UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 3
Fall 2023

1. **Uncorrelatedness and Independence**

   a. Show that if $X_1, \ldots, X_n$ are pairwise uncorrelated, then

   $$\operatorname{var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \operatorname{var}(X_i).$$

   b. Find an example where a pair of random variables are uncorrelated but not independent.

2. **Galton–Watson Branching Process**

Consider a population of N individuals for some positive integer N. Let ξ be a random variable taking values in $\mathbb{N}$ with $\mathbb{E}(\xi) = \mu$ and $\text{var}(\xi) = \sigma^2$. At the end of each year, each individual, independently of all other individuals and generations, leaves behind a number of offspring which has the same distribution as ξ. For each $n \in \mathbb{N}$, let X_n denote the size of the population at the end of the nth year.

a. Compute $\mathbb{E}(X_n)$.

b. Compute $\text{var}(X_n | X_{n-1})$. Then, write $\text{var}(X_n)$ in terms of $\text{var}(X_{n-1})$.

3. **Minimum and Maximum of Exponentials**

   Let $\lambda_1, \lambda_2 > 0$, and $X_1 \sim \text{Exponential}(\lambda_1)$, $X_2 \sim \text{Exponential}(\lambda_2)$ are independent. Also, define $U := \min(X_1, X_2)$ and $V := \max(X_1, X_2)$. Show that U and $V - U$ are independent.