## Problem 0: Homework checklist

✓I didn't talk with any one about this homework.
✓Source-code are included at the end of this document.

## Problem 1

1. $\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 5 & 0 \\ 2 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 5 & -5 \\ 2 & -2 \\ 0 & 0 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

## Problem 2:

1. For example, The matrix

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \tag{1}$$

has two different decomposition. But in both cases they have the same singular value.

2.

3. Suppose $\boldsymbol{A}$ is a matrix with eigenvalues $\{\sigma_i\}$
The SVD of $\boldsymbol{A}$ is $\boldsymbol{A} = \boldsymbol{U}\Sigma\boldsymbol{V}^T$ where $\boldsymbol{U}$ and $\boldsymbol{V}$ are both orthogonal matrix.

$$\boldsymbol{A}^{-1} = (\boldsymbol{U}\Sigma\boldsymbol{V}^T)^{-1} \tag{2}$$
$$= (\boldsymbol{V}^T)^{-1}\Sigma^{-1}\boldsymbol{U}^{-1} \tag{3}$$
$$= (\boldsymbol{V}^T)^T\Sigma^{-1}\boldsymbol{U}^T \tag{4}$$
$$= \boldsymbol{V}\Sigma^{-1}\boldsymbol{U}^T \tag{5}$$

Then $\boldsymbol{A}^{-1}$ is a matrix with eigenvalues $\{\frac{1}{\sigma_i}\}$

$$\|\boldsymbol{A}^{-1}\| = max\{\frac{1}{\sigma_i}\} \tag{6}$$

$$= \frac{1}{\sigma_{min}} \tag{7}$$

$$\tag{8}$$

4. The SVD of $\boldsymbol{Q}$ is $\boldsymbol{Q} = \boldsymbol{U}\Sigma\boldsymbol{V}^T = \boldsymbol{Q}\boldsymbol{I}\boldsymbol{I}^T$ where $\boldsymbol{I}$ is an identity matrix and it is also an orthogonal matrix.
In particular, the singular value are all 1.