



# STAT 453: Introduction to Deep Learning and Generative Models

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Ben Lengerich

Lecture 19: Recurrent Neural Networks

November 10, 2025

Reading: See course homepage



# Our semester

Week	Lecture Dates	Topic
Module 1: Introduction and Foundations		
1	9/3	Course Introduction
2	9/8, 9/10	A Brief History of DL, Statistics / linear algebra / calculus review
3	9/15, 9/17	Single-layer networks Parameter Optimization and Gradient Descent
4	9/22, 9/24	Automatic differentiation with PyTorch, Cluster and cloud computing resources
Module 2: Neural Networks		
5	9/29, 10/1	Multinomial logistic regression, Multi-layer perceptrons and backpropagation
6	10/6, 10/8	Regularization Normalization / Initialization
7	10/13, 10/15	Optimization, Learning Rates CNNs
8	10/20, 10/22	Review, <b>Midterm Exam</b>

Module 3: Intro to Generative Models			
9	10/27, 10/29	A Linear Intro to Generative Models, Factor Analysis, Autoencoders, VAEs	
10	11/3, 11/5	Generative Adversarial Networks, Diffusion Models	Project Midway Report
Module 4: Large Language Models			
11	11/10, 11/12	Sequence Learning with RNNs Attention, Transformers	HW4
12	11/17, 11/19	GPT Architectures, Unsupervised Training of LLMs	
13	11/24, 11/26	Supervised Fine-tuning of LLMs, Prompts and In-context learning	HW5
14	12/1, 12/3	Foundation models, alignment, explainability Open directions in LLM research	
15	12/8, 12/10	<b>Project Presentations</b>	Project Final Report
16	12/17	<b>Final Exam</b>	Final Exam



# A quick vote...

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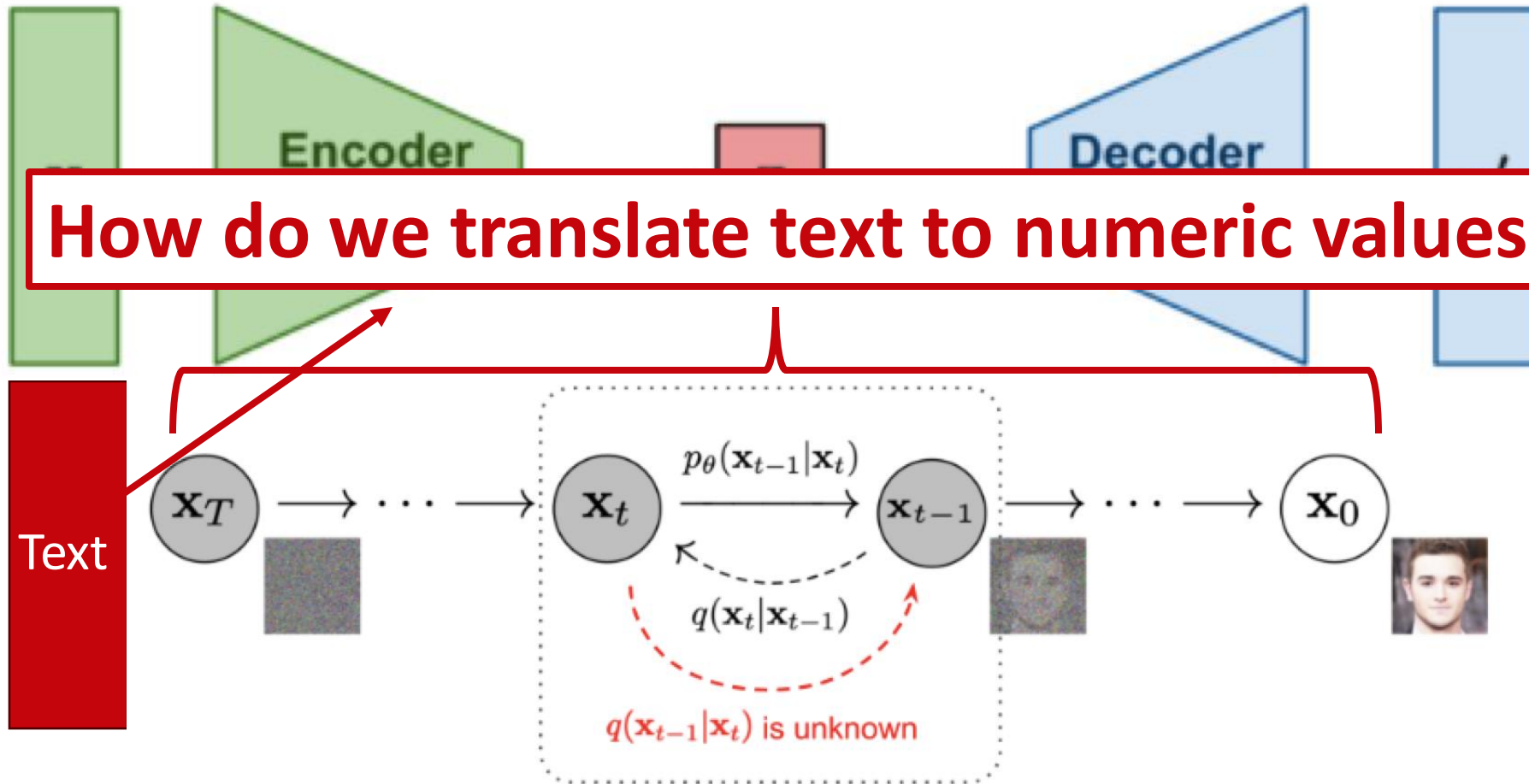




# HW4

- Released on [the website](#)
- Due next Friday
- Auto-encoder (4 parts) + bonus GAN
- We recommend Colab

# Recall “Conditioning on Text”...



# Challenges with text

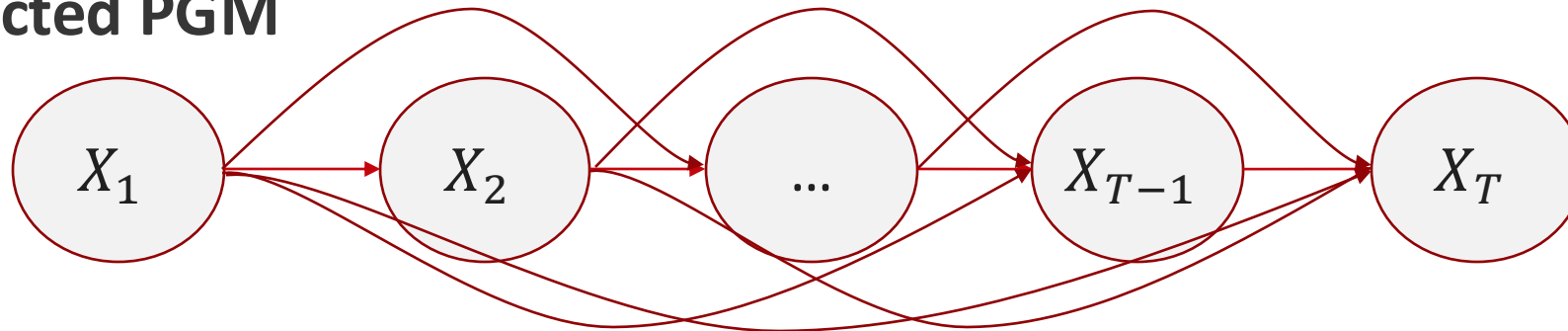
- Variable length input
- Long-range dependencies
- Want a generative model
- Scale
- Emergent properties of large language models

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# Where we're going

GPT = Auto-regressive probabilistic model

- **Directed PGM**



$$P_{\theta}(X) = \prod_i \prod_t P_{\theta}(X_{i,t} | X_{i,<t})$$

- **Probabilistic objective:** Max log-likelihood of observed seqs

$$\max_{\theta} \sum_i \sum_t \log P_{\theta}(X_{i,t} | X_{i,<t})$$

[Radford et al., [Improving Language Understanding by Generative Pre-Training](#)]



# Today

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- **Different Ways to Model Text**
- Sequence Modeling with RNNs
- Different Types of Sequence Modeling Tasks
- Backpropagation Through Time
- Long-Short Term Memory (LSTM)
- Many-to-one Word RNNs



# A classic approach: Bag-of-words

## "Raw" training dataset

$\mathbf{x}^{[1]}$  = "The sun is shining"

$\mathbf{x}^{[2]}$  = "The weather is sweet"

$\mathbf{x}^{[3]}$  = "The sun is shining,  
the weather is sweet, and  
one and one is two"

$\mathbf{y} = [0, 1, 0]$

class labels

vocabulary = {  
'and': 0,  
'is': 1  
'one': 2,  
'shining': 3,  
'sun': 4,  
'sweet': 5,  
'the': 6,  
'two': 7,  
'weather': 8,  
}

Training set as design matrix

$$\mathbf{X} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 2 & 3 & 2 & 1 & 1 & 1 & 2 & 1 & 1 \end{bmatrix}$$

$\mathbf{y} = [0, 1, 0]$

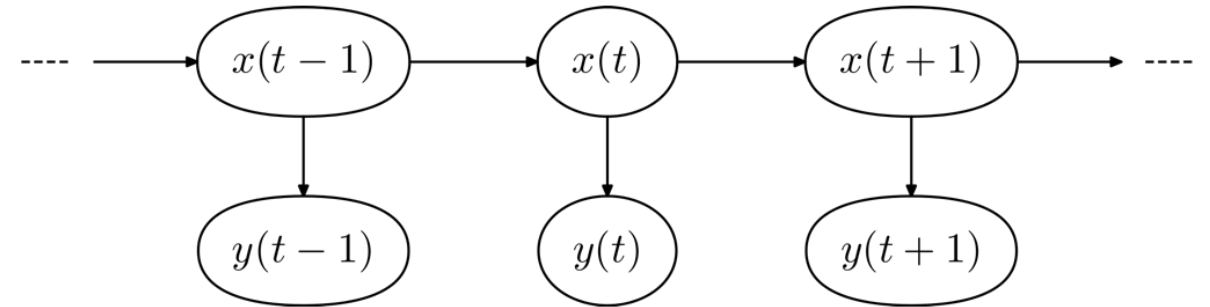
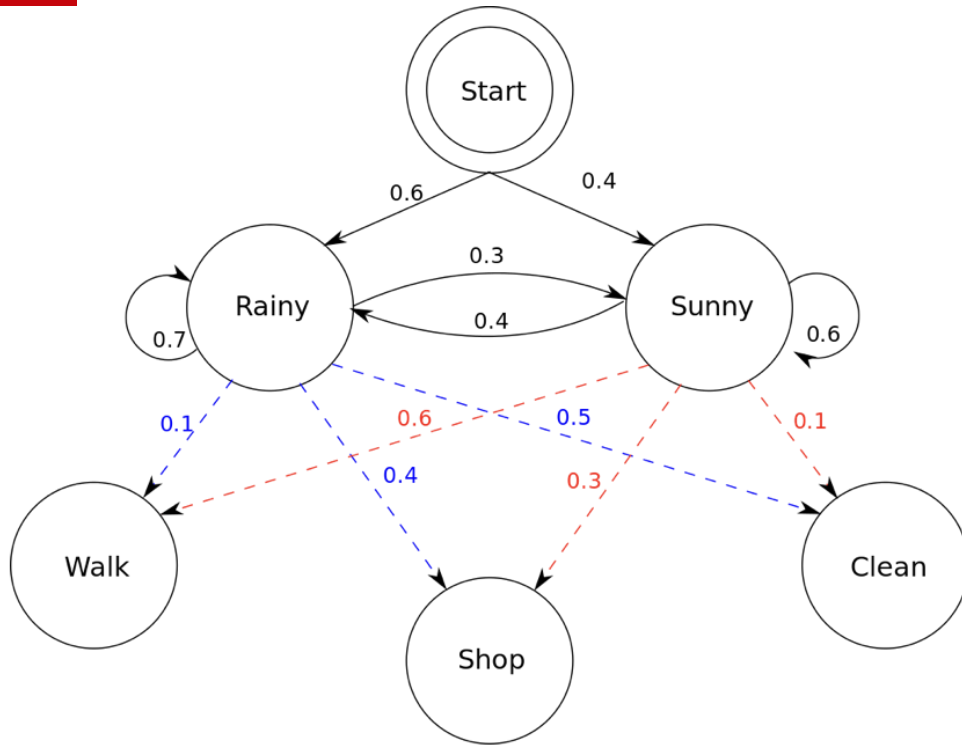
class labels

training

Classifier

(e.g., logistic regression, MLP, ...)

