Problem 1: Gauss Seidel Method

$$\boldsymbol{A} = \begin{bmatrix} 5 & -1 & 0 & -1 & 0 & 0 \\ -1 & 5 & -1 & 0 & -1 & 0 \\ 0 & -1 & 5 & 0 & 0 & -1 \\ -1 & 0 & 0 & 5 & -1 & 0 \\ 0 & -1 & 0 & -1 & 5 & -1 \\ 0 & 0 & -1 & 0 & -1 & 5 \end{bmatrix} \tag{1}$$

$$\mathbf{b} = \begin{bmatrix} -5 \\ 1 \\ 1 \\ -2 \\ 1 \\ 2 \end{bmatrix} \tag{2}$$

$$\mathbf{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

For the first 10 iterations:
residual: 1.27057
residual: 0.189021
residual: 0.0518507
residual: 0.0175355
residual: 0.00437263
residual: 0.00103081
residual: 0.000240774
residual: 5.61515e-05
residual: 1.30917e-05
residual: 3.0522e-06

The residual is defined as $\|x_{n+1} - x_n\|_2$

$$\mathbf{x} = \begin{bmatrix} -1.09909 \\ 0.0840332 \\ 0.317577 \\ -0.579482 \\ 0.20168 \\ 0.503851 \end{bmatrix} \tag{4}$$

Problem 2: Power Method

1.

$$\boldsymbol{A} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 1 & -1 & 2 \end{bmatrix} \tag{5}$$

When the initial guess is

$$\mathbf{x}_0 = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \tag{6}$$

The results of first 21 iterations are:

```
0  [0.5472,0.7295,0.4104]  Eigenvalue: 3.58835758835759  Convergence: 3.58835758835759
1  [0.591,0.788,0.1724]  Eigenvalue: 3.90661006670709  Convergence: 0.318252478349506
2  [0.5996,0.7994,0.03747]  Eigenvalue: 3.98970198158839  Convergence: 0.083091914881297
3  [0.5997,0.7996,-0.03123]  Eigenvalue: 4.00429268292683  Convergence: 0.0145907013384377
4  [0.5987,0.7983,-0.06548]  Eigenvalue: 4.00449237171945  Convergence: 0.000199688792621444
5  [0.598,0.7973,-0.08253]  Eigenvalue: 4.00282729038406  Convergence: 0.00166508133539001
6  [0.5975,0.7967,-0.09103]  Eigenvalue: 4.00155802744283  Convergence: 0.00126926294122853
7  [0.5973,0.7964,-0.09527]  Eigenvalue: 4.00081498421819  Convergence: 0.000743043224646023
8  [0.5971,0.7962,-0.09739]  Eigenvalue: 4.00041646827872  Convergence: 0.00039851593946949
9  [0.5971,0.7961,-0.09845]  Eigenvalue: 4.00021047607401  Convergence: 0.000205992204711514
10 [0.5971,0.7961,-0.09897]  Eigenvalue: 4.00010579825402  Convergence: 0.000104677819981802
11 [0.597,0.7961,-0.09924]  Eigenvalue: 4.00005303914774  Convergence: 5.27591062837729e-05
12 [0.597,0.796,-0.09937]  Eigenvalue: 4.00002655457485  Convergence: 2.64845728921159e-05
13 [0.597,0.796,-0.09944]  Eigenvalue: 4.00001328603714  Convergence: 1.32685377050024e-05
14 [0.597,0.796,-0.09947]  Eigenvalue: 4.00000664520594  Convergence: 6.64083120671677e-06
15 [0.597,0.796,-0.09949]  Eigenvalue: 4.0000033231498  Convergence: 3.32205613595704e-06
16 [0.597,0.796,-0.0995]  Eigenvalue: 4.00000166171161  Convergence: 1.66143819235742e-06
17 [0.597,0.796,-0.0995]  Eigenvalue: 4.00000083088998  Convergence: 8.30821626607303e-07
18 [0.597,0.796,-0.0995]  Eigenvalue: 4.00000041545353  Convergence: 4.15436447021023e-07
19 [0.597,0.796,-0.0995]  Eigenvalue: 4.0000002077289  Convergence: 2.07724630385542e-07
20 [0.597,0.796,-0.0995]  Eigenvalue: 4.00000010386499  Convergence: 1.0386391746664e-07
```

