Latent Semantic Analysis & Topic Models

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Review

- Recall
- Precision
- Mean Average Precision (MAP)
- Normalized Discounting Cumulated Gain (NDCG)

Introduction

- Classic IR might lead to poor retrieval due to:
 - Relevant documents that do not contain at least one index term are not retrieved
 - Synonymy (同義詞) and polysemy (一詞多義) are crucial for IR
 - Car vs. Automobile

The prevalence of synonyms tends to decrease the **recall** performance of retrieval systems

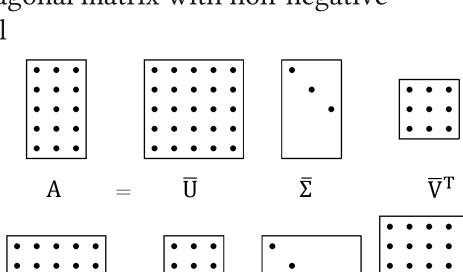
Bank

Polysemy is one factor underlying poor **precision**

- Retrieval based on index terms is vague and noisy
 - The user information need is more related to **concepts** and ideas than to **index terms**

Singular Value Decomposition

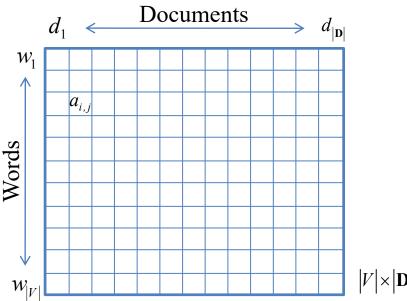
- In linear algebra, the singular-value decomposition (SVD) is a factorization of a real or complex matrix
- Formally, the SVD of an $m \times n$ matrix A is a factorization of the form $\overline{U}\overline{\Sigma}\overline{V}^T$
 - $-\overline{U}$ is an $m \times m$ unitary matrix (i.e., $\overline{U}\overline{U}^T = I = \overline{U}^T\overline{U}$)
 - $\bar{\Sigma}$ is an $m \times n$ rectangular diagonal matrix with non-negative real numbers on the diagonal
 - $-\overline{V}$ is an $n \times n$ unitary matrix



Introduction - LSA

- Latent Semantic Analysis also called
 - Latent Semantic Indexing (LSI)
 - Latent Semantic Mapping (LSM)
 - Two-Mode Factor Analysis
- The LSA paradigm operates under the assumption that there is some underlying **latent semantic structure** in the data
 - Algebraic and/or statistical techniques are brought to bear to estimate this latent structure and get rid of the obscuring "noise"

- A given document collection can be represented as a worddocument matrix
 - Row: composed of **words** (terms), which are the individual components making up a document
 - Column: composed of **documents** which are of a predetermined size of text such as paragraphs, collections of paragraphs, sentences, etc.



- In the word-by-document, each element $a_{i,j}$ is represented the importance of word w_i in document d_i $d_1 \leftarrow \overline{\text{Documents}} \rightarrow d_{|\mathbf{D}|}$
 - The TF-IDF score
 - The Entropy-based method

$$a_{i,j} = (1 - \varepsilon_i) \frac{c(w_i, d_j)}{|d_j|}$$

$$a_{i,j} = (1 - \varepsilon_i) \frac{\varepsilon(w_i, u_j)}{|d_j|}$$

$$\varepsilon_i = -\frac{1}{\log |\mathbf{D}|} \sum_{j=1}^{|\mathbf{D}|} \left(\frac{c(w_i, d_j)}{\sum_{j'=1}^{|\mathbf{D}|} c(w_i, d_{j'})} \log \frac{c(w_i, d_j)}{\sum_{j'=1}^{|\mathbf{D}|} c(w_i, d_{j'})} \right)$$

- $-0 \le \varepsilon_i \le 1$
 - $\varepsilon_i = 1 \Rightarrow \forall d_j$, $c(w_i, d_j) = \frac{\sum_{j'=1}^{|\mathbf{D}|} c(w_i, d_{j'})}{|\mathbf{D}|}$: the word distributed across many documents throughout the corpus
 - $\varepsilon_i = 0 \Rightarrow \exists d_j$, $c(w_i, d_j) \approx \sum_{j'=1}^{|\mathbf{D}|} c(w_i, d_{j'})$: the word is present only in a few specific documents

Latent Semantic Analysis – 2... $a_{i,j} = (1 - \varepsilon_i) \frac{c(w_i, d_j)}{|d_j|}$

$$\varepsilon_{i} = -\frac{1}{\log |\mathbf{D}|} \sum_{j=1}^{|\mathbf{D}|} \left(\frac{c(w_{i}, d_{j})}{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})} \log \frac{c(w_{i}, d_{j})}{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})} \right)$$

• $\varepsilon_i = 1 \Rightarrow \forall d_j$, $c(w_i, d_j) = \frac{\sum_{j'=1}^{|D|} c(w_i, d_{j'})}{|D|}$: the word distributed across many documents throughout the corpus

$$\begin{split} \varepsilon_{i} &= -\frac{1}{\log |\mathbf{D}|} \sum_{j=1}^{|\mathbf{D}|} \left(\frac{c\left(w_{i}, d_{j}\right)}{\sum_{j'=1}^{|\mathbf{D}|} c\left(w_{i}, d_{j'}\right)} \log \frac{c\left(w_{i}, d_{j}\right)}{\sum_{j'=1}^{|\mathbf{D}|} c\left(w_{i}, d_{j'}\right)} \right) \\ &= -\frac{1}{\log |\mathbf{D}|} \sum_{j=1}^{|\mathbf{D}|} \left(\frac{\sum_{j'=1}^{|\mathbf{D}|} c\left(w_{i}, d_{j'}\right)}{|\mathbf{D}|} \log \frac{\sum_{j'=1}^{|\mathbf{D}|} c\left(w_{i}, d_{j'}\right)}{|\mathbf{D}|} \right) \\ &= -\frac{1}{\log |\mathbf{D}|} \sum_{j=1}^{|\mathbf{D}|} \left(\frac{1}{|\mathbf{D}|} \log \frac{1}{|\mathbf{D}|} \right) = -\frac{1}{\log |\mathbf{D}|} \left(\log \frac{1}{|\mathbf{D}|} \right) = -\frac{1}{\log |\mathbf{D}|} \left(-\log |\mathbf{D}| \right) = 1 \end{split}$$

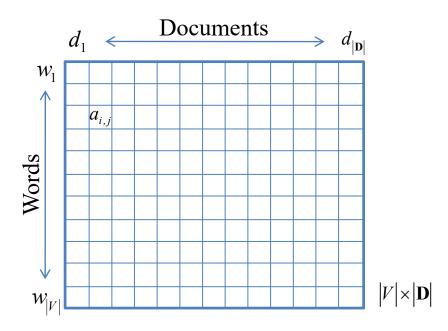
Latent Semantic Analysis – 2... $a_{i,j} = (1 - \varepsilon_i) \frac{c(w_i, d_j)}{|d_j|}$

$$\varepsilon_{i} = -\frac{1}{\log |\mathbf{D}|} \sum_{j=1}^{|\mathbf{D}|} \left(\frac{c(w_{i}, d_{j})}{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})} \log \frac{c(w_{i}, d_{j})}{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})} \right)$$

• $\varepsilon_i = 0 \Rightarrow \exists d_i$, $c(w_i, d_i) \approx \sum_{i'=1}^{|\mathbf{D}|} c(w_i, d_{i'})$: the word is present only in a few specific documents

$$\begin{split} \varepsilon_{i} &= -\frac{1}{\log |\mathbf{D}|} \sum_{j=1}^{|\mathbf{D}|} \left(\frac{c(w_{i}, d_{j})}{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})} \log \frac{c(w_{i}, d_{j})}{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})} \right) \\ &= -\frac{1}{\log |\mathbf{D}|} \times (|\mathbf{D}| - 1) \times \left(\frac{0}{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})} \log \frac{0}{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})} \right) \\ &- \frac{1}{\log |\mathbf{D}|} \times \left(\frac{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})}{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})} \log \frac{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})}{\sum_{j'=1}^{|\mathbf{D}|} c(w_{i}, d_{j'})} \right) \\ &= 0 \end{split}$$

