ECE 580 Spring 2016

FunWork #1

Due on February 03

INSTRUCTIONS: The assignment must be typed. Clearly identify the steps you have taken to solve each problem. Your grade depends on the completeness and clarity of your work as well as the resulting answer.

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1. Generate an example of proof by contraposition—see page 5 in the textbook. That is, state a theorem and prove it by contraposition. Do not use examples from our textbook. You are allowed to use examples from other books. Make sure to cite completely sources that you have used; no citation, no credit.

2. Generate an example of proof by contradiction—see page 5 in the textbook. That is, state a theorem and prove it by contradiction. Do not use examples from our textbook. You are allowed to use examples from other books. Make sure to cite completely sources that you have used; no citation, no credit.

3. Generate an example of proof by induction—see page 5 in the textbook. That is, state a theorem and prove it by induction. Do not use examples from our textbook. You are allowed to use examples from other books. Make sure to cite completely sources that you have used; no citation, no credit.

4. Consider the following systems of equations,

(a)

$$\left.\begin{array}{rcl} x_1 + x_2 + 2x_3 + x_4 & = & 1 \\ x_1 + 2x_2 + 4x_3 - 2x_4 & = & 0 \end{array}\right\}$$

(b)

$$\left.\begin{array}{rcl} 2x_1 + x_2 + 2x_3 + x_4 & = & 0 \\ 6x_1 + 3x_2 + 6x_3 + 3x_4 & = & 1 \end{array}\right\}$$

Use Theorem 2.1, on page 17, to check if the system has a solution. If there is a solution, use the method of the proof of Theorem 2.2, on page 18, to find a general solution to the system.

5. Determine the condition under which the system of linear equations,

$$\left.\begin{array}{rcl} cx_1 + ax_3 & = & b \\ cx_2 + bx_3 & = & a \\ bx_1 + ax_2 & = & c \end{array}\right\}$$

has a unique solution. Then find the solution.

6. Compute

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}^{57}.$$

7. Let

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}, \quad f_1(x) = x^2 - 2x + 5, \quad \text{and} \quad f_2(x) = 7x + 5.$$

Find $5f_1(\boldsymbol{A}) - 3f_2(\boldsymbol{A})$.

8. For the function

$$f = f(x_1, x_2) = x_1^2 + x_2^2 + 4x_1x_2 + \frac{2}{3}x_2^3 - 2x_2 + 7,$$

(a) find points that satisfy the first-order necessary conditions for the extremum;

(b) which point is a strict local minimizer? Justify your answer.

9. Represent the following quadratic forms,

(a) $f(x_1, x_2, x_3, x_4) = 7x_1^2 + x_3^2 - 2x_1x_3 + x_1x_4$;

(b) $f(x_1, x_2, x_3) = x_2^2 - 3x_1x_2$;

(c) $f(x_1, x_2, x_3) = 2x_1^2 - 5x_2^2 + 2x_1x_2$,

as

$$f = \frac{1}{2}\boldsymbol{x}^\top \boldsymbol{Q}\boldsymbol{x},$$

where $\boldsymbol{Q} = \boldsymbol{Q}^\top$.