When the initial guess is

$$\mathbf{x}_0 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \tag{7}$$

The results of first 21 iterations are:

```
0  [0.6114,0.6988,0.3712]  Eigenvalue: 3.69194086790653  Convergence: 3.69194086790653
1  [0.6485,0.7411,0.1737]  Eigenvalue: 3.92356175405659  Convergence: 0.231620886150058
2  [0.6571,0.751,0.06454]  Eigenvalue: 3.9856105201556  Convergence: 0.0620487660990081
3  [0.6585,0.7525,0.008819]  Eigenvalue: 3.99901487197656  Convergence: 0.0134043518209652
4  [0.6584,0.7524,-0.0191]  Eigenvalue: 4.00106691546963  Convergence: 0.00205204349306154
5  [0.6581,0.7522,-0.03305]  Eigenvalue: 4.00092262021984  Convergence: 0.000144295249783433
6  [0.658,0.752,-0.04002]  Eigenvalue: 4.00055841300695  Convergence: 0.000364207212891543
7  [0.6579,0.7519,-0.0435]  Eigenvalue: 4.00030345259712  Convergence: 0.000254960409828087
8  [0.6578,0.7518,-0.04524]  Eigenvalue: 4.00015778373466  Convergence: 0.000145668862463033
9  [0.6578,0.7518,-0.04611]  Eigenvalue: 4.00008040569154  Convergence: 7.73780431222448e-05
10 [0.6578,0.7518,-0.04655]  Eigenvalue: 4.00004058123348  Convergence: 3.9824458060167e-05
11 [0.6578,0.7518,-0.04677]  Eigenvalue: 4.00002038520503  Convergence: 2.01960284478275e-05
12 [0.6578,0.7517,-0.04688]  Eigenvalue: 4.0000102162485  Convergence: 1.01689565275009e-05
13 [0.6578,0.7517,-0.04693]  Eigenvalue: 4.00000511403561  Convergence: 5.10221289040658e-06
14 [0.6578,0.7517,-0.04696]  Eigenvalue: 4.00000255849563  Convergence: 2.5555399822963e-06
15 [0.6578,0.7517,-0.04697]  Eigenvalue: 4.00000127961727  Convergence: 1.27887836143259e-06
16 [0.6578,0.7517,-0.04698]  Eigenvalue: 4.000000639901  Convergence: 6.39716270178781e-07
17 [0.6578,0.7517,-0.04698]  Eigenvalue: 4.00000031997359  Convergence: 3.19927408121146e-07
18 [0.6578,0.7517,-0.04698]  Eigenvalue: 4.00000015999257  Convergence: 1.59981022207489e-07
19 [0.6578,0.7517,-0.04698]  Eigenvalue: 4.00000007999773  Convergence: 7.99948400853623e-08
20 [0.6578,0.7517,-0.04698]  Eigenvalue: 4.00000003999922  Convergence: 3.9998503176264e-08
```

The results is converging using both initial guess.

2.

$$A = \begin{bmatrix} 25 & -5 & 15 & -5 \\ -57 & 25 & -15 & 5 \\ -5 & 5 & 5 & 5 \\ 5 & -5 & 15 & 15 \end{bmatrix} \tag{8}$$

When the initial guess is

$$\mathbf{x}_0 = \begin{bmatrix} 7 \\ 8 \\ 9 \\ 6 \end{bmatrix} \tag{9}$$

The results of first 21 iterations are:

```
0 [0.7047,0.1762,0.2349,0.6459]  Eigenvalue: 22.7586206896552  Convergence: 22.7586206896552
1 [0.7296,0.02516,0.07547,0.6793]  Eigenvalue: 21.6455696202532
Convergence: 1.11305106940201
2 [0.7294,-0.03473,0.01158,0.6831]  Eigenvalue: 20.8310991957105
Convergence: 0.814470424542705
3 [0.7276,-0.06109,-0.01666,0.6831]  Eigenvalue: 20.4133251079581
Convergence: 0.417774087752402
4 [0.7263,-0.07345,-0.02992,0.6828]  Eigenvalue: 20.2057125943466
Convergence: 0.207612513611448
5 [0.7256,-0.07943,-0.03635,0.6825]  Eigenvalue: 20.1025771140672
Convergence: 0.103135480279413
6 [0.7253,-0.08237,-0.03951,0.6824]  Eigenvalue: 20.0512139775591
Convergence: 0.0513631365080656
7 [0.7251,-0.08383,-0.04108,0.6823]  Eigenvalue: 20.02558777141
Convergence: 0.0256262061491732
8 [0.725,-0.08456,-0.04186,0.6823]  Eigenvalue: 20.0127890113521
Convergence: 0.0127987600578301
9 [0.7249,-0.08492,-0.04225,0.6823]  Eigenvalue: 20.0063932784323
Convergence: 0.00639573291977769
10 [0.7249,-0.0851,-0.04245,0.6823]  Eigenvalue: 20.0031963313293
Convergence: 0.00319694710308838
11 [0.7249,-0.08519,-0.04254,0.6822]  Eigenvalue: 20.0015980885588
Convergence: 0.00159824277049125
12 [0.7249,-0.08524,-0.04259,0.6822]  Eigenvalue: 20.0007990249862
Convergence: 0.000799063572586078
13 [0.7249,-0.08526,-0.04262,0.6822]  Eigenvalue: 20.0003995076677
Convergence: 0.000399517318484754
14 [0.7249,-0.08527,-0.04263,0.6822]  Eigenvalue: 20.0001997526272
Convergence: 0.000199755040455329
15 [0.7249,-0.08527,-0.04263,0.6822]  Eigenvalue: 20.0000998760119
Convergence: 9.98766153053054e-05
16 [0.7249,-0.08528,-0.04264,0.6822]  Eigenvalue: 20.0000499379305
Convergence: 4.99380813963057e-05
17 [0.7249,-0.08528,-0.04264,0.6822]  Eigenvalue: 20.0000249689464
Convergence: 2.49689841247402e-05
18 [0.7249,-0.08528,-0.04264,0.6822]  Eigenvalue: 20.0000124844685
Convergence: 1.24844779172406e-05
19 [0.7249,-0.08528,-0.04264,0.6822]  Eigenvalue: 20.0000062422331
Convergence: 6.24223542544655e-06
20 [0.7249,-0.08528,-0.04264,0.6822]  Eigenvalue: 20.0000031211162
Convergence: 3.12111683697935e-06
```

