{1}{3}}\ ,\ d = bL \tag{10}$$

Where a and b are proportionality coefficient, while d denotes the distance when one wing flutters for one time. According theorem of momentum, we have:

$$\frac{Ft_0}{2} = Mu \tag{11}$$

$$W = Fd \quad , \quad P_1 = \frac{W}{t_0} \tag{12}$$

$$q_1 = P_1 t_1 \tag{13}$$

Where F denotes the resistance that the wing receives and t_0 denotes the periodicity of the fluttering. From (10), (11), (12) and (13), we determine the value of q_1 as below:

$$q_1 = \frac{2abut_1 M^{\frac{4}{3}}}{t_0^2} \tag{14}$$

Where:

1. According to the data from Andean condor on Wikipedia [4], we can receive that $a = 0.6m \cdot kg^{-\frac{1}{3}}$.[5]
2. With reference to the dragon's performance on the book and the TV series, we set a rational value that $t_0 = 0.5s$ and $t_1 = 365 \times 1 \times 3600s$.
3. We assume that $b = 0.5$ according to the situation in reality.
4. With reference to the flying speed of other creatures [5], we assume that $u = 12.5m \cdot s^{-1}$.[6]

③ The study of q_2

To determine the value of q_2, we assume that the flame is a cylinder of radius R and length h, the energy dissipation of flame is approximated by blackbody radiation, therefore, we have:

$$S = 2\pi Rh + \pi R^2 \tag{15}$$

$$P_2 = \sigma T_0^{\ 4} S \tag{16}$$

$$q_2 = t_2 P_2 \tag{17}$$

Where t_2 denotes the time that last for each fire and T_0 denotes the temperature of the flame. From (15), (16) and (17), we determine the value of q_2 as below:

$$q_2 = t_2 \sigma T_0^{\ 4} (2\pi Rh + \pi R^2) \tag{18}$$

Where:
1. With reference to the dragon's performance on the book and the TV series, we set a rational value that $t_2 = 0s$ before the steady state.
2. According to the flame image on TV and its description in the book, we assume that $R = 0.5m$ and $h = 5m$.
3. According to the actual situation, we assume that $T_0 = 800K$.

---

[5] According to the statistic shown on Andean condor, we take the data as the initial wing length was 1.3m(because the bird's weight was similar to that of the dragon). Then we are able to read the value of a.

[6] According to our reference, a typical, unhurried soaring speed in golden eagles is around 45–52 kilometers per hour (28–32 mph). We take the minimum.

Growth model before steady state

After analyzing both positive factors and negative factors, from (1), (5), (6), (7), (14), (18), we have:

$$\frac{dM}{dt} = rM\left[1 - \frac{\frac{P_0}{m_0} \times M \times t + \frac{2abut_1 M^{\frac{4}{3}}}{t_0^2} + t_2 \sigma T_0^{\,4}(2\pi Rh + \pi R^2)}{365 \times k\eta c M}\right]$$

This equation is denoted as (*)

Together with (*), (2), (3), we can determined the value of r:

$$r = 4.5 year^{-1} \tag{19}$$

After these steps, all the parameters' value in (*) are confirmed except M and t, therefore, we can study the relations between weight and time before steady state through this equation.

5.2.2 Growth model after steady state

Basic equation

After the dragon reaches the steady state, the basic equation will be different. In this period, when the dragon suffered from long distance migration, tremendous trauma or extra firing, it will consume more energy. At this time (t_n), the negative factor will increase, but slowly recover over time, and this additional negative factor will decrease with time index. At the same time, the dragon will feed more to promote recovery accordingly, so the positive factor will increase, and this additional growth factor will also decrease exponentially over time. After a certain period of time, the dragon will be back to steady state. Therefore, after steady state the equation becomes:

$$\frac{dM}{dt} = \mathrm{r}M\left(1 - \frac{q^* e^{-\lambda(t-t_n)} + \sum q_i}{q^* e^{-\mu(t-t_n)} + q}\right) \tag{20}$$

Where:

1. q^* denotes the extra energy consumption in t_n.

2. λ and μ are two constant parameter. $\frac{\lambda}{\mu}$ represents the recovery capability of the dragon.

3. We find that $\lambda = 1$ and $\mu = 20$.[7]

[7] To reach the steady state again, we set two parameters in our model, denoted by λ and μ. Through simulation on our computer program, we let $\lambda = 1$ and $\mu = 20$.

4. In this period, $q^{*}e^{-\mu(t-t_n)}+q$ denotes positive factors and $q^{*}e^{-\lambda(t-t_n)}+\sum q_i$ denotes negative factors.

