UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Homework 08
Fall 2023

1. **Random Walk on an Undirected Graph**

   Consider a random walk on an undirected connected finite graph (that is, define a Markov chain where the state space is the set of vertices of the graph, and at each time step, transition to a vertex chosen uniformly at random out of the neighborhood of the current vertex). What is the stationary distribution π? Your answer may depend on $\deg(v)$ (i.e., the degree of a vertex v) for some v. *Hint*: assume first that the chain is *reversible*.

2. **Recurrence and Transience of Random Walks**

"A drunk man will find his way home, but a drunk bird may get lost forever."
— Shizuo Kakutani

Consider the symmetric random walk $S_n = X_1 + \cdots + X_n$ in d dimensions, in which we start at the origin, and in each time step jump to an adjacent point on the d-dimensional lattice $\mathbb{Z}^d$ with uniform probability. That is,

$$X_i \sim_{\mathsf{i.i.d.}} \text{Uniform}\{\pm e_1, \ldots, \pm e_d\},$$

where $\{e_1, \ldots, e_d\}$ are the unit vectors in $\mathbb{Z}^d$.

a. Show that if $\sum_{n=0}^{\infty} \mathbb{P}(S_n = 0) = \infty$, then the random walk is recurrent.
   Hint: Let N be the number of times the random walk visits the origin. It may help to notice that $\mathbb{E}(N) = \infty$ is equivalent to recurrence of the random walk.
b. Use part a to show that the random walk for $d = 1$ is recurrent. You may use Stirling's approximation, where $f(n) \sim g(n)$ indicates $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1$:
   $$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n.$$
c. Use part b to show that the random walk for $d = 2$ is recurrent.
   Hint: consider two independent 1-dimensional random walks in orthogonal directions.
d. **Optional**. Show that the random walk for $d = 3$ is transient.
e. Use part d to show that the random walk for any $d > 3$ is also transient.

3. **Customers in a Store**

   Consider two independent Poisson processes with rates λ_1 and λ_2, which measure the number of customers arriving in store 1 and 2.

   a. What is the probability that a customer arrives in store 1 before any arrives in store 2?
   b. What is the probability that in the first hour, a total of exactly 6 customers arrive in the two stores?
   c. Given that exactly 6 have arrived in total at the two stores, what is the probability that exactly 4 went to store 1?