# Another classic approach: Hidden Markov Model

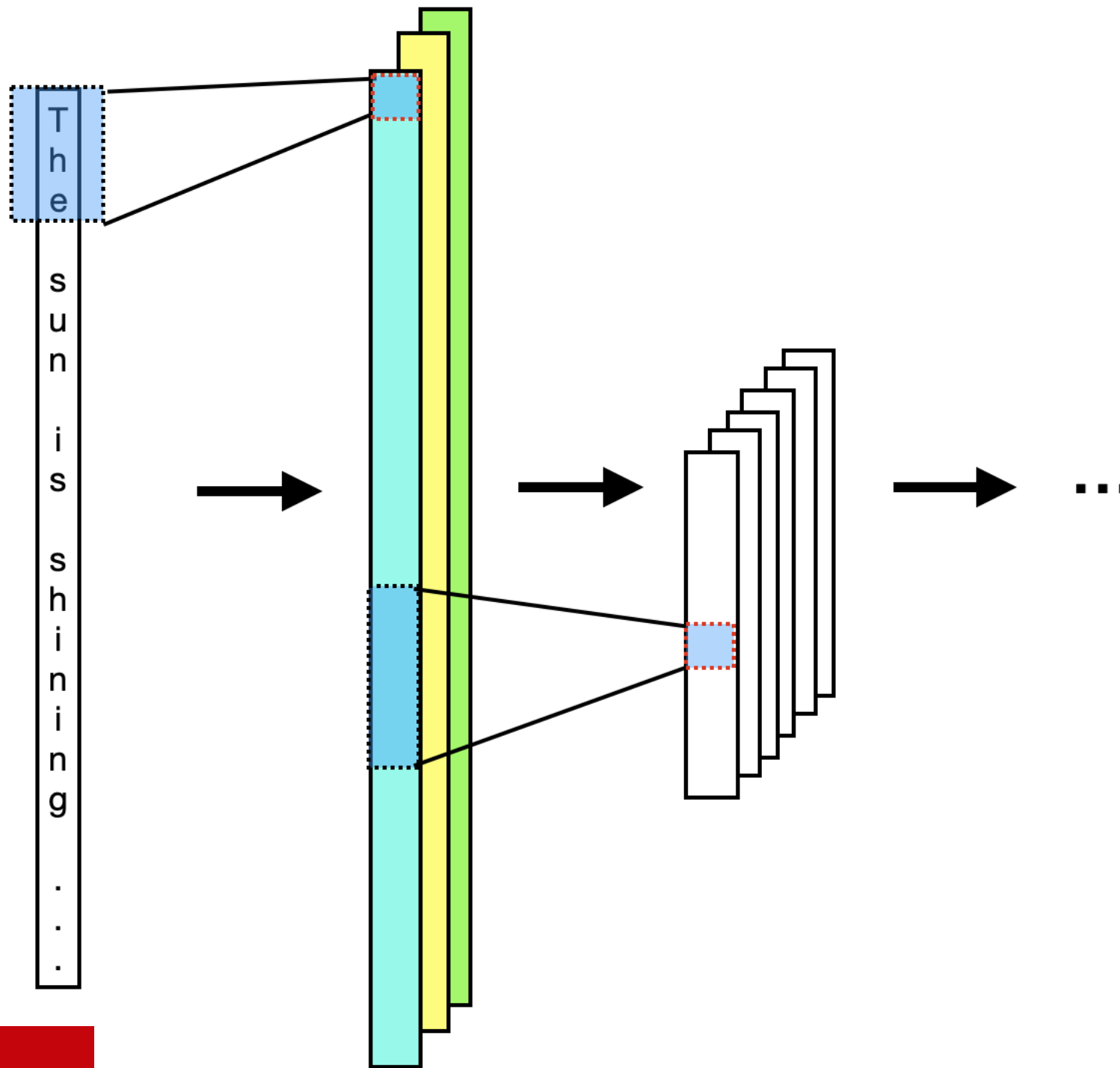


Wikipedia example: each day, weather follows a Markov chain, and activities are observables

$$\mathbb{P}(Y_n = y | X_1 = x_1, \dots, X_n = x_n) = \mathbb{P}(Y_n = y | X_n = x_n)$$

# Another approach: CNNs

Can't handle variable  
length input,  
→ need padding to max  
input length





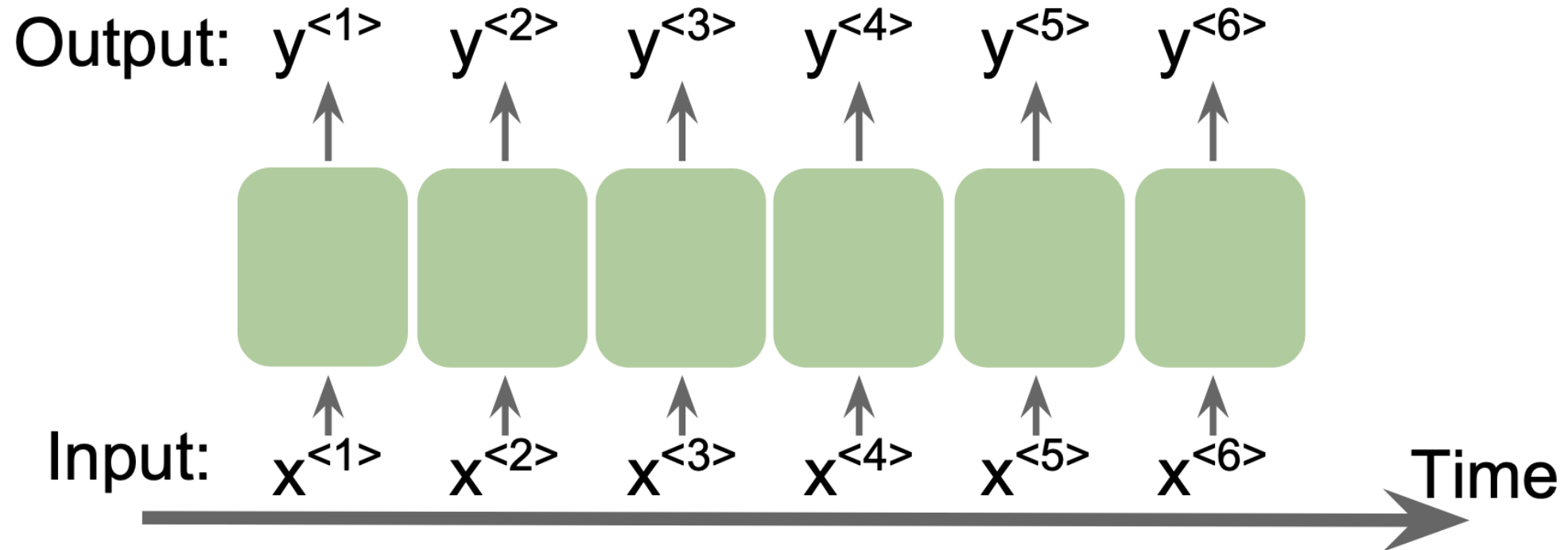
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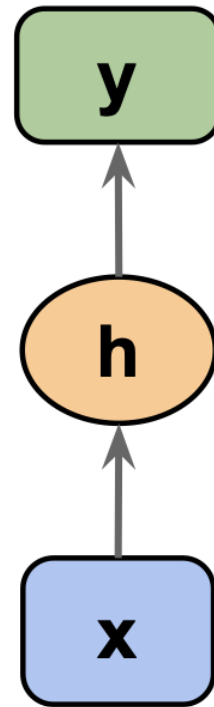
# Sequence data: order matters

The movie my friend has not seen is good  
 The movie my friend has seen is not good



# Recurrent Neural Networks (RNNs)

Networks we used previously: also called feedforward neural networks



Recurrent Neural Network (RNN)

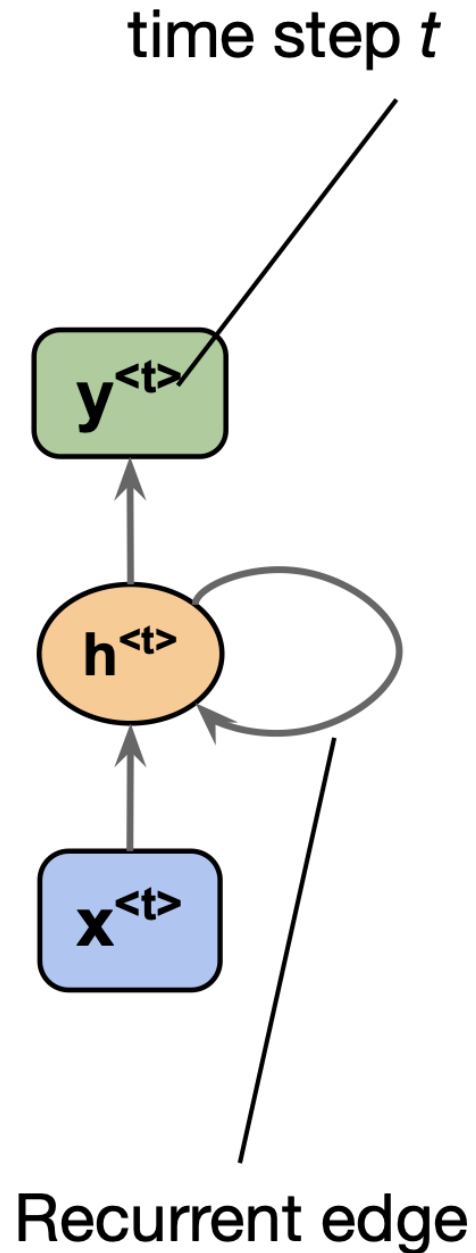


Image source: Sebastian Raschka, Vahid Mirjalili. Python Machine Learning. 3rd Edition. Packt, 2019

