PURDUE UNIVERSITY · CS 51500
MATRIX COMPUTATIONS

HOMEWORK
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Problem 0: Homework checklist

✓I didn't talk with any one about this homework.
✓Source-code are included at the end of this document.

Problem 1: Prove or disprove

For the proof below, I will assume all the matrices are $n \times n$ square matrices.

1. The product of two diagonal matrices is diagonal.
   $\boldsymbol{A}$ and $\boldsymbol{B}$ are diagonal matrices, so
   $\boldsymbol{A}_{ij} = 0, if\ i \neq j$
   $\boldsymbol{B}_{ij} = 0, if\ i \neq j$
   $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$
   $\boldsymbol{C}_{ij} = \sum_{k=1}^{n} \boldsymbol{A}_{ik}\boldsymbol{B}_{kj}$
   Only if $i = j = k$, $\boldsymbol{C}_{ij}$ will not be zero, which mean $\boldsymbol{C}$ is also a diagonal matrix.

2. The product of two upper triangular matrices is upper triangular $\boldsymbol{A}$ and $\boldsymbol{B}$ are two upper triangular matrices, so

   $\boldsymbol{A}_{ij} = 0, if\ i \geqslant j$
   $\boldsymbol{B}_{ij} = 0, if\ i \geqslant j$
   $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$
   $\boldsymbol{C}_{ij} = \sum_{k=1}^{n} \boldsymbol{A}_{ik}\boldsymbol{B}_{kj}$
   When $i \geqslant j$, $\boldsymbol{C}_{ij} = 0$ because one of $\boldsymbol{A}_{ik}$ and $\boldsymbol{B}_{kj}$ must be zero.

3. The product of two symmetric matrices is symmetric. $\boldsymbol{A}$ and $\boldsymbol{B}$ are two symmetric matrices, so

   $\boldsymbol{A}_{ij} = \boldsymbol{A}_{ji}, if\ i = j$
   $\boldsymbol{B}_{ij} = \boldsymbol{B}_{ji}, if\ i = j$
   $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$
   $\boldsymbol{C}_{ij} = \sum_{k=1}^{n} \boldsymbol{A}_{ik}\boldsymbol{B}_{kj}$
   Then $\boldsymbol{C}_{ij}$ is not necessary to be equal to $\boldsymbol{C}_{ji}$
   Counter-example:
   $\boldsymbol{A} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ is a symmetric matrix;
   $\boldsymbol{B} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is also a symmetric matrix.
   $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$ which is not a symmetric matrices.

4. The product of two orthogonal matrices is orthogonal.

5. The product of two square, full rank matrices is full rank This statement is incorrect.
   Counter-example:
   $\boldsymbol{A} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ is a full rank matrix;
   $\boldsymbol{B} = \begin{bmatrix} 3 & 5 \\ 1 & 1 \end{bmatrix}$ is also a full rank matrix.

$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$ which is not a full matrices.

Problem 2

Problem 3

$$f(\boldsymbol{A}) = \max_{i,j} |A_{ij}| \tag{1}$$

1.
   - $f(A) > 0, when \boldsymbol{A} \neq 0,$
     $f(A) = 0 \; iff \boldsymbol{A} = 0$
   - For and scalar k,
     $f(k\boldsymbol{A})$
     $= k \max_{i,j} |A_{ij}|$
     $= \|k\| \| \max_{i,j} |A_{ij}| \|$
     $= \|k\| \|f(\boldsymbol{A})\|$
   - $\|f(\boldsymbol{A}) + f(\boldsymbol{B})\|$
     $= \| \max_{i,j} |A_{ij}| + \max_{i,j} |B_{ij}| \|$
     $\leq \| \max_{i,j} |A_{ij}| \| + \| \max_{i,j} |B_{ij}| \|$
     $= \|f(\boldsymbol{A})\| + \|f(\boldsymbol{B})\|$
2. $\|f(\boldsymbol{A}) f(\boldsymbol{B})\|$
   $= \| \max_{i,j} |A_{ij}| \max_{i,j} |B_{ij}| \|$
   $\leq \| \max_{i,j} |A_{ij}| \| \| \max_{i,j} |B_{ij}| \|$
   $= \|f(\boldsymbol{A})\| \|f(\boldsymbol{B})\|$