When the initial guess is

$$\mathbf{x}_0 = \begin{bmatrix} 4 \\ 9 \\ 2 \\ 7 \end{bmatrix} \tag{10}$$

The results of first 21 iterations are:

```
0  [0.1983,0.8327,0.2776,0.4362]   Eigenvalue: 20.5660377358491   Convergence: 20.5660377358491
1  [0.1341,0.8622,0.3257,0.364]   Eigenvalue: 20.3964757709251   Convergence: 0.169561964923943
2  [0.1032,0.8724,0.3471,0.3283]   Eigenvalue: 20.2217529039071   Convergence: 0.174722867018037
3  [0.08811,0.8765,0.3571,0.3107]   Eigenvalue: 20.1161290322581   Convergence: 0.105623871649012
4  [0.08068,0.8783,0.3619,0.302]   Eigenvalue: 20.0592998788497   Convergence: 0.0568291534083514
5  [0.07699,0.8791,0.3643,0.2976]   Eigenvalue: 20.0299491340105   Convergence: 0.0293507448392241
6  [0.07516,0.8795,0.3655,0.2955]   Eigenvalue: 20.0150481659801   Convergence: 0.0149009680304069
7  [0.07424,0.8797,0.366,0.2944]   Eigenvalue: 20.0075423331937   Convergence: 0.00750583278642836
8  [0.07378,0.8798,0.3663,0.2938]   Eigenvalue: 20.0037757104826   Convergence: 0.00376662271109396
9  [0.07355,0.8798,0.3665,0.2936]   Eigenvalue: 20.0018889888813   Convergence: 0.00188672160124881
10 [0.07344,0.8799,0.3665,0.2934]   Eigenvalue: 20.0009447775593   Convergence: 0.000944211321961319
11 [0.07338,0.8799,0.3666,0.2934]   Eigenvalue: 20.0004724595229   Convergence: 0.00047231803640102
12 [0.07335,0.8799,0.3666,0.2933]   Eigenvalue: 20.0002362474427   Convergence: 0.000236212080206855
13 [0.07334,0.8799,0.3666,0.2933]   Eigenvalue: 20.0001181281411   Convergence: 0.000118119301625086
14 [0.07333,0.8799,0.3666,0.2933]   Eigenvalue: 20.0000590651754   Convergence: 5.90629656898045e-05
15 [0.07333,0.8799,0.3666,0.2933]   Eigenvalue: 20.0000295328639   Convergence: 2.95323115047097e-05
16 [0.07333,0.8799,0.3666,0.2933]   Eigenvalue: 20.000014766501   Convergence: 1.47663629093131e-05
17 [0.07332,0.8799,0.3666,0.2933]   Eigenvalue: 20.0000073832678   Convergence: 7.38323323901113e-06
18 [0.07332,0.8799,0.3666,0.2933]   Eigenvalue: 20.0000036916382   Convergence: 3.69162957269964e-06
19 [0.07332,0.8799,0.3666,0.2933]   Eigenvalue: 20.0000018458202   Convergence: 1.84581801576655e-06
20 [0.07332,0.8799,0.3666,0.2933]   Eigenvalue: 20.0000009229104   Convergence: 9.22909823231066e-07
```

The results is converging using both initial guess.

Problem 3: Power Method

1.