- For the word-by-document matrix, it should be noted that
 - the dimensions and can be extremely large
 - the vectors and are typically very sparse
 - the two spaces (for words and documents) are distinct from one other



- In order to explore the latent semantic space, to project word and document vectors in the space, and to reduce the size of the vectors, the **Singular Value Decomposition** (SVD) can be employed
 - K ≤ min(|V|, $|\mathbf{D}|$): low-rank approximation

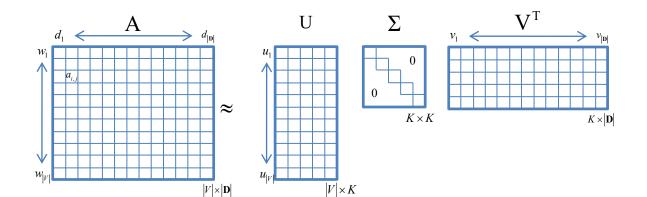
$$\mathbf{A}_{|V|\times|\mathbf{D}|} = \overline{\mathbf{U}}_{|V|\times|V|} \overline{\mathbf{\Sigma}}_{|V|\times|\mathbf{D}|} \overline{\mathbf{V}}_{|\mathbf{D}|\times|\mathbf{D}|}^{\mathrm{T}} \approx \mathbf{U}_{|V|\times K} \mathbf{\Sigma}_{K\times K} \mathbf{V}_{K\times|\mathbf{D}|}^{\mathrm{T}} = \mathbf{A}_{|V|\times|\mathbf{D}|}'$$

- The objective function is



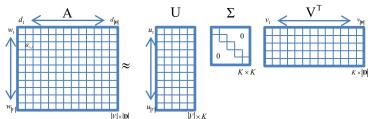


 $min \|A - A'\|_F^2$, for a given K



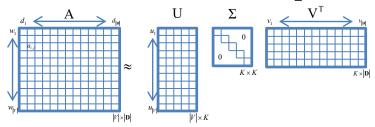
$$\|\mathbf{B}\|_F^2 = \sum_{i} \sum_{j} b_{i,j}^2$$

- Properties of SVD decomposition
 - Both left and right singular matrices and are columnorthonormal
 - $U^TU = V^TV = I$
 - Values (nonnegative real numbers) in diagonal matrix are square roots of the eigenvalues of A^TA
 - $\Sigma^2 = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_K\}$
 - $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_K \ge 0$



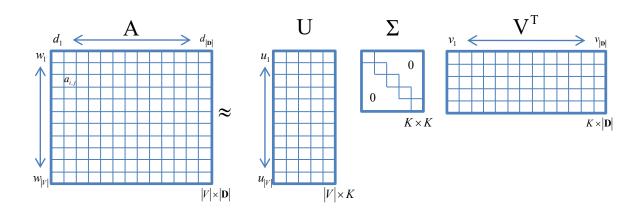
- The column vectors of U define an orthonormal basis for d_j
 - $A \approx U\Sigma V^T \Rightarrow A^TU \approx (U\Sigma V^T)^TU = V\Sigma U^TU = V\Sigma \Rightarrow U^TA = \Sigma V^T$
- The column vectors of V define an orthonormal basis for w_i
 - $A \approx U\Sigma V^T \Rightarrow AV \approx (U\Sigma V^T)V = U\Sigma V^TV = U\Sigma \Rightarrow V^TA^T = \Sigma U^T$

- New representations
 - For each words, the new vector representation is Σu_i^{T}
 - For each document, the new vector representation is Σv_i^{T}



- While the original high-dimensional vectors are sparse, the corresponding low-dimensional latent vectors will typically not be sparse
 - It is possible to compute meaningful association values between pairs of documents, even if the documents do not have any terms in common
 - The hope is that terms having a common meaning (synonyms), are roughly mapped to the same direction in the latent space

- By using the decomposition
 - Compare two words
 - $A \approx U\Sigma V^T \Rightarrow AA^T \approx U\Sigma V^T (U\Sigma V^T)^T = U\Sigma V^T V\Sigma^T U^T = U\Sigma (U\Sigma)^T$
 - Compare two documents
 - $A \approx U\Sigma V^T \Rightarrow A^T A \approx (U\Sigma V^T)^T U\Sigma V^T = V\Sigma^T U^T U\Sigma V^T = V\Sigma (V\Sigma)^T$
 - Compare words and documents
 - $A \approx U \Sigma V^T$



- For a given query (as a document), a low-dimensional representation should be inferred
 - The low-dimensional representation can be obtained by using the **fold-in** strategy $B = U\Sigma V^{T}$

$$(\vec{q}'^{\mathrm{T}})_{1\times K} = (\vec{q}^{\mathrm{T}})_{1\times |V|} \mathbf{U}_{|V|\times K} \Sigma_{K\times K}^{-1}$$

 $\Rightarrow B^{T} = (U\Sigma V^{T})^{T} = V\Sigma^{T}U^{T}$ $\Rightarrow B^{T}U = V\Sigma^{T}$ $\Rightarrow B^{T}U\Sigma^{-1} = V\Sigma\Sigma^{-1} = V$

Weighted sum of the word vectors

Each dimension has its own weight

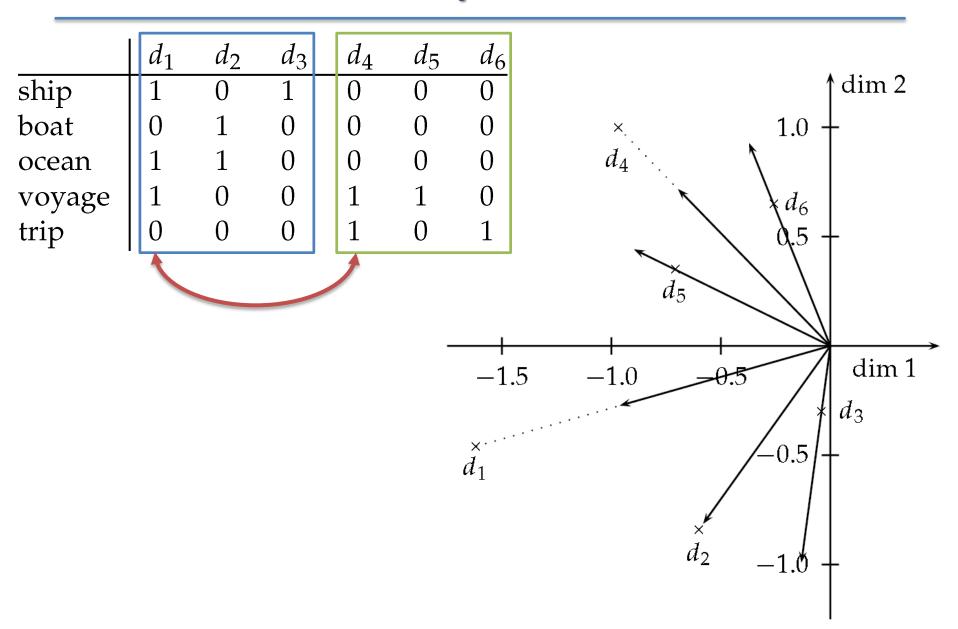
- For a new document, the representation can also be derived by the fold-in strategy
- Consequently, the relevance degree can be computed:

$$sim(q, d_j) = cos(\Sigma \vec{q}', \Sigma \vec{d}'_j) = \frac{\Sigma \vec{q}' \cdot \Sigma \vec{d}'_j}{|\Sigma \vec{q}| |\Sigma \vec{d}'_j|}$$

 $sim(q,d_j) = cos(\vec{q}',\vec{d}'_j)$

Example – 1.

Example – 1...



Example – 2.

c1: Human machine interface for Lab ABC computer applications

c2: A survey of user opinion of computer system response time

c3: The EPS user interface management system

c4: System and human system engineering testing of EPS

c5: Relation of user-perceived response time to error measurement

m1: The generation of random, binary, unordered trees

m2: The intersection graph of paths in trees

m3: Graph minors IV: Widths of trees and well-quasi-ordering

m4: Graph minors: A survey

| İ | Terms | Documents | | | | | | | | |
|-----|------------|-----------|----|----|----|----|----|----|----|----|
| - 1 | | c1 | c2 | c3 | c4 | c5 | m1 | m2 | m3 | m4 |
| 1 | human | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | interface | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | computer | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | user | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 5 | system | 0 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 0 |
| 6 | response | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 7 | time | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 8 | EPS | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 9 | survey | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 10] | trees | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 11 | graph | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 12 | minors | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

Example – 2..

