UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: PROBABILITY AND RANDOM PROCESSES

Discussion 4
Fall 2023

1. **Covariance Matrix**

   For a random vector $X = [X_1, X_2, \ldots, X_n]^\mathsf{T}$, its covariance matrix Σ is defined with entries $\Sigma_{ij} = \mathrm{Cov}(X_i, X_j)$. Suppose that $\mathbb{E}[X] = 0$.

   a. Show that Σ is positive semi-definite, i.e. for all $v \in \mathbb{R}^n$, we have $v^\mathsf{T} \Sigma v \geq 0$.

   b. Show that if the X_i's are *pairwise* independent, then Σ is diagonal.

   c. Give an example of two random variables X_1, X_2 with a diagonal covariance matrix, but such that X_1, X_2 are not independent.