HOMEWORK
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Problem 4:

(a)

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 3 & 0 & 3 & -4 \\ 1 & 1 & 1 & 2 \\ 2 & 3 & 1 & 3 \end{bmatrix}$$
$$\mathbf{b} = \begin{bmatrix} 0 \\ 7 \\ 6 \\ 6 \end{bmatrix}$$

(i) Partial pivoting The output from my code is

$$\mathbf{x} = \begin{bmatrix} 4 \\ -3 \\ 1 \\ 2 \end{bmatrix}$$

(ii) Scaled pivoting The output from my code is

$$\mathbf{x} = \begin{bmatrix} 4 \\ -3 \\ 1 \\ 2 \end{bmatrix}$$

(b)

$$\boldsymbol{A} = \begin{bmatrix} 0.2115 & 2.296 & 2.715 & 3.215 \\ 0.4371 & 3.916 & 1.683 & 2.852 \\ 6.099 & 4.324 & 23.20 & 1.578 \\ 4.623 & 0.8926 & 15.32 & 5.305 \end{bmatrix}$$
$$\mathbf{b} = \begin{bmatrix} 8.438 \\ 8.8888 \\ 35.20 \\ 26.14 \end{bmatrix}$$

(i) Partial pivoting The output from my code is

$$\mathbf{x} = \begin{bmatrix} 0.999081 \\ 0.999913 \\ 1.00021 \\ 1.0001 \end{bmatrix}$$

(ii) Scaled pivoting The output from my code is

$$\mathbf{x} = \begin{bmatrix} 0.999081 \\ 0.999913 \\ 1.00021 \\ 1.0001 \end{bmatrix}$$

Problem 5:

$$\boldsymbol{A} = \begin{bmatrix} -9 & 11 & -21 & 63 & -252 \\ 70 & -69 & 141 & -421 & 1684 \\ -575 & 575 & -1149 & 3451 & -13801 \\ 3891 & -3891 & 7782 & -23345 & 93365 \\ 1024 & -1024 & 2048 & -6144 & 24572 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} -356 \\ 2385 \\ -19551 \\ 132274 \\ 34812 \end{bmatrix}$$

The output of scaled pivoting result is

$$\widetilde{\mathbf{x}} = \begin{bmatrix} 1 \\ -0.864667 \\ 0.0848896 \\ 5.27044 \\ 2.64978 \end{bmatrix}$$

$$\mathbf{e} = \widetilde{\mathbf{x}} - \mathbf{x} = \begin{bmatrix} 0 \\ 0.1353 \\ -0.9151 \\ 6.2704 \\ 1.6498 \end{bmatrix}$$

$$\frac{\|\mathbf{e}\|}{\|\mathbf{x}\|} = 2.929$$

$$\boldsymbol{A}\widetilde{\mathbf{x}} - \mathbf{b} = \begin{bmatrix} -0.0009 \\ 0.0057 \\ -0.0470 \\ 0.3181 \\ 0.0837 \end{bmatrix}$$

$$\frac{\|\mathbf{r}\|}{\|\mathbf{b}\|} = \frac{\|\boldsymbol{A}\widetilde{\mathbf{x}} - \mathbf{b}\|}{\|\mathbf{b}\|} = \frac{0.3323}{1.3819e+05} = 2.4046e-06$$

Therefore, the estimated condition number is $\frac{2.929}{2.4046e-06} = 1.2181e+06$

Problem 6:

We can construct 12 equation if there are no external forces, 2 for each node:

$$
\begin{aligned}
\sum F_{1x} &= F_1 \cos\alpha + F_3 \cos\gamma + F_H && = 0 \\
\sum F_{1y} &= F_v + F_3 \cos\gamma + F_1 \sin\alpha && = 0 \\
\sum F_{2x} &= F_8 \cos\beta + F_9 \cos\alpha - F_2 \cos\beta - F_1 \cos\alpha && = 0 \\
\sum F_{2y} &= F_5 + F_2 \sin\beta + F_8 \sin\beta - F_1 \sin\alpha - F_9 \sin\alpha && = 0 \\
\sum F_{3x} &= F_4 \cos\beta + F_2 \cos\beta - F_3 \cos\gamma && = 0 \\
\sum F_{3y} &= F_4 \sin\beta - F_2 \sin\beta - F_3 \sin\gamma && = 0 \\
\sum F_{4x} &= F_6 \cos\beta - F_4 \cos\beta && = 0 \\
\sum F_{4y} &= -F_4 \sin\beta - F_6 \sin\beta - F_5 && = 0 \\
\sum F_{5x} &= F_7 \cos\gamma - F_6 \cos\beta - F_8 \cos\beta && = 0 \\
\sum F_{5y} &= F_6 \sin\beta - F_8 \sin\beta - F_7 \sin\gamma && = 0 \\
\sum F_{6x} &= -F_7 \cos\gamma - F_9 \cos\alpha && = 0 \\
\sum F_{6y} &= F_R + F_7 \sin\gamma + F_9 \sin\alpha && = 0
\end{aligned}
$$