Query="human computer interaction"

c1: Human machine interface for Lab ABC computer applications

c2: A survey of user opinion of computer system response time

c3: The EPS user interface management system

c4: System and human system engineering testing of EPS

c5: Relation of user-perceived response time to error measurement

m1: The generation of random, binary, unordered trees

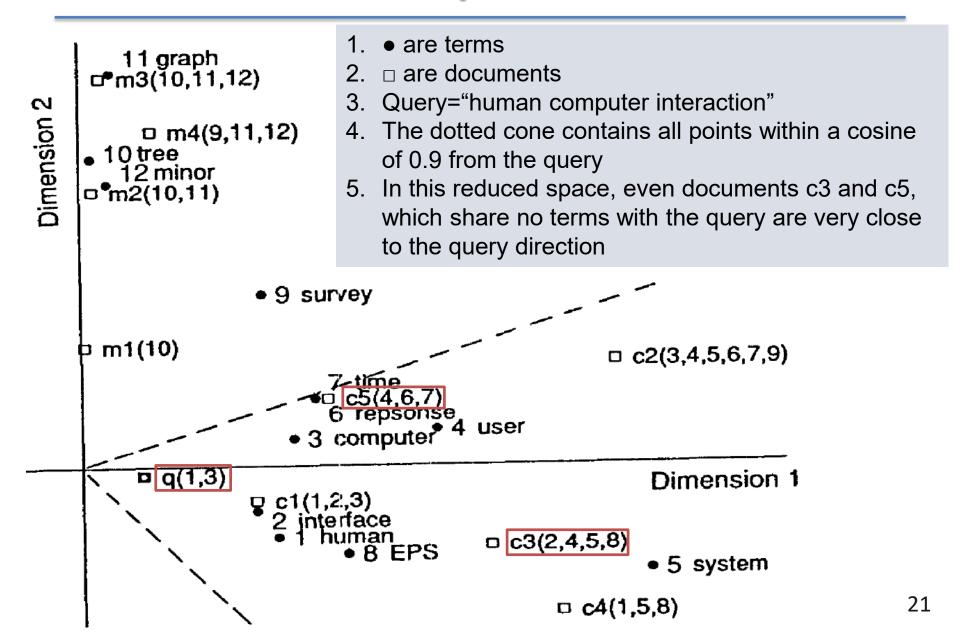
m2: The intersection graph of paths in trees

m3: Graph minors IV: Widths of trees and well-quasi-ordering

m4: Graph minors: A survey

| | Terms | Documents | | | | | | | | |
|-----|-----------|-----------|----|----|----|----|----|----|----|------|
| | | c1 | c2 | с3 | c4 | c5 | m1 | m2 | m3 | m4 { |
| _1_ | human | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | interface | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | computer | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | user | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 5 | system | 0 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 0 |
| 6 | response | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 7 | time | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 8 | EPS | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 9 | survey | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 10 | trees | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 11 | graph | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 12 | minors | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

Example – 2..



Pros and Cons

Advantages

- As we reduce *K*, **recall tends to increase**, as expected
- Most surprisingly, a value of *K* in the low hundreds can actually increase precision on some query benchmarks
- Finding a new space for words and documents

Disadvantages

- The computational cost of the SVD is significant
- Irrelevant or Antonymous
- The reconstruction has negative entities

LSA-based Language Modeling - 1

- A goal of statistical language modeling is to learn the joint probability function of sequences of words in a language
 - − By using *n*-gram model

$$P(w_1, w_2, ..., w_T) \approx \prod_{t=1}^{T} P(w_t | w_{t-n+1}, ..., w_{t-1})$$

By incorporating *n*-gram model and LSA-based model

$$P(w_1, w_2, \dots, w_T) \approx \prod_{t=1}^T P(w_t | H_{t-1}^{n,l}) = \prod_{t=1}^T P(w_t | H_{t-1}^n, H_{t-1}^l)$$
Lexical Semantic Information

LSA-based Language Modeling – 2

• The probability can further be decomposed:

$$P(w_t|H_{t-1}^n, H_{t-1}^l) = \frac{P(w_t, H_{t-1}^l|H_{t-1}^n)}{\sum_{w_i \in V} P(w_i, H_{t-1}^l|H_{t-1}^n)}$$

• Expanding and rearranging, the numerator is seen to be:

$$\begin{split} P \Big(w_t, H_{t-1}^l \big| H_{t-1}^n \Big) &= \frac{P \Big(w_t, H_{t-1}^l, H_{t-1}^n \Big)}{P \big(H_{t-1}^n \big)} \\ &= \frac{P \Big(w_t, H_{t-1}^l, H_{t-1}^n \big) P \big(w_t, H_{t-1}^n \big)}{P \big(H_{t-1}^n \big) P \big(w_t, H_{t-1}^n \big)} \\ &= P \big(w_t \big| H_{t-1}^n \big) P \Big(H_{t-1}^l \big| w_t, H_{t-1}^n \big) \\ &= P \big(w_t \big| w_{t-n+1}, \dots, w_{t-1} \big) P \Big(H_{t-1}^l \big| w_{t-n+1}, \dots, w_{t-1}, w_t \big) \\ &= P \big(w_t \big| w_{t-n+1}, \dots, w_{t-1} \big) P \Big(H_{t-1}^l \big| w_t \Big) \end{split}$$

We assume the probability of the document history given the current word is not affected by other context words

LSA-based Language Modeling – 3

• Consequently, we can obtain:

$$\begin{split} P\big(w_{t}|H_{t-1}^{n,l}\big) &= P\big(w_{t}|H_{t-1}^{n},H_{t-1}^{l}\big) = \frac{P\big(w_{t}|w_{t-n+1},\ldots,w_{t-1}\big)P\big(H_{t-1}^{l}|w_{t}\big)}{\sum_{w_{i}\in V}P\big(w_{i}|w_{t-n+1},\ldots,w_{t-1}\big)P\big(H_{t-1}^{l}|w_{i}\big)} \\ &= \frac{P\big(w_{t}|w_{t-n+1},\ldots,w_{t-1}\big)\frac{P\big(w_{t}|H_{t-1}^{l}\big)}{P\big(w_{t}\big)}}{\sum_{w_{i}\in V}P\big(w_{i}|w_{t-n+1},\ldots,w_{t-1}\big)\frac{P\big(w_{i}|H_{t-1}^{l}\big)}{P\big(w_{i}\big)}} \\ &= \frac{\sum_{w_{i}\in V}P\big(w_{i}|w_{t-n+1},\ldots,w_{t-1}\big)\frac{P\big(w_{i}|H_{t-1}^{l}\big)}{P\big(w_{i}\big)}}{\sum_{w_{i}\in V}P\big(w_{i}|w_{t-n+1},\ldots,w_{t-1}\big)\frac{P\big(w_{i}|H_{t-1}^{l}\big)}{P\big(w_{i}\big)}} \end{split}$$

• H_{t-1}^l can be represented by a vector in the semantic space

$$\left(\overline{H_{t-1}^{l}}^{\mathsf{T}}\right)_{1\times K} = \left(\overline{H_{t-1}^{l}}^{\mathsf{T}}\right)_{1\times |V|} \mathsf{U}_{|V|\times K} \Sigma_{K\times K}^{-1}$$

• Thus, the semantic smoothing factor can be estimated by:

$$P(w_t|H_{t-1}^l) \propto \cos(\Sigma^{\frac{1}{2}} \overline{H_{t-1}^l}', \Sigma^{\frac{1}{2}} u_{w_t}^{\mathrm{T}})$$