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 0.5 \\ 1 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & 2.5 \end{bmatrix} \tag{11}$$

When the initial guess is

$$\mathbf{x}_0 = \begin{bmatrix} 4 \\ 9 \\ 2 \\ 7 \end{bmatrix} \tag{12}$$

The results of first 21 iterations are:

```
0  [0.1648,0.3532,-0.07063,0.9182]   Eigenvalue: 2.41130820399113
Convergence: 2.41130820399113
1  [0.1787,0.2429,-0.3987,0.8661]   Eigenvalue: 2.17711045779084
Convergence: 0.234197746200286
2  [0.1977,0.2793,-0.4822,0.8065]   Eigenvalue: 1.93329992603712
Convergence: 0.243810531753723
3  [0.2084,0.3106,-0.481,0.7929]   Eigenvalue: 1.88369147371446   Convergence: 0.0496084523226599
```

```
4 [0.2104,0.3211,-0.4664,0.7969]  Eigenvalue: 1.90310405160878
Convergence: 0.0194125778943175
5 [0.2094,0.3199,-0.4594,0.8017]  Eigenvalue: 1.92327442979667
Convergence: 0.0201703781878948
6 [0.2084,0.3173,-0.4589,0.8033]  Eigenvalue: 1.92916071816977
Convergence: 0.00588628837309368
7 [0.2082,0.3162,-0.4601,0.8031]  Eigenvalue: 1.92802317901948
Convergence: 0.00113753915029013
8 [0.2083,0.3162,-0.4608,0.8027]  Eigenvalue: 1.92632308164457
Convergence: 0.00170009737490284
9 [0.2083,0.3165,-0.4609,0.8025]  Eigenvalue: 1.92572469180958
Convergence: 0.000598389834998381
10 [0.2084,0.3166,-0.4608,0.8025]  Eigenvalue: 1.92576820408011
Convergence: 4.35122705355617e-05
11 [0.2084,0.3166,-0.4607,0.8026]  Eigenvalue: 1.92590654837988
Convergence: 0.000138344299764892
12 [0.2084,0.3166,-0.4607,0.8026]  Eigenvalue: 1.92596491265444
Convergence: 5.83642745597412e-05
13 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92596601612052
Convergence: 1.10346608672707e-06
14 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92595511127207
Convergence: 1.09048484520091e-05
15 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92594962272772
Convergence: 5.48854435256096e-06
16 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.9259491062206
Convergence: 5.1650711685447e-07
17 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92594993215116
Convergence: 8.25930560610644e-07
18 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92595043270496
Convergence: 5.00553793303382e-07
19 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92595051284897
Convergence: 8.01440145359322e-08
20 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92595045349808
Convergence: 5.93508875379456e-08
```

When the initial guess is

$$\mathbf{x}_0 = \begin{bmatrix} 3 \\ 5 \\ 2 \\ 1 \end{bmatrix} \tag{13}$$

The results of first 21 iterations are:

```
[\tiny\arabic{VerbboxLineNo}\small\hspace{3ex}]
0 [0.07454,0.5217,0.5217,0.6708]  Eigenvalue: 1.46111111111111
Convergence: 1.46111111111111
1 [0.145,0.1772,-0.2095,0.9506]  Eigenvalue: 2.50051921079958  Convergence: 1.03940809968847
2 [0.1814,0.2367,-0.4765,0.827]  Eigenvalue: 2.01000037808613  Convergence: 0.490518832713456
3 [0.2051,0.2951,-0.4979,0.7892]  Eigenvalue: 1.86471050679047
Convergence: 0.145289871295662
4 [0.2115,0.3214,-0.4763,0.7906]  Eigenvalue: 1.87649374009219
Convergence: 0.0117832333017227
5 [0.2106,0.3233,-0.4606,0.7993]  Eigenvalue: 1.91401340973697
Convergence: 0.0375196696447804
6 [0.2088,0.3188,-0.4575,0.8034]  Eigenvalue: 1.93000163879722
Convergence: 0.0159882290602549
```

```
7 [0.2081,0.3163,-0.4592,0.8036]  Eigenvalue: 1.93024861702547
Convergence: 0.000246978228242822
8 [0.2082,0.316,-0.4606,0.8029]  Eigenvalue: 1.92722964069017  Convergence: 0.00301897633529835
9 [0.2083,0.3163,-0.461,0.8025]  Eigenvalue: 1.92572794507759  Convergence: 0.00150169561257751
10 [0.2084,0.3165,-0.4609,0.8025]  Eigenvalue: 1.92559210996346
Convergence: 0.000135835114126959
11 [0.2084,0.3166,-0.4607,0.8025]  Eigenvalue: 1.92582074753743
Convergence: 0.000228637573965518
12 [0.2084,0.3166,-0.4607,0.8026]  Eigenvalue: 1.92595794413598
Convergence: 0.000137196598550382
13 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92597946332219
Convergence: 2.15191862100728e-05
14 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92596297824214
Convergence: 1.64850800483674e-05
15 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92595079093439
Convergence: 1.21873077487678e-05
16 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92594812925657
Convergence: 2.6616778183719e-06
17 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92594923085227
Convergence: 1.1015956911109e-06
18 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92595028510293
Convergence: 1.0542506609923e-06
19 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92595057892723
Convergence: 2.93824303332357e-07
20 [0.2084,0.3165,-0.4607,0.8026]  Eigenvalue: 1.92595051437343
Convergence: 6.45538023036352e-08
```

The results is converging using both initial guess.

