Problem 1: Gauss Seidel Method

$$\boldsymbol{A} = \begin{bmatrix} 5 & -1 & 0 & -1 & 0 & 0 \\ -1 & 5 & -1 & 0 & -1 & 0 \\ 0 & -1 & 5 & 0 & 0 & -1 \\ -1 & 0 & 0 & 5 & -1 & 0 \\ 0 & -1 & 0 & -1 & 5 & -1 \\ 0 & 0 & -1 & 0 & -1 & 5 \end{bmatrix} \tag{1}$$

$$\mathbf{b} = \begin{bmatrix} -5 \\ 1 \\ 1 \\ -2 \\ 1 \\ 2 \end{bmatrix} \tag{2}$$

$$\mathbf{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

For the first 10 iterations:
residual: 1.27057
residual: 0.189021
residual: 0.0518507
residual: 0.0175355
residual: 0.00437263
residual: 0.00103081
residual: 0.000240774
residual: 5.61515e-05
residual: 1.30917e-05
residual: 3.0522e-06

The residual is defined as $\|x_{n+1} - x_n\|_2$

$$\mathbf{x} = \begin{bmatrix} -1.09909 \\ 0.0840332 \\ 0.317577 \\ -0.579482 \\ 0.20168 \\ 0.503851 \end{bmatrix} \tag{4}$$