LSA-based Language Modeling – Appendix

 By using the entropy-based method to score each element in the vector, a fast strategy can be derived for sequential data

$$\overrightarrow{H_{t}^{l}} = \frac{\left|H_{t}^{l}\right| - 1}{\left|H_{t}^{l}\right|} + \frac{1 - \varepsilon_{w_{t}}}{\left|H_{t}^{l}\right|} \begin{vmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{vmatrix}$$

$$a_{i,j} = (1 - \varepsilon_i) \frac{c(w_i, d_j)}{|d_j|}$$

Statistical Topic Models

From LSA to Probabilistic Topic Models

- Three important claims made for LSA
 - The semantic information can derived from a word-document co-occurrence matrix
 - The dimension reduction is an essential part of its derivation
 - Words and documents can be represented as points in the Euclidean space
- Probabilistic topic models are consistent with the first two claims, but differs in the third one
 - The semantic properties of words and documents are expressed in terms of probabilistic topics

Probabilistic Latent Semantic Analysis

- Probabilistic Latent Semantic Analysis also called
 - Probabilistic Latent Semantic Indexing (PLSI)
 - Aspect Model
- PLSA is a probabilistic counterpart of LSA
 - $P(d_i)$: the probability of selecting document d_i
 - $P(w_i|T_k)$: the probability of word w_i condition on a latent factor/topic T_k
 - Aspect!
 - $P(T_k|d_j)$: the probability of a latent factor/topic T_k generated by document d_j

PLSA – 1

The PLSA model is a latent variable model for co-occurrence data (i.e., each pair of word w_i and document d_i) which associates an unobserved class variable (i.e., latent factor T_k)

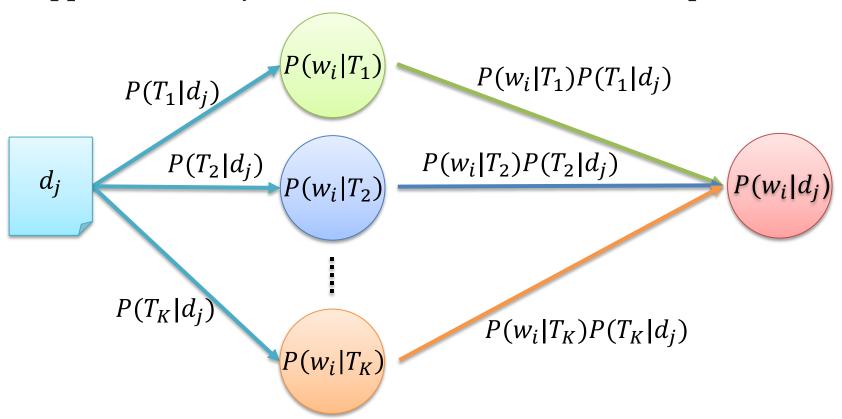
$$P(w_i, d_j) = P(d_j)P(w_i|d_j) = P(d_j)\sum_{k=1}^{K} P(w_i|T_k)P(T_k|d_j)$$

$$P(w_{i}|d_{j}) = \sum_{k=1}^{K} P(w_{i}, T_{k}|d_{j}) = \sum_{k=1}^{K} \frac{P(w_{i}, T_{k}, d_{j})}{P(d_{j})}$$

$$= \sum_{k=1}^{K} \frac{P(w_{i}, d_{j}|T_{k})P(T_{k})}{P(d_{j})}$$

$$= \sum_{k=1}^{K} \frac{P(w_{i}|T_{k})P(d_{j}|T_{k})P(T_{k})}{P(d_{j})}$$
Conditional Independence Assumption document and word are independent conditioned on the state of the associated latent variable
$$= \sum_{k=1}^{K} \frac{P(w_{i}|T_{k})P(d_{j},T_{k})}{P(d_{j})} = \sum_{k=1}^{K} P(w_{i}|T_{k})P(T_{k}|d_{j})$$
30

• Thus, the modeling goal is to identify conditional probability mass functions $P(w_i|T_k)$ such that the document-specific word distributions $P(w_i|d_j)$ are as faithfully as possible approximated by convex combinations of these aspects



- The training objective is defined to maximize the total loglikelihood of a given training collection
 - The model parameters are $P(d_j)$, $P(w_i|T_k)$, and $P(T_k|d_j)$

$$\mathcal{L} = \sum_{w_i \in V} \sum_{d_j \in \mathbf{D}} c(w_i, d_j) log P(w_i, d_j)$$

$$= \sum_{w_i \in V} \sum_{d_j \in \mathbf{D}} c(w_i, d_j) log \left(P(d_j) \sum_{k=1}^K P(w_i | T_k) P(T_k | d_j) \right)$$

- By using the Expectation-Maximization algorithm
 - E-step

$$P(T_k|w_i,d_j) = \frac{P(w_i|T_k)P(T_k|d_j)}{\sum_{k=1}^K P(w_i|T_k)P(T_k|d_j)}$$

- M-step

$$P(w_i|T_k) = \frac{\sum_{d_j \in \mathbf{D}} c(w_i, d_j) P(T_k | w_i, d_j)}{\sum_{i'=1}^{|V|} \sum_{d_j \in \mathbf{D}} c(w_{i'}, d_j) P(T_k | w_{i'}, d_j)}$$

$$P(T_k|d_j) = \frac{\sum_{i=1}^{|V|} c(w_i, d_j) P(T_k|w_i, d_j)}{\sum_{i'=1}^{|V|} c(w_{i'}, d_j)}$$

 Consequently, for a given pair of query and document, the relevance degree can be determined by combining unigram model and PLSA model

$$P(q|d_j) \approx \prod_{\substack{i=1\\|q|}}^{|q|} P(w_i|d_j)$$

$$= \prod_{\substack{i=1\\|q|}}^{|q|} \left(\alpha \cdot P(w_i|d_j) + (1-\alpha) \cdot P_{PLSA}(w_i|d_j)\right)$$

$$= \prod_{\substack{i=1\\|q|}}^{|q|} \left[\alpha \cdot P(w_i|d_j) + (1-\alpha) \cdot \left(\sum_{k=1}^K P(w_i|T_k)P(T_k|d_j)\right)\right]$$

 In order to incorporate the general information, the background model can also be integrated

$$P(q|d_{j}) = \prod_{i=1}^{|q|} \left[\alpha \cdot P(w_{i}|d_{j}) + \beta \cdot \left(\sum_{k=1}^{K} P(w_{i}|T_{k}) P(T_{k}|d_{j}) \right) + (1 - \alpha - \beta) \cdot P_{BG}(w_{i}) \right]_{34}$$

- For a new document d_m , the **fold-in** strategy can be perform to obtain the topic distribution $P(T_k|d_m)$ for the document
 - The word distribution for each topic $P(w_i|T_k)$ is fixed
 - E-step

$$P(T_k|w_i, d_m) = \frac{P(w_i|T_k)P(T_k|d_m)}{\sum_{k=1}^{K} P(w_i|T_k)P(T_k|d_m)}$$

- M-step

$$P(T_k|d_m) = \frac{\sum_{i=1}^{|V|} c(w_i, d_m) P(T_k|w_i, d_m)}{\sum_{i'=1}^{|V|} c(w_{i'}, d_m)}$$

- In addition to the query likelihood measure, we can combine PLSA with the vector-space model
 - Query can be treated as a document, thus the fold-in strategy can be perform to obtain $P(T_k|q)$
 - The topic distributions for document and query are vector representations
 - The similarity degree can be estimated under the semantic space

$$sim(q, d_j) = cos(\vec{q}, \vec{d}_j) = \frac{\sum_{k=1}^K P(T_k | q) P(T_k | d_j)}{\sqrt{\sum_{k=1}^K P(T_k | q)^2} \sqrt{\sum_{k=1}^K P(T_k | d_j)^2}}$$

Link PLSA and LSA

Another derivation of PLSA model

$$P(w_i, d_j) = \sum_{k=1}^{K} P(w_i, d_j, T_k)$$

$$= \sum_{k=1}^{K} P(w_i | d_j, T_k) P(d_j, T_k)$$

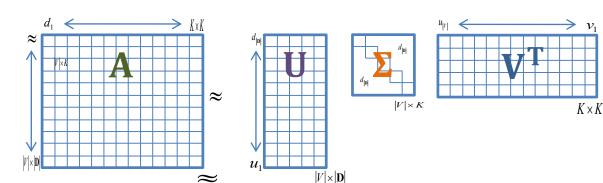
$$= \sum_{k=1}^{K} P(w_i | T_k) P(d_j, T_k)$$

$$= \sum_{k=1}^{K} P(w_i | T_k) P(T_k) P(d_j | T_k)$$

Conditional Independence Assumption

document and word are independent conditioned on the state of the associated latent variable

$$P(T_k)P(d_j|T_k) = P(T_k)\frac{P(d_j,T_k)}{P(T_k)}$$
$$= P(d_j,T_k) = P(T_k|d_j)P(d_j)$$