2.

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} \tag{14}$$

When the initial guess is

$$\mathbf{x}_0 = \begin{bmatrix} 3 \\ 5 \\ 2 \\ 1 \end{bmatrix} \tag{15}$$

The results of first 21 iterations are:

```
[0.8078,0.5655,0.1616,0.04039]  Eigenvalue: 1.67781402936378  Convergence: 1.67781402936378
1 [0.8996,0.4261,0.0947,0.01184]  Eigenvalue: 1.50882723833543
Convergence: 0.16898679102835
2 [0.937,0.3436,0.06247,0.003904]  Eigenvalue: 1.40193747046539
Convergence: 0.106889767870046
3 [0.9563,0.2891,0.04448,0.00139]  Eigenvalue: 1.33184804678218
Convergence: 0.0700894236832097
4 [0.9676,0.2502,0.03337,0.0005213]  Eigenvalue: 1.28277184402293
Convergence: 0.0490762027592542
5 [0.9749,0.221,0.026,0.0002031]  Eigenvalue: 1.24653596109163
Convergence: 0.0362358829312992
6 [0.98,0.1981,0.02085,8.145e-05]  Eigenvalue: 1.21867187872359
Convergence: 0.0278640823680367
```

7 [0.9836,0.1796,0.01711,3.341e-05] Eigenvalue: 1.19656181341414
Convergence: 0.0221100653094486
8 [0.9863,0.1644,0.01429,1.396e-05] Eigenvalue: 1.17857690133378
Convergence: 0.0179849120803597
9 [0.9884,0.1516,0.01213,5.921e-06] Eigenvalue: 1.16365173907519
Convergence: 0.0149251622585884
10 [0.99,0.1407,0.01042,2.544e-06] Eigenvalue: 1.15106019058575
Convergence: 0.0125915484894468
11 [0.9913,0.1313,0.009053,1.105e-06] Eigenvalue: 1.14029012826214
Convergence: 0.0107700623236096
12 [0.9924,0.1231,0.007939,4.846e-07] Eigenvalue: 1.13096982588814
Convergence: 0.00932030237399961
13 [0.9932,0.1158,0.007019,2.142e-07] Eigenvalue: 1.12282279323
Convergence: 0.00814703265813899
14 [0.994,0.1094,0.006251,9.539e-08] Eigenvalue: 1.11563901400056
Convergence: 0.00718377922943669
15 [0.9946,0.1037,0.005603,4.275e-08] Eigenvalue: 1.1092560322768
Convergence: 0.00638298172376039
16 [0.9951,0.0985,0.005051,1.927e-08] Eigenvalue: 1.10354616262974
Convergence: 0.00570986964706544
17 [0.9956,0.09384,0.004577,8.731e-09] Eigenvalue: 1.09840762663035
Convergence: 0.00513853599938585
18 [0.996,0.0896,0.004167,3.974e-09] Eigenvalue: 1.09375827591746
Convergence: 0.00464935071288952
19 [0.9963,0.08572,0.00381,1.817e-09] Eigenvalue: 1.08953106061156
Convergence: 0.00422721530590087
20 [0.9966,0.08218,0.003497,8.337e-10] Eigenvalue: 1.08567070095044
Convergence: 0.00386035966112308

The results is converging, but not so fast.

$$\mathbf{x}_0 = \begin{bmatrix} 6 \\ 9 \\ 1 \\ 4 \end{bmatrix} \tag{16}$$

The results of first 21 iterations are:

