UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 6
Fall 2023

1. **Chernoff Bound**

   Prove that

   - $P(X > t) \leq \frac{M_X(\lambda)}{e^{\lambda t}}$ $\forall \lambda > 0$, and $P(X < t) \leq \frac{M_X(\lambda)}{e^{\lambda t}}$ $\forall \lambda < 0$.
   - For $X \sim \mathcal{N}(\mu, \sigma^2)$, upper bound the probability of deviation from mean $P(|X - \mu| > t)$.

2. **Exponential Bounds**

   Let $X \sim \text{Exponential}(\lambda)$. For $x > \lambda^{-1}$, find bounds on $\mathbb{P}(X \geq x)$ using Markov's inequality, Chebyshev's inequality, and the Chernoff bound.

3. **Almost Sure Convergence Implies Convergence in Probability**

   For random variables $X_1, X_2, \ldots$ and X on a common probability space $(\Omega, \mathcal{F}, P)$, prove that if $X_n \overset{\text{a.s.}}{\to} X$ then $X_n \overset{\text{p}}{\to} X$.