UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Homework 09
Fall 2023

1. **System Shocks**

   For a positive integer n, let $X_1, \ldots, X_n$ be independent Exponentially distributed random variables, each with mean 1. Let $\gamma > 0$. A system experiences shocks at times $k = 1, \ldots, n$, and the size of the shock at time k is X_k.

   a. Suppose that the system fails if any shock exceeds the value γ. What is the probability of system failure?
   b. Suppose instead that the effect of the shocks is cumulative, i.e. the system fails when the total amount of shock received exceeds γ. What is the probability of system failure?

2. **Basketball II**

   Captain America and Superman are playing an untimed basketball game in which the two players score points according to independent Poisson processes with rates λ_C and λ_S respectively. The game is over when one player has scored k more points than the other.

   a. Suppose $\lambda_C = \lambda_S$, and suppose Captain America has a head start of $m < k$ points. Find the probability that Captain America wins.
   Hint: if $\alpha_i = \frac{1}{2}\alpha_{i-1} + \frac{1}{2}\alpha_{i+1}$, then $\alpha_{i+1} - \alpha_i = \alpha_i - \alpha_{i-1}$.

   b. Keeping the assumptions, find the expected time $\mathbb{E}(T)$ it will take for the game to end.
   Hint: consider the telescoping sum $\beta_j = \beta_0 + (\beta_1 - \beta_0) + \cdots + (\beta_j - \beta_{j-1})$.

3. **Illegal U-Turns**

   Each morning, as you pull out of your driveway, you would like to make a U-turn rather than drive around the block. Unfortunately, U-turns are illegal, and police cars drive by according to a Poisson process with rate λ. You decide to make a U-turn once you see that the road has been clear of police cars for time $\tau > 0$. Let N be the number of police cars you see before you make a U-turn.

   a. Find $\mathbb{E}(N)$.

   b. Let $n \geq 2$. Find the conditional expectation of the time elapsed between police cars $n-1$ and n, given that $N \geq n$.

   c. Find the expected time that you wait until you make a U-turn.