Suppose $\boldsymbol{x} \in \mathbb{R}^{D_x}$, and $\boldsymbol{y} \in \mathbb{R}^{D_y}$ is a noisy measurement of $\boldsymbol{x}$. Assume the following prior of $\boldsymbol{x}$ and likelihood $p(\boldsymbol{y}|\boldsymbol{x})$:

$$\boxed{\begin{array}{rcl} p(\boldsymbol{x}) & = & \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu_x}, \boldsymbol{\Sigma_x}) \\ p(\boldsymbol{y}|\boldsymbol{x}) & = & \mathcal{N}(\boldsymbol{y}|\boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}, \boldsymbol{\Sigma_y}) \end{array}}$$

Then the posterior $p(\boldsymbol{x}|\boldsymbol{y})$ can be written as

$$\boxed{\begin{array}{rcl} p(\boldsymbol{x}|\boldsymbol{y}) & = & \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu_{x|y}}, \boldsymbol{\Sigma_{x|y}}) \\ \boldsymbol{\Sigma_{x|y}^{-1}} & = & \boldsymbol{\Sigma_x^{-1}} + \boldsymbol{A^T\Sigma_y^{-1}A} \\ \boldsymbol{\mu_{x|y}} & = & \boldsymbol{\Sigma_{x|y}}\left[\boldsymbol{A^T\Sigma_y^{-1}(y-b)} + \boldsymbol{\Sigma_x^{-1}\mu_x}\right] \end{array}}$$