0 [0.8421,0.5263,0.05263,0.1053] Eigenvalue: 1.50969529085873
Convergence: 1.50969529085873
1 [0.925,0.3769,0.03426,0.03426] Eigenvalue: 1.39260563380282
Convergence: 0.117089657055909
2 [0.9554,0.294,0.0245,0.01225] Eigenvalue: 1.31140285071268 Convergence: 0.0812027830901387
3 [0.97,0.2425,0.01865,0.004663] Eigenvalue: 1.25781233010764
Convergence: 0.0535905206050344
4 [0.9781,0.2075,0.01482,0.001853] Eigenvalue: 1.22051147419464
Convergence: 0.0373008559129997
5 [0.9832,0.1821,0.01214,0.0007586] Eigenvalue: 1.19316300166967
Convergence: 0.0273484725249777
6 [0.9866,0.1627,0.01017,0.0003179] Eigenvalue: 1.17225319092724
Convergence: 0.0209098107424217
7 [0.989,0.1475,0.008676,0.0001356] Eigenvalue: 1.15572782176051
Convergence: 0.0165253691667298
8 [0.9908,0.1351,0.007506,5.864e-05] Eigenvalue: 1.1423178748547
Convergence: 0.0134099469058173

```
9 [0.9922,0.1248,0.006571,2.567e-05]  Eigenvalue: 1.13120062126532
Convergence: 0.0111172535893758
10 [0.9932,0.1162,0.005808,1.134e-05]  Eigenvalue: 1.12182038990716
Convergence: 0.00938023135816501
11 [0.9941,0.1087,0.005177,5.056e-06]  Eigenvalue: 1.11378866668672
Convergence: 0.0080317232204401
12 [0.9947,0.1023,0.004648,2.27e-06]  Eigenvalue: 1.10682569520657
Convergence: 0.00696297148014557
13 [0.9953,0.09659,0.0042,1.025e-06]  Eigenvalue: 1.10072488050872
Convergence: 0.00610081469784651
14 [0.9958,0.09157,0.003815,4.657e-07]  Eigenvalue: 1.09533028617996
Convergence: 0.00539459432876099
15 [0.9962,0.08708,0.003483,2.126e-07]  Eigenvalue: 1.09052194802356
Convergence: 0.00480833815640436
16 [0.9965,0.08305,0.003194,9.747e-08]  Eigenvalue: 1.08620601707797
Convergence: 0.00431593094559179
17 [0.9968,0.07939,0.002941,4.487e-08]  Eigenvalue: 1.08230797831406
Convergence: 0.00389803876390316
18 [0.9971,0.07607,0.002717,2.073e-08]  Eigenvalue: 1.07876788165995
Convergence: 0.00354009665411481
19 [0.9973,0.07304,0.002519,9.607e-09]  Eigenvalue: 1.07553692169126
Convergence: 0.00323095996869371
20 [0.9975,0.07025,0.002342,4.466e-09]  Eigenvalue: 1.07257494091909
Convergence: 0.00296198077217058
```

The results is converging, but not so fast.

Problem 4: Power Method

1.

$$A = \begin{bmatrix} 0 & 3 & 0 & 3 & 0 \\ 3 & 0 & 3 & 0 & 3 \\ 0 & 3 & 0 & 3 & 0 \\ 3 & 0 & 3 & 0 & 3 \\ 0 & 3 & 0 & 3 & 0 \end{bmatrix} \tag{17}$$

The actual eigenvalues are : $[-7.3485, 0, 0, 73485]$. When the initial guess is

$$\mathbf{x}_0 = \begin{bmatrix} 6 \\ 9 \\ 1 \\ 4 \\ 5 \end{bmatrix} \tag{18}$$

The results of first 21 iterations are:

```
0 [0.4611,0.4256,0.4611,0.4256,0.4611]  Eigenvalue: 7.06415094339623
Convergence: 7.06415094339623
1 [0.3475,0.5647,0.3475,0.5647,0.3475]  Eigenvalue: 7.06415094339623
Convergence: 0
2 [0.4611,0.4256,0.4611,0.4256,0.4611]  Eigenvalue: 7.06415094339623
Convergence: 0
3 [0.3475,0.5647,0.3475,0.5647,0.3475]  Eigenvalue: 7.06415094339623
Convergence: 0
4 [0.4611,0.4256,0.4611,0.4256,0.4611]  Eigenvalue: 7.06415094339623
Convergence: 0
```

```
5 [0.3475,0.5647,0.3475,0.5647,0.3475]  Eigenvalue: 7.06415094339623
Convergence: 0
6 [0.4611,0.4256,0.4611,0.4256,0.4611]  Eigenvalue: 7.06415094339623
Convergence: 0
7 [0.3475,0.5647,0.3475,0.5647,0.3475]  Eigenvalue: 7.06415094339623
Convergence: 0
8 [0.4611,0.4256,0.4611,0.4256,0.4611]  Eigenvalue: 7.06415094339623
Convergence: 0
9 [0.3475,0.5647,0.3475,0.5647,0.3475]  Eigenvalue: 7.06415094339623
Convergence: 0
10 [0.4611,0.4256,0.4611,0.4256,0.4611]  Eigenvalue: 7.06415094339623
Convergence: 0
11 [0.3475,0.5647,0.3475,0.5647,0.3475]  Eigenvalue: 7.06415094339623
Convergence: 0
12 [0.4611,0.4256,0.4611,0.4256,0.4611]  Eigenvalue: 7.06415094339623
Convergence: 0
13 [0.3475,0.5647,0.3475,0.5647,0.3475]  Eigenvalue: 7.06415094339623
Convergence: 0
14 [0.4611,0.4256,0.4611,0.4256,0.4611]  Eigenvalue: 7.06415094339623
Convergence: 0
15 [0.3475,0.5647,0.3475,0.5647,0.3475]  Eigenvalue: 7.06415094339623
Convergence: 0
16 [0.4611,0.4256,0.4611,0.4256,0.4611]  Eigenvalue: 7.06415094339623
Convergence: 0
17 [0.3475,0.5647,0.3475,0.5647,0.3475]  Eigenvalue: 7.06415094339623
Convergence: 0
18 [0.4611,0.4256,0.4611,0.4256,0.4611]  Eigenvalue: 7.06415094339623
Convergence: 0
19 [0.3475,0.5647,0.3475,0.5647,0.3475]  Eigenvalue: 7.06415094339623
Convergence: 0
20 [0.4611,0.4256,0.4611,0.4256,0.4611]  Eigenvalue: 7.06415094339623
Convergence: 0
```