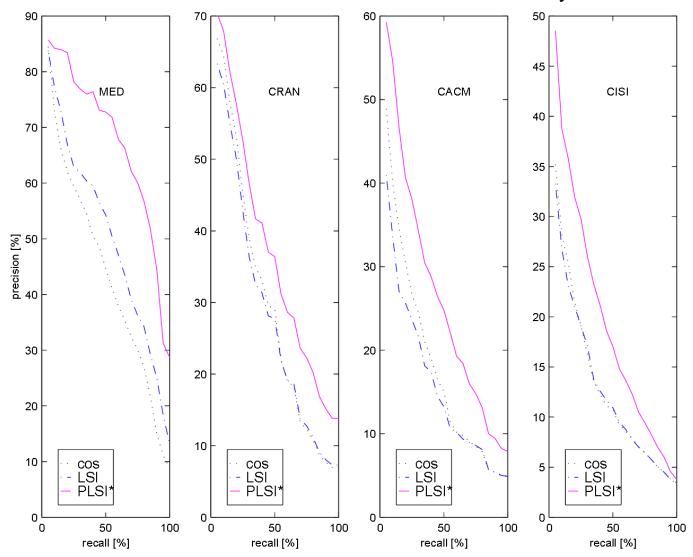


Comparisons – PLSA & LSA

- Decomposition/Approximation
 - LSA: least-squares criterion measured on the L2- or Frobenius norms of the word-by-document matrix
 - PLSA: maximization of the collection likelihood, which implies to minimize the KL-divergence measure
- Computational complexity
 - LSA: SVD decomposition
 - PLSA: EM training
 - The model complexity of Both LSA and PLSA grows linearly with the number of training documents
 - There is no general way to estimate or predict the vector representation (of LSA) or the model parameters (of PLSA) for a newly observed document
 - Fold-in strategy

Comparisons – Experiments

• All of the results are based on cosine similarity measure



Comparisons – Factors/Topics

- Factors from a 128 factor decomposition of the TDT-1 corpus
 - Factors are represented by their 10 most probable words, i.e., the words are ordered according to $P(w_i|T_k)$
- There is no obvious interpretation of the directions in the LSA latent space, while the directions in the PLSA space are interpretable as multinomial word distributions

| "plane" | "space shuttle" | "family" | "Hollywood" | "Bosnia" | "Iraq" | "Rwanda" | "Kobe" |
|------------------------|------------------------|----------|-----------------------|-----------------------|-----------|-------------------------|-----------------------|
| plane | space | home | film | un | iraq | m refugees | building |
| airport | $_{ m shuttle}$ | family | movie | bosnian | iraqi | aid | city |
| crash | mission | like | music | serbs | sanctions | rwanda | people |
| flight | astronauts | love | new | bosnia | kuwait | relief | rescue |
| safety | launch | kids | best | serb | un | people | buildings |
| aircraft | station | mother | hollywood | sarajevo | council | camps | $\mathbf{workers}$ |
| air | crew | life | love | nato | gulf | zaire | kobe |
| passenger | nasa | happy | actor | peacekeepers | saddam | camp | victims |
| board | ${f satellite}$ | friends | entertainment | nations | baghdad | food | area |
| airline | earth | cnn | star | peace | hussein | rwandan | earthquake |

Comparisons – Polysemy

- Many words in natural language are polysemous, having multiple senses; their semantic ambiguity can only be resolved by other words in the context

Topic 77

| word | prob. |
|---------|-------|
| MUSIC | .090 |
| DANCE | .034 |
| SONG | .033 |
| PLAY | .030 |
| SING | .026 |
| SINGING | .026 |
| BAND | .026 |
| PLAYED | .023 |
| SANG | .022 |
| SONGS | .021 |
| DANCING | .020 |
| PIANO | .017 |
| PLAYING | .016 |
| RHYTHM | .015 |
| ALBERT | .013 |
| MUSICAL | .013 |

Topic 82

| word | prob. |
|-------------|-------|
| LITERATURE | .031 |
| POEM | .028 |
| POETRY | .027 |
| POET | .020 |
| PLAYS | .019 |
| POEMS | .019 |
| PLAY | .015 |
| LITERARY | .013 |
| WRITERS | .013 |
| DRAMA | .012 |
| WROTE | .012 |
| POETS | .011 |
| WRITER | .011 |
| SHAKESPEARE | .010 |
| WRITTEN | .009 |
| STAGE | .009 |
| | |

Topic 166

| word | prob. |
|----------|-------|
| PLAY | .136 |
| BALL | .129 |
| | |
| GAME | .065 |
| PLAYING | .042 |
| HIT | .032 |
| PLAYED | .031 |
| BASEBALL | .027 |
| GAMES | .025 |
| BAT | .019 |
| RUN | .019 |
| THROW | .016 |
| BALLS | .015 |
| TENNIS | .011 |
| HOME | .010 |
| CATCH | .010 |
| FIELD | .010 |

Revisiting the Objective Function

$$\mathcal{L} = \sum_{w_i \in V} \sum_{d_j \in \mathbf{D}} c(w_i, d_j) \log P(w_i, d_j) \qquad KL(T||E) = \sum_{x \in \mathbf{X}} T(x) \log \frac{T(x)}{E(x)}$$

$$= \sum_{w_i \in V} \sum_{d_j \in \mathbf{D}} c(w_i, d_j) \log \left(P(d_j) \sum_{k=1}^K P(w_i|T_k) P(T_k|d_j) \right)$$

$$= \sum_{d_j \in \mathbf{D}} \sum_{w_i \in V} c(w_i, d_j) \left[\log P(d_j) + \log \left(\sum_{k=1}^K P(w_i|T_k) P(T_k|d_j) \right) \right]$$

$$= \sum_{d_j \in \mathbf{D}} \sum_{w_i \in V} |d_j| \frac{c(w_i, d_j)}{|d_j|} \left[\log P(d_j) + \log \left(\sum_{k=1}^K P(w_i|T_k) P(T_k|d_j) \right) \right]$$

$$= \sum_{d_j \in \mathbf{D}} |d_j| \sum_{w_i \in V} P(w_i|d_j) [\log P(d_j) + \log P_{PLSA}(w_i|d_j)]$$

$$= \sum_{d_j \in \mathbf{D}} |d_j| \sum_{w_i \in V} \frac{(P(w_i|d_j) \log P(d_j) + P(w_i|d_j) \log P_{PLSA}(w_i|d_j))}{\mathbf{Constant}}$$
KL-Divergence

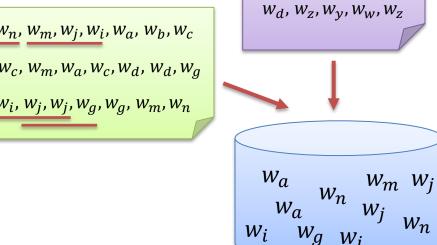
- WTM is a novel extension of the PLSA model
 - The model parameters of PLSA grow linearly with the size of the corpus
 - For WTM, the number of parameters is fixed
 - PLSA explores the co-occurrence relationship between words and documents
 - WTM focuses on the word-word co-occurrence relationship
 - For a new document, fold-in strategy is time consuming
 - For WTM, a linear combination technique can be applied

A given document collection can be represented as a wordby-pseudo-document matrix

Row: composed of words (terms)

Column: composed of pseudo-documents

 $W_i, W_n, W_m, W_i, W_i, W_a, W_b, W_c$ $W_c, W_c, W_m, W_a, W_c, W_d, W_d, W_g$ $W_i, W_i, W_j, W_j, W_g, W_g, W_m, W_n$