The determination of q^{*}

We discuss a situation after the dragon experienced a war between countries. To determine the value of q^{*}, we assume that $q_3^{*}=4.32\times 10^{8}J$, we have:

$$q^{*}=q_1^{*}+q_2^{*}+q_3^{*} \tag{21}$$

We assume that the dragon flies for 3 hours, fires for 0.5 hours, so we have:

$$q_1^{*}=3\times 3600\times P_1 \ , \ q_2^{*}=30\times 60\times P_2 \tag{22}$$

From (21), (22), we have:

$$q^{*}=2.5\times 10^{9}J \tag{23}$$

Growth model after steady state

After analyzing the additional positive factors and negative factors in this period, from (5), (6), (7), (14), (18), (19), (20), (23) we have:

$$\frac{dM}{dt}=rM\left[1-\frac{q^{*}e^{-\lambda(t-t_n)}+\frac{P_0}{m_0}\times M\times t+\frac{2abut_1M^{\frac{4}{3}}}{t_0^{2}}+t_2\sigma T_0^{\ 4}(2\pi Rh+\pi R^{2})}{q^{*}e^{-\mu(t-t_n)}+365\times k\eta cM}\right]$$

This equation is denoted as (**)

After these steps, all the parameters' value in (**) are confirmed, therefore, we can study the relations between weight and time after steady state in different situation through this equation.

5.3 Results

5.3.1 Model before steady state

According to equation (*), by applying a Matlab program, we manage to creating the following image. According to our result, the dragon will use almost 5 years to reach the steady state with a weight about 500kg.

Our program for model before steady state is demonstrated in appendix two.

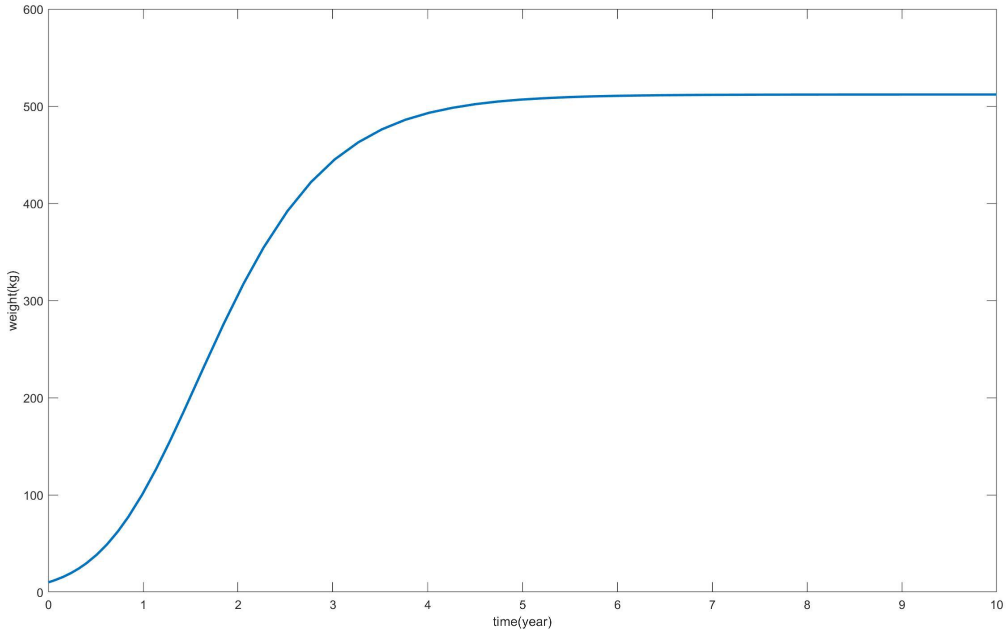

Fig3. The relations between weight and time before steady state

5.3.2 Model after steady state

According to equation (**), by applying a Matlab program, we manage to create the following image. According to our result, the dragon can fully recover from the incident (like a war) within 3-4 years.

Our program for model after steady state is demonstrated in appendix three.