Then the linear equation $\boldsymbol{A}\mathbf{x} = \mathbf{b}$ becomes:

$$
\begin{bmatrix}
\cos\alpha & 0 & \cos\gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\sin\alpha & 0 & \cos\gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
-\cos\alpha & -\cos\beta & 0 & 0 & 0 & 0 & 0 & \cos\beta & \cos\alpha & 0 & 0 & 0 \\
-\sin\alpha & \sin\beta & 0 & 0 & 1 & 0 & 0 & \sin\beta & -\sin\alpha & 0 & 0 & 0 \\
0 & \cos\beta & -\cos\gamma & \cos\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\sin\beta & -\sin\gamma & \sin\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\cos\beta & 0 & \cos\beta & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\sin\beta & -1 & -\sin\beta & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\cos\beta & \cos\gamma & -\cos\beta & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sin\beta & -\sin\gamma & -\sin\beta & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\cos\gamma & 0 & \cos\alpha & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sin\gamma & 0 & \sin\alpha & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \\ F_R \\ F_V \\ F_H
\end{bmatrix}
= \mathbf{b}
$$

We know:

$$
\begin{aligned}
\sin\alpha &= 0.447 \\
\cos\alpha &= 0.894 \\
\sin\beta &= 0.316 \\
\cos\beta &= 0.949 \\
\sin\gamma &= 0.707 \\
\cos\gamma &= 0.707
\end{aligned}
$$

Then matrix A becomes:

$$
\boldsymbol{A} = \begin{bmatrix}
0.894 & 0 & 0.707 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0.447 & 0 & 0.707 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
-0.894 & -0.949 & 0 & 0 & 0 & 0 & 0 & 0.949 & 0.894 & 0 & 0 & 0 \\
-0.447 & 0.316 & 0 & 0 & 1 & 0 & 0 & 0.316 & -0.316 & 0 & 0 & 0 \\
0 & 0.949 & -0.707 & 0.949 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.316 & -0.707 & 0.316 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.949 & 0 & 0.949 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.316 & -1 & -0.316 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.949 & 0.707 & -0.949 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.316 & -0.707 & -0.316 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.707 & 0 & 0.894 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.707 & 0 & 0.447 & 1 & 0 & 0
\end{bmatrix}
$$

For different configurations:

$$
\mathbf{b}_a = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 500 \\ 0 \\ 1000 \\ 0 \\ 500 \\ 0 \\ 0 \end{bmatrix} \quad
\mathbf{b}_b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 500 \\ 0 \\ 1000 \\ 0 \\ 1500 \\ 0 \\ 0 \end{bmatrix} \quad
\mathbf{b}_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1500 \\ 0 \\ 1000 \\ 0 \\ 500 \\ 0 \\ 0 \end{bmatrix} \quad
\mathbf{b}_d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1000 \\ 0 \\ -500 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad
\mathbf{b}_e = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 500 \\ 0 \\ 1000 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

The results are shown:

$$
F_a = \begin{bmatrix} -784.006 \\ -51.402 \\ -991.374 \\ -687.167 \\ -565.711 \\ -687.167 \\ -991.374 \\ -51.402 \\ -784.006 \\ 1051.35 \\ 1051.35 \\ 1401.8 \end{bmatrix}
F_b = \begin{bmatrix} -2162.2 \\ -334.561 \\ -611.893 \\ -121.296 \\ -923.341 \\ -121.296 \\ -1672.99 \\ -1125.07 \\ -1323.05 \\ 1774.21 \\ 1399.11 \\ 2365.61 \end{bmatrix}
F_c = \begin{bmatrix} -189.822 \\ -610.785 \\ -2362.23 \\ -1149.06 \\ -273.791 \\ -1149.06 \\ -1301.13 \\ 179.729 \\ -1028.97 \\ 1379.85 \\ 1754.95 \\ 1839.8 \end{bmatrix}
F_d = \begin{bmatrix} 727.085 \\ -684.362 \\ 211.065 \\ -212.136 \\ 300.561 \\ -739.007 \\ -495.589 \\ 369.796 \\ -391.926 \\ 525.572 \\ -474.23 \\ -799.237 \end{bmatrix}
F_e = \begin{bmatrix} -727.085 \\ 157.491 \\ -211.065 \\ -314.734 \\ 32.4208 \\ 212.136 \\ 495.589 \\ -896.666 \\ 391.926 \\ -525.572 \\ 474.23 \\ 799.237 \end{bmatrix}
$$

For the direction of the forces: negative values indicate forces point outward. Positive direction of F_V, F_H, F_R are as shown in the figure.