When the initial guess is

$$\mathbf{x}_0 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 9 \\ 6 \end{bmatrix} \tag{19}$$

The results of first 21 iterations are:

```
0 [0.4636,0.4214,0.4636,0.4214,0.4636]  Eigenvalue: 7.03374777975133
Convergence: 7.03374777975133
1 [0.3441,0.5678,0.3441,0.5678,0.3441]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
2 [0.4636,0.4214,0.4636,0.4214,0.4636]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
3 [0.3441,0.5678,0.3441,0.5678,0.3441]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
4 [0.4636,0.4214,0.4636,0.4214,0.4636]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
5 [0.3441,0.5678,0.3441,0.5678,0.3441]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
6 [0.4636,0.4214,0.4636,0.4214,0.4636]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
```

```
7 [0.3441,0.5678,0.3441,0.5678,0.3441]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
8 [0.4636,0.4214,0.4636,0.4214,0.4636]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
9 [0.3441,0.5678,0.3441,0.5678,0.3441]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
10 [0.4636,0.4214,0.4636,0.4214,0.4636]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
11 [0.3441,0.5678,0.3441,0.5678,0.3441]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
12 [0.4636,0.4214,0.4636,0.4214,0.4636]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
13 [0.3441,0.5678,0.3441,0.5678,0.3441]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
14 [0.4636,0.4214,0.4636,0.4214,0.4636]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
15 [0.3441,0.5678,0.3441,0.5678,0.3441]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
16 [0.4636,0.4214,0.4636,0.4214,0.4636]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
17 [0.3441,0.5678,0.3441,0.5678,0.3441]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
18 [0.4636,0.4214,0.4636,0.4214,0.4636]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
19 [0.3441,0.5678,0.3441,0.5678,0.3441]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
20 [0.4636,0.4214,0.4636,0.4214,0.4636]  Eigenvalue: 7.03374777975133
Convergence: 2.66453525910038e-15
```

The results does not converge using both initial guess.

2.

$$\boldsymbol{A} = \begin{bmatrix} 0 & 3 & 0 & 3 & 0 \\ 3 & 0 & 3 & 0 & 3 \\ 0 & 3 & 0 & 3 & 0 \\ 3 & 0 & 3 & 0 & 3 \\ 0 & 3 & 0 & 3 & 0 \end{bmatrix} \tag{20}$$

The actual eigenvalues are : $[1, 2 + 2.45i, 2 - 2.45i]$. When the initial guess
is