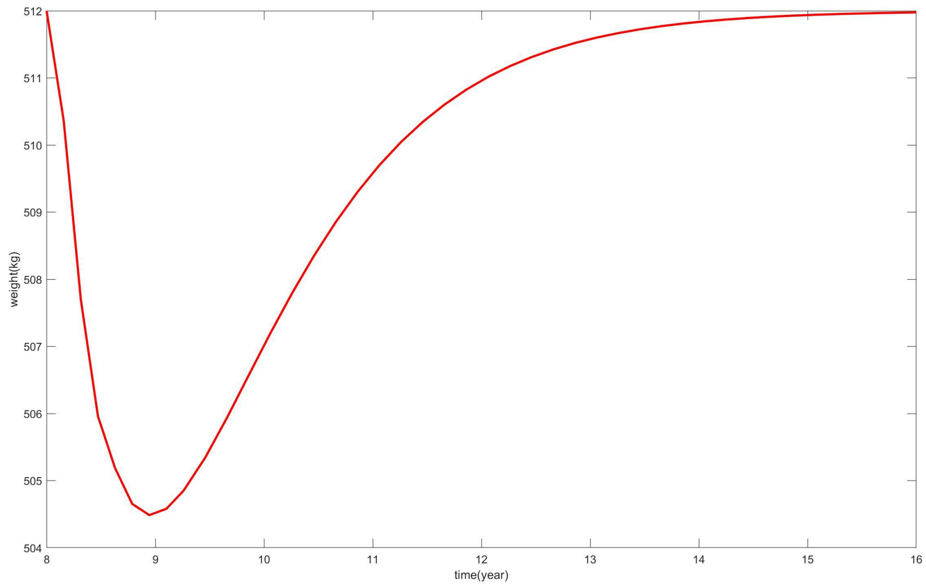

Fig4. The relations between weight and time after experience a war in steady state

5.3.3 Growth pattern

Integrating the conclusions above, we have the growth path for the dragon.

Our program for model before steady state is demonstrated in appendix four.

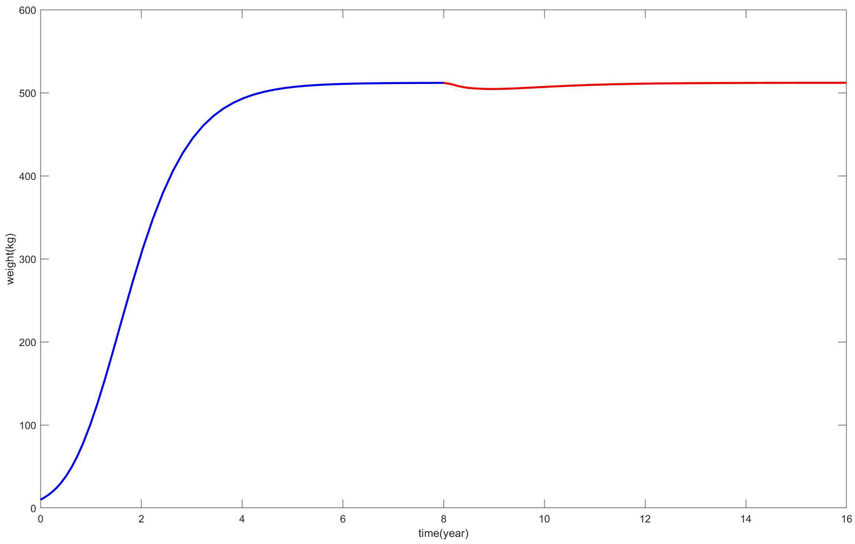

Fig5. The relations between weight and time

5.4 Answer to Task one

5.4.1 Answer to question one

Q1. What is the ecological impact and requirements of the dragons?

Ecological impact: In a battle, if the fire-breathing time of all three dragons is 30min, the flame coverage area can reach 3*105m^2, and most creatures in this area will be burnt.

Ecological requirements: Adequate food sources, moderate climatic conditions, and moderate external harm.

5.4.2 Answer to question two

Q2. What are the energy expenditures of the dragons, and what are their caloric intake requirements?

All the following calculations are based on our model.

Annual basal metabolic energy consumption: $2*10^{10}$ J

Flame energy consumption: $2.4* 10^7$J

Energy dissipation against trauma: $6*10^7$J

Long-distance migration energy consumption: $1.32*10^{10}$ J

5.4.3 Answer to question three

Q3. How much area is required to support the three dragons?

In this part, we only consider the area needed for grass and people, the activity area of cattle and dragons is ignored. And we assume the only source of food for dragon is beef.

The annual food intake of the three dragons in the steady state is about 22.4 tons according to our model and calculation. According to statistics that 50% of the killed cattle will be sold on the market. So the quality of the cattle to be killed each year is about 45 tons. Suppose the assimilation rate from grass to cattle is 10%, then the mass of grass needed is about 450 tons.

Let the planting density of herbage be 1kg/m^2. The area is needed is approximately 450000 km^2.

The per capita GDP in the US is 59532 dollars, the population density is 33.27 / km^2 and the price of beef is $12 /kg. Assuming that each person uses 15% of his income to raise the dragon, then each person can buy 744.15kg of beef every year. Therefore, the number of people needed is about 30.

To sum up, the amount of required area is about 1.4 km^2.

5.4.4 Answer to question four

Q4. How large a community is necessary to support a dragon for varying levels of assistance that can be provided to the dragons?

The conditions necessary to support the basic growth of the dragon is shown in question 3. Considering the damage to the dragon and the firing consumption, a larger area is needed:

For injuries, assuming that the area of each wound of the dragon is 1m^2 and the depth is 2mm, the volume of the injured part is $2*10^{-3}$m^3. Assuming the density of the dragon is $1.2*10^3$kg/m3, the mass of the injured part is 2.4kg. According to our model, the amount of food needed for this part is 2.4kg/80%=3kg, namely 3kg of beef. We can find that area needed for this consumption of is about 181m^2.

