HOMEWORK
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Problem 0: Homework checklist

✓I asked RUN CHEN about Lanczos.
✓Source-code are included at the end of this document.

Problem 1: Compare CG as described in class to the standard CG iteration

1. The CGL code is shown as below:

```
function [res, x] = CGL(A, b, n, tol, num)
T = zeros(n, n);
v = zeros(n, n+1);
res = zeros(n);
alpha = zeros(n, 1);
beta = zeros(n+1,1);
beta(1) = norm(b);
v0 = zeros(n, 1);
v_temp = b/beta(1);
v(1:n, 1) = v_temp;

for j = 1 : num
    wj = A*v_temp;
    alpha(j)  = transpose(v_temp)*wj;
    T(j, j) = alpha(j);
    wj = wj - beta(j)*v0 -alpha(j) *v_temp;
    beta(j+1) = norm(wj);
    v0 = v_temp;
    v_temp = wj/beta(j+1);
    if j ~= n
        T(j, j+1) = beta(j+1);
        T(j+1, j) = beta(j+1);
        v(1:n, j+1) = v_temp;
    end

    e1 = zeros(j, 1);
    e1(1) = 1;
    z = T(1:j, 1:j)\(norm(b)*e1);
    x = v(1:n, 1:j)*z;
    res(j) =norm(beta(j+1)*z)-norm(b);
    if res(j) < tol
        break
    end
end

return
```

