PURDUE UNIVERSITY · CS 51500
MATRIX COMPUTATIONS

HOMEWORK
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Problem 0: Homework checklist

✓I didn't talk with any one about this homework.
✓Source-code are included at the end of this document.

Problem 1: Operations

1. $\begin{bmatrix}1 & 1 & 2\\ 3 & 5 & 8\\ 13 & 21 & 34\end{bmatrix}\begin{bmatrix}1 & -2 & 3\\ -4 & 5 & -6\\ 7 & -8 & 9\end{bmatrix} = \begin{bmatrix}11 & -13 & 15\\ 39 & -45 & 51\\ 167 & -193 & 219\end{bmatrix}$

2. $\mathbf{x} =$ `ones(1000,1)` $\mathbf{y} =$ `[1:1000]'` $\mathbf{x}^T\mathbf{y} = 500500.0$

3. Assume $\mathbf{e} = \begin{bmatrix}1\\0\\0\end{bmatrix}$

   $\mathbf{x} = \begin{bmatrix}2 & 4 & -1\end{bmatrix}^T$.
   $\mathbf{e}\mathbf{x}^T = \begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}2 & 4 & -1\end{bmatrix} = \begin{bmatrix}2 & 4 & -1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$
   $\mathbf{x}\mathbf{e}^T = \begin{bmatrix}2\\4\\-1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\end{bmatrix} = \begin{bmatrix}2 & 0 & 0\\ 4 & 0 & 0\\ -1 & 0 & 0\end{bmatrix}$

4. $\mathbf{x} = \begin{bmatrix}1 & -18 & 3\end{bmatrix}^T$.
   Assume $\mathbf{e}_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$
   $\mathbf{e}_1\mathbf{x}^T = \begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1 & -18 & 3\end{bmatrix} = \begin{bmatrix}1 & -18 & 3\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$
   Assume $\mathbf{e}_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$
   $\mathbf{x}\mathbf{e}_3^T = \begin{bmatrix}1\\-18\\3\end{bmatrix}\begin{bmatrix}0 & 0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 1 & -18 & 3\end{bmatrix}$

```
% Problem 1
% Part 1
A = [1,1,2;  3,5,8; 13,21,34]
B = [1,-2,3; -4,5,-6;7,-8,9]
C= A*B

% Part 2
x = ones(1000,1)
y = [1:1000]'
z = x'*y

% Part 3
x= [2, 4, -1]'
```

```
e = [1;0;0]
y = e*x'
y2 = x*e'

% Part4
x= [1, -18, 3]'
e1 = [1; 0;  0]
y1= e1*x'
e3 = [0;0;1]
y2 = x*e3'
```

Problem 2: A proof

1. Proof:

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

   Therefore the inverse of $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ is $\begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix}$

2. Proof:

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{A} \\ 0 & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{A} \\ 0 & \boldsymbol{I} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & 0 \\ 0 & \boldsymbol{I} \end{bmatrix}.$$

   Therefore the inverse of $\begin{bmatrix} \boldsymbol{I} & \boldsymbol{A} \\ 0 & \boldsymbol{I} \end{bmatrix}$ is $\begin{bmatrix} \boldsymbol{I} & -\boldsymbol{A} \\ 0 & \boldsymbol{I} \end{bmatrix}$

3. Let's start from a matrix

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{A}^{-1}\boldsymbol{B} \\ 0 & \boldsymbol{I} \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{A}^{-1}\boldsymbol{B} \\ 0 & \boldsymbol{I} \end{bmatrix}^{-1} = \left[\begin{bmatrix} \boldsymbol{A}^{-1} & 0 \\ 0 & \boldsymbol{C}^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ 0 & \boldsymbol{C} \end{bmatrix}\right]^{-1} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ 0 & \boldsymbol{C} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{A}^{-1} & 0 \\ 0 & \boldsymbol{C}^{-1} \end{bmatrix}^{-1}$$
$$= \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ 0 & \boldsymbol{C} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{A} & 0 \\ 0 & \boldsymbol{C} \end{bmatrix}$$

   Also from conclusion of part2 we know

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{A}^{-1}\boldsymbol{B} \\ 0 & \boldsymbol{I} \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{A}^{-1}\boldsymbol{B} \\ 0 & \boldsymbol{I} \end{bmatrix}$$

