



第5章 时变电磁场和平面电磁波

Time-Varying Fields and Plane EM Waves

1、时谐电磁场的复数表示

复数形式的场方程

复数形式的能量关系

2、平面电磁波在不同媒质中传播特性的分析。

3、电磁波的极化



小 结

• 复振幅、复矢量

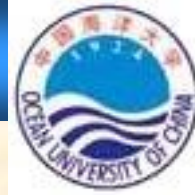
- **复振幅** $\bar{E}(t) = \hat{x}E_x(t) + \hat{y}E_y(t) + \hat{z}E_z(t)$

$$E_x(t) = E_{xm} \cos(\omega t + \phi_x)$$

$$E_x(t) = \text{Re}[(E_{xm} e^{j\phi_x}) e^{j\omega t}] = \text{Re}[\dot{E}_x e^{j\omega t}]$$

$$\dot{E}_x = E_{xm} e^{j\phi_x}$$

$$\dot{E}_x: \text{复振幅或相量}$$



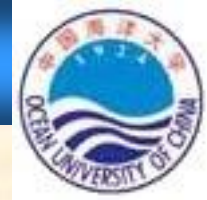
复振幅的求导运算

$$\frac{\partial}{\partial t} E_x(t) = \operatorname{Re} \left[\frac{\partial}{\partial t} (\dot{E}_x e^{j\omega t}) \right] = \operatorname{Re} [j\omega \dot{E}_x e^{j\omega t}]$$

$$\frac{\partial^2}{\partial t^2} E_x(t) = \operatorname{Re} \left[\frac{\partial^2}{\partial t^2} (\dot{E}_x e^{j\omega t}) \right] = \operatorname{Re} [-\omega^2 \dot{E}_x e^{j\omega t}]$$

因此 $\frac{\partial}{\partial t} E_x(t) \leftrightarrow j\omega \dot{E}_x$

$E_x(t)$ 对时间的微分可化为对复振幅 \dot{E}_x 乘以 $j\omega$ 的代数运算



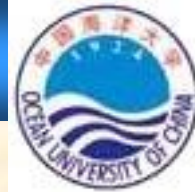
·复矢量 $\dot{\bar{E}}$

$$\begin{aligned}\bar{E}(t) &= \hat{x}E_{xm} \cos(\omega t + \phi_x) + \hat{y}E_{ym} \cos(\omega t + \phi_y) + \hat{z}E_{zm} \cos(\omega t + \phi_z) \\ &= \text{Re} \left[\left(\hat{x}E_{xm} e^{j\phi_x} + \hat{y}E_{ym} e^{j\phi_y} + \hat{z}E_{zm} e^{j\phi_z} \right) e^{j\omega t} \right] \\ &= \text{Re} \left[\left(\hat{x}\dot{E}_x + \hat{y}\dot{E}_y + \hat{z}\dot{E}_z \right) e^{j\omega t} \right] \\ &= \text{Re} \left[\dot{\bar{E}} e^{j\omega t} \right]\end{aligned}$$

$$\bar{E}(t) = \text{Re} \left[\dot{\bar{E}} e^{j\omega t} \right]$$

·复矢量 $\dot{\bar{E}} = \hat{x}E_{xm} e^{j\phi_x} + \hat{y}E_{ym} e^{j\phi_y} + \hat{z}E_{zm} e^{j\phi_z} = \hat{x}\dot{E}_x + \hat{y}\dot{E}_y + \hat{z}\dot{E}_z$

$\dot{\bar{E}}$ 只是 (x, y, z) 的函数, $\bar{E}(t)$ 是 (x, y, z, t) 的函数。从而将4维问题化为3维问题。



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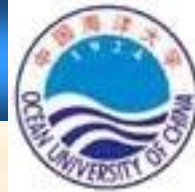
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§ 5.2 复数形式Maxwell方程组

Complex Maxwell's Equations

本节内容

- Maxwell方程组的复数形式
- 本构关系和边界条件的复数形式



$$\nabla \times \bar{E}(x, y, z, t) = - \frac{\partial \bar{B}(x, y, z, t)}{\partial t}$$

