HOMEWORK

Jun Cheng

January 18, 2016

Problem 1:

Problem 2:

Problem 3:

Problem 4:

1.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 4 & -2 \end{bmatrix}$$
$$\mathbf{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$rank(\boldsymbol{A}) = 2$$
$$rank(\boldsymbol{A}, \mathbf{b}) = 2$$

Based on Theorem 2.1, $rank(\boldsymbol{A}) = rank(\boldsymbol{A}, \mathbf{b})$, then the system has a solution.

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 - 2d_3 - d_4 \\ -4x_3 - 2x_4 \end{bmatrix}$$
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 - 2d_3 - d_4 \\ -4x_3 - 2x_4 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ -1 - 2d_3 \end{bmatrix}$$

2.

$$\boldsymbol{A} = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 6 & 3 & 6 & 3 \end{bmatrix}$$
$$\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$rank(\boldsymbol{A}) = 1$$
$$rank(\boldsymbol{A}, \mathbf{b}) = 2$$

Based on Theorem 2.1, $rank(\boldsymbol{A}) \neq rank(\boldsymbol{A}, \mathbf{b})$, then the system has no solutions.

Problem 5:

$$\boldsymbol{A} = \begin{bmatrix} c & 0 & a \\ 0 & b & b \\ b & a & 0 \end{bmatrix}$$
$$\mathbf{b} = \begin{bmatrix} b \\ a \\ c \end{bmatrix}$$
$$\boldsymbol{A}^{-1} =][$$

Problem 6:

$$\begin{aligned}
&\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} \\
&= \begin{bmatrix} \cos x\cos y - \sin x \sin y & \cos x \sin y + \sin x \cos y \\ -\sin x \cos y - \sin y \cos x & \cos x \cos y - \sin x \sin y \end{bmatrix} \\
&= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix}
\end{aligned}$$

Then

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}^2 = \begin{bmatrix} \cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$$

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}^{57} = \begin{bmatrix} \cos 57x & \sin 57x \\ -\sin 57x & \cos 57x \end{bmatrix}$$

Problem 7:

Problem 8:

1. $f(x_1, x_2) = x_1^2 + x_2^2 + 4x_1x_2 + \frac{2}{3}x_2^3 - 2x_2 + 7$

$$\begin{aligned}
\frac{\partial f}{\partial x_1} &= 2x_1 + 4x_2 = 0 \\
\frac{\partial f}{\partial x_2} &= 2x_2 + 4x_1 + 2x_2^2 - 2 = 0 \\
x_1 &= -6.6 \\
x_2 &= 3.3 \\
&\mathit{or} \\
x_1 &= 0.6 \\
x_2 &= -0.3
\end{aligned}$$

   So there are two extremum points $(-6.6, 3.3)$ and $(0.6, -0.3)$

2.

Problem 9:

1. $f(x_1, x_2, x_3, x_4) = 7x_1^2 + x_3^2 - 2x_1x_3 + x_1x_4$

$$\begin{aligned}
f &= \frac{1}{2}\mathbf{x}^T \boldsymbol{Q} \mathbf{x} \\
&= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} \begin{bmatrix} 14 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ -2 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}
\end{aligned}$$

2. $f(x_1, x_2, x_3) = x_2^2 - 3x_1x_2$

$$\begin{aligned}
f &= \frac{1}{2}\mathbf{x}^T \boldsymbol{Q} \mathbf{x} \\
&= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 0 & -3 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
\end{aligned}$$

3. $f(x_1, x_2, x_3) = 2x_1^2 - 5x_2^2 + 2x_1x_2$

$$\begin{aligned} f &= \frac{1}{2}\mathbf{x}^T\boldsymbol{Q}\mathbf{x} \\ &= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}\begin{bmatrix} 4 & 2 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \end{aligned}$$