UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 12
Fall 2023

1. **Generating Erdős–Rényi Random Graphs**

   Let G_1 and G_2 be independent Erdős–Rényi random graphs on n vertices with probabilities p_1 and p_2 respectively. Let G be $G_1 \cup G_2$, that is, the graph generated by combining the edges in G_1 and G_2.

   a. Is G an Erdős–Rényi random graph on n vertices with probability $p_1 + p_2$?
   b. Is G an Erdős–Rényi random graph?

2. **Voltage MAP**

You are trying to detect whether voltage V_1 or voltage V_2 was sent over a channel with independent Gaussian noise $Z \sim N(V_3, \sigma^2)$. Assume that both voltages are equally likely to be sent.

   a. Derive the MAP detector for this channel.
   b. Using the Gaussian Q-function, determine the average error probability for the MAP detector.
   c. Suppose that the average transmit energy is $(V_1^2 + V_2^2)/2$ and that the average transmit energy is constrained such that it cannot be more than $E > 0$. What voltage levels V_1, V_2 should you choose to meet this energy constraint but still minimize the average error probability?

3. **Poisson Process MAP**

   Customers arrive to a store according to a Poisson process with rate 1. The store manager learns of a rumor that one of the employees is sending every other customer to the rival store, so that *deterministically*, every odd-numbered customer $1, 3, 5, \ldots$ is sent away.

   Let $X = 1$ be the hypothesis that the rumor is true and $X = 0$ the rumor is false, assuming that both hypotheses are equally likely. Suppose a customer arrives to the store at time 0. After that, the manager observes $T_1, \ldots, T_n$, where T_i is the time of the ith subsequent sale, $i = 1, \ldots, n$. Derive the MAP rule to determine whether the rumor was true or not.