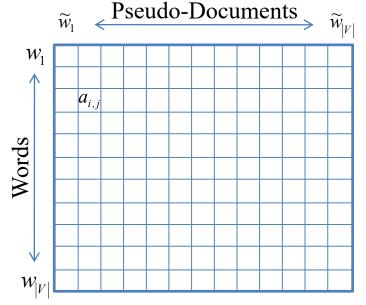


pseudo-document \widetilde{w}_i

 W_h, W_a, W_a, W_j, W_a

 W_a , W_b , W_c , W_c , W_c

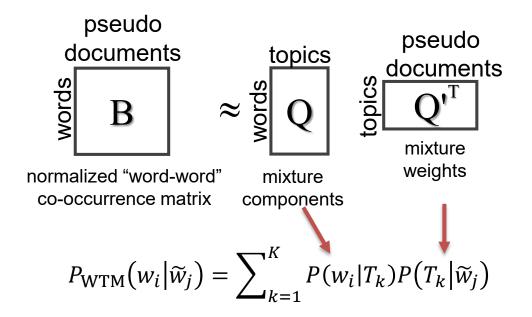
 W_j, W_n, W_m, W_a, W_i



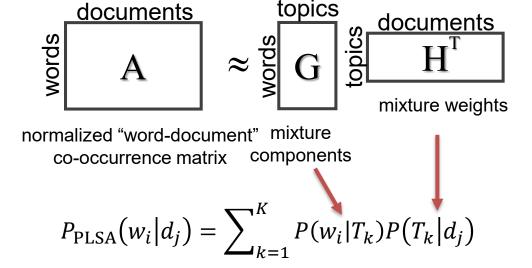
$$|V| \times |V|$$

- The window size is set to 3
- $a_{i,j} = 2$ $a_{m,j} = 1$

WTM



PLSA



 For a new document, we can linearly combine the associated WTM models of the words occurring in the document to form a composite WTM model

$$P_{\text{WTM}}(w_{i}|d_{j}) = \frac{1}{|d_{j}|} \sum_{j'=1}^{|d_{j}|} \sum_{k=1}^{K} P(w_{i}|T_{k}) P(T_{k}|\widetilde{w}_{j'})$$

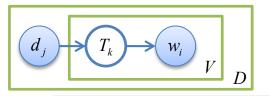
$$= \sum_{k=1}^{K} P(w_{i}|T_{k}) \sum_{j'=1}^{|d_{j}|} \frac{P(T_{k}|\widetilde{w}_{j'})}{|d_{j}|}$$

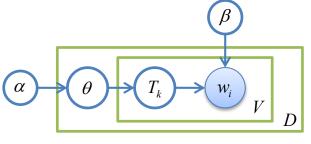
$$= \sum_{k=1}^{K} P(w_{i}|T_{k}) \widehat{P}(T_{k}|d_{j})$$

From PLSA to Latent Dirichelet Allocation

- In traditional topic models, there are several problems:
 - The model parameters grow linearly with the size of the corpus
 - EM is time-consuming
 - It is not clear how to assign probability to a document outside of the training set
 - Fold-in is a compromising strategy
 - Retrain the model is time-consuming







PLSA

 PLSA assumes that the model parameters are fixed and unknown

$$\mathcal{L} = \prod_{w_i \in V} \prod_{d_j \in \mathbf{D}} P(w_i, d_j)^{c(w_i, d_j)} = \prod_{d_j \in \mathbf{D}} \prod_{i=1}^{|d_j|} P(w_i, d_j)$$

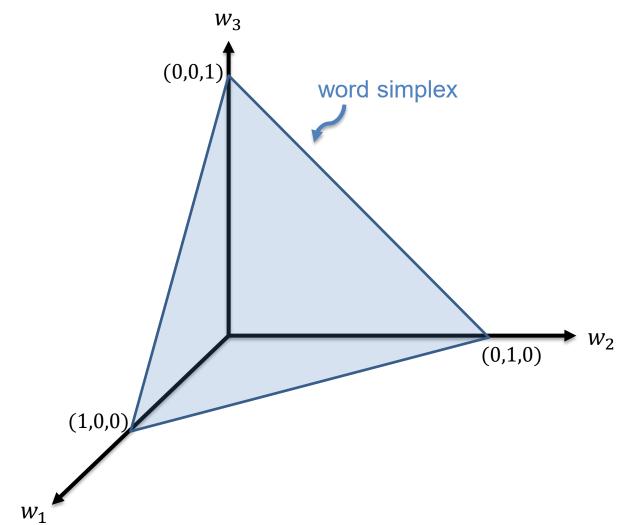
$$= \prod_{d_j \in \mathbf{D}} \prod_{i=1}^{|d_j|} \left(P(d_j) \sum_{k=1}^K P(w_i | T_k) P(T_k | d_j) \right)$$

LDA

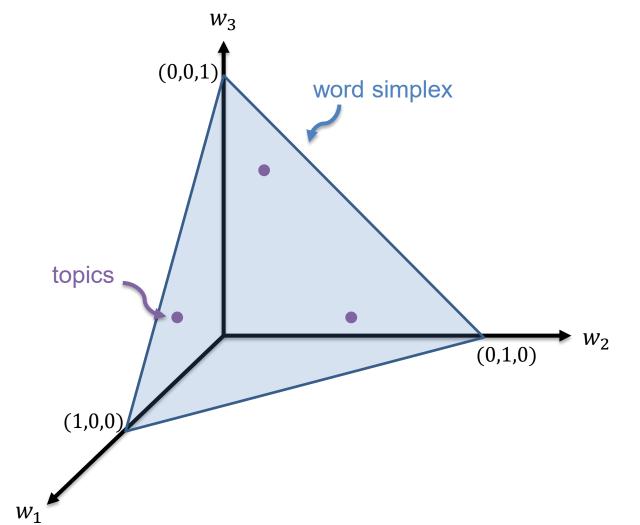
- LDA places a priori constraints on the model parameters
 - Dirichelet distribution

$$\mathcal{L} = \prod_{d_j \in \mathbf{D}} \int P(\theta_{d_j} | \alpha) \left(\prod_{i=1}^{|d_j|} \left(\sum_{k=1}^K P(w_i | T_k, \beta) P\left(T_k \middle| \theta_{d_j} \right) \right) \right) d\theta_{d_j}$$

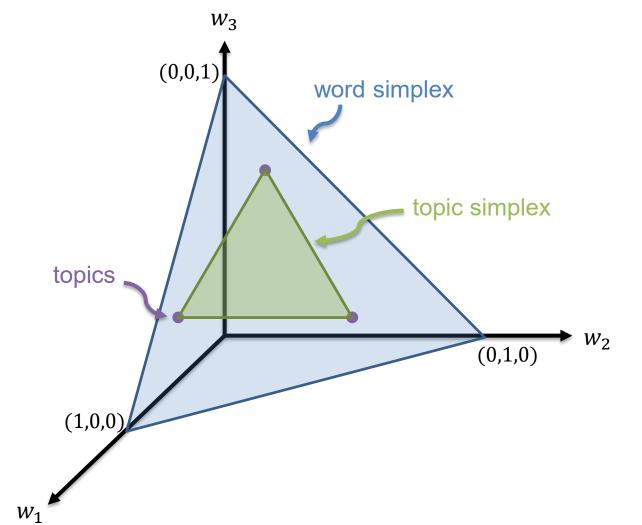
• The topic simplex for three topics embedded in the word simplex for three words



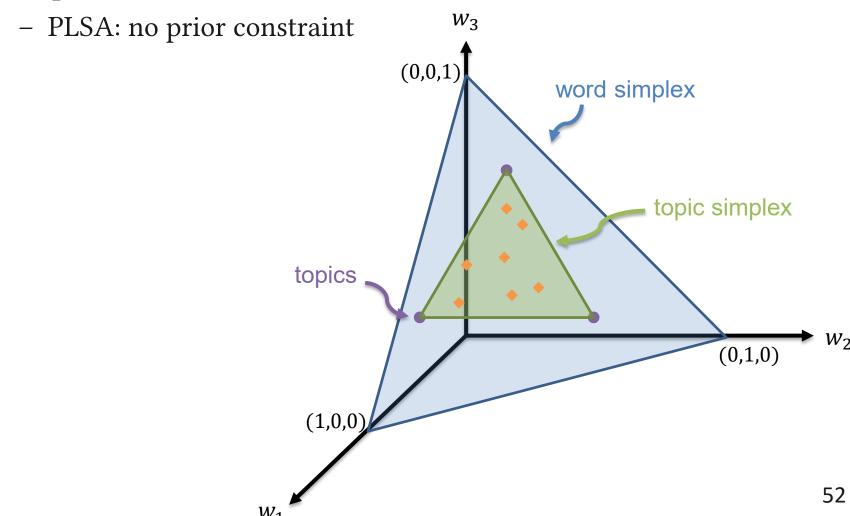
 The topic simplex for three topics embedded in the word simplex for three words