一、复数形式Maxwell方程组的导出

$$\nabla \times \bar{E} = - \frac{\partial \bar{B}}{\partial t} \quad \nabla \times \operatorname{Re} [\dot{\bar{E}} e^{j\omega t}] = - \frac{\partial \left[\operatorname{Re} (\dot{\bar{B}} e^{j\omega t}) \right]}{\partial t} = - \operatorname{Re} [j\omega \dot{\bar{B}} e^{j\omega t}]$$

$$\nabla \times \operatorname{Re} [\dot{\bar{E}} e^{j\omega t}] = - \operatorname{Re} [j\omega \dot{\bar{B}} e^{j\omega t}]$$

$$\text{即 } \operatorname{Re} [\nabla \times \dot{\bar{E}} e^{j\omega t}] = \operatorname{Re} [-j\omega \dot{\bar{B}} e^{j\omega t}]$$

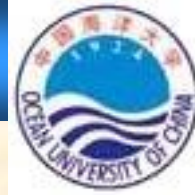
$$\text{故 } \nabla \times \dot{\bar{E}} = -j\omega \dot{\bar{B}} \quad (\text{a})$$

$$\text{同理, } \nabla \times \dot{\bar{H}} = \dot{\bar{J}} + j\omega \dot{\bar{D}} \quad (\text{b})$$

$$\nabla \cdot \dot{\bar{D}} = \dot{\rho}_v \quad (\text{c})$$

$$\nabla \cdot \dot{\bar{B}} = 0 \quad (\text{d})$$

$$\text{电荷连续性方程: } \nabla \cdot \dot{\bar{J}} = -j\omega \dot{\rho}_v \quad (\text{e})$$



二、本构关系和边界条件复数形式

$$\begin{aligned}\dot{\mathbf{D}} &= \varepsilon \dot{\mathbf{E}} \\ \dot{\mathbf{B}} &= \mu \dot{\mathbf{H}} \\ \dot{\mathbf{J}} &= \sigma \dot{\mathbf{E}}\end{aligned}$$

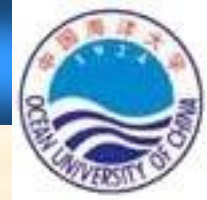
限定形式复Maxwell方程组

$$\nabla \times \dot{\mathbf{E}} = -j\omega\mu\dot{\mathbf{H}} \quad (a)$$

$$\nabla \times \dot{\mathbf{H}} = \dot{\mathbf{J}} + j\omega\varepsilon\dot{\mathbf{E}} \quad (b)$$

$$\nabla \cdot \dot{\mathbf{E}} = \frac{\dot{\rho}_v}{\varepsilon} \quad (c)$$

$$\nabla \cdot \dot{\mathbf{H}} = 0 \quad (d)$$



齐次复矢量波动方程(无源区: $\dot{\mathbf{J}} = 0, \dot{\rho}_v = 0$)

$$\text{由(a), } \nabla \times \nabla \times \dot{\mathbf{E}} = -j\omega\mu \nabla \times \dot{\mathbf{H}}$$

$$\text{将(b)代入, 有 } \nabla(\nabla \cdot \dot{\mathbf{E}}) - \nabla^2 \dot{\mathbf{E}} = \omega^2 \mu \epsilon \dot{\mathbf{E}}$$

将(c)代入, 得

$$\nabla^2 \dot{\mathbf{E}} + k^2 \dot{\mathbf{E}} = 0$$

$$k = \omega \sqrt{\mu \epsilon}$$

同理,

$$\nabla^2 \dot{\mathbf{H}} + k^2 \dot{\mathbf{H}} = 0$$

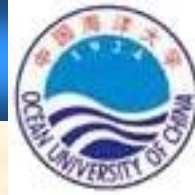
复矢量边界条件

$$\hat{n} \times (\dot{\mathbf{E}}_1 - \dot{\mathbf{E}}_2) = 0$$

$$\hat{n} \times (\dot{\mathbf{H}}_1 - \dot{\mathbf{H}}_2) = \dot{\mathbf{J}}_s$$

