$\boxed{\text{Problem \#3}}$

For $y = 0$, $\begin{cases} y' = 0, \\ y^{1/3} = 0 \end{cases} \Rightarrow y = y^{1/3}$

For $y = \left(\frac{2t}{3}\right)^{3/2}$ $\begin{cases} y' = \frac{3}{2} \cdot \frac{2}{3} \left(\frac{2t}{3}\right)^{\frac{1}{2}} = \left(\frac{2t}{3}\right)^{1/2} \\ y^{1/3} = \left(\frac{2t}{3}\right)^{1/2} \end{cases} \Rightarrow y' = y^{1/3}$

(a) If the initical condition $y = 0$ we will have solution $y = 0$

If the initial condition $y = 1 \times 10^{-16}$, we will have solution $y = \left(\frac{2t}{3}\right)^{3/2}$

Problem #2

$$\begin{cases} y_1' = \alpha_1 y_1 - \beta_1 y_2 y_1 \\ y_2' = -\alpha_2 y_2 + \beta_2 y_2 y_1 \end{cases} \quad \text{with} \begin{cases} \alpha_1 = 1 \\ \alpha_2 = 0.5 \end{cases} \begin{cases} \beta_1 = 0.1 \\ \beta_2 = 0.02 \end{cases}$$

$$\Rightarrow \cancel{\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \left(\quad \right)} \qquad y_1(0) = 100$$

$$y_2(0) = 10$$

$$y_1' = y_1 - 0.1\, y_2 y_1$$

$$y_2' = -0.5\, y_2 + 0.02\, y_2 y_1$$

(a) See attachment

(b) see attachment

(c) To find non zero initial population that never change.

$$\begin{cases} 0 = y_1 - 0.1\, y_2 y_1 \\ 0 = -0.5\, y_2 + 0.02\, y_2 y_1 \end{cases}$$

$$\Rightarrow \begin{cases} y_1 = 25 \\ y_2 = 10 \end{cases}$$

Problem #1

(a) $y'' = 16.81\, y(t)$

$y(t) = C_1 e^{4.1t} + C_2 e^{-4.1t}$

$\begin{cases} y(t=0) = 1 \\ y'(t=0) = -4.1 \end{cases} \Rightarrow \begin{cases} C_1 = 0 \\ C_2 = 1 \end{cases}$

$y(t) = e^{-4.1t}$

(b) I anticipate some dificulties. In numerical integration we can not get a corret constant value for $C_1 = 0$. Instead of 0 we may get some very small value.

(c) $y'' = 16.81\, y(t)$

$x_1 = y$

$x_2 = x_1' = y'$

$\Rightarrow x_1' = y' = x_2$

$x_2' = y'' = 16.81\, x_1$

See attachment

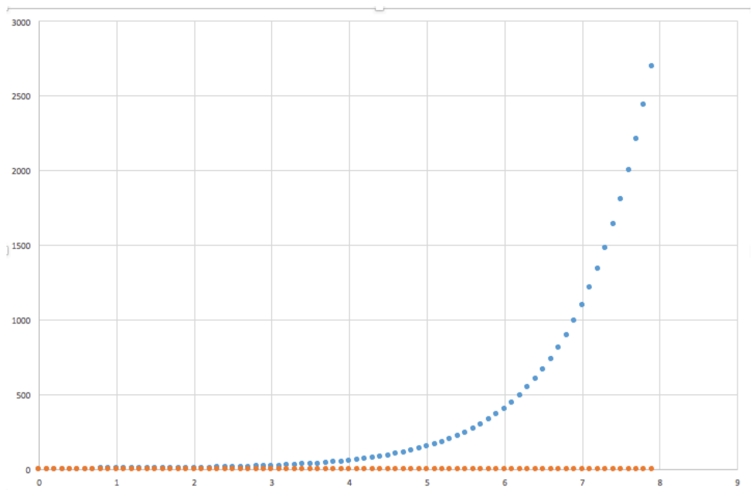

Figure 1: Red indicates the analytical answer and blue curve indicates the numerical answers

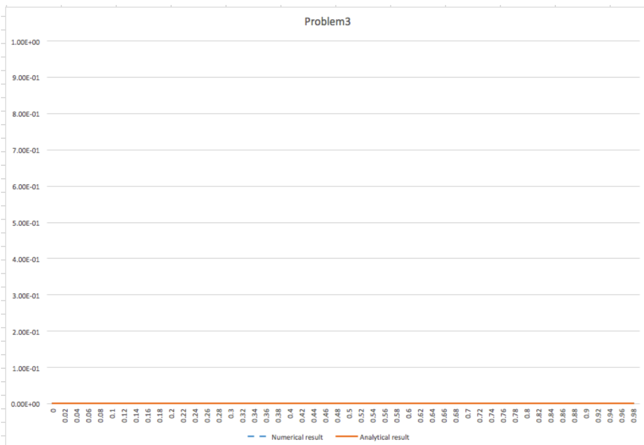

Figure 2: Problem 3 when the initial condition is $y = 0$

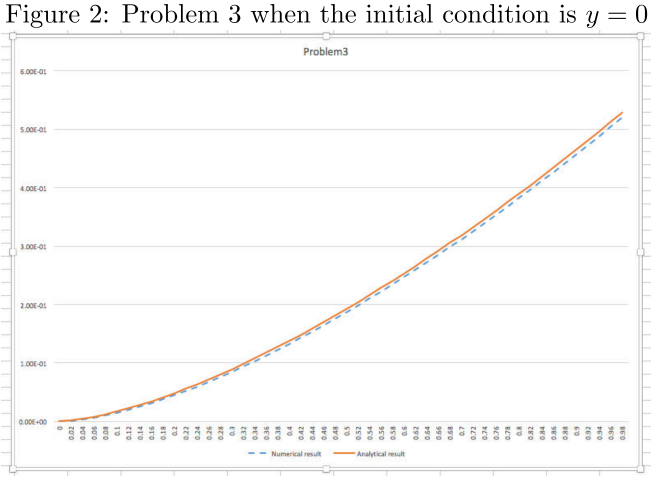

Figure 3: Problem 3 when the initial condition is $y = 1.0 \times 10^{-6}$