# Recurrent Neural Networks (RNNs)

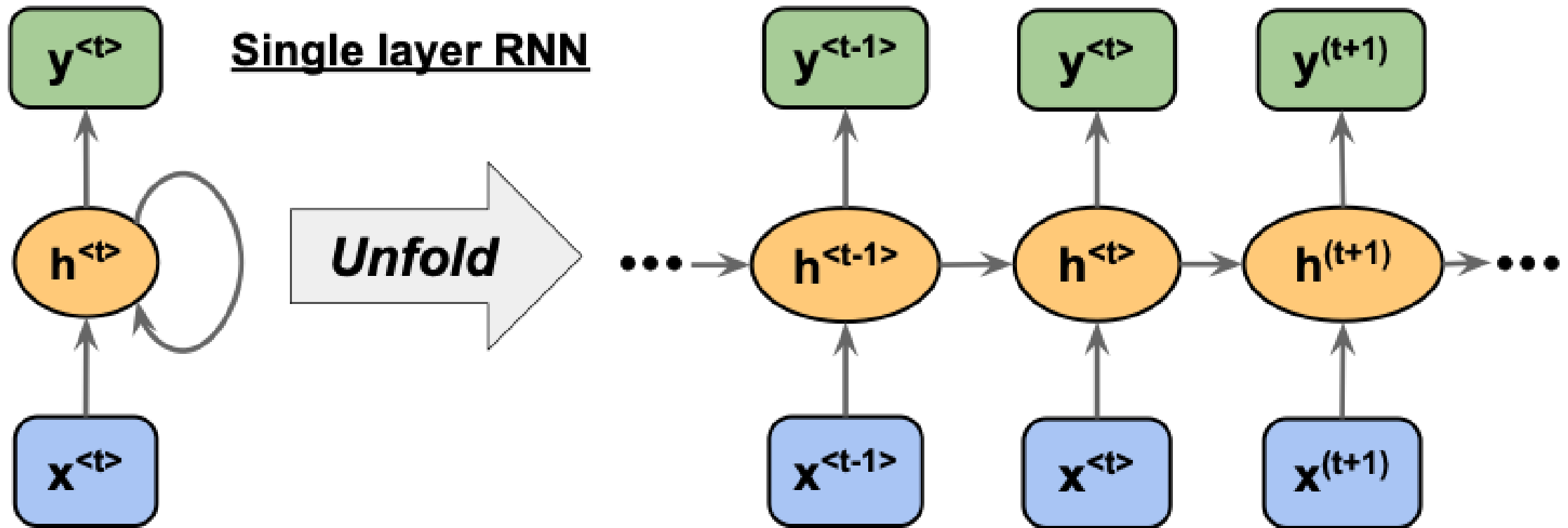


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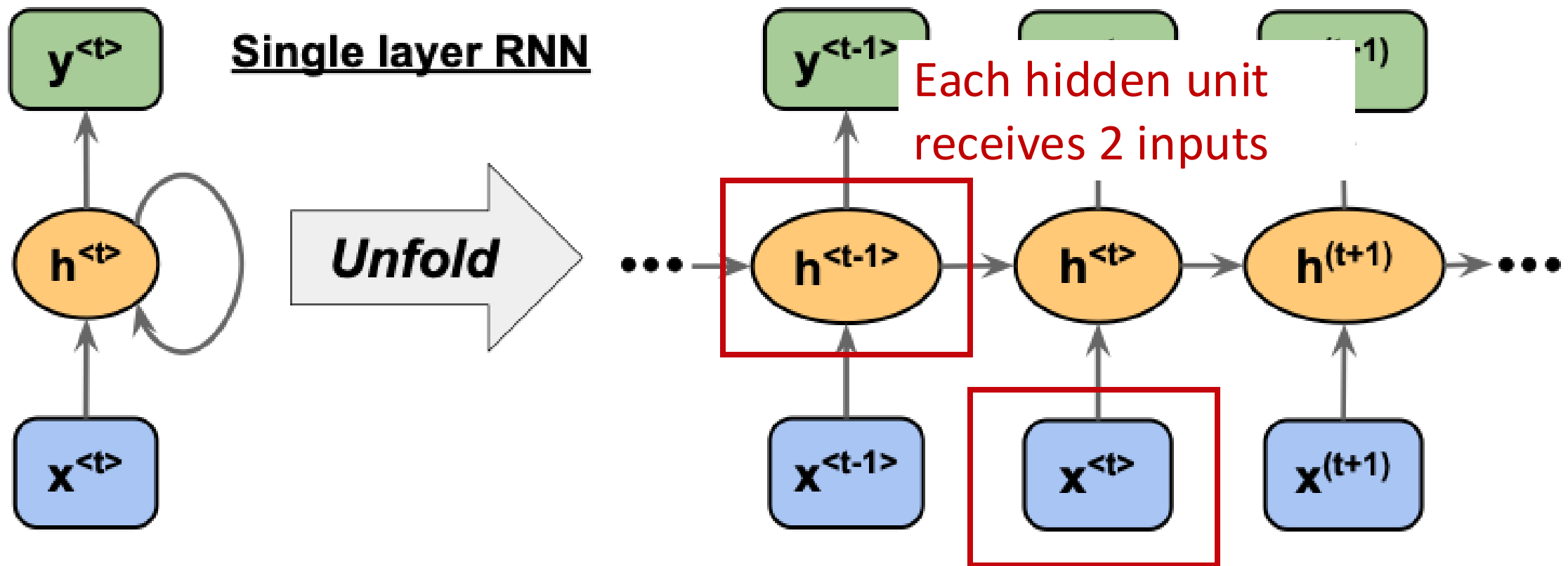


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# Multilayer RNNs

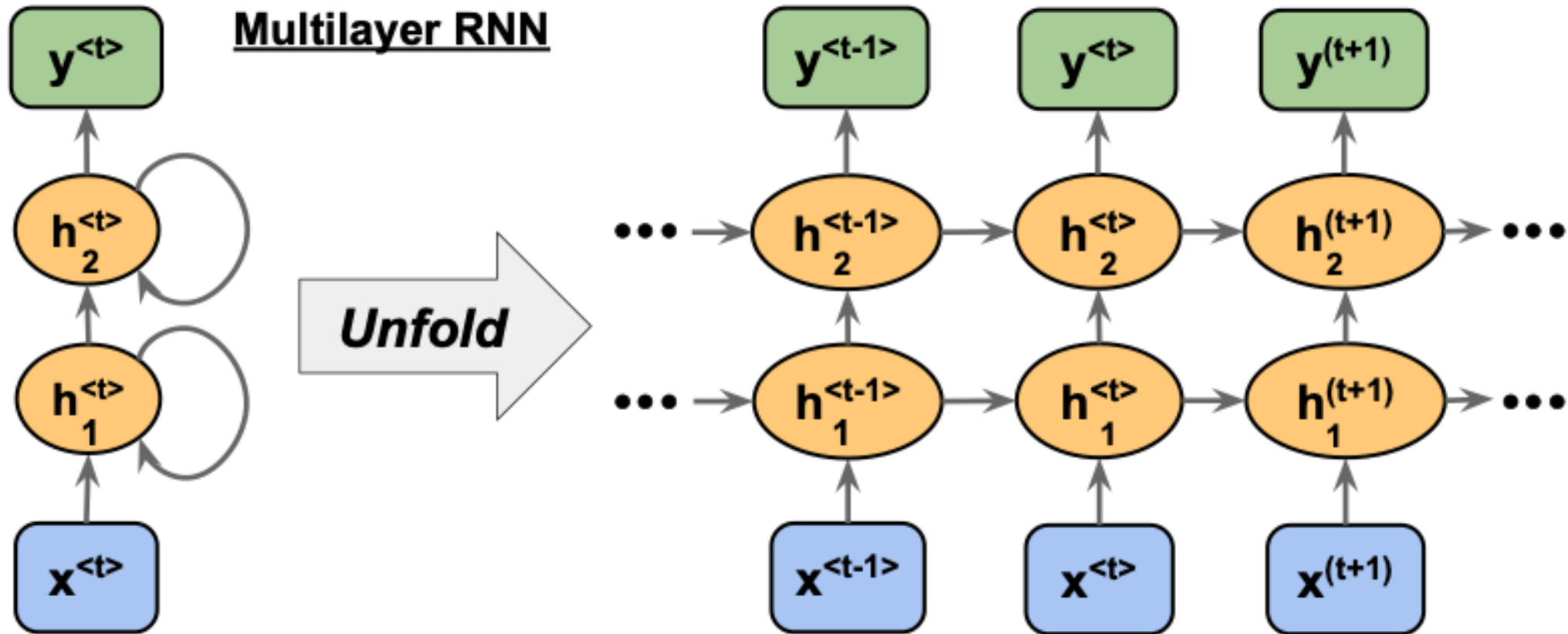


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# Recurrence unlocks many types of sequence tasks

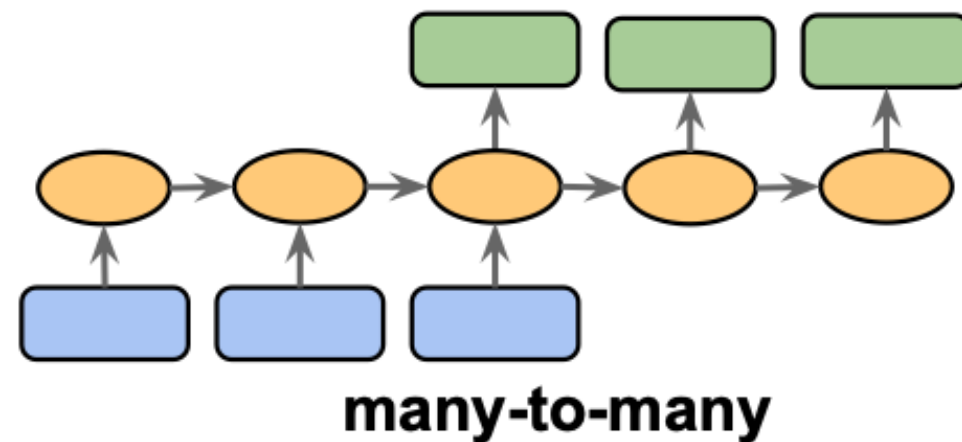
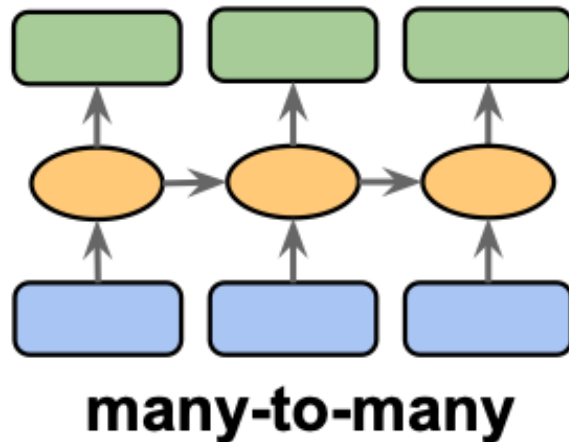
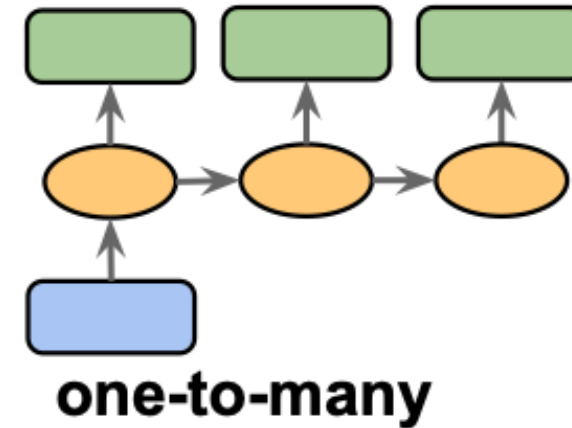
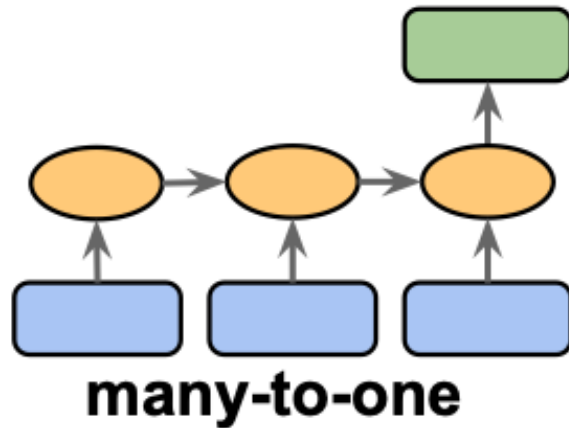


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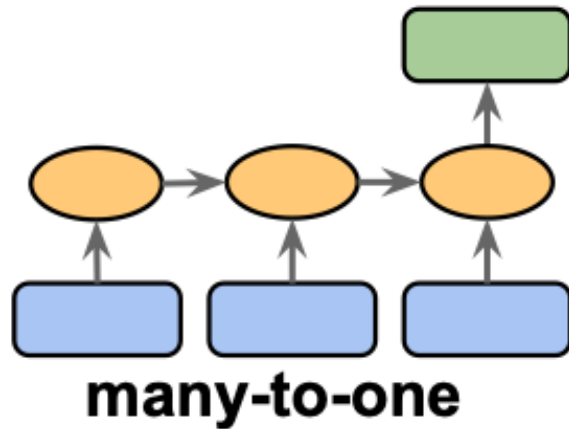


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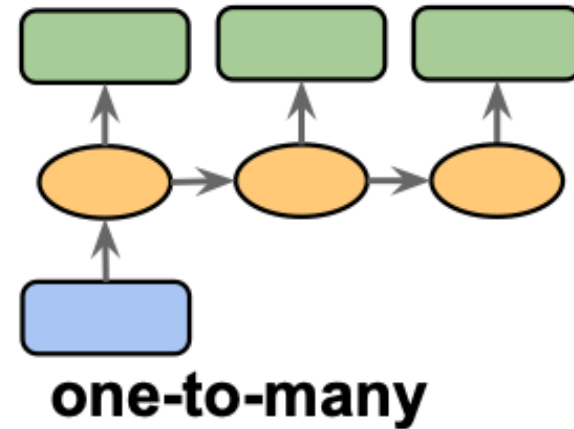
# Recurrence unlocks many types of sequence tasks



**Many-to-one:** The input data is a sequence, but the output is a fixed-size vector, not a sequence.

**Example:** sentiment analysis, the input is some text, and the output is a class label.

# Recurrence unlocks many types of sequence tasks



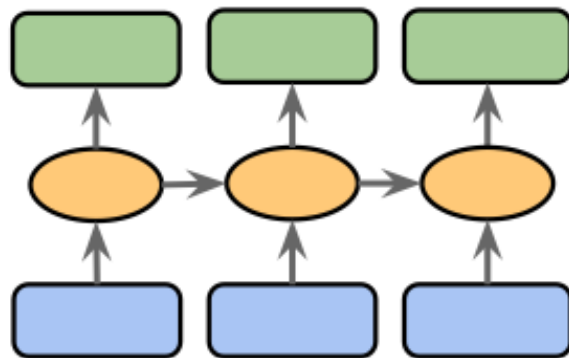
**One-to-many:** Input data is in a standard format (not a sequence), the output is a sequence.

