HOMEWORK

Jun Cheng

December 11, 2015

Problem 0: Homework checklist

✓I didn't talk with any one about this homework.
✓Source-code are included at the end of this document.

Problem 1:

1. The convergence plots are shown in Figure 1.

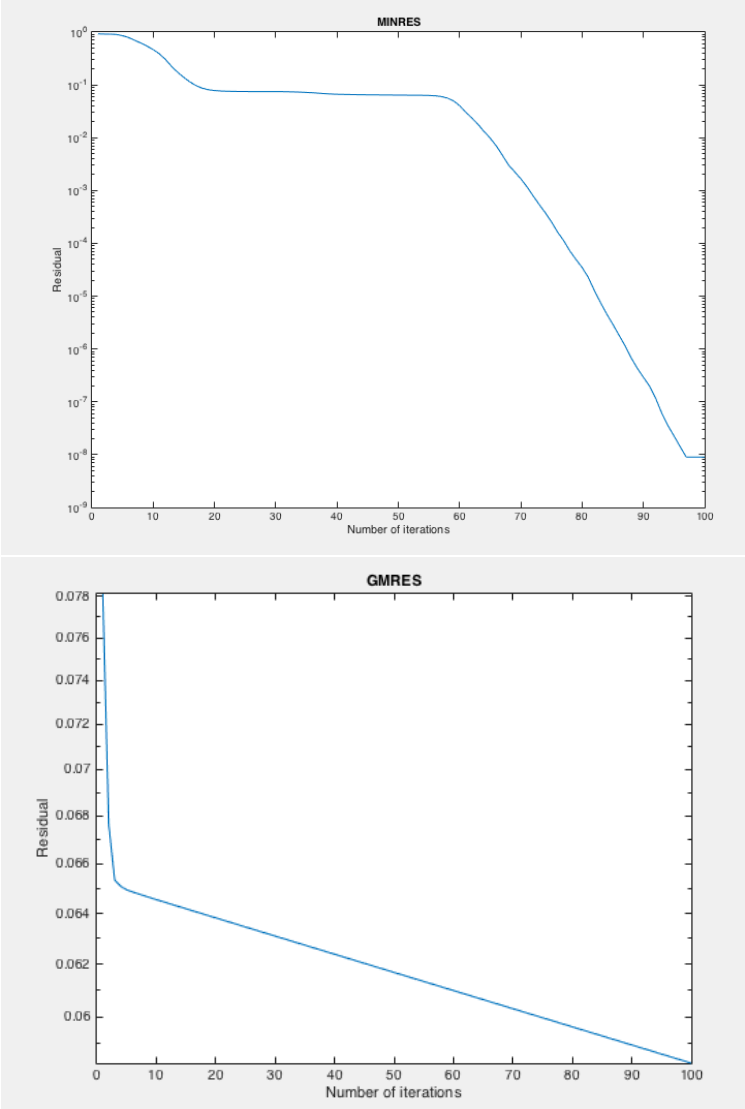

Figure 1: Convergence for both MIRES and GMRES

2. If we use a *ichol* incomplete Cholesky preconditioner it will take 319 iterations to converge. MINRES doesn't benefit from and incomplete Cholesky

preconditioner. Figure 2 shows the convergence of GMRES with incomplete Cholesky preconditioner.