2. Plots are shown below:

3. When the tolerance is 10^{-8} , the size of residual is 50. This is half of the size $n = 100$.

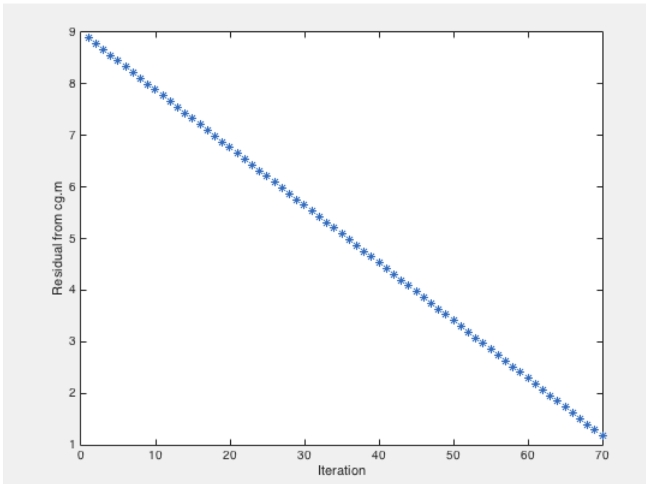

Figure 1: Problem 1.2. Residual vs iteration for cg.m code

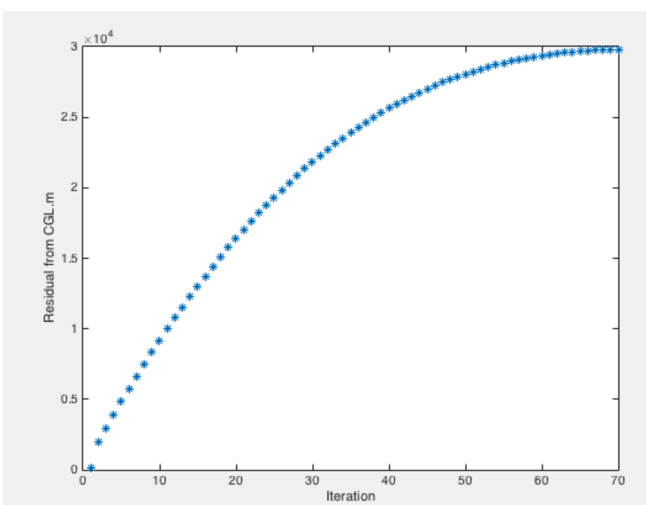

Figure 2: Problem 1.2. Residual vs iteration for CGL.m code

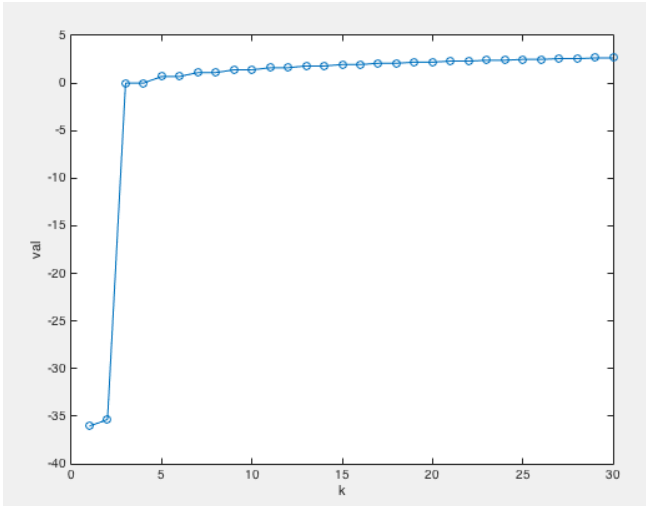

Figure 3: Problem 2.1. A fixed starting point

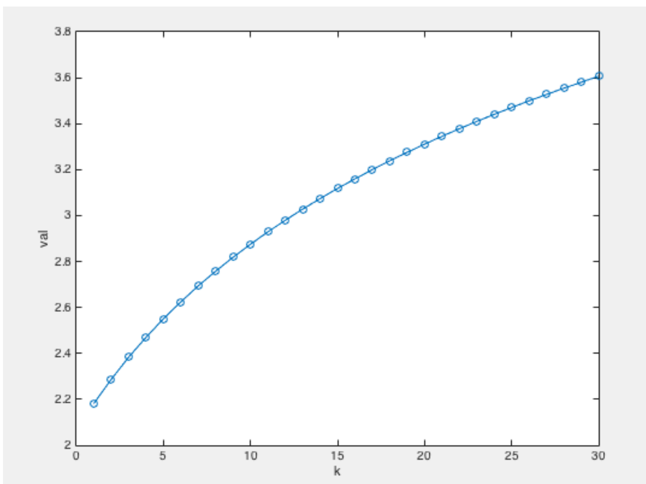

Figure 4: Problem 2.1. A random starting point

Problem 2: Orthogonality of Lanczos

1. I should find that the quantity is increasing converge to some value, and it doesn't depend on the starting vector. Actually I find that.

2. The plots are shown below:

3. Because matrix $\boldsymbol{T}$ is a tridiagonal matrix, then $\beta_{3,1}$ is supposed to be zero. But I got $\beta_{3,1} = 1.4726e^{-8}$

4. Plots are shown below:

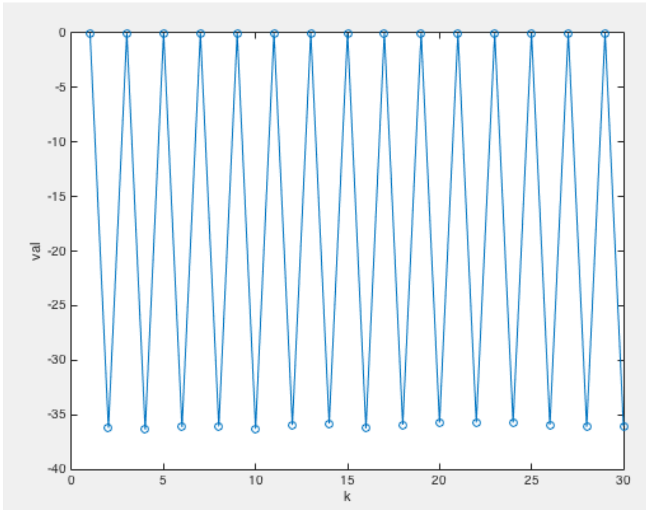

Figure 5: Problem 2.2. A fix starting point of $log(|\mathbf{v}_1^T\mathbf{v}_k| + 10^{-20})$

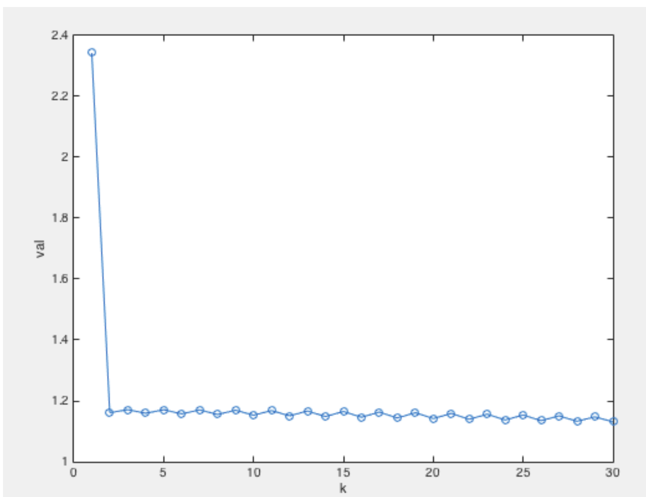

Figure 6: Problem 2.2. A random starting point of $log(|\mathbf{v}_1^T\mathbf{v}_k| + 10^{-20})$

