Problem 1

For equation $f(x) = e^{-x} - x = 0$
$f(0) = 1 > 0$ and $f(1) = e^{-1} - 1 < 0$. Also function $f(x)$ is continuous monotonic decreasing function.Therefore there must be a root on the interval $(0, 1)$
For the first 4 iterations. interval [a, b] becomes:

- iter0: [0, 1]
- iter1: [0.5, 1]
- iter2: [0.5, 0.75]
- iter3: [0.5, 0.625]
- iter4: [0.5625, 0.625]

Therefore $p_3 = 0.625$ and $(a_4, b_4) = (0.5625, 0.625)$

Problem 2

For equation $f(x) = x_6 - 3 = 0$
$f(1) = -2 < 0$ and $f(2) = 61 > 0$. Also function $f(x)$ is continuous monotonic increasing function.Therefore there must be a root on the interval $(1, 2)$

Output from bisection code:

- iter0 [1, 2]
- iter1 [1, 1.5] :actual error $|1.5 - \sqrt[6]{3}| = 0.299 < 0.5$
- iter2 [1, 1.25] :actual error $|1.25 - \sqrt[6]{3}| = 0.049 < 0.25$
- iter3 [1.125, 1.25] :actual error $|1.125 - \sqrt[6]{3}| = 0.0759 < 0.125$
- iter4 [1.1875, 1.25] :actual error $|1.1875 - \sqrt[6]{3}| = 0.0134 < 0.0625$
- iter5 [1.1875, 1.21875] :actual error $|1.21875 - \sqrt[6]{3}| = 0.0178 < 0..03125$

We can see that each approximation satisfies the theoretical error, but the actual error does not steadily decrease. Sometimes it is large and sometimes it is small.

Problem 3

For each step the error would become half of the interval. So

$$error_n = \frac{(b-a)}{2^n} \tag{1}$$
$$\epsilon > error_n \tag{2}$$
$$\epsilon > \frac{(b-a)}{2^n} \tag{3}$$
$$n > \log_2 \frac{(b-a)}{\epsilon} \tag{4}$$
$$\tag{5}$$

Therefore n should be the integer bigger than $\log_2 \frac{(b-a)}{\epsilon}$

Problem 4

1. See output of attached code. The result is 1.73205.
2. For the first 5 iterations:
   $|p_n - p_{n-1}|$, $|p_{n-1} - p|$, $|p_n - p|$
   0.277778, 0.232051, 0.045727
   0.0444171, 0.045727, 0.00130986
   0.00130874, 0.00130986, 1.12184e-06
   1.12184e-06, 1.12184e-06, 8.24008e-13
   8.23785e-13, 8.24008e-13, 2.22045e-16
3. The ratios of $|p_n - p|/|p_{n-1} - p|^2$:
   - 0.849193
   - 0.62644
   - 0.653856
   - 0.65474

   Which is approaches to $|f''(p)/2f'(p)| = 0.654701$

Problem 5

The true value is : 2.35134
The estimated value is 2.351

Problem 6

The true value (from Wolfram Alpha) is : 1.45757030926521
The estimated value is 1.45757

Problem 7

1. $f(x) = e^x + x^2 - x - 4$
   $x = 1.28868$

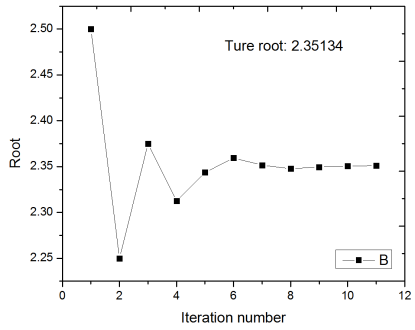

Figure 1: Problem 5. Root vs interation number

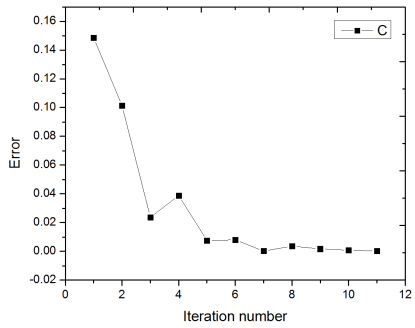

Figure 2: Problem 5. Error vs interation number

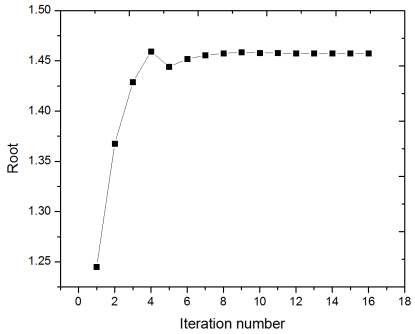

Figure 3: Problem 6. Root vs interation number

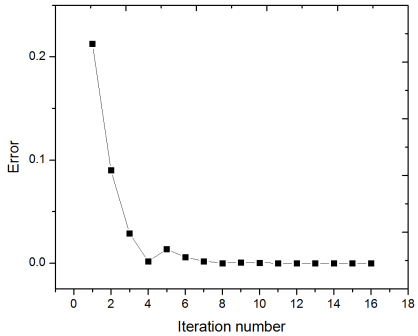

Figure 4: Problem 6. Error vs interation number

2. $f(x) = x^3 - x^2 - 10x + 7$
   $x = 0.68522$

3. $f(x) = 1.05 - 1.04x + ln(x)$
   $x = 1.10971$