That is to say, we need to expand an area of about 181m^2 to help a dragon recover from each one of its wound.

As for fire-breathing, each flame need to consume energy for $2.4 * 10^5$ J, sets the heat of combustion of the grass to be $2 * 10^5$ kJ/kg.

Through calculation, we find that the dragon needs to expand an area of about 3.6m^2 to reset the influence of each time for its firing.

VI. Task two: Model improvement for climate change

6.1 Preparation for improvement

6.1.1The Foundation of our improvement

In this part of our paper, we take the climate condition into consideration of our model. We study the impact of different climates have on the positive factor and negative factors in our previous model. In our discussion, we divided the change in climate into mainly two aspects: the change in temperature represented by T and the change in precipitation represented by R . We use mathematical model to understand how these changes will affect the growth of dragon.

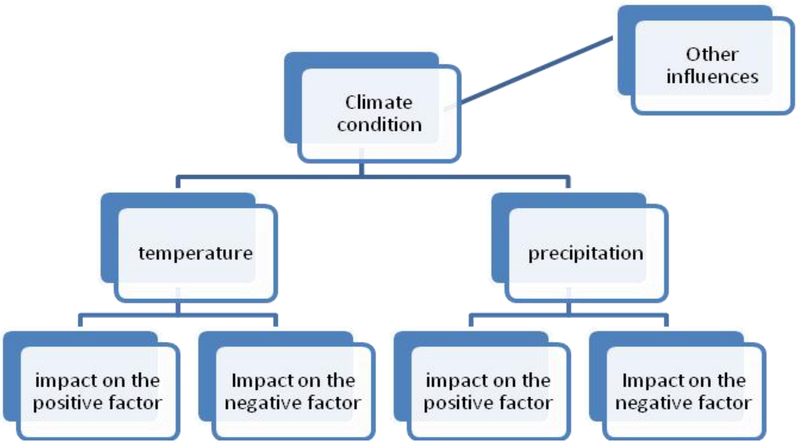

Fig6. How climate condition affect our previous model

6.1.2 The assumptions we add for model improvement

1. In our model, the influence of climate is reflected by the changing of P_m.

2. We assume that the most suitable climate for the dragon is temperate region(because the climate is one of the world's most widely distributed), the optimum temperature $T_s = 10$ ℃ and precipitation $R_s = 400\,mm$.

3. The dragon can adapt to the temperature whose range is - 80 ℃ to 60 ℃.
4. We suppose when the temperature is higher than 70 ℃ or lower than -90 ℃, the dragon dies

6.2 Model improvement

According to our assumptions, P_m is in relation with both T and R .

6.2.1 Adding the parameter T

The curve of $P_m - T$ for the thermostatic animals was valley-like, and the metabolic lever was at the lowest in the ther-moneutral zone.[8] At the same time, the influence of T on P_m is dominant, so the following differential expression can be obtained:

$$\frac{\partial P_m}{\partial T} = -\beta_1 e^{\alpha_1 (T_s - T)} + \beta_2 e^{\alpha_2 (T - T_s)} \tag{24}$$

Where:

[8] Because in that state, metabolic lever was only controlled by skin vasomotor or bristle

1. α_1 and α_2 are temperature index. We determine that $\alpha_1 = \frac{1}{10-(-80)}$ and $\alpha_2 = \frac{1}{60-10}$ according to our assumption about the temperature setting for dragon's survival in 6.1.2(3.).

2. β_1 and β_2 are parameters we set to satisfy our assumptions in 6.1.2(4). Through calculation[9], we can reach their value as: $\beta_1 = \beta_2 = \frac{3}{350}$.

From (24) we have:

$$P_m = \frac{\beta_1}{\alpha_1} e^{\alpha_1(T_s - T)} + \frac{\beta_2}{\alpha_2} e^{\alpha_2(T - T_s)} + P_R \tag{25}$$

Where P_R denotes the part in relation with R.

6.2.2 Adding the parameter R

The decreasing rate of P_m with the increase of R is related to P_m, and the smaller P_m is, the slower the reduction of P_m will be until it reaches a stable value denoted by P_T, so we have:

$$\frac{\partial P_m}{\partial R} = -\kappa(P_m - P_T) \tag{26}$$

Where:

1. When P_m reaches the stable value of P_T, the impact of R on P_m can be neglect. So in this state, P_m is affected by T.
2. κ is a parameter we set to satisfy our assumptions in 6.1.2(4). Through calculation, we can reach its value as: $\kappa = \frac{1}{R_s} = \frac{1}{400}$.

