UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: PROBABILITY AND RANDOM PROCESSES

Discussion 1
Fall 2023

1. **Independence**

   Events $A, B \in \mathcal{F}$ are said to be **independent** if $\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$.

   a. Show that if events A, B are independent, then the probability exactly one of the events occurs is
   $$\mathbb{P}(A) + \mathbb{P}(B) - 2\,\mathbb{P}(A)\,\mathbb{P}(B).$$

   b. Show that if the event A is independent of itself, then $\mathbb{P}(A) = 0$ or 1.

2. **Balls and Bins**

Suppose n bins are arranged from left to right. You sequentially throw n balls; each ball lands in a bin chosen uniformly at random, independent of all other balls.

a. Formulate an appropriate probability space for modelling the outcome of this experiment.

b. Let A_i denote the event that exactly i bins are empty, $i = 0, \dots, n$. Compute the probability of the event

$$\{\text{all empty bins sit to the left of all bins containing at least one ball}\}$$

in terms of the $\mathbb{P}(A_i)$'s.

c. Practice your CS70 skills by computing $\mathbb{P}(A_1)$.

3. **Coin Flipping and Symmetry**

   Alice and Bob have $2n+1$ fair coins, $n \geq 1$. Bob tosses $n+1$ coins, while Alice tosses the remaining n coins. A fair coin lands on heads with probability $\frac{1}{2}$; assume that coin tosses are independent.

   a. Formulate this scenario in terms of a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Describe explicitly what the outcomes are and what the probability measure of any event is defined to be.

   b. Show that the probability that after all coins have been tossed, Bob will have gotten more heads than Alice is $\frac{1}{2}$.

      Hint: Consider the event $A = \{\text{more heads in the first } n+1 \text{ tosses than the last } n \text{ tosses}\}$.