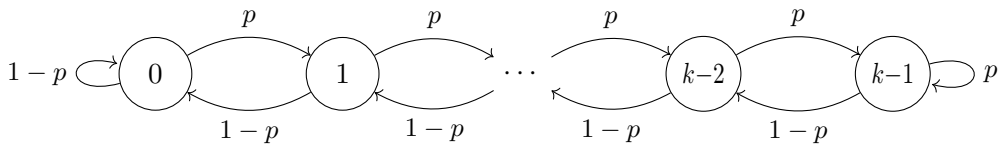
UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 9
Fall 2023

1. **Finite Random Walk**

   Let $0 < p < 1$, and consider the following finite *random walk* with bias p on $\mathcal{X} = \{0, \ldots, k-1\}$, also known as the finite *birth-death chain*.



   a. Find the stationary distribution π.
      Hint: Write $q = 1 - p$ and define $r \coloneqq \frac{p}{q}$. Be careful when $r = 1$.
   b. Find the limit of $\pi(0)$ and $\pi(k-1)$, as functions of k, as $k \to \infty$.

2. **Moving Books Around**

You have N books labelled $1, \ldots, N$ on your shelf. At each time step, you pick a book i with probability $\frac{1}{N}$, place it on the left of all others on the shelf, then repeat this process, each step independent of any other step. Construct a suitable Markov chain which takes values in the set of all $N!$ permutations of the books.

a. Find the transition probabilities of the Markov chain.

b. Find its stationary distribution.

Hint: You can guess the stationary distribution before computing it.

3. **Product of Rolls of a Die**

A fair die with labels 1 through 6 is rolled until the product of the last two rolls is 12. What is the expected number of rolls?

Hint: You can model this process as a Markov chain with 3 states, choosing your states according to the outcome of last roll. For example, assign one state if its outcome was 1 or 5, which is useless if you want the product to be 12. If the outcome was 2, 3, 4 or 6, it's useful and can be assigned to another state. Assign a third state to the case when the product of the last two outcomes was 12.