From (26) we have:

$$P_m = \theta e^{-\kappa R} + P_T \tag{27}$$

Where:

1. P_T denotes the part in relation with T.
2. θ is an integration constant. According to our basic assumption 9 and 10, through calculation, we find that

[9] When $R \geq 400$, we have $P_m = \frac{\beta_1}{\alpha_1} e^{\alpha_1(T_s - T)} + \frac{\beta_2}{\alpha_2} e^{\alpha_2(T - T_s)}$, let $T = T_s$, $P_m = 1.2W \cdot kg^{-1}$, we have $\beta_1 = \beta_2 = \frac{3}{350}$.

$\theta = 1.8$.[10]

Integrating (25) and (27), we have:

$$P_m = (\frac{\beta_1}{\alpha_1} e^{\alpha_1 (T_s - T)} + \frac{\beta_2}{\alpha_2} e^{\alpha_2 (T - T_s)} + P_c) + \theta e^{-\kappa R} \tag{28}$$

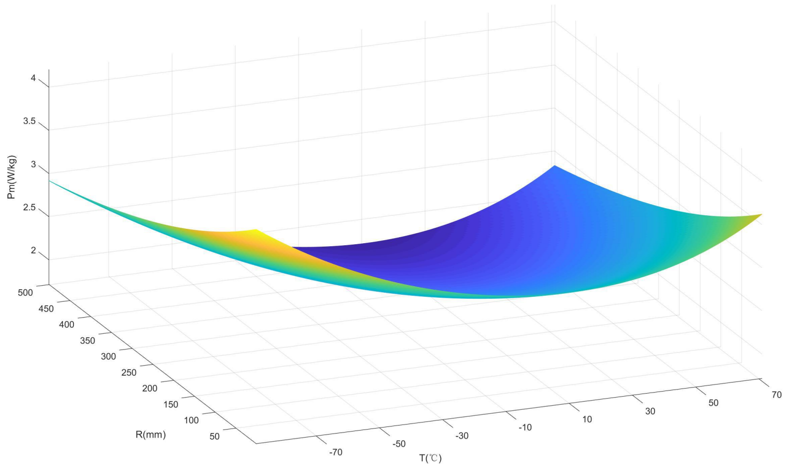

Fig7. The relations between P_m , T and R

Our program for model after steady state is demonstrated in appendix five.

With equation (28), we manage to understand the influence of climate condition to our previous model.

6.3 Results: migration simulations

To demonstrate the influence of climate condition to the performance of the dragon, we make two simulations: from temperate region to arid region and from temperate region to arctic region:

6.3.1 from temperate region to arid region

Assume that the dragon migrate from a temperate region with the optimum temperature $T_s = 10$ ℃ and precipitation $R_s = 400\,mm$ to a arid region with the temperature $T = 20$ ℃ and precipitation $R = 50mm$. So according to equation (28), $P_m = 2.8W \cdot kg^{-1}$. We assume $q^* = 1.32 \times 10^{10} J$ for a 30-hours extra long distance flying.

According to equation (**), by applying a Matlab program, we manage to create the following image.

[10] Under this value of θ, the dragon's weight will drop dramatically when $R = 0$.

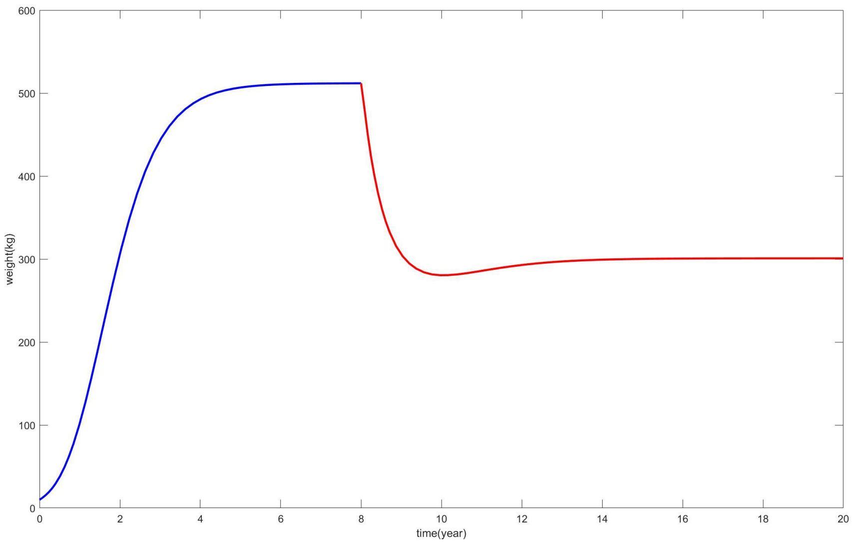

Fig8. Migration simulation from temperate region to arid region

Our program for model after steady state is demonstrated in appendix six.