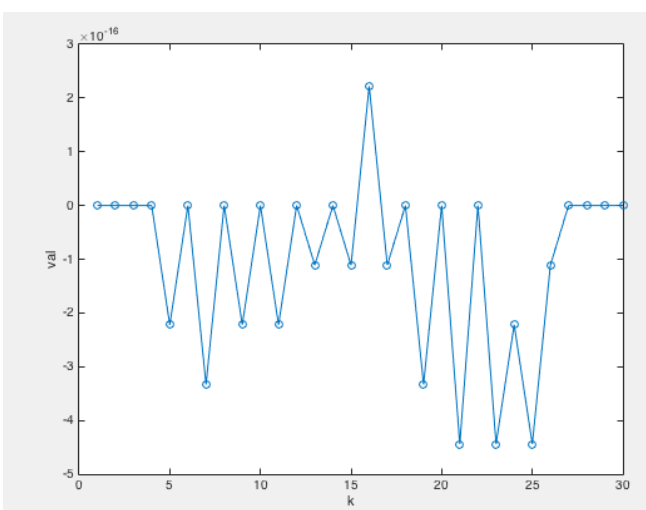

Figure 7: Problem 2.2. A fix starting point of $log(|\mathbf{v}_{k-2}^T\mathbf{v}_k| + 10^{-20})$

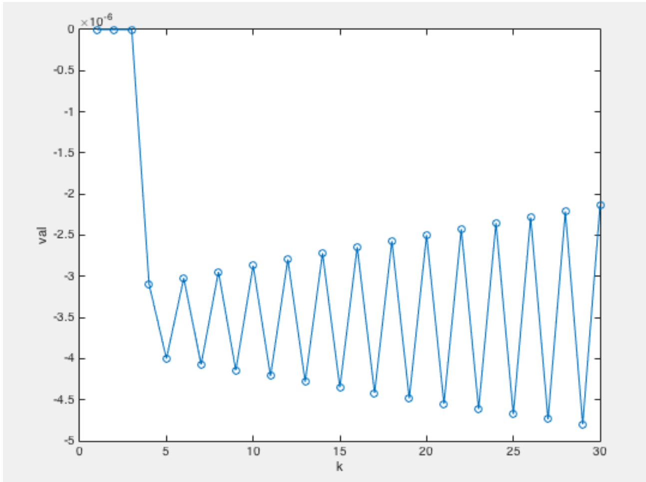

Figure 8: Problem 2.2. A random starting point of $log(|\mathbf{v}_{k-2}^T\mathbf{v}_k| + 10^{-20})$

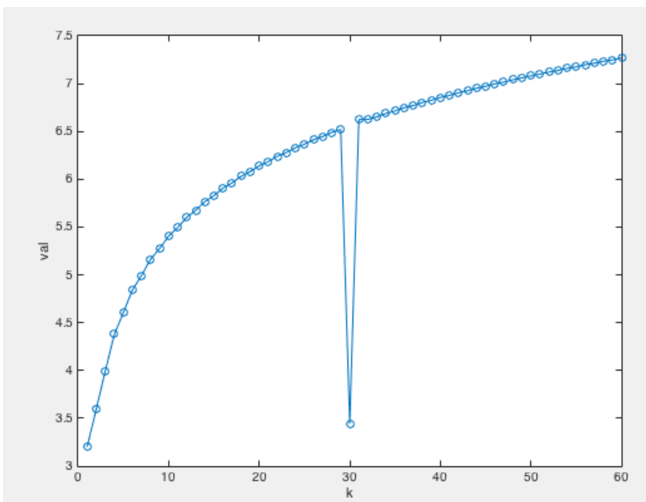

Figure 9: Problem 2.4. A fix starting point.

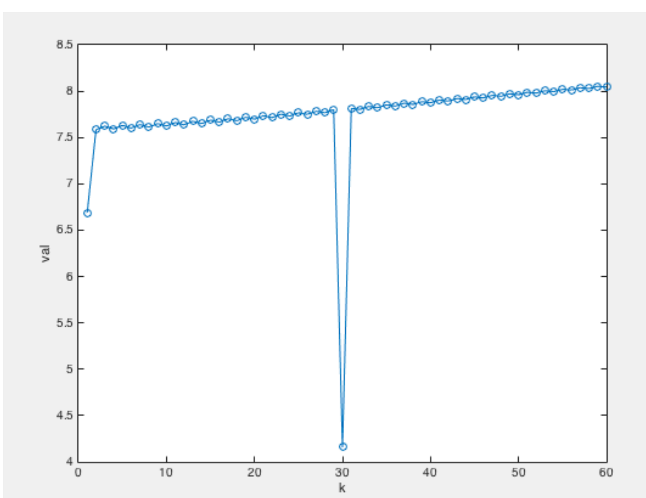

Figure 10: Problem 2.4. A random starting point.