HOMEWORK

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Problem 0: Homework checklist

✓I didn't talk with any one about this homework.
✓Source-code are included at the end of this document.

Problem 1

1. $\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
2. $\begin{bmatrix} 5 & 0 \\ 2 & 0 \end{bmatrix}$
3. $\begin{bmatrix} 5 & -5 \\ 2 & -2 \\ 0 & 0 \end{bmatrix}$
4. $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

Problem 2:

1. For example, The matrix

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \tag{1}$$

has two different decomposition. But in both cases they have the same singular value.

2.

3. Suppose $\boldsymbol{A}$ is a matrix with eigenvalues $\{\sigma_i\}$
The SVD of $\boldsymbol{A}$ is $\boldsymbol{A} = \boldsymbol{U}\Sigma\boldsymbol{V}^T$ where $\boldsymbol{U}$ and $\boldsymbol{V}$ are both orthogonal matrix.

$$\begin{aligned} \boldsymbol{A}^{-1} &= (\boldsymbol{U}\Sigma\boldsymbol{V}^T)^{-1} & (2)\\ &= (\boldsymbol{V}^T)^{-1}\Sigma^{-1}\boldsymbol{U}^{-1} & (3)\\ &= (\boldsymbol{V}^T)^T\Sigma^{-1}\boldsymbol{U}^T & (4)\\ &= \boldsymbol{V}\Sigma^{-1}\boldsymbol{U}^T & (5) \end{aligned}$$

Then $\boldsymbol{A}^{-1}$ is a matrix with eigenvalues $\{\frac{1}{\sigma_i}\}$

$$\begin{aligned} \|\boldsymbol{A}^{-1}\| &= max\{\frac{1}{\sigma_i}\} & (6)\\ &= \frac{1}{\sigma_{min}} & (7)\\ & & (8) \end{aligned}$$

4. The SVD of $\boldsymbol{Q}$ is $\boldsymbol{Q} = \boldsymbol{U}\Sigma\boldsymbol{V}^T = \boldsymbol{Q}\boldsymbol{I}\boldsymbol{I}^T$ where $\boldsymbol{I}$ is an identity matrix and it is also an orthogonal matrix.
In particular, the singular value are all 1.

Problem 3:

1. If $m < n$, we can use $\boldsymbol{A}^T = (\boldsymbol{U}\Sigma\boldsymbol{V}^T)^T = \boldsymbol{V}^T\Sigma\boldsymbol{U}^T$