6.3.2 from temperate region to arctic region

Assume that the dragon migrate from a temperate region with the optimum temperature $T_s = 10$ ℃ and precipitation $R_s = 400\,mm$ to a artic region with the temperature $T = $ -20 ℃ and precipitation $R = 100\,mm$. So according to equation (28), $P_m = 2.7W \cdot kg^{-1}$. We assume $q^* = 1.32 \times 10^{10} J$ for a 30-hours extra long distance flying.

According to equation (**), by applying a Matlab program, we manage to create the following image.

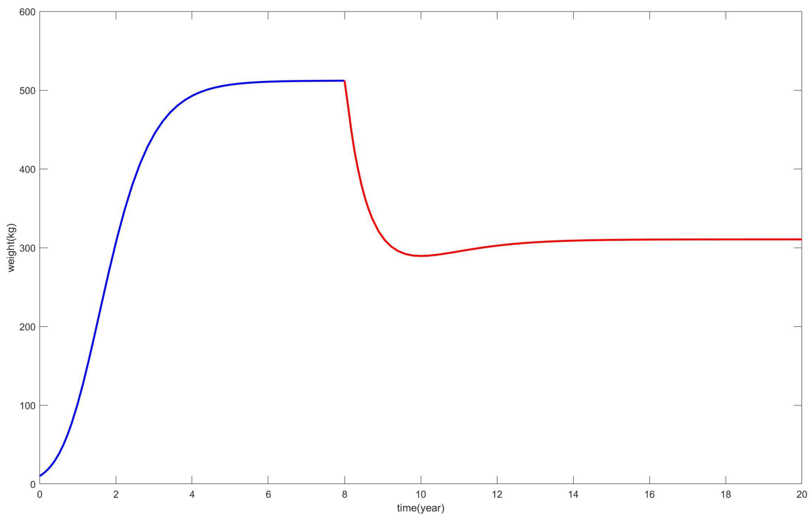

Fig9. Migration simulation from temperate region to arctic region

Our program for model after steady state is demonstrated in appendix six.

VII. Task three: Our Letter

7.1 Preparation

In this part, we will write a letter to the author of *A Song of Ice and Fire*, George R.R. Martin. Our purpose of writing the letter is to provide guidance about how to maintain the realistic ecological underpinning of the story, especially with respect to the movement of dragons from arid regions to temperate regions and to arctic regions. In order to achieve this goal, we need to use our model and our result from the previous section. Therefore, we need to know the temperature and the precipitation in the three mentioned area.

	temperature	precipitation
Arid region	High	Extremely low
Temperate region	Moderate	Moderate
Arctic region	Extremely low	Low

Fig10. Temperature and precipitation arid, temperate and arctic region

7.2 Our letter

See the letter to George R.R. Martin in appendix one.

VIII. Task Four: Model promotion

8.1 Instruction

8.1.1 The reasons for promotion

Although our analysis about dragon does not apply to a real physical situation, the mathematical models we build are based on realistic data and features. As a result, our model can actually explain many other object in the reality realm aside of dragons' growth.

8.1.2 The subject we can apply our model to

Our model can be applied to explain the development of objects that has identical characteristics with the dragon, especially for subjects with feedback mechanism, for example:

Animals: like eagle, shake and etc.

Objects: like aircraft and etc.

Concepts: like ecosystem, economic and etc.

8.2 Promotion

In order to demonstrate how to apply our model outside of the realm of fictional dragons, we take "ecosystem" for instance. We use the analogy method to understand how the ecosystem develop according to our model.

variable	dragon	ecosystem
M	The weight of the dragon	The size of the ecosystem
q	The energy absorbed by the dragon	Solar energy absorbed by ecosystem
q_i	Daily energy consumed by dragons	Daily energy consumed by the livings in ecosystem
q*	Extra energy consumption after steady state	Energy used for recovery from man-made destruction
……	……	……

Fig11. Analogy

By analogy, we can find that the develop pattern of ecosystem is similar to the grown path of dragon, which we modeling for. Therefore, if we can change the value of the parameter in our conclusions to fit the situations in ecosystem, we will be able to determine the develop pattern of the ecosystem and we can the study the impact of man-made destruction on the ecosystem.

IX. Evaluation of the model

9.1 Strength

1. Although the dragon is a virtual animal, we have referred to the relevant data of many real species on earth to describe the characteristics of the dragon, which makes our model and conclusions have practical significance.
2. We use simple models to simulate the characteristics of the dragon, such as treating the dragon as a cuboid,

treating the flame as a cylinder and making it conform to the characteristics of the black body.
3. In the models after steady state, to deal with factors like long-distance migration and fire-breathing, we introduced a feedback mechanism.
4. Our model can be applied to many different subjects.

