UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Homework 10
Fall 2023

1. **Reversibility of CTMCs**

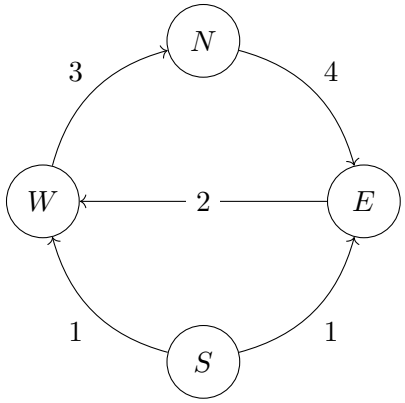
   We say that a CTMC with transition rate matrix Q and distribution π is *reversible* if π and Q satisfy the detailed balance equations

   $$\pi(i) \cdot q(i,j) = \pi(j) \cdot q(j,i) \quad \forall i, j \in S.$$

   Show that if π is a reversible distribution for a CTMC, then π is also a stationary distribution for the chain, and moreover the embedded jump chain is also reversible. *Remark*: the converse is also true — the CTMC is reversible if and only if the embedded chain is reversible.

2. **Jump Chain Stationary Distribution**

   Use properties of transient states and the jump chain to find the stationary distribution of this CTMC.



3. $M/M/2$ **Queue**

A queue has Poisson arrivals with rate λ and two servers with i.i.d. Exponential(μ) service times. The two servers work in parallel: when there are at least two customers in the queue, two are being served; when there is only one customer, only one server is active. Let X_t be the number of customers either in the queue or in service at time t.

a. Argue that the process $(X_t)_{t\geq 0}$ is a Markov process, and draw its state transition diagram.

b. Find the range of values of μ for which the Markov chain is positive recurrent. For this range of values, calculate the stationary distribution of the Markov chain.