**Example:** Image captioning, where the input is an image, the output is a text description of that image

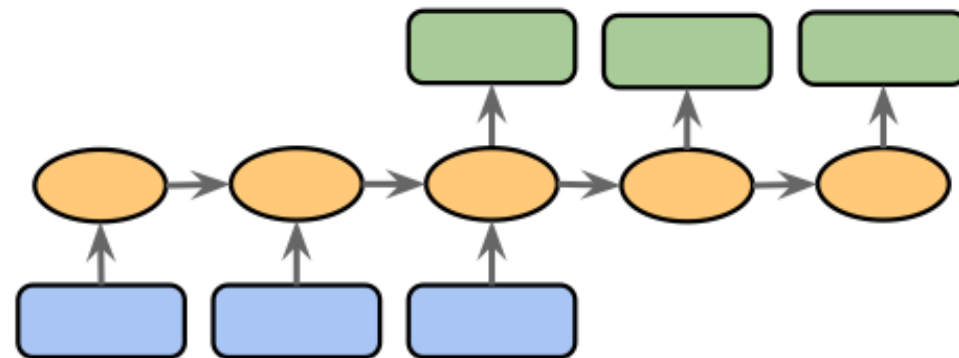
# Recurrence unlocks many types of sequence tasks

**Many-to-many:** Both inputs and outputs are sequences. Can be direct or delayed.

**Example:** Video-captioning, i.e., describing a sequence of images via text (direct). Translation.



**many-to-many**



**many-to-many**

Image source: Sebastian Raschka, Vahid Mirjalili. Python Machine Learning. 3rd Edition. Packt, 2019



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# Under the hood: weight matrices in an RNN

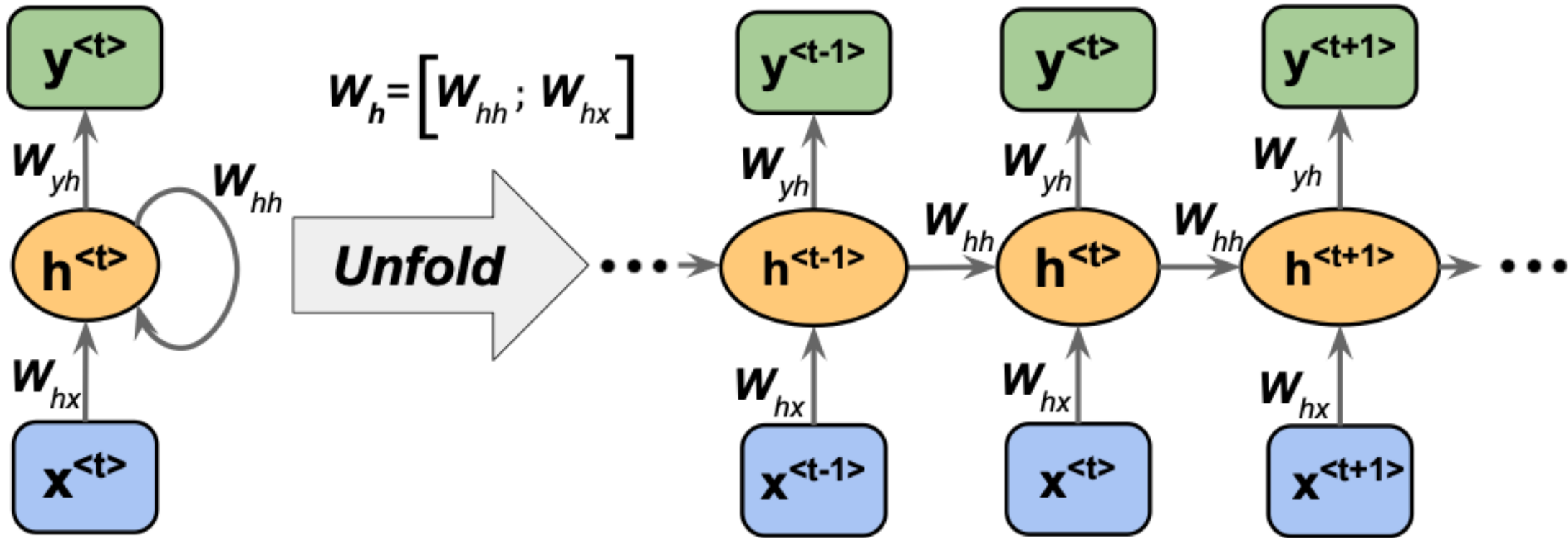
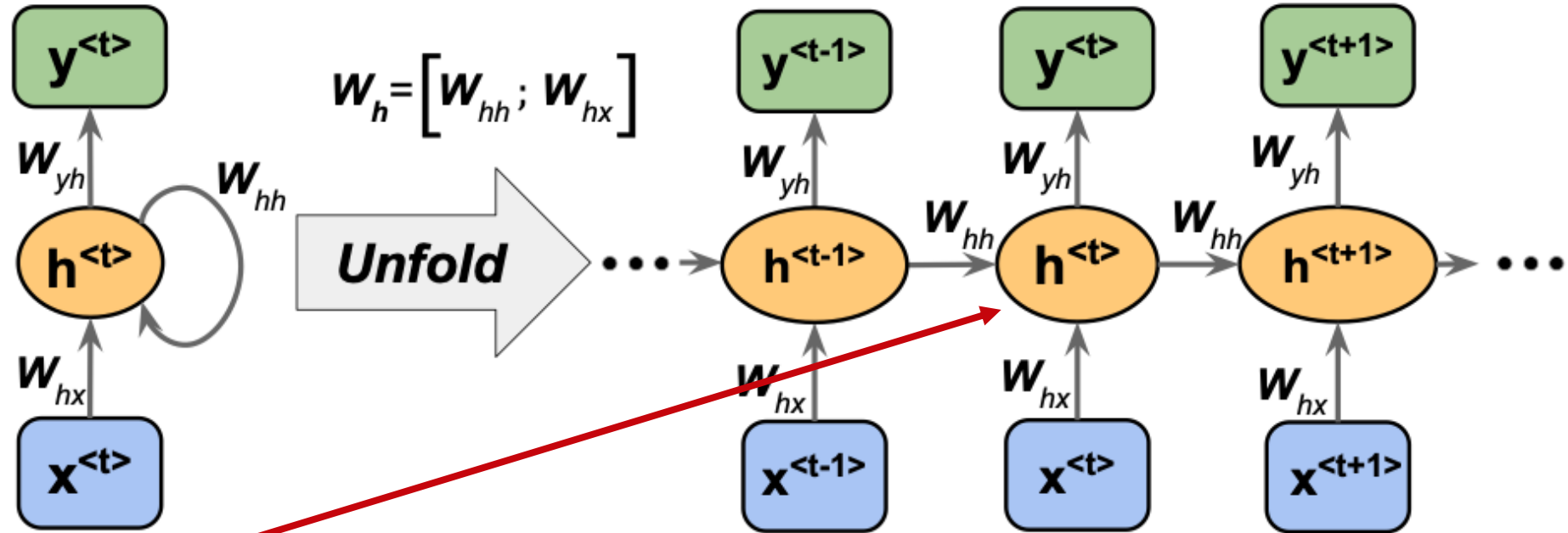


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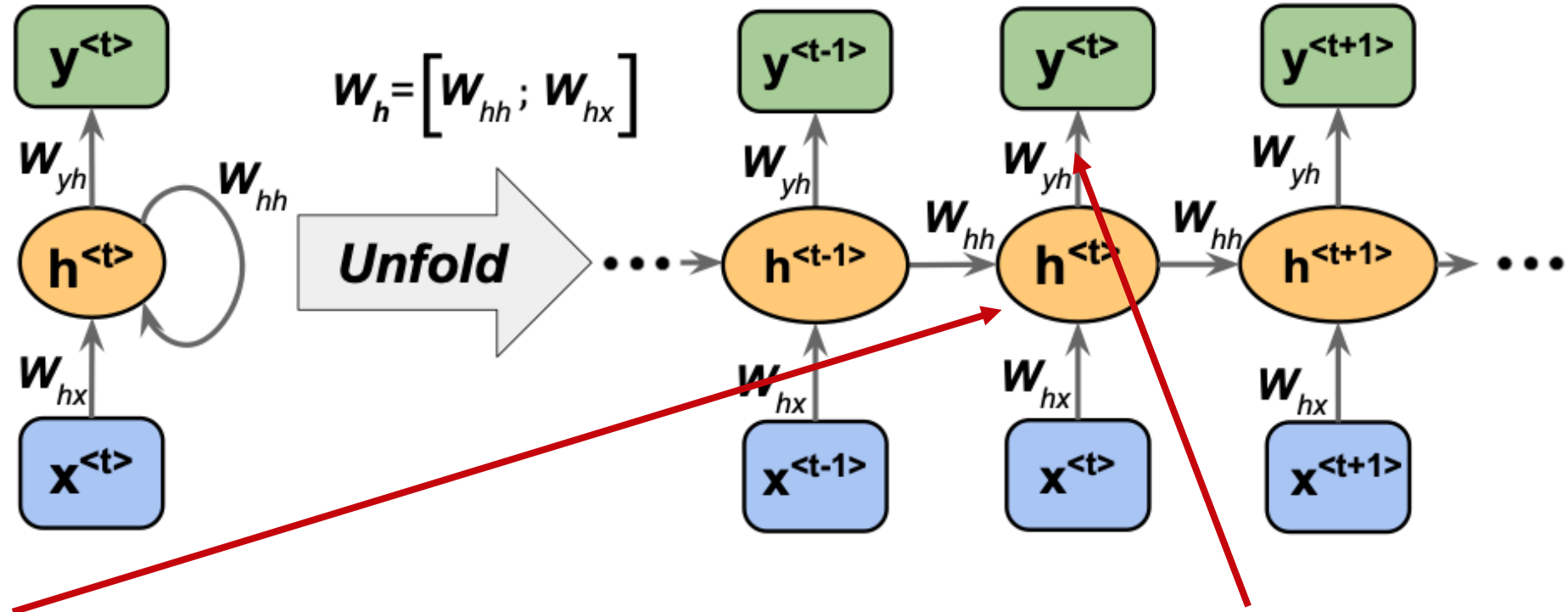
# Under the hood: weight matrices in an RNN



Net input:  $\mathbf{z}_h^{(t)} = \mathbf{W}_{hx} \mathbf{x}^{(t)} + \mathbf{W}_{hh} \mathbf{h}^{(t-1)} + \mathbf{b}_h$

Activation:  $\mathbf{h}^{(t)} = \sigma_h(\mathbf{z}_h^{(t)})$

# Under the hood: weight matrices in an RNN



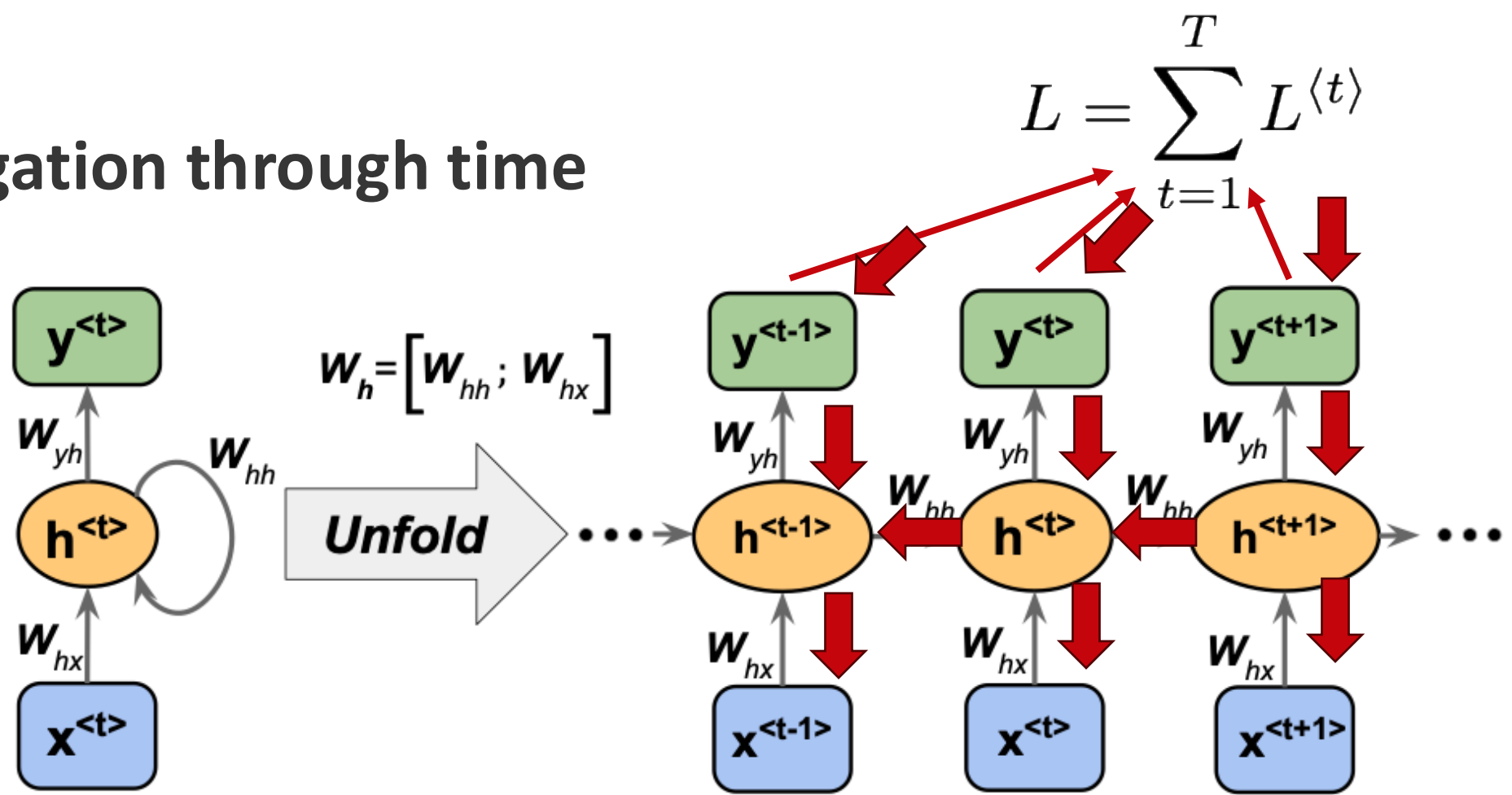
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Net input:  $\mathbf{z}_y^{(t)} = \mathbf{W}_{yh}\mathbf{h}^{(t)} + \mathbf{b}_y$