9.2 weakness

1. Most of the data we refer to are from the encyclopedia, which is not detailed enough. So there is a little lack of more in-depth and accurate analysis.
2. We do not strictly demonstrate that certain characteristics of dragons are comparable to the reliability of certain animals.
3. We do not study the effects of injury, fire breathing, long-distance migration and climate change on the growth of dragon before they reach a stable weight.

X. References

[1] Diansheng Wang, Xingzhou Wang, "Relationship between forage intake, body weight, milk yield and type of diet in black and white dairy cows", *Journal of Heilongjiang animal husbandry and veterinary medicine*, vol. 04, 1989

[2] Sogo Uencyclopedia, "*calorific value*" [online]. Available: http://baike.sogou.com/v11039.htm

[3] Yun Jiang, *"How much energy we consume every day"*, [J]. Science world, 2015(2):13-13.

[4] *"Andean condor"* Wikipedia. [Online]. Available: https://en.wikipedia.org/wiki/Andean_condor

[5] *"golden eagle"* Wikipedia. [Online]. Available: https://en.wikipedia.org/wiki/Golden_eagle

XI. Appendix

Appendix One: A letter to George R.R. Martin

Dear George:

I write to you on behave of the MCM team 1917768. We believe that our group's study towards dragons can provide you some suggestions about how to maintain the realistic ecological underpinning of the story. In this letter, we will illustrate our idea about dragon's growth pattern. More importantly, we will show you the movement of dragons from acid regions to temperate regions and to arctic regions.

In your book of *A Song of Ice and Fire*, three dragons are raised by Daenerys Targaryen, the "Mother of Dragons. As dragon only exists in our imagination, its performance in your book is also mainly based on your assumptions. We really enjoy the image of dragon you show to the audience, however, there are some details contradict with the realistic ecological underpinning.

First of all, the image of the dragon should follow the pattern of its growth.

According to our common sense, with time passing by, the dragon's weight will enter a steady state where it stays the same. The story shows how the dragons enter this steady state reasonably. The growing speed of the dragon's weigh is rational. However, after the steady state, the weight of dragon should be flowing when accident like tremendous trauma occurs. In this period, when the dragon suffered from long distance migration, tremendous trauma or extra firing, it will consume more energy. At this time, its weight should decease but slowly recover over time. By eating more to promote recovery accordingly, after a certain period of time, the dragon will be back to steady state.

The part of growing before steady state in your book is perfect while the part after steady state needs some modifications. After the dragon grows up, whenever it suffers from tremendous trauma or long distance migration and etc., the appearance of dragons in your book should be different. To overcome from it, its diet or behavior should change too. For example, during a war, the dragon breathes a large amount of fire and suffers from many tremendous traumas. Then its weight should decrease and should be thin and weak in your description. The dragon should seek more food or having more rest to recover from the incident, after which, its appearance will be the same as before.

More importantly, in your story, the dragons' performance doesn't seem to be affected by different climates, while our study shows the opposite.

When dragons migrate from one region to another, there are at least two aspects of environment will be altered, the temperature in the environment and the precipitation in the region. The dragon's performance will be influenced by those changes.

Arid region is a region with scarce precipitation, large evaporation and little flow. Lack of water is a major factor limiting plant growth and animal growth. Temperate region is a region suitable for plant growth, as a result of which, it contain a large quantity of food for animals. The arctic region is covered by floating ice with a very low temperature, which is only suitable for certain creature's survival.

Our group studied the impact of climate to the dragons from two aspects, the temperature of the region and the precipitation in the region.

When dragons migrate from acid regions to temperate regions, the temperature in the environment for dragon decrease to a moderate level and the precipitation in the environment for it raised to a moderate level. In this case, the performance of the dragon will be affected by the long distance and the amelioration in both temperature and precipitation. According to our stimulation, the weight of the dragon will increase. Such increase is mainly due to the improvement for the living environment. Although the energy consumption of a long distance of migration will cause the weight of dragon to decrease, it will soon recover from the influence. What's more, because of the amelioration of the environment, the dragon's weight will overpass its origin level and reach a higher steady state and remain that way. To apply this scientific characteristic in the fiction, you may try to mention the change of the dragon's condition. For example, when a dragon move from a acid regions to a temperate region, due to the long distance of flying, it will first appear a sense of tiring. However, after recovering from it, it will start to grow stronger, its description in this period should be more powerful than the previous, maybe reflected in it volume of its attack performance.