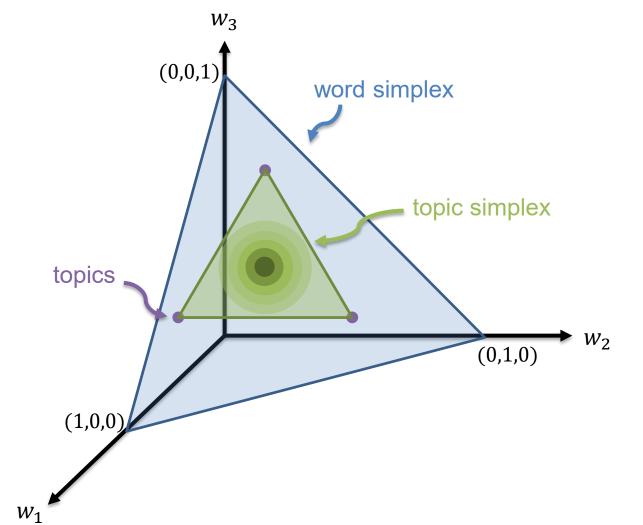
• The topic simplex for three topics embedded in the word simplex for three words



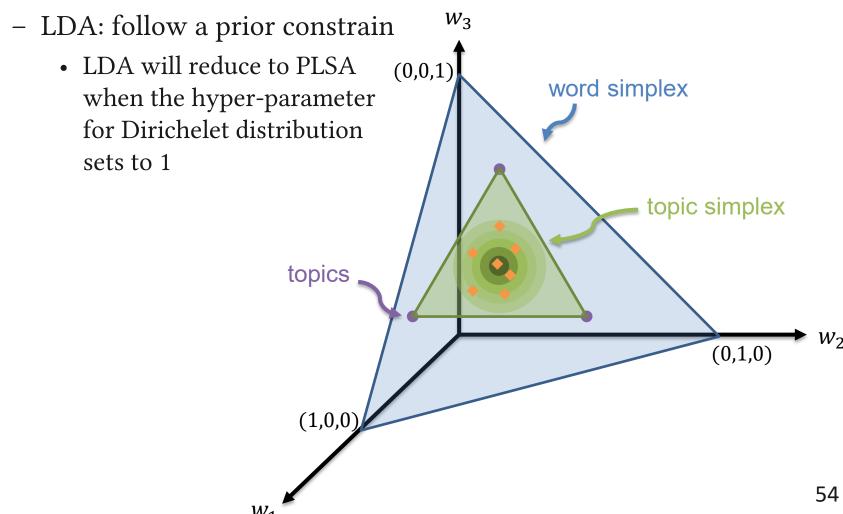
 The topic simplex for three topics embedded in the word simplex for three words

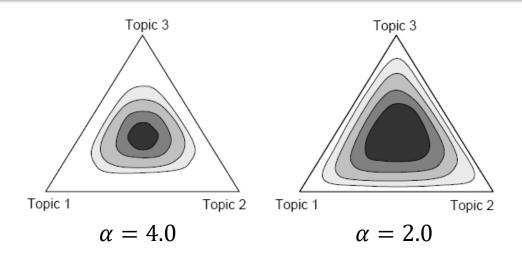


• The topic simplex for three topics embedded in the word simplex for three words



• The topic simplex for three topics embedded in the word simplex for three words





• Dirichlet priors on the topic distributions can be interpreted as forces on the topic combinations with higher α moving the topics away from the corners of the simplex, leading to more smoothing

$$P(\boldsymbol{\theta}|\boldsymbol{\alpha}) = \frac{1}{B(\boldsymbol{\alpha})} \prod_{k=1}^{K} \theta_k^{\alpha_k - 1} = \frac{\Gamma(\sum_{k=1}^{K} \alpha_k)}{\prod_{k=1}^{K} \Gamma(\alpha_k)} \prod_{k=1}^{K} \theta_k^{\alpha_k - 1}$$

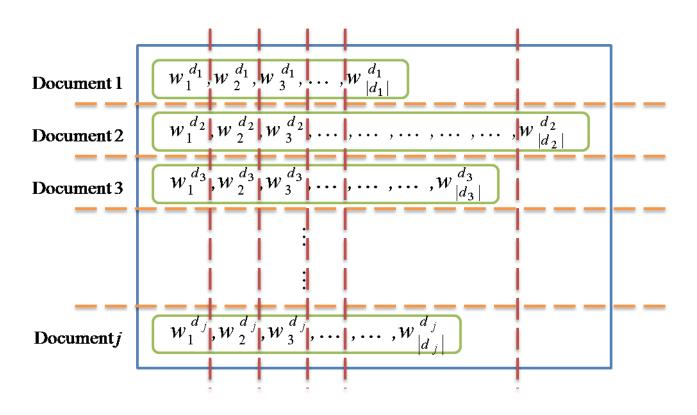
LDA – Experiments

- QL: query likelihood measure
- CBDM: cluster-based model (simplified variant of PLSA)
- LBDM: LDA model

| Collection | QL | CBDM | LBDM | %chg over QL | %chg over CBDM |
|------------|--------|--------|--------|-----------------|----------------------|
| AP | 0.2179 | 0.2326 | 0.2651 | +21.64* | +13.97* |
| FT | 0.2589 | 0.2713 | 0.2807 | +7.54* | +3.46* |
| SJMN | 0.2032 | 0.2171 | 0.2307 | +13.57* | +6.26* |
| LA | 0.2468 | 0.2590 | 0.2666 | $+8.02^{2}$ | +2.93 |
| WSJ | 0.2958 | 0.2984 | 0.3253 | +9.97* | +9.01* |

PLSA, LDA and WTM – 1

- PLSA, LDA and WTM can be analyzed from several perspectives
 - Explore the latent information from different points of view
 - "between words and documents" vs. "between words"



PLSA, LDA and WTM – 2

- Use different methods when calculating the topic mixture weights
 - "time-consuming" vs. "simple combination"
- Model parameters

| Model | PLSA | LDA | WTM |
|-------------------|--|--------------------|-------------------------|
| Relationships | doc-word | doc-word | word-word |
| No. of parameters | $ V \times K + \mathbf{D} \times K$ | $K + V \times K$ | $2 \times V \times K$ |

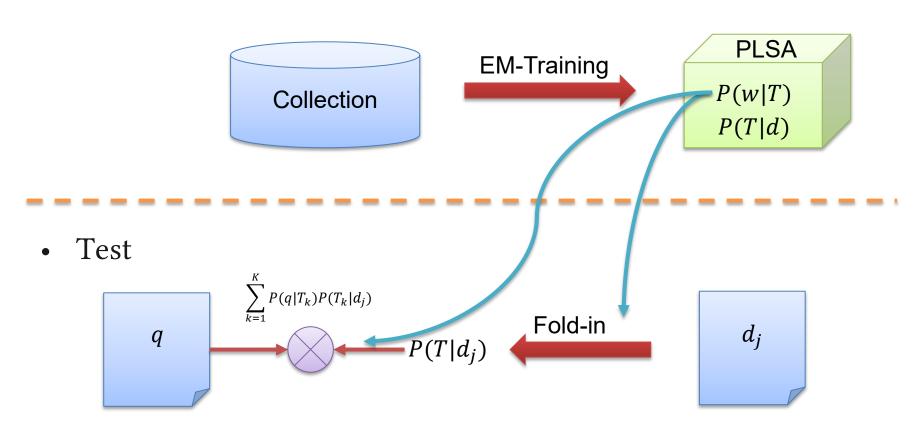
Homework 3 – Description

- In this project, you will have
 - 16 Short Queries
 - 2265 Documents
 - A Background Language Model (BGLM.txt)
 - A Set of Documents for Topic Model Training (Collection.txt)
- Our goal is to implement the PLSA model, and incorporate the PLSA and query likelihood measure for retrieval
 - Thus, the ultimate goal is to enhance the estimation of each document language model

$$P(q|d_j) \approx \prod_{i=1}^{|q|} P'(w_i|d_j)$$
 Obtained by using fold-in strategy
$$P'(w_i|d_j) = \alpha \cdot P(w_i|d_j) + \beta \cdot \sum_{k=1}^K P(w_i|T_k)P(T_k|d_j) + (1 - \alpha - \beta) \cdot P(w_i|BG)$$

