

Problem 1:

Problem 2:

Problem 3:

Problem 4:

1.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 4 & -2 \end{bmatrix} \quad (1)$$

$$\mathbf{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (2)$$

$$\text{rank}(\mathbf{A}) = 2 \quad (3)$$

$$\text{rank}(\mathbf{A}, \mathbf{b}) = 2 \quad (4)$$

$$(5)$$

Based on Theorem 2.1, $\text{rank}(\mathbf{A}) = \text{rank}(\mathbf{A}, \mathbf{b})$, then the system has a solution.

Problem 5:

Problem 6:

$$\begin{aligned} & \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} \\ &= \begin{bmatrix} \cos x \cos y - \sin x \sin y & \cos x \sin y + \sin x \cos y \\ -\sin x \cos y - \sin y \cos x & \cos x \cos y - \sin x \sin y \end{bmatrix} \\ &= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} \end{aligned}$$

Then

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}^2 = \begin{bmatrix} \cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$$

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}^{57} = \begin{bmatrix} \cos 57x & \sin 57x \\ -\sin 57x & \cos 57x \end{bmatrix}$$