When dragons migrate from acid regions to arctic regions, the temperature in the environment alter from a high level to a low level and the precipitation in the environment stays low. Under this circumstance, the performance of the dragon will be affected by the long distance migration and the changing of temperature. According to our stimulation, the weight of the dragon will change depending on the level of the environment's precipitation. As we known, both the precipitation in acid region and arctic region are both the most suitable level for dragon's growing. Therefore, if the dragon prefer the precipitation in acid regions, the dragon's weight will seem to decrease due to this migration and stay at a relatively lower level of steady state; if the dragon prefer the precipitation in arctic regions, the dragon's weight will seem to increase due to this migration and stay at a relatively higher level of steady state. So such adjustments should appear in the description of the dragon after migration. In some situations, the dragon may need additional food supply outside its living area because of the severe environment.

Above all, we really like your fictions, the dragons in the story provide us a sense of excitement. It would be better if the dragon can acquire more realistic features. We do hope our study can provide guidance about how to maintain the realistic ecological underpinning of the story, especially our advice for the migration situation when it occurs to the movement from arid regions to temperate regions and to arctic regions.

Sincerely yours,
MCM 2019 Team 1917768

Appendix Two: matlab program for growth model before steady state

①A function file named "M.m":

```
function dMdt=M(t,x)
dMdt=4.5*x.*(1-(38*x+39.*x.^(4/3))./(350*x));
```

②A script file named "Mt.m":

```
clear;
clc;
[t,x]=ode45('M',[0,12],10);
plot(t,x,'linewidth',2);
xlabel('time(year)');
ylabel('weight(kg)');
```

Appendix three: matlab program for growth model after steady state

①A function file named "M2.m":

```
function dMdt=M2(t,x)
dMdt=4.5*x.*(1-(38*x+39.*x.^(4/3)+2500*exp(-t+8))./(350*x+2500*exp(-20*t+160)));
```

②A script file named "Mt2.m":

```
clear;
clc;
[t,x]=ode45('M2',[8,16],512);
plot(t,x,'linewidth',2,'color','r');
xlabel('time(year)');
ylabel('weight(kg)');
```

Appendix four: matlab program for growth path

①A function file named "M.m":

```
function dMdt=M(t,x)
dMdt=4.5*x.*(1-(38*x+39.*x.^(4/3))./(350*x));
```

②A function file named "M2.m":

```
function dMdt=M2(t,x)
dMdt=4.5*x.*(1-(38*x+39.*x.^(4/3)+2500*exp(-t+8))./(350*x+2500*exp(-20*t+160)));
```

③A script file named"Mt12.m":

```
clear;
clc;
[t,x]=ode45('M',[0,8],10);
plot(t,x,'linewidth',2,'color','b');
```

```
hold on;

[t,x]=ode45('M2',[8,16],512);
plot(t,x,'linewidth',2,'color','r');
hold on;
xlabel('time(year)');
ylabel('weight(kg)');
```

Appendix five: matlab program for the figure of P_m

A script file named"PTR.m":

```
clear;
clc;
f=@(x,y)((27/35)*exp((10-x)/90)+(3/7)*exp((x-10)/50)+1.8*exp(-y/400))
x=-90:1:70;
y=0:1:500;
[X,Y]=meshgrid(x,y);
z=f(X,Y);
mesh(z)
axis('tight')
set(gca,'xticklabel',-70:20:90)
xlabel('T(°C)')
ylabel('R(mm)')
zlabel('Pm(W/kg)')
```

Appendix six: matlab program for migration simulations

Figure 8

①A function file named "M.m":

```
function dMdt=M(t,x)
dMdt=4.5*x.*(1-(38*x+39.*x.^(4/3))./(350*x));
```

②A function file named "M3.m":

```
function dMdt=M3(t,x)
dMdt=4.5*x.*(1-(2.333*38*x+39.*x.^(4/3)+13200*exp(-(t-8)))./(350*x+13200*exp(-20*(t-8))));
```

③A script file named"Mt13.m":

```
clear;
clc;
[t,x]=ode45('M',[0,8],10);
plot(t,x,'linewidth',2,'color','b');
hold on;
```

```
[t,x]=ode45('M3',[8,20],512);
plot(t,x,'linewidth',2,'color','r');
hold on;
xlabel('time(year)');
ylabel('weight(kg)');
```

Figure 9

①A function file named “M.m”:

```
function dMdt=M(t,x)
dMdt=4.5*x.*(1-(38*x+39.*x.^(4/3))./(350*x));
```

②A function file named “M4.m”:

```
function dMdt=M4(t,x)
dMdt=4.5*x.*(1-(2.261*38*x+39.*x.^(4/3)+13200*exp(-(t-8)))./(350*x+13200*exp(-20*(t-8))));
```

③A script file named“Mt14.m”:

```
clear;
clc;
[t,x]=ode45('M',[0,8],10);
plot(t,x,'linewidth',2,'color','b');
hold on;

[t,x]=ode45('M4',[8,20],512);
plot(t,x,'linewidth',2,'color','r');
hold on;
xlabel('time(year)');
ylabel('weight(kg)');
```