UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 8
Fall 2023

1. **Information Loss**

   Suppose we have discrete random variables X and Y, which represent the input message and received message respectively. Let n be the number of distinct values X can take. Our estimate of X from Y is $\hat{X} = g(Y)$, where g is some decoding function. Now define $E = \mathbb{1}\{X \neq \hat{X}\}$ to be the indicator of estimation error, and define the probability of error $p_e := \mathbb{P}(X \neq \hat{X})$.

   a. Show that $H(\hat{X} \mid Y) = 0$.
   b. Show that $H(E, X \mid \hat{X}) = H(X \mid \hat{X})$.
   c. Show that $H(X \mid Y) \leq p_e \log_2(n-1) + H(E)$.
   (You may use the fact that $H(X \mid Y) \leq H(X \mid \hat{X})$.)

   Hint. The chain rule for entropy can be generalized to three random variables:

   $$H(A, B \mid C) = H(A \mid C) + H(B \mid A, C).$$

2. **Hitting Time with Coins**

   Consider a sequence of fair coin flips.

   a. What is the expected number of flips until we first see two heads in a row?
   b. What is the expected number of flips until we see a head followed immediately by a tail?

3. **Before Absorption**

Consider the Markov chain in Figure 1. Suppose that $X(0) = 1$. Calculate the expected number of times that the chain is in state 1 before being absorbed in state 3. ($X(0) = 1$ is included in this number.)

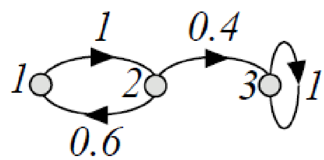

Figure 1: A Markov chain.