PURDUE UNIVERSITY · ECE 580
OPTIMIZATION METHODS FOR SYS-
TEMS AND CONTROL

HOMEWORK
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Problem 1:

Problem 2:

Problem 3:

Problem 4:

1.

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 4 & -2 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$rank(A) = 2$$

$$rank(A, \mathbf{b}) = 2$$

Based on Theorem 2.1, $rank(A) = rank(A, \mathbf{b})$, then the system has a solution.

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 - 2d_3 - d_4 \\ -4x_3 - 2x_4 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 - 2d_3 - d_4 \\ -4x_3 - 2x_4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 - 2d_3 \end{bmatrix}$$

2.

$$A = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 6 & 3 & 6 & 3 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$rank(A) = 1$$

$$rank(A, \mathbf{b}) = 2$$

Based on Theorem 2.1, $rank(A) \neq rank(A, \mathbf{b})$, then the system has no solutions.

Problem 5:

$$A = \begin{bmatrix} c & 0 & a \\ 0 & c & b \\ b & a & 0 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} b \\ a \\ c \end{bmatrix}$$

$$A|\mathbf{b} = \begin{bmatrix} c & 0 & a \\ 0 & c & b \\ 0 & 0 & -\frac{2ab}{c} \end{bmatrix} \Big| \begin{bmatrix} b \\ a \\ \frac{c^2 - a^2 - b^2}{c} \end{bmatrix}$$

The one unique solution for this linear system is:

$$a \neq 0$$
$$b \neq 0$$
$$c \neq 0$$

The solution is :

$$x_1 = \frac{b^2 + c^2 - a^2}{2bc}$$
$$x_2 = \frac{a^2 + c^2 - b^2}{2ac}$$
$$x_3 = \frac{a^2 + b^2 - c^2}{2bc}$$

Problem 6:

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y & \cos x \sin y + \sin x \cos y \\ -\sin x \cos y - \sin y \cos x & \cos x \cos y - \sin x \sin y \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix}$$

Then

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}^2 = \begin{bmatrix} \cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$$

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}^{57} = \begin{bmatrix} \cos 57x & \sin 57x \\ -\sin 57x & \cos 57x \end{bmatrix}$$

Problem 7:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$
$$f_1(x) = x_2 - 2x + 5$$
$$f_2(x) = 7x + 5$$

Then

$$f_1(A) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} - 2 \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -2 \\ -3 & 11 \end{bmatrix}$$

$$f_2(A) = 7 \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 14 \\ 21 & 5 \end{bmatrix}$$

$$5f_1(A) - 3f_2(A) = \begin{bmatrix} 14 & -62 \\ 78 & 40 \end{bmatrix}$$

Problem 8:

1. $f(x_1, x_2) = x_1^2 + x_2^2 + 4x_1x_2 + \frac{2}{3}x_2^3 - 2x_2 + 7$

$$\frac{\partial f}{\partial x_1} = 2x_1 + 4x_2 = 0$$
$$\frac{\partial f}{\partial x_2} = 2x_2 + 4x_1 + 2x_2^2 - 2 = 0$$
$$x_1 = -6.6$$
$$x_2 = 3.3$$
$$\text{or}$$
$$x_1 = 0.6$$
$$x_2 = -0.3$$

So there are two extremum points $(-6.6, 3.3)$ and $(0.6, -0.3)$

2.

$$\frac{\partial^2 f}{\partial x_1^2} = 2 > 0$$
$$\frac{\partial^2 f}{\partial x_2^2} = 2 + 4x_2 > 0$$

Then we need $x_2 > -0.5$, and both points satisfies these conditions. There-fore both points are local minimizers.

Problem 9:

1. $f(x_1, x_2, x_3, x_4) = 7x_1^2 + x_3^2 - 2x_1x_3 + x_1x_4$

$$f = \frac{1}{2}\mathbf{x}^T Q\mathbf{x}$$

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} \begin{bmatrix} 14 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ -2 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

2. $f(x_1, x_2, x_3) = x_2^2 - 3x_1x_2$

$$f = \frac{1}{2}\mathbf{x}^T Q\mathbf{x}$$

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 0 & -3 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

3. $f(x_1, x_2, x_3) = 2x_1^2 - 5x_2^2 + 2x_1x_2$

$$f = \frac{1}{2}\mathbf{x}^T Q\mathbf{x}$$

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 4 & 2 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$