Output:  $\mathbf{y}^{(t)} = \sigma_y(\mathbf{z}_y^{(t)})$

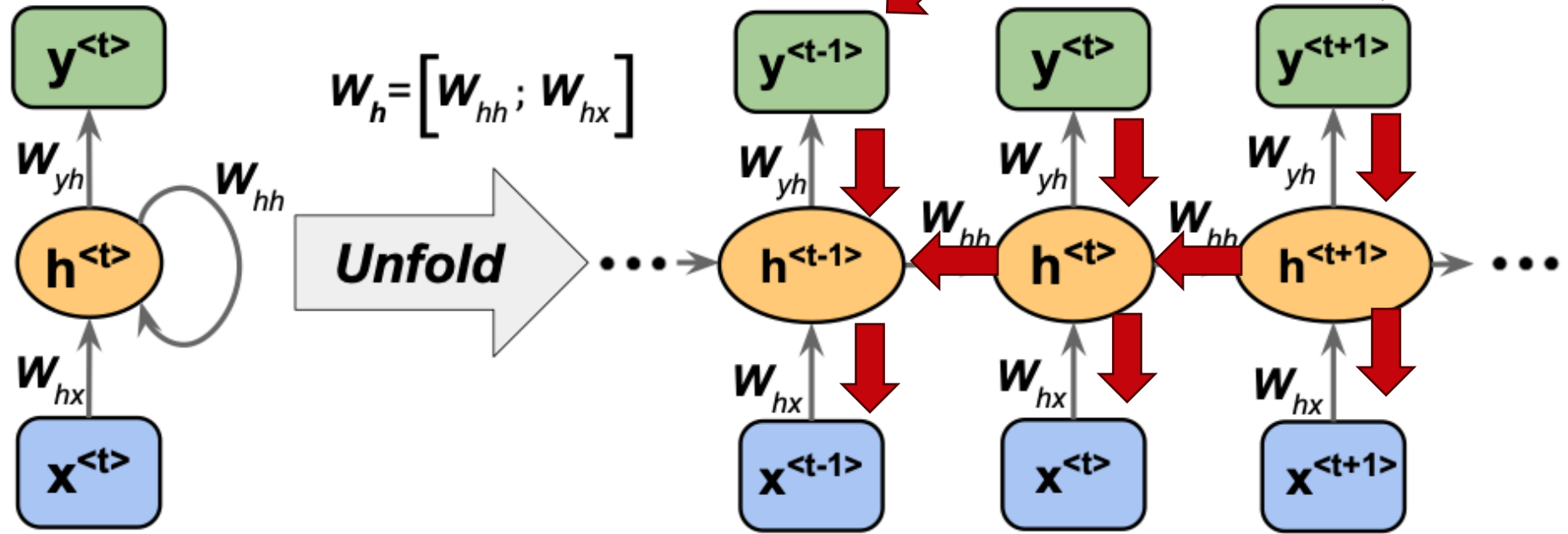
# Backpropagation through time



The overall loss can be computed as the sum over all time steps

# Backpropagation through time

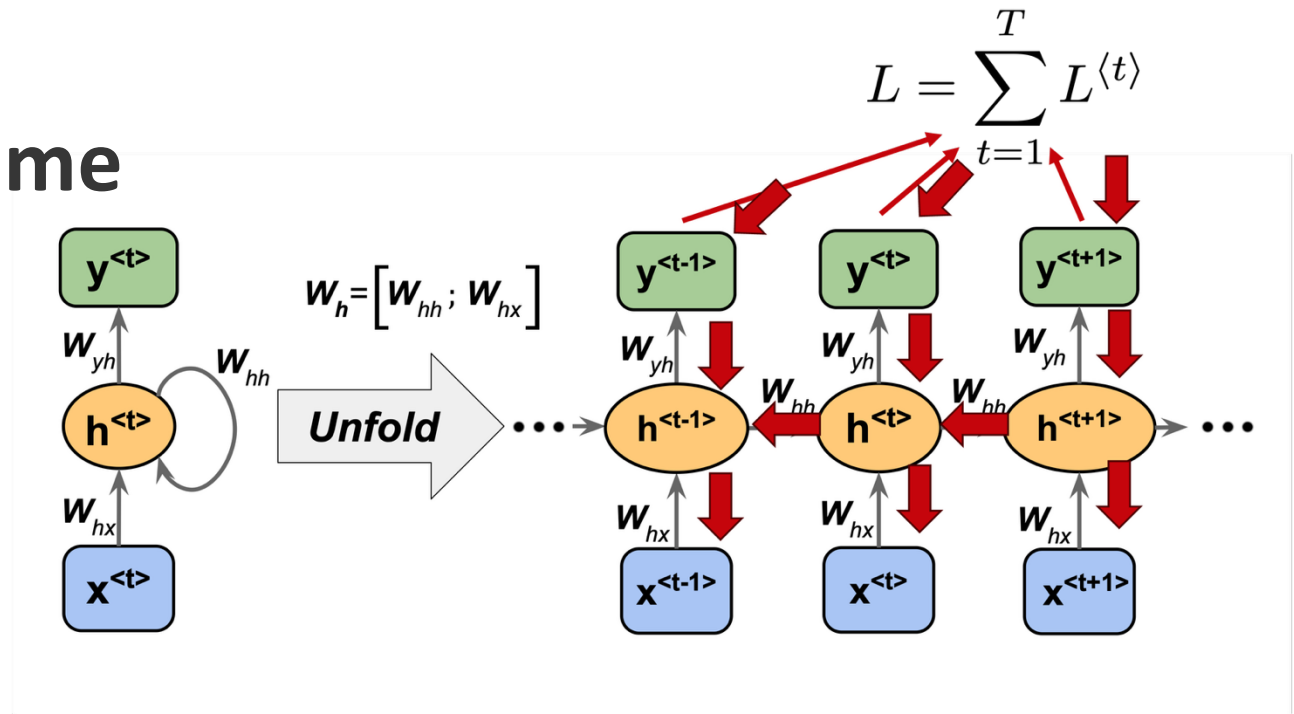
$$L = \sum_{t=1}^T L^{(t)}$$



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$$\frac{\partial L^{(t)}}{\partial \mathbf{W}_{hh}} = \frac{\partial L^{(t)}}{\partial y^{(t)}} \cdot \frac{\partial y^{(t)}}{\partial \mathbf{h}^{(t)}} \cdot \left( \sum_{k=1}^t \frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(k)}} \cdot \frac{\partial \mathbf{h}^{(k)}}{\partial \mathbf{W}_{hh}} \right)$$

# Backpropagation through time



Computed as a multiplication of adjacent time steps:

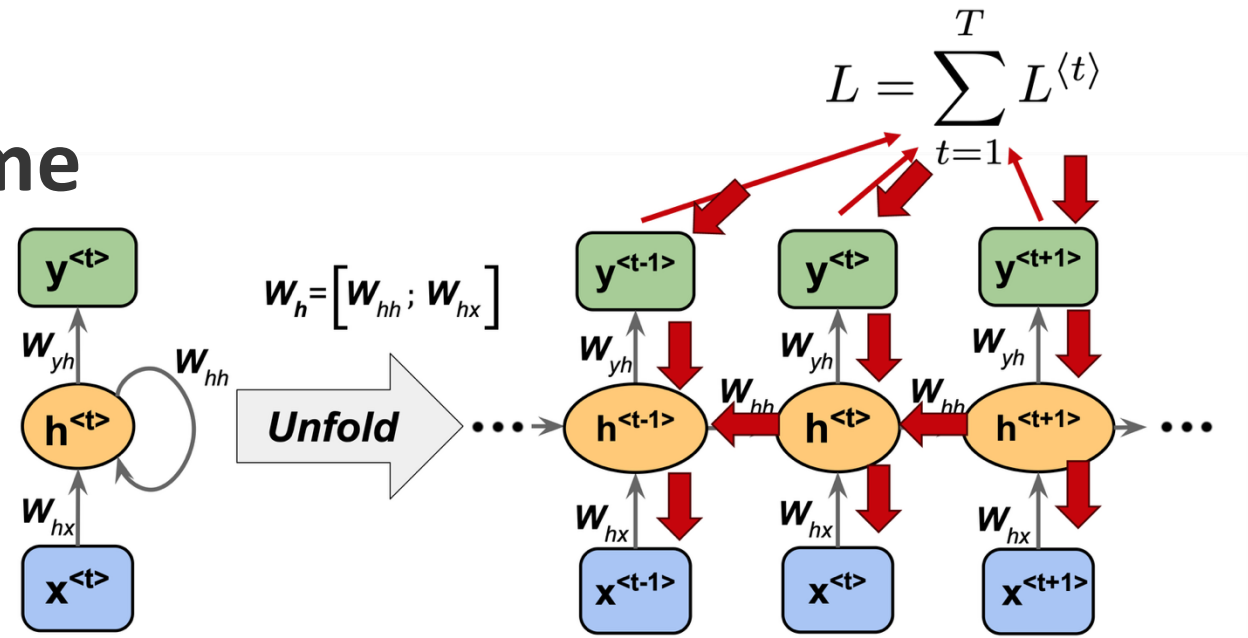
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$$\frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(k)}} = \prod_{i=k+1}^t \frac{\partial \mathbf{h}^{(i)}}{\partial \mathbf{h}^{(i-1)}}$$

# Backpropagation through time

Straightforward, but problematic:  
vanishing / exploding gradients!



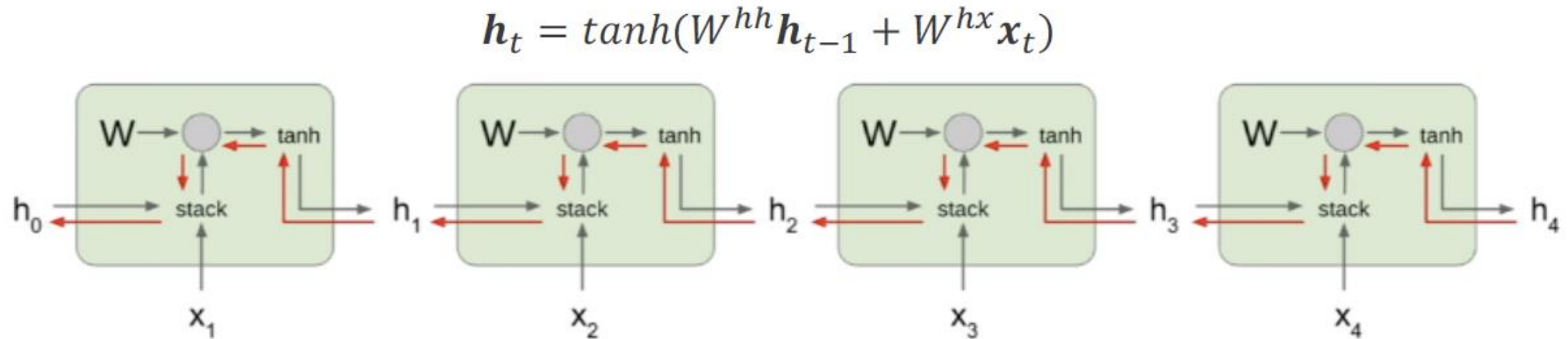
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$$\frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(k)}} = \prod_{i=k+1}^t \frac{\partial \mathbf{h}^{(i)}}{\partial \mathbf{h}^{(i-1)}}$$

# A challenge: Vanishing / exploding gradients



Computing gradient of  $h_0$  involves many factors of  $W$  (and repeated  $\tanh$ )