   Therefore,

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ 0 & \boldsymbol{C} \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{A}^{-1}\boldsymbol{B} \\ 0 & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{A}^{-1} & 0 \\ 0 & \boldsymbol{C}^{-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}^{-1} & -\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{C}^{-1} \\ 0 & \boldsymbol{C}^{-1} \end{bmatrix}$$

Problem 3: A statistical test

1. My initial guess is that the rank of C is also 1.

2. Output of my Matlab code shows matrix C and rank:

```
C =

  183.9073 -164.5903   43.4759   -6.3856  -15.1367   28.9753
  -39.0918  -44.8619 -106.7406  -67.2886  -65.7232  -92.5014
   26.0308   39.4461   -3.3600   34.7701  108.6805  -30.6203
  129.2102   95.8134  -55.5014 -120.9416  -64.0614 -129.8827
  101.5532   43.5142 -211.3501  -39.0490   66.1217   33.0372
   47.6437  168.3017  -72.2175 -261.1961  -63.8599 -104.6600
```

```
r =

     6
```

Therefore it is a full rank matrix.

3. The result is different from my initial guess, and I think the result from my code should be correct. The reason is that we you have two rank-1 matrix, the sum matrix is not necessary to be a rank-1 matrix. For example, $\boldsymbol{A} = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 & 5 \\ 2 & 10 \end{bmatrix}$ are two rank-1 matrix.
   But the sum $\boldsymbol{A} + \boldsymbol{B} = \begin{bmatrix} 3 & 8 \\ 6 & 16 \end{bmatrix}$ is a full rank matrix. Code for Problem 3:

```
% Part 2

N = 10000           % sampling numbers of Monte Carlo simulation
n = 6           % dimension of vector
C = zeros(n, n)    % initial an n by n matrix C
for i=[1:N]
    x = randn(n,1)
    y = randn(n,1)
    B = x*y'
    C= C+ B
end

r = rank(C)
```

Problem 4: Image downsampling

1. $\mathbf{y} = \boldsymbol{A}\mathbf{x}$ so A must be a 4×16 matrix. $y_i = \sum_{j=1}^{16} A_{ij} x_j$
   $y_1 = (x_1 + x_2 + x_5 + x_6)/4$
   $y_2 = (x_3 + x_4 + x_7 + x_8)/4$
   $y_3 = (x_9 + x_{10} + x_{13} + x_{14})/4$
   $y_4 = (x_{11} + x_{12} + x_{15} + x_{16})/4.$

   Then
   $\boldsymbol{A}_{11} = \boldsymbol{A}_{12} = \boldsymbol{A}_{15} = \boldsymbol{A}_{16} = 0.25$
   $\boldsymbol{A}_{23} = \boldsymbol{A}_{24} = \boldsymbol{A}_{27} = \boldsymbol{A}_{28} = 0.25$
   $\boldsymbol{A}_{39} = \boldsymbol{A}_{3,10} = \boldsymbol{A}_{3,13} = \boldsymbol{A}_{3,14} = 0.25$
   $\boldsymbol{A}_{4,11} = \boldsymbol{A}_{4,12} = \boldsymbol{A}_{4,15} = \boldsymbol{A}_{4,16} = 0.25$

$$\boldsymbol{A} = \begin{bmatrix} 0.25 & 0.25 & 0 & 0 & 0.25 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0.25 & 0 & 0 & 0.25 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0.25 & 0 & 0 & 0.25 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0.25 & 0 & 0 & 0.25 & 0.25 \end{bmatrix}$$

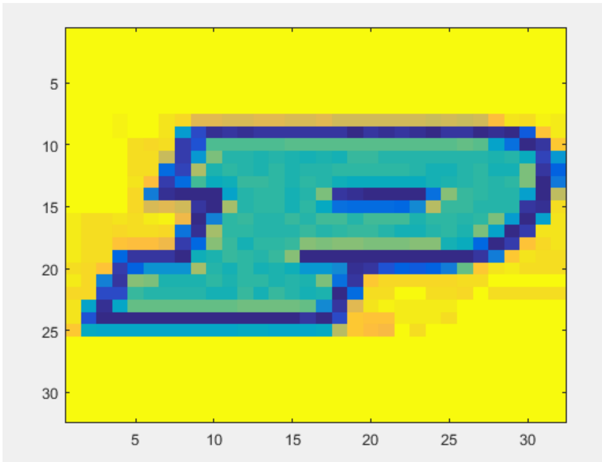
2. The sum of diagonal elements of X is 24.2686