$$\mathbf{x}_0 = \begin{bmatrix} 15 \\ 1 \\ 5 \end{bmatrix} \tag{21}$$

The results of first 21 iterations are:

```
0 [0.6349,-0.6984,0.3302]  Eigenvalue: 5.19593613933237  Convergence: 5.19593613933237
1 [0.6272,-0.7575,0.1809]  Eigenvalue: 2.6052403373278  Convergence: 2.59069580200457
2 [0.6874,-0.7241,0.05591]  Eigenvalue: 0.428518861718814  Convergence: 2.17672147560899
3 [0.7759,-0.1323,-0.6169]  Eigenvalue: -4.04686794760419  Convergence: 4.47538680932301
4 [-0.5271,0.8042,-0.2748]  Eigenvalue: 4.27542038170068  Convergence: 8.32228832930487
5 [-0.6321,0.7637,-0.1311]  Eigenvalue: 1.73336702097138  Convergence: 2.5420533607293
6 [-0.7354,0.6749,0.06072]  Eigenvalue: -1.45721860053175  Convergence: 3.19058562150312
7 [0.3099,-0.8091,0.4994]  Eigenvalue: 7.79583478293618  Convergence: 9.25305338346792
8 [0.5868,-0.7851,0.1983]  Eigenvalue: 2.92373843349758  Convergence: 4.87209634943859
9 [0.6697,-0.7394,0.06973]  Eigenvalue: 0.666411793654987  Convergence: 2.2573266398426
```

```
10 [0.8145,-0.3581,-0.4564]  Eigenvalue: -5.09246548154954  Convergence: 5.75887727520452
11 [-0.5112,0.807,-0.2957]  Eigenvalue: 4.63895721535941  Convergence: 9.73142269690895
12 [-0.6253,0.7673,-0.142]  Eigenvalue: 1.92592112782745  Convergence: 2.71303608753196
13 [-0.7212,0.6921,0.02905]  Eigenvalue: -0.965796036715803  Convergence: 2.89171716454326
14 [0.2025,-0.7863,0.5838]  Eigenvalue: 8.69370644312446  Convergence: 9.65950247984026
15 [0.5776,-0.7886,0.211]  Eigenvalue: 3.14891923028427  Convergence: 5.54478721284018
16 [0.6616,-0.7452,0.08354]  Eigenvalue: 0.903813723757681  Convergence: 2.24510550652659
17 [0.8102,-0.4926,-0.3176]  Eigenvalue: -4.50211235064026  Convergence: 5.40592607439794
18 [-0.4898,0.8107,-0.3209]  Eigenvalue: 5.0705958102215  Convergence: 9.57270816086176
19 [-0.6181,0.771,-0.1529]  Eigenvalue: 2.11801117916707  Convergence: 2.95258463105443
20 [-0.7086,0.7056,0.003037]  Eigenvalue: -0.549322585739546  Convergence: 2.66733376490661
```

When the initial guess is

$$\mathbf{x}_0 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \tag{22}$$

The results of first 21 iterations are:

```
0 [0.6086,-0.6694,0.426]  Eigenvalue: 6.8  Convergence: 6.8
1 [0.6088,-0.7654,0.2087]  Eigenvalue: 3.09954614220878  Convergence: 3.70045385779122
2 [0.6672,-0.7395,0.08896]  Eigenvalue: 0.995022414592672  Convergence: 2.1045237276161
3 [0.8079,-0.5251,-0.2676]  Eigenvalue: -4.05483747609943  Convergence: 5.0498598906921
4 [-0.4778,0.8131,-0.3326]  Eigenvalue: 5.26978422235133  Convergence: 9.32462169845076
5 [-0.615,0.7727,-0.1572]  Eigenvalue: 2.19370990124548  Convergence: 3.07607432110585
6 [-0.704,0.7101,-0.005898]  Eigenvalue: -0.403816508158983  Convergence: 2.59752640940446
7 [-0.06386,-0.6729,0.737]  Eigenvalue: 8.73485432351693  Convergence: 9.13867083167591
8 [0.5633,-0.7935,0.2302]  Eigenvalue: 3.48985458763121  Convergence: 5.24499973588571
9 [0.6511,-0.7522,0.1011]  Eigenvalue: 1.20883302243031  Convergence: 2.2810215652009
10 [0.783,-0.5921,-0.1909]  Eigenvalue: -3.24504405350103  Convergence: 4.45387707593135
11 [-0.4502,0.815,-0.3648]  Eigenvalue: 5.80544454471056  Convergence: 9.05048859821159
12 [-0.6079,0.776,-0.1681]  Eigenvalue: 2.3876989724285  Convergence: 3.41774557228206
13 [-0.6934,0.7201,-0.02665]  Eigenvalue: -0.0615311075496359  Convergence: 2.44923007997814
14 [-0.3702,-0.4451,0.8154]  Eigenvalue: 5.18789466016809  Convergence: 5.24942576771772
15 [0.5515,-0.7972,0.2457]  Eigenvalue: 3.76263206496699  Convergence: 1.4252625952011
16 [0.6438,-0.7568,0.113]  Eigenvalue: 1.41552593496256  Convergence: 2.34710613000443
17 [0.7629,-0.6334,-0.1296]  Eigenvalue: -2.45474541366359  Convergence: 3.87027134862615
18 [-0.4105,0.8165,-0.4059]  Eigenvalue: 6.4641923188112  Convergence: 8.91893773247479
19 [-0.6002,0.7795,-0.1794]  Eigenvalue: 2.58712425680872  Convergence: 3.87706806200248
20 [-0.6837,0.7284,-0.04471]  Eigenvalue: 0.241114528468301  Convergence: 2.34600972834042
```

The results does not converge using both initial guess.