Largest singular value  $> 1$ :  
**Exploding gradients**

Largest singular value  $< 1$ :  
**Vanishing gradients**

**Gradient clipping:** Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
    grad *= (threshold / grad_norm)
```

*Bengio et al., 1994 "Learning long-term dependencies with gradient descent is difficult"*  
*Pascanu et al., 2013 "On the difficulty of training recurrent neural networks"*

# Solutions to Vanishing / Exploding Gradients

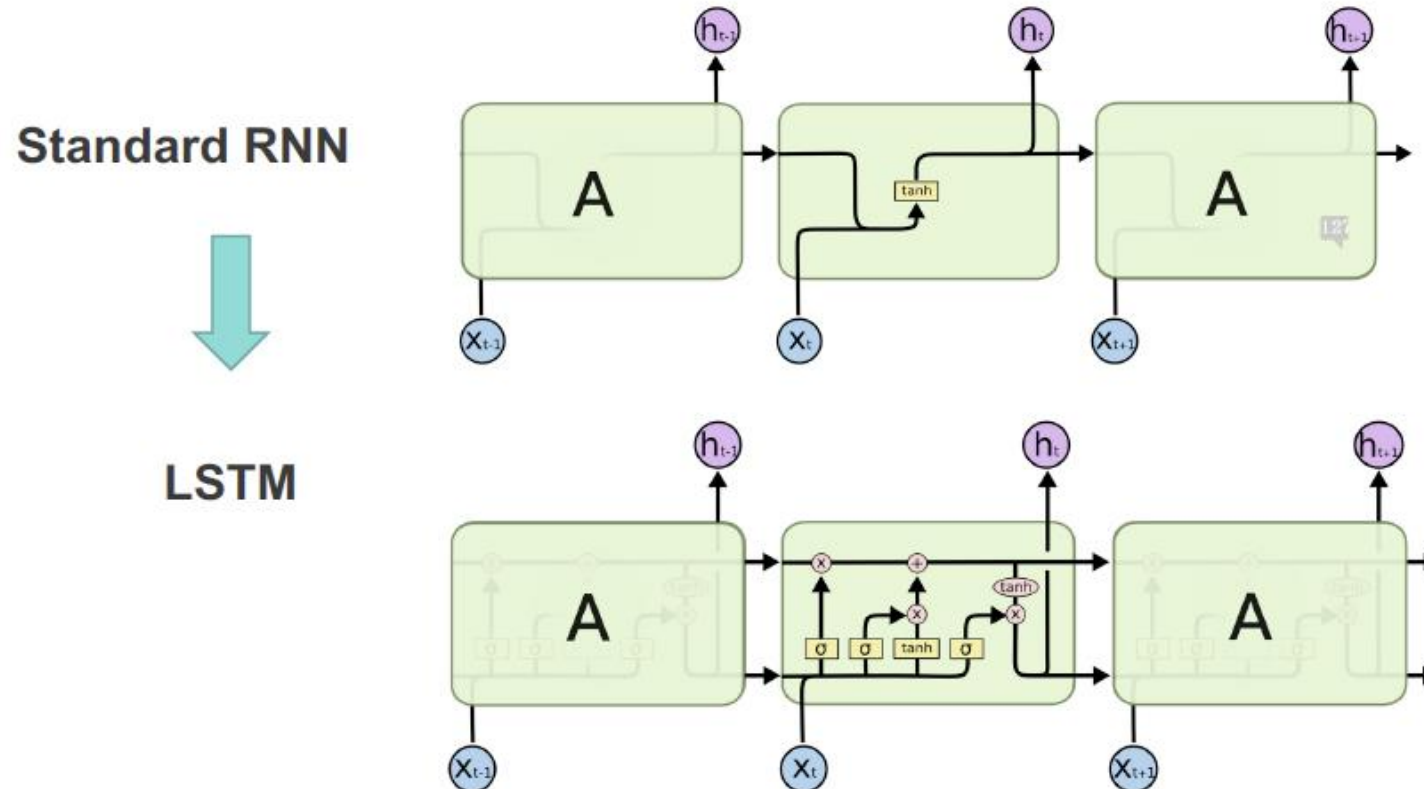
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- **Gradient Clipping:** set a max value for gradients if they grow to large (solves only exploding gradient problem)
- **Truncated backpropagation through time (TBPTT):** limit the number of time steps the signal can backpropagate after each forward pass. E.g., even if the sequence has 100 elements/steps, we may only backpropagate through 20 or so.



# Solutions to Vanishing / Exploding Gradients

**Long short-term memory (LSTM):** uses a *memory cell* for modeling long-range dependencies and avoid vanishing gradient problems





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- Many-to-one Word RNNs

# Long-short term memory (LSTM)

- Not an oxymoron: **2 paths** of memory

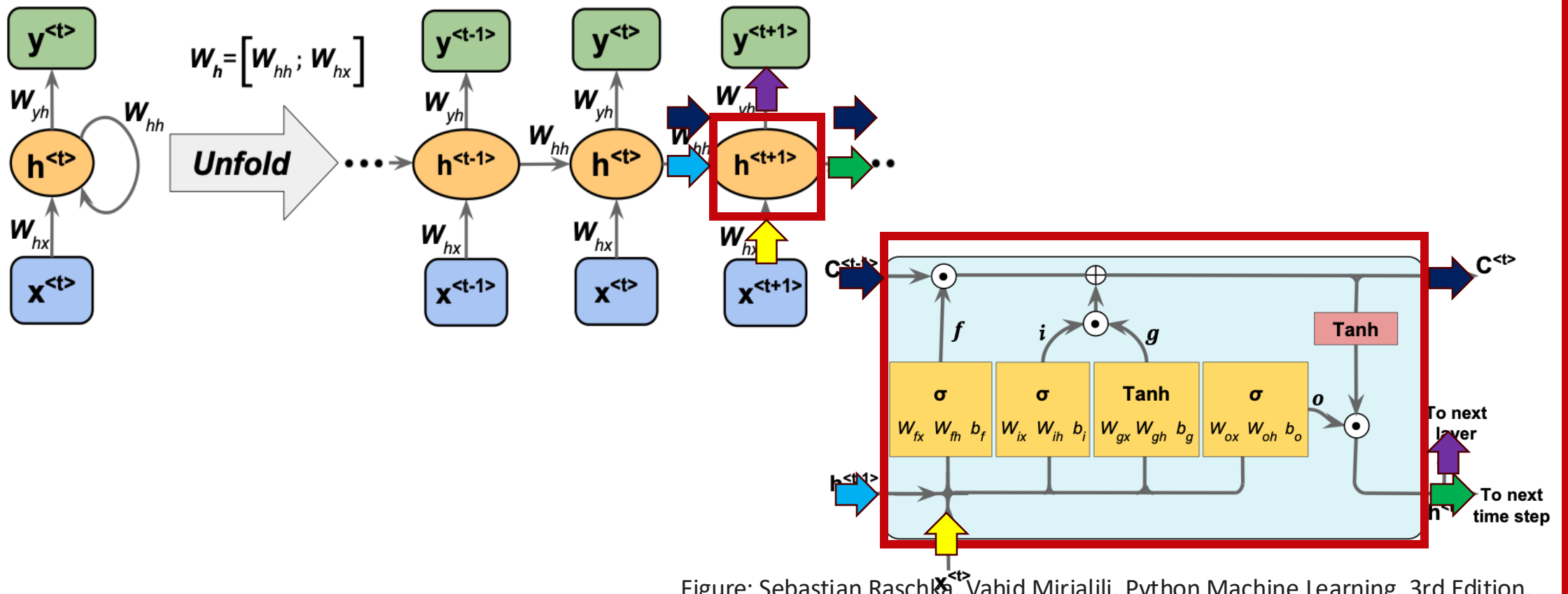


Figure: Sebastian Raschka, Vahid Mirjalili. Python Machine Learning. 3rd Edition. Birmingham, UK: Packt Publishing, 2019

# Long-short term memory (LSTM)

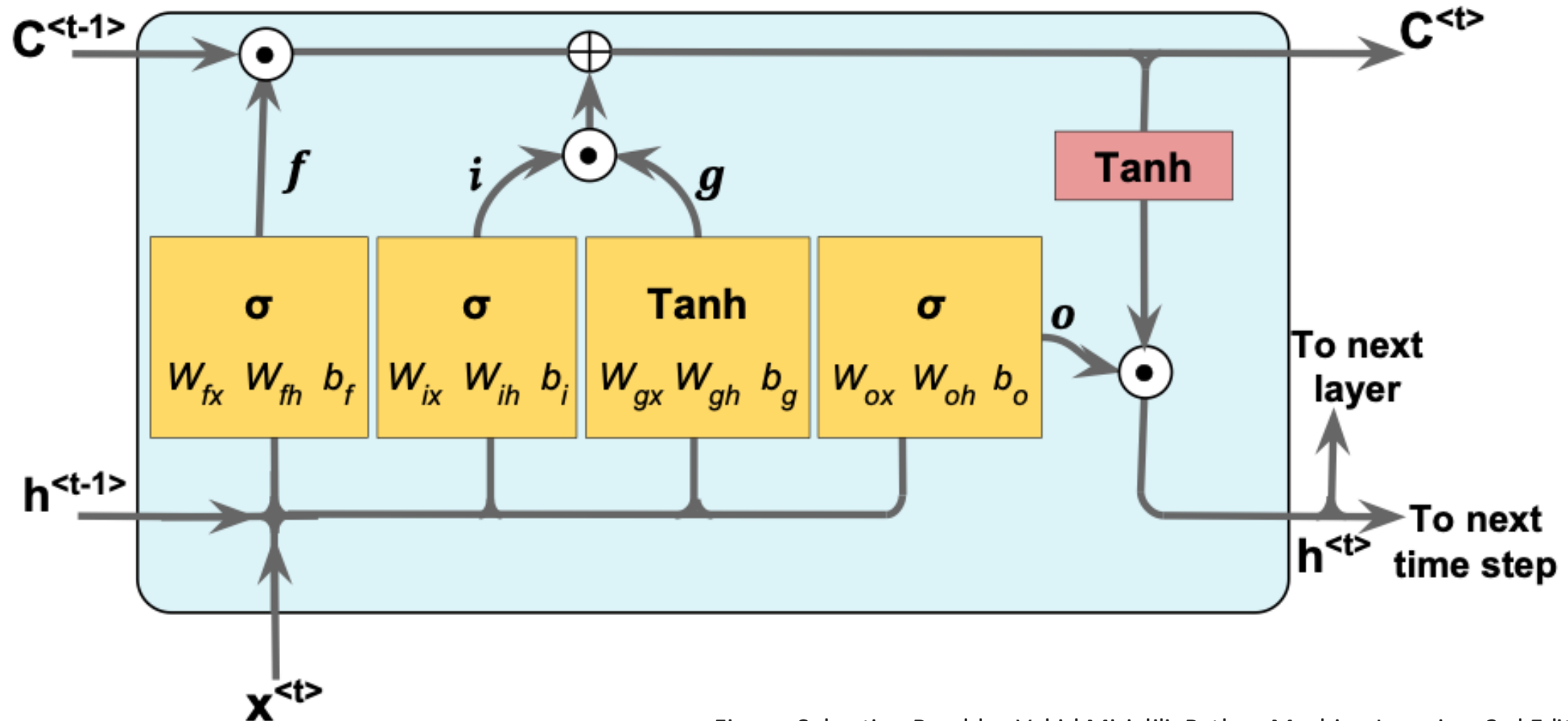


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# Inside LSTM

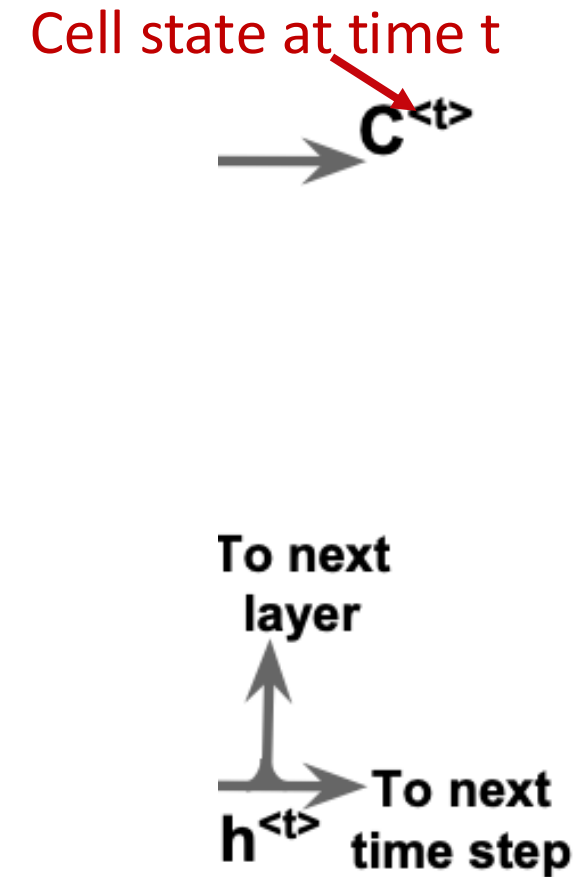
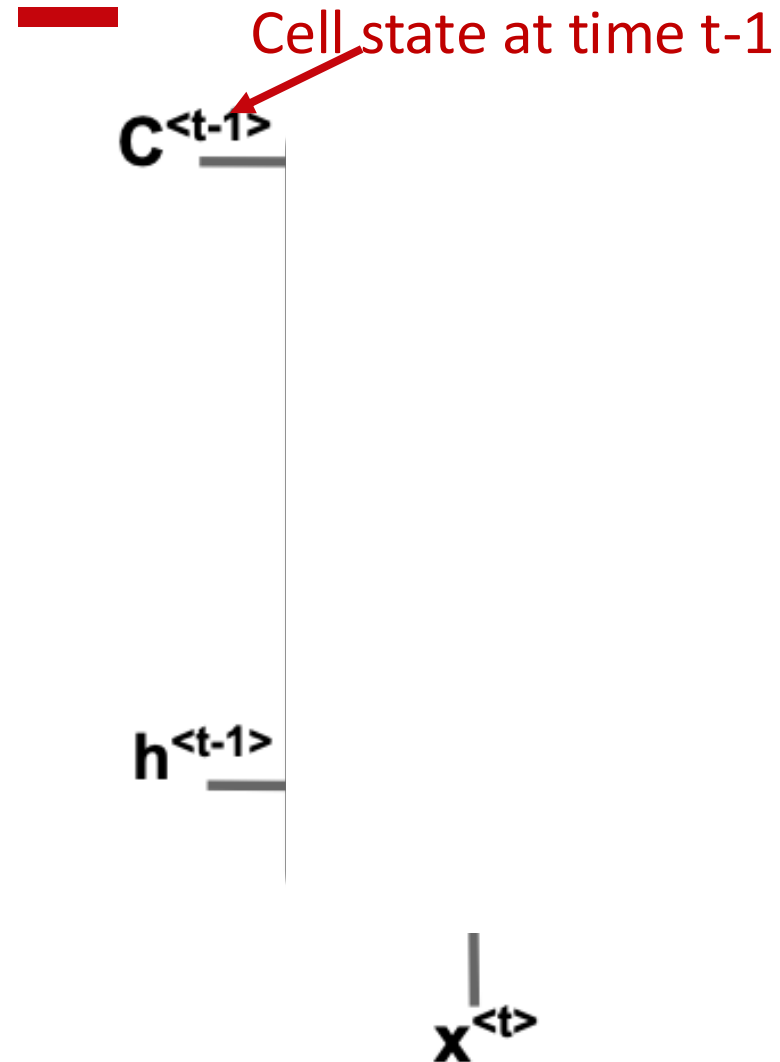


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# Inside LSTM



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# Inside LSTM

“Forget gate”: controls which information is remembered and which is forgotten

$$f_t = \sigma \left( \mathbf{W}_{fx} \mathbf{x}^{\langle t \rangle} + \mathbf{W}_{fh} \mathbf{h}^{\langle t-1 \rangle} + \mathbf{b}_f \right)$$