$$\hat{n} \cdot (\dot{\mathbf{D}}_1 - \dot{\mathbf{D}}_2) = \dot{\rho}_s$$

$$\hat{n} \cdot (\dot{\mathbf{B}}_1 - \dot{\mathbf{B}}_2) = 0$$



例1 设自由空间某点电磁场的电场强度为

$$\bar{E}(t) = \hat{x}E_0 \sin(\omega t - kz) \quad (V/m)$$

求 (a) 电场强度复矢量 $\dot{\bar{E}}$

(b) 磁场强度 $\bar{H}(t)$

[解] (a) $\dot{\bar{E}} = \hat{x}E_0 e^{-jkz - j\frac{\pi}{2}} = \hat{x}\dot{E}_x$

$$\nabla \times \dot{\bar{E}} = -j\omega \mu \dot{\bar{H}}$$

$$(b) \dot{\bar{H}} = -\frac{1}{j\omega \mu_0} \nabla \times \dot{\bar{E}} = j \frac{1}{\omega \mu_0} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \dot{E}_x & 0 & 0 \end{vmatrix}$$

$$= \hat{y} \frac{j}{\omega \mu_0} \frac{\partial}{\partial z} \dot{E}_x = \hat{y} \frac{k}{\omega \mu_0} E_0 e^{-jkz - j\frac{\pi}{2}}$$

$$\bar{H}(t) = \text{Re}[\dot{\bar{H}} e^{j\omega t}] = \hat{y} \frac{k}{\omega \mu_0} E_0 \cos(\omega t - kz - \frac{\pi}{2})$$

$$= \hat{y} \frac{E_0}{\eta_0} \sin(\omega t - kz)$$

$$\frac{\omega \mu_0}{k} = \frac{\omega \mu_0}{\omega \sqrt{\mu_0 \epsilon_0}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \eta_0$$



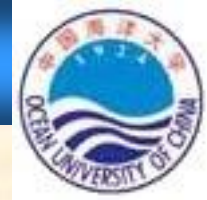
例2 某卫星地面站在空中某点形成频率为5GHz的时谐电磁场，其磁场强度复矢量为

$$\dot{\vec{H}} = \hat{y}0.01e^{-j(100\pi/3)z} \quad (\mu A/m)$$

求：(a) 磁场强度瞬时值 $\vec{H}(t)$
 (b) 电场强度瞬时值 $\vec{E}(t)$

[解] (a) 磁场强度在 \hat{y} 方向, 振幅 $0.01\mu A/m$, 角频率为 $\omega = 2\pi f = 2\pi \times 5.0 \times 10^9 = 10^{10}\pi$

$$\begin{aligned} \vec{H}(t) &= \text{Re}[\dot{\vec{H}}e^{j\omega t}] \\ &= \text{Re}[\hat{y}0.01e^{-j(100\pi/3)z+j10^{10}\pi t}] \\ &= \hat{y}0.01 \cos[(10^{10}\pi t - \frac{100}{3}\pi z)] \quad (\mu A/m) \end{aligned}$$



(b) 在无源区内 $\nabla \times \dot{\vec{H}} = j\omega \epsilon_0 \dot{\vec{E}}$

$$\begin{aligned} \dot{\vec{E}} &= \frac{-j}{\omega \epsilon_0} \nabla \times \dot{\vec{H}} \\ &= \frac{-j}{10^{10} \pi \times \frac{1}{36\pi} \times 10^{-9}} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0.01e^{-j\frac{100\pi}{3}z} & 0 \end{vmatrix} \end{aligned}$$

$$= +j3.6\hat{x}0.01e^{-j(100\pi/3)z} \left(-j\frac{100\pi}{3} \right) = \hat{x}1.2\pi e^{-j(100\pi/3)z}$$

$$\begin{aligned} \bar{E}(t) &= \text{Re}[\dot{\vec{E}}e^{j\omega t}] \\ &= \text{Re}[\hat{x}1.2\pi e^{-j(100\pi/3)z + j\pi 10^{10}t}] \\ &= \hat{x}1.2\pi \cos(10^{10}\pi t - \frac{100}{3}\pi z) \quad (\mu V/m) \end{aligned}$$