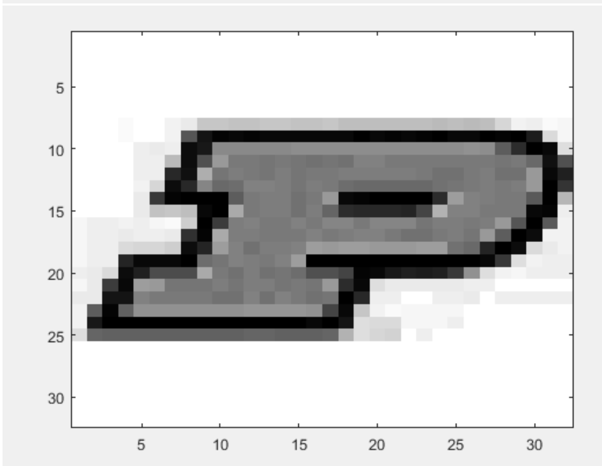
3. Color image and grey image:

4. Resahpe command will reshape the input array into a $m \times n$ matrix, and return the new matrix.

5. Please see the attached code

6. This image looks correct, shown as below. It looks correct.



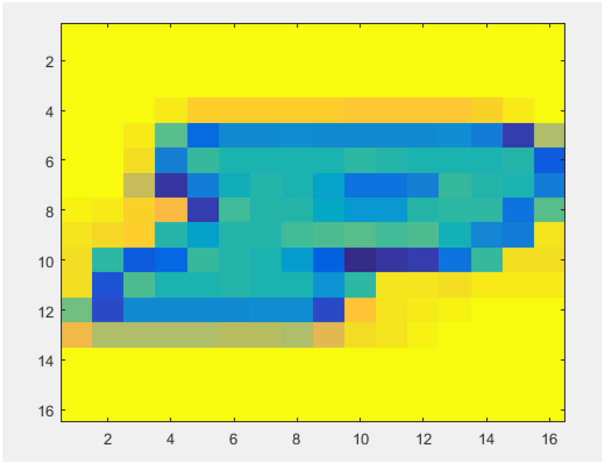


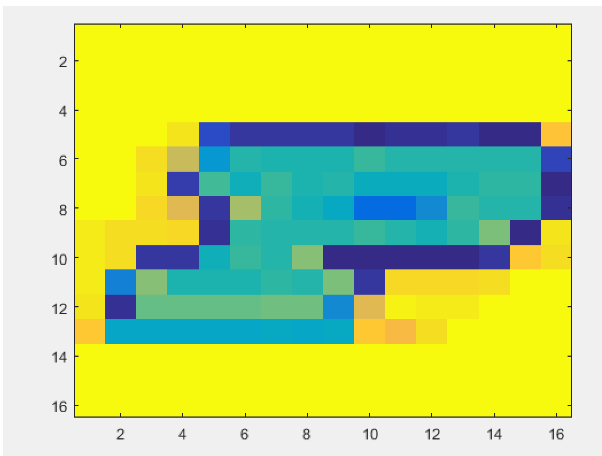
7. After applying 'interp2' function. The image would be like this which is very similar with what we got from

8. Here are all the code for Problem 7.

```
%% Part 2
load smallicon.txt
t = trace(smallicon)
%% Part 3
imagesc(smallicon)
colormap(gray)
%% Part 4
%% Part 5
load smallicon.txt
A = zeros(16*16,32*32);     % initialize A matrix
x = zeros(32*32,1);      % initialize B matrix

NX = [32, 1]; % the map between pixel indices and linear ...
    indices for X
NY = [16,1]; % the map between pixel indices and linear ...
    indices for Y
 for i=1:32
     for j=1:32
         xi = dot(NX,[i-1,j]); % the index of the pixel i,j ...
             in the vector x
         yij = [floor(0.5*i+0.6)-1, floor(0.5*j+0.6)] ; % ...
             the resulting location of pixel in the matrix Y
         yi = dot( NY, yij); % the index of the linear pixel ...
             in the vector y
         x(xi) = smallicon(i,j);    % fill in the linear ...
             vector x
```






```
            A(yi,xi) = 1/4; % place the entry of the matrix
        end
    end

  y = A*x;     % Apply matrix on vector x
  Y = reshape(y,16,16)';    % Reshape the linear indexed ... vector to be an 16 by 16 matrix.
imagesc(Y)    % Show the downsampled image.

%% Part 7

load smallicon.txt
X = smallicon
Ym = interp2(X,-1)
imagesc(Ym)
```