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# Inside LSTM

“Input gate”:  $\mathbf{i}_t = \sigma \left( \mathbf{W}_{ix} \mathbf{x}^{(t)} + \mathbf{W}_{ih} \mathbf{h}^{(t-1)} + \mathbf{b}_i \right)$

“Input node”:  $\mathbf{g}_t = \tanh \left( \mathbf{W}_{gx} \mathbf{x}^{(t)} + \mathbf{W}_{gh} \mathbf{h}^{(t-1)} + \mathbf{b}_g \right)$

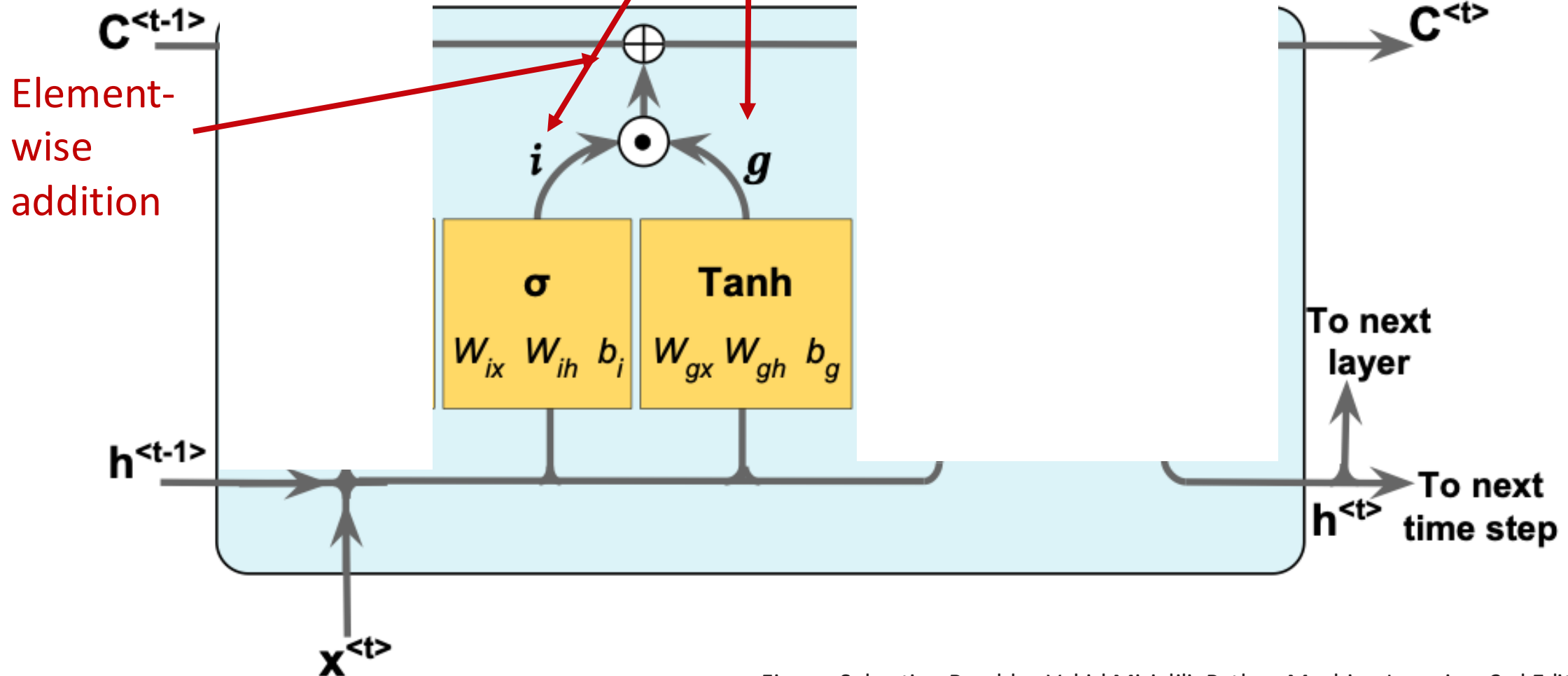


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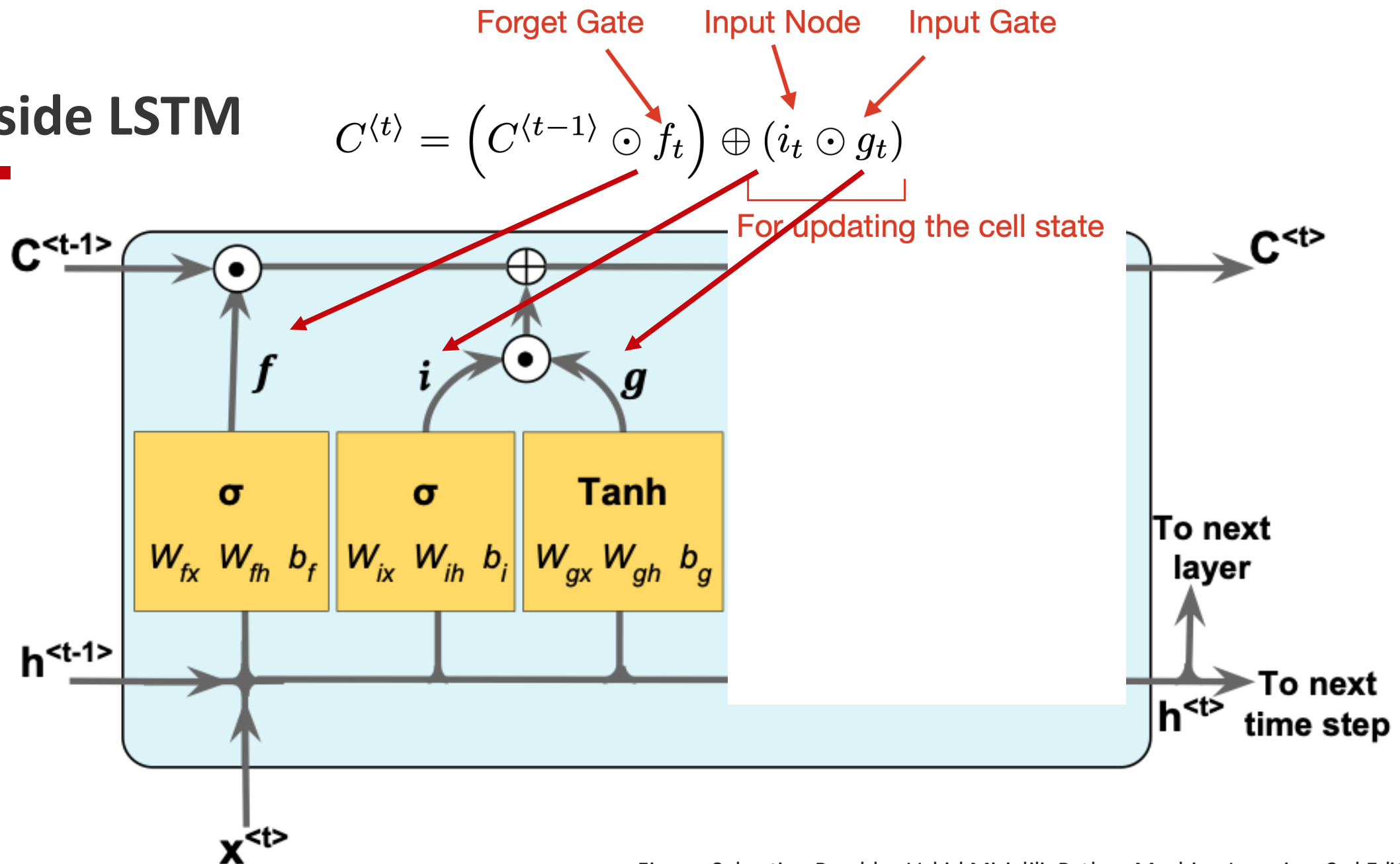


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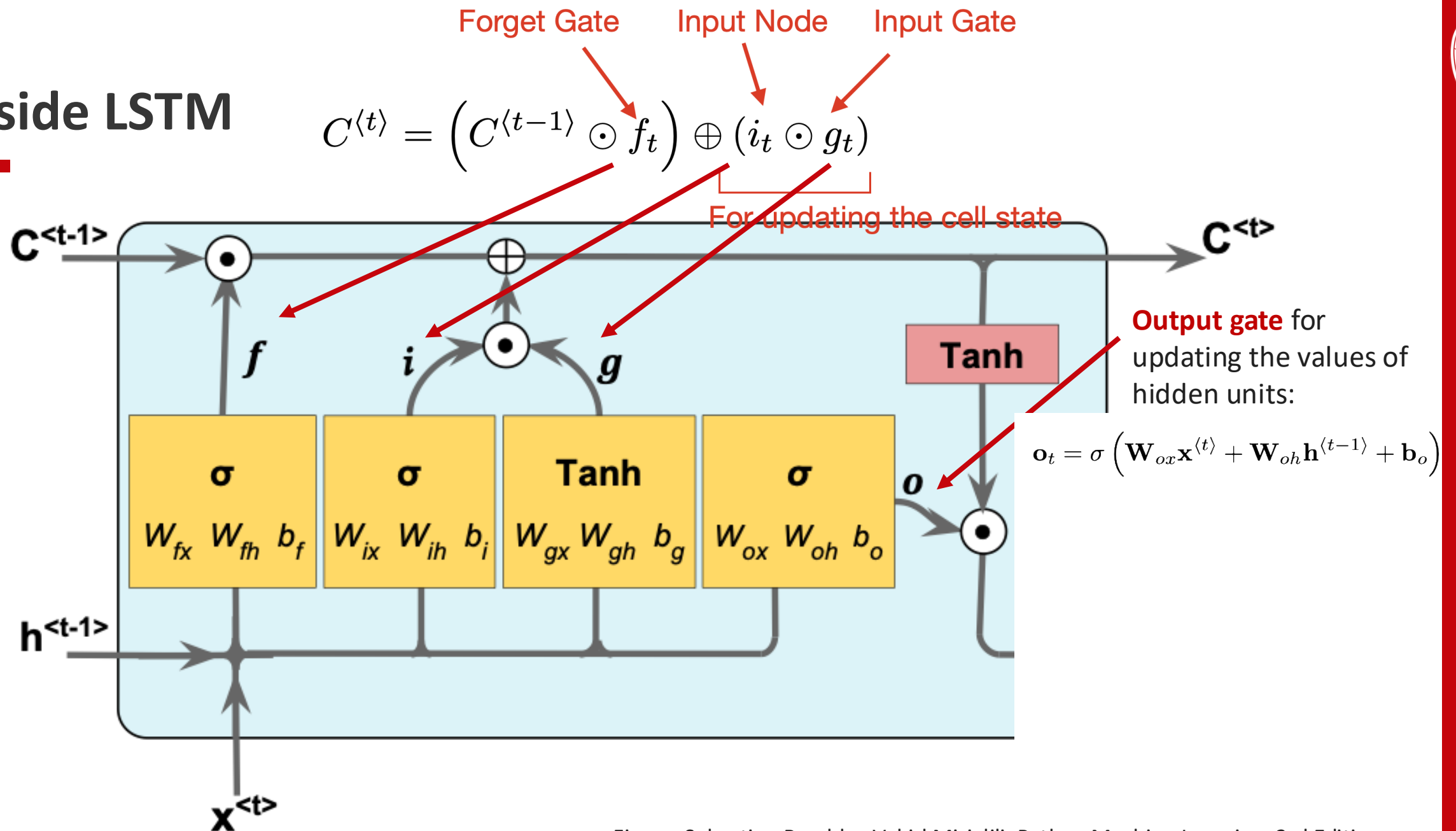


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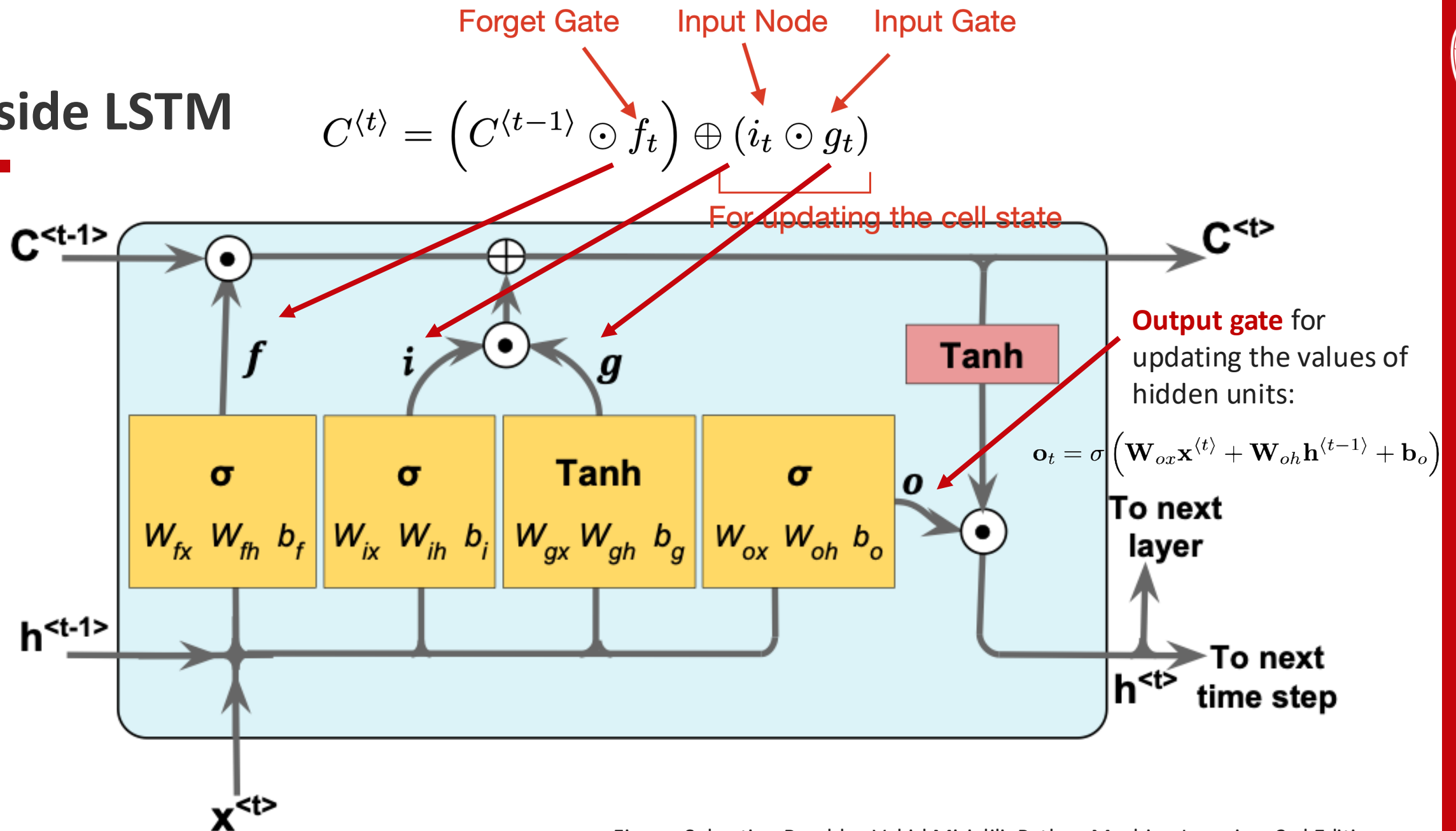


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# RNN Step 1: Build Vocabulary

## "Raw" training dataset

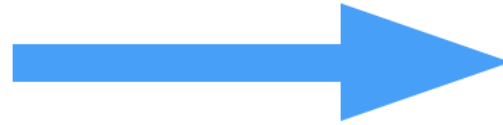
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$\mathbf{y}$  = [0, 1, 0]

class labels



vocabulary = {  
    '<unk>': 0,  
    'and': 1,  
    'is': 2  
    'one': 3,  
    'shining': 4,  
    'sun': 5,  
    'sweet': 6,  
    'the': 7,  
    'two': 8,  
    'weather': 9,  
    '<pad>': 10 }

# RNN Step 2: Convert text to indices

## "Raw" training dataset

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$\mathbf{x}^{[1]}$  = "The sun is shining"

[7 5 2 4 ... 10 10 10]

$\mathbf{x}^{[2]}$  = "The weather is sweet"

[7 9 2 6 ... 10 10 10]

$\mathbf{x}^{[3]}$  = "The sun is shining,  
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one and one is two"

[7 5 2 4 ... 3 2 8]

# RNN Step 3: Convert indices to one-hot representation

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'one': 3,  
'shining': 4,  
'sun': 5,  
'sweet': 6,  
'the': 7,  
'two': 8,  
'weather': 9,  
'<pad>': 10  
}

[7	[0	0	0	0	0	0	1	0	0	0]
5	[0	0	0	0	1	0	0	0	0	0]
2	[0	0	1	0	0	0	0	0	0	0]
4	[0	0	0	1	0	0	0	0	0	0]
...	...									
10	[0	0	0	1	0	0	0	0	0	1]
10	[0	0	0	1	0	0	0	0	0	1]
10]	[0	0	0	1	0	0	0	0	0	1]



## RNN Step 4: Convert one-hot to embeddings

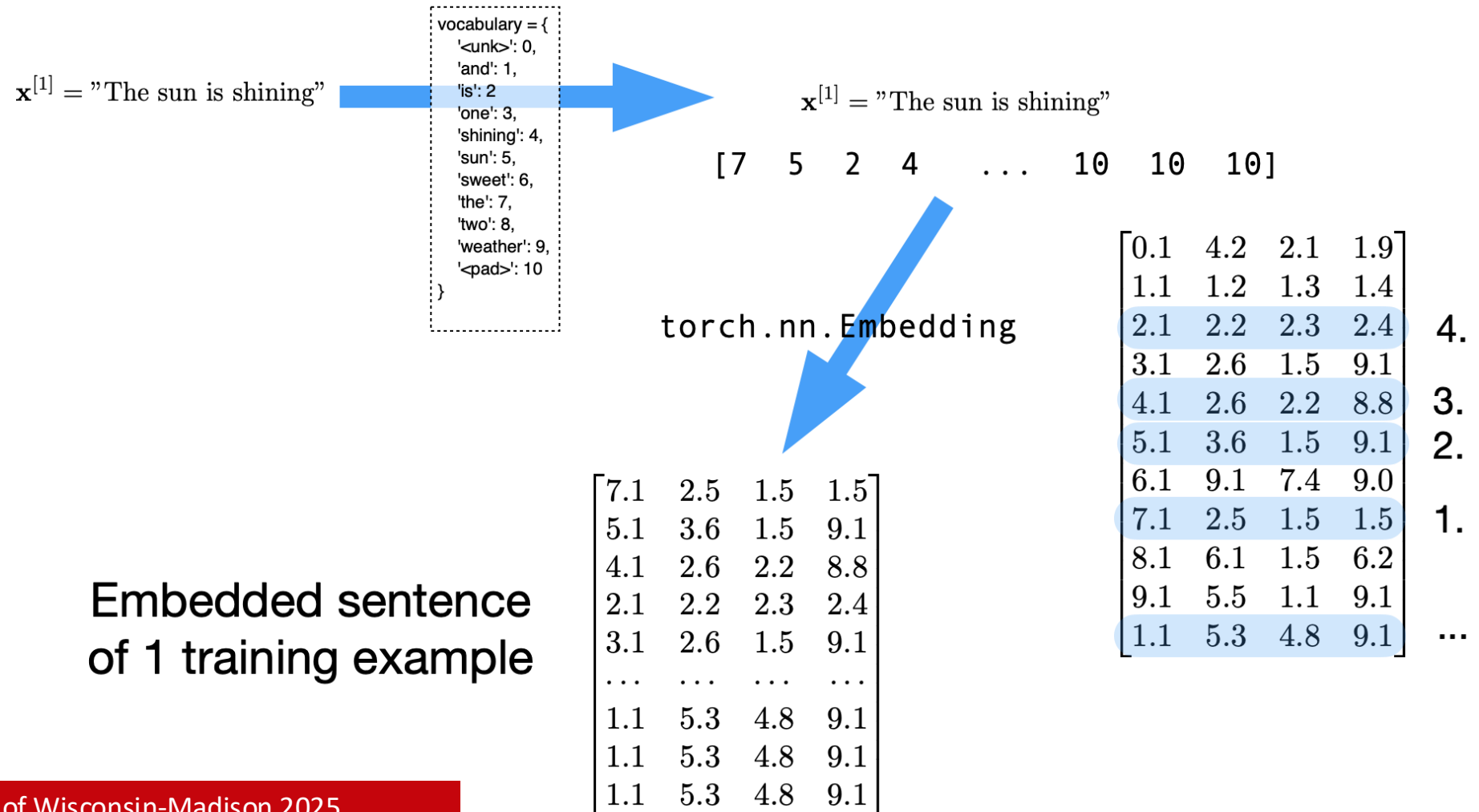
One-hot vector

