UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 10
Fall 2023

1. **Poisson Process Arrival Times**

   Consider a Poisson process $(N_t)_{t\geq 0}$ with rate 1. Let T_k be the time of the kth arrival, $k \geq 1$.

   a. Find $\mathbb{E}(T_3 \mid N_1 = 2)$.
   b. Given $T_3 = s$, where $s > 0$, find the joint distribution of T_1 and T_2.
   c. Find $\mathbb{E}(T_2 \mid T_3 = s)$.

2. **Poisson Process Practice**

Let $(N_t)_{t \geq 0}$ be a Poisson process with rate λ. Let T_k, $k \geq 1$ denote the time of the kth arrival. Given $0 \leq s < t$, we write $N(s,t) \coloneqq N(t) - N(s)$. Compute the following:

a. $\mathbb{P}(N(1) + N(2,4) + N(3,5) = 0)$.
b. $\mathbb{E}(N(1,3) \mid N(1,2) = 3)$.
c. $\mathbb{E}(T_2 \mid N(2) = 1)$.

3. **Poisson Process Warmup**

   Give an interpretation of the following fact in terms of a Poisson process with rate λ. If N is Geometric with parameter p and $(X_k)_{k\in\mathbb{N}}$ are i.i.d. Exponential(λ), then $X_1 + \cdots + X_N$ has an Exponential distribution with parameter λp.