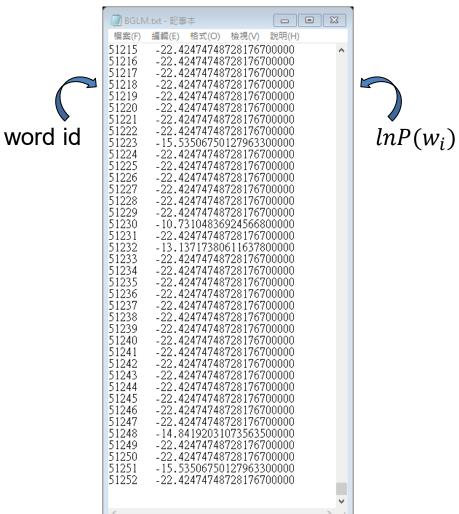
Homework 3 – Flowchart

Training



Homework 3 – BGLM

- The vocabulary size is 51253
 - The word id starts from 0 to 51252



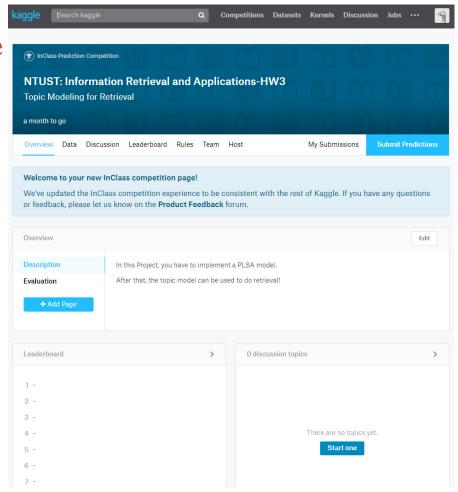
Homework 3 – Collection

- In the collection file, each line refers to a document
 - Thus, we have about 18 thousand documents

Collection.txt 906 596 27799 1027 1985 906 596 27799 1027 1985 25607 1405 1250 2188 25607 3483 2343 26763 25460 29626 24310 3015 26273 2572 29978 38527 4165 8 11164 18329 644 35313 28940 27960 16906 17541 32882 38527 43039 9 2280 3068 457 26763 27631 14645 732 4332 2690 2923 20646 38527 27352 28727 19721 39250 2572 1583 12620 38527 2619 2091 2054 612 25607 1860 11963 50171 3100 1715 612 1407 2986 1715 612 27647 31743 26254 38527 25607 1164 2262 2343 8221 9966 24702 29 25607 2055 3941 4171 18316 855 20529 33697 7546 30066 3015 30802 346 47495 8994 2220 50171 38925 2572 10736 33266 20258 24 27647 353 2016 11408 16761 18964 37448 11708 38527 34616 612 1407 2986 1715 16761 18964 7609 41399 3015 44939 38506 44962 41611 2914 3383 15438 32565 38527 48455 1407 2986 25607 488 2495 1474 612 28 21413 1416 2386 39936 23728 43347 29602 38527 34616 612 1407 2986 21413 1416 3981 2786 1376 1109 25394 39936 30 15912 17639 16665 24022 10054 18586 25197 33229 26088 38527 17639

Homework 3 – Kaggle

- Please login our competition page at Kaggle
 - https://www.kaggle.com/t/3a0b466580294899a666e4c088be1074
- Your team name is ID_Name
 - M123456_陳冠宇

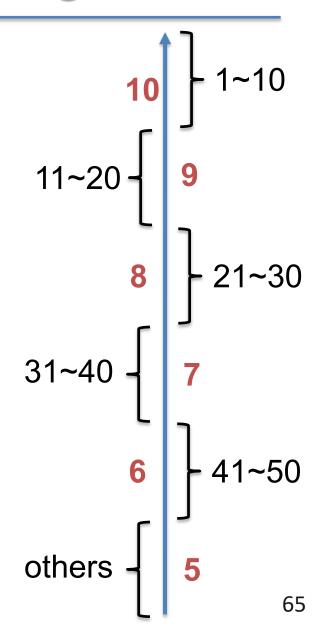


Homework 3 – Submission Format

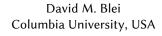
submission.txt 2 20001.query, VOM19980619.0700.0347 VOM19980225.0700.0510 VOM19980317.0900.0192 VOM19980317.0900.0330 VOM1998022 . 00.0173 VOM19980302.0700.0241 VOM19980303.0700.2287 VOM19980530.0730.0166 VOM19980404.0700.2088 VOM19980616.09 216 VOM19980614.0700.0357 VOM19980626.0700.0409 VOM19980403.0700.0489 VOM19980523.0730.0220 VOM19980524.0730.0 . VOM19980624.0900.0077 VOM19980625.0700.0363 VOM19980605.0730.0152 VOM19980602.0730.0102 VOM19980603.0730.0280 . 9980522.0730.0037 VOM19980228.0700.0327 VOM19980414.0900.0260 VOM19980223.0700.0765 VOM19980505.0700.0529 VOM1 503.0730.0136 VOM19980319.0900.3416 VOM19980620.0730.0034 VOM19980302.0700.0209 VOM19980302.0900.2091 VOM19980 0900.0207 VOM19980305.0900.1926 VOM19980521.0730.0029 VOM19980504.0700.0376 VOM19980314.0700.0239 VOM19980619 .0137 VOM19980611.0700.0150 VOM19980326.0700.2112 VOM19980522.0900.0269 VOM19980503.0700.0412 VOM19980428.0900 VOM19980422.0900.0021 VOM19980605.0700.0194 VOM19980611.0700.0046 VOM19980223.0700.2728 VOM19980614.0730.026 . M19980303.0700.0696 VOM19980326.0900.0149 VOM19980505.0700.0481 VOM19980614.0730.0034 VOM19980226.0900.1964 V . 80523.0730.0083 VOM19980316.0700.0356 VOM19980609.0900.0009 VOM19980314.0700.2300 VOM19980302.0700.2137 VOM199 . 4.0700.0458 VOM19980319.0900.2169 VOM19980305.0700.2126 VOM19980515.0700.0472 VOM19980403.0700.0129 VOM199806 . 30.0142 VOM19980618.0700.0234 VOM19980319.0900.0647 VOM19980527.0700.0528 VOM19980607.0730.0033 VOM19980305.09 3 20002.query, VOM19980530.0730.0101 VOM19980611.0900.0216 VOM19980506.0900.0089 VOM19980624.0700.0434 VOM199803 . 00.2021 VOM19980604.0900.0246 VOM19980606.0700.0562 VOM19980303.0900.2085 VOM199802 171 VOM19980220.0900.1979 VOM19980305.0700.0763 VOM19980627.0700.0360 VOM19980225.0700.0302 VOM19980529.0700.0 . VOM19980612.0730.0192 VOM19980319.0700.2737 VOM19980630.0700.0071 VOM19980526.0730.0131 VOM1998

Homework 3 – Scoring

- The evaluation measure is MAP
- You can upload ten results each day
 - Turning the parameters
 - Please **Do Not** register several teams
- The **hard** deadline is 11/9 11:00
- You should also upload source codes and a mini report to moodle
 - TA will ask you to demo your program
 - In this HW, you can only leverage PLSA to do retrieval
 - If you use other models, you will get 0



The Evolution



Thomas Hofmann ETH Zurich, Switzerland



2003 Latent Dirichlet Allocation

1999 Probabilistic Latent Semantic Analysis

1998 Language Modeling Approaches

1994 Best Match Models (Okapi Systems)

1988 Latent Semantic Analysis

1976 Probabilistic Model

1975 Vector Space Model

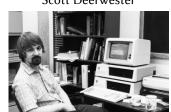
1973 Boolean Model

1972 Inverse Document Frequency

1957 Term Frequency







Questions?



kychen@mail.ntust.edu.tw