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0.1 & 4.2 & 2.1 & 1.9 \\ 1.1 & 1.2 & 1.3 & 1.4 \\ 2.1 & 2.2 & 2.3 & 2.4 \\ 3.1 & 2.6 & 1.5 & 9.1 \\ 4.1 & 2.6 & 2.2 & 8.8 \\ 5.1 & 3.6 & 1.5 & 9.1 \\ 6.1 & 9.1 & 7.4 & 9.0 \\ 7.1 & 2.5 & 1.5 & 1.5 \\ 8.1 & 6.1 & 1.5 & 6.2 \\ 9.1 & 5.5 & 1.1 & 9.1 \\ 1.1 & 5.3 & 4.8 & 9.1 \end{bmatrix} = \begin{bmatrix} 7.1 & 2.5 & 1.5 & 1.5 \end{bmatrix}$$

Hidden layer output

# PyTorch: Skip steps 3 and 4. Instead...

use a lookup function (`torch.nn.Embedding`)



# LSTMs in PyTorch

<https://pytorch.org/docs/stable/generated/torch.nn.LSTM.html>

## Parameters

- **input\_size** – The number of expected features in the input  $x$
- **hidden\_size** – The number of features in the hidden state  $h$
- **num\_layers** – Number of recurrent layers. E.g., setting `num_layers=2` would mean stacking two LSTMs together to form a *stacked LSTM*, with the second LSTM taking in outputs of the first LSTM and computing the final results. Default: 1
- **bias** – If `False`, then the layer does not use bias weights  $b_{ih}$  and  $b_{hh}$ . Default: `True`
- **batch\_first** – If `True`, then the input and output tensors are provided as (batch, seq, feature). Default: `False`
- **dropout** – If non-zero, introduces a *Dropout* layer on the outputs of each LSTM layer except the last layer, with dropout probability equal to `dropout`. Default: 0
- **bidirectional** – If `True`, becomes a bidirectional LSTM. Default: `False`
- **proj\_size** – If `> 0`, will use LSTM with projections of corresponding size. Default: 0

## Examples:

```
>>> rnn = nn.LSTM(10, 20, 2)
>>> input = torch.randn(5, 3, 10)
>>> h0 = torch.randn(2, 3, 20)
>>> c0 = torch.randn(2, 3, 20)
>>> output, (hn, cn) = rnn(input, (h0, c0))
```

# Good reading

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- [The Unreasonable Effectiveness of Recurrent Neural Networks](#) by Andrej Karpathy
- [On the difficulty of training recurrent neural networks](#) by Razvan Pascanu, Tomas Mikolov, Yoshua Bengio

Questions?