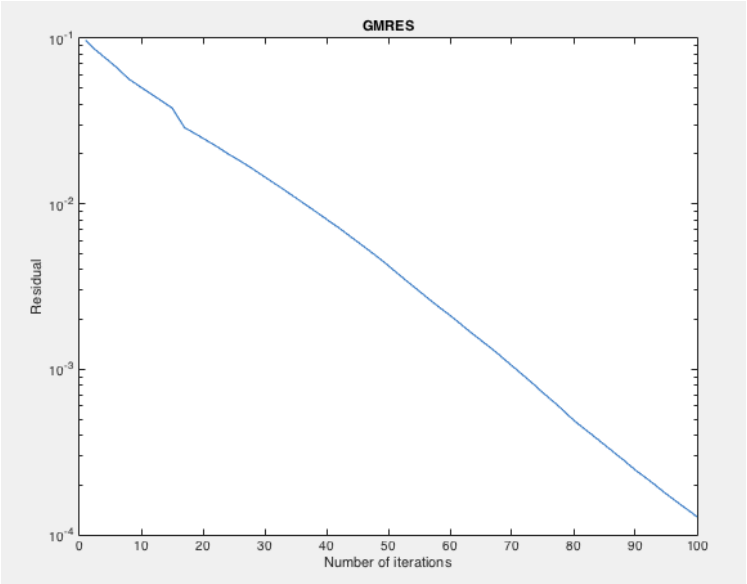

Figure 2: GMRES with *ichol*

3. Eigenvalues before and after preconditioning are show in Figure 3

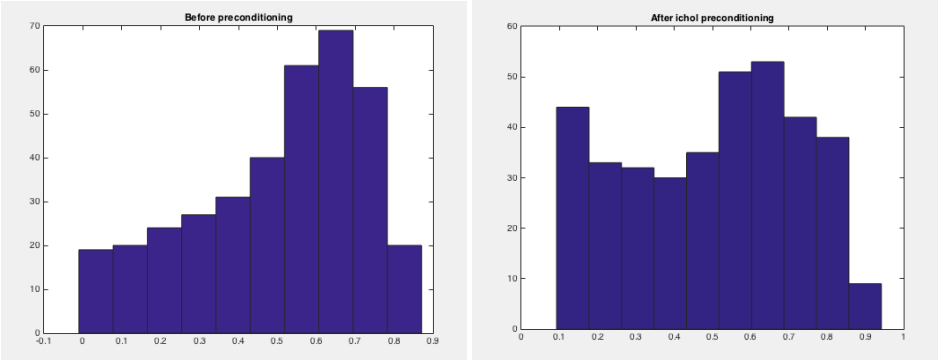

Figure 3: Eigenvalues before and after preconditioning

We can see see some clustering before preconditioning.

Problem 2:

1. In the following expressions, $g_0$indicates the old residual and g_1 indicates the new residual.

$$
\begin{aligned}
g_0 &= \boldsymbol{A}x_0 - b \\
\mu &= \frac{g_0^T g}{g_0^T \boldsymbol{A} g_0} \\
x_1 &= x_0 - \mu g_0 \\
g_1 &= \boldsymbol{A}x_1 - b = \boldsymbol{A}(x_0 - \mu g_0) - b \\
&= \boldsymbol{A}[x_0 - \frac{g_0^T g}{g_0^T \boldsymbol{A} g_0} g_0] - b \\
g_0^T g_1 &= g_0^T [\boldsymbol{A}x_0 - b - \boldsymbol{A}\frac{g_0^T g}{g_0^T \boldsymbol{A} g_0} g_0] \\
&= g_0^T (\boldsymbol{A}x_0 - b) - g_0^T g_0 \\
&= g_0^T g_0 - g_0^T g_0 \\
&= 0
\end{aligned}
$$

Problem 3:

For a linear system of equation with

$$\boldsymbol{A} = \boldsymbol{I} + \mathbf{e}\mathbf{e}^T$$

The number of steps to converge is $rank(\mathbf{e}\mathbf{e}^T) + 1 = 1$

Problem 4:

To solve this problem we should use the simultaneous iteration.

Pick $\hat{Q}^{(0)} \in \mathbb{R}^{\mathrm{m}\times\mathrm{n}}$ with orthonormal columns.
for k = 1, 2, ...
$Z = A\hat{Q}^{(k-1)}$
$\hat{Q}^{(k)}\hat{R}^{(k)} = Z$

After several iterations, the matrix $\hat{Q}$ would contain the orthonormal basis of the 8-dimensional eigenspace.