



STAT 453: Introduction to Deep Learning and Generative Models

Ben Lengerich

Lecture 19: Recurrent Neural Networks

November 10, 2025

Reading: See course homepage



Our semester

Week	Lecture Dates	Topic
Module 1: Introduction and Foundations		
1	9/3	Course Introduction
2	9/8, 9/10	A Brief History of DL, Statistics / linear algebra / calculus review
3	9/15, 9/17	Single-layer networks Parameter Optimization and Gradient Descent
4	9/22, 9/24	Automatic differentiation with PyTorch, Cluster and cloud computing resources
Module 2: Neural Networks		
5	9/29, 10/1	Multinomial logistic regression, Multi-layer perceptrons and backpropagation
6	10/6, 10/8	Regularization Normalization / Initialization
7	10/13, 10/15	Optimization, Learning Rates CNNs
8	10/20, 10/22	Review, Midterm Exam

Module 3: Intro to Generative Models			
9	10/27, 10/29	A Linear Intro to Generative Models, Factor Analysis, Autoencoders, VAEs	
10	11/3, 11/5	Generative Adversarial Networks, Diffusion Models	Project Midway Report
Module 4: Large Language Models			
11	11/10, 11/12	Sequence Learning with RNNs Attention, Transformers	HW4
12	11/17, 11/19	GPT Architectures, Unsupervised Training of LLMs	
13	11/24, 11/26	Supervised Fine-tuning of LLMs, Prompts and In-context learning	HW5
14	12/1, 12/3	Foundation models, alignment, explainability Open directions in LLM research	
15	12/8, 12/10	Project Presentations	Project Final Report
16	12/17	Final Exam	Final Exam



A quick vote...

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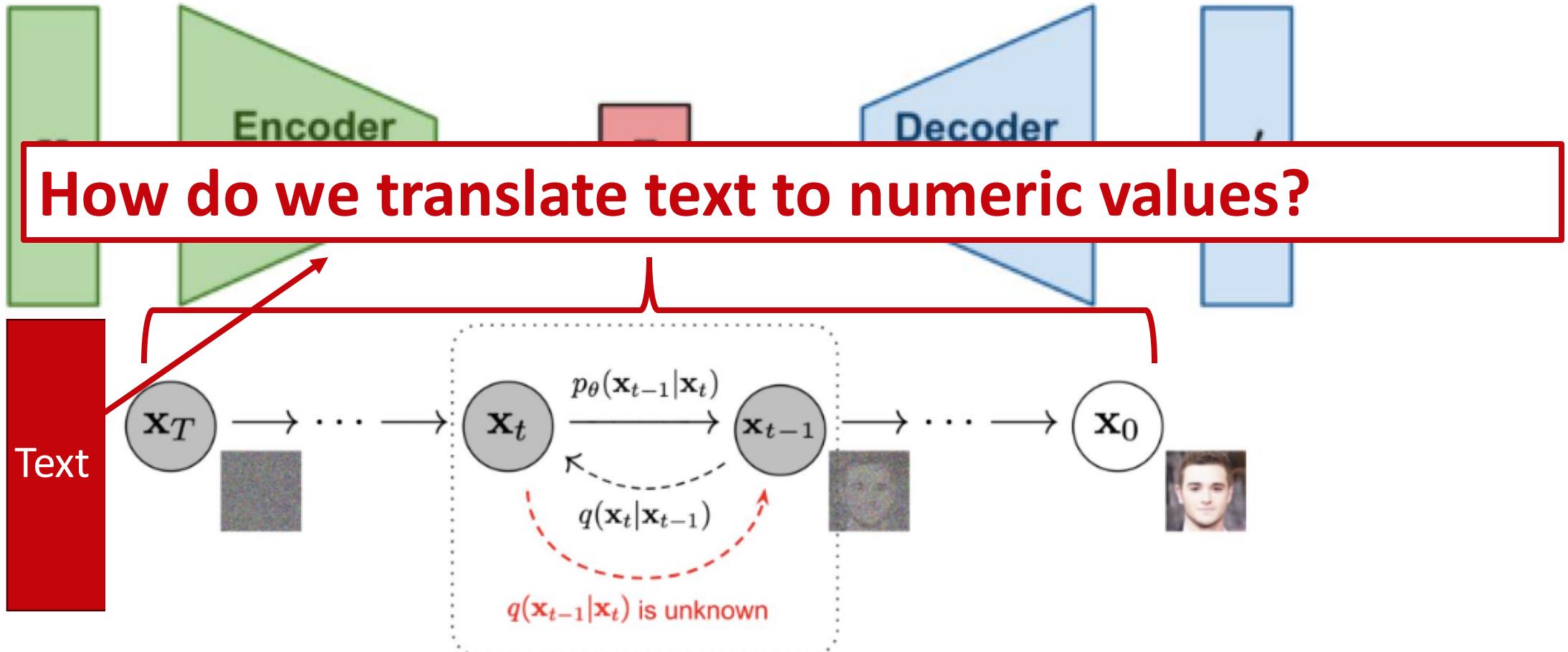
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HW4

- Released on [the website](#)
- Due next Friday
- Auto-encoder (4 parts) + bonus GAN
- We recommend Colab

Recall “Conditioning on Text”...





Challenges with text

- Variable length input
- Long-range dependencies
- Want a generative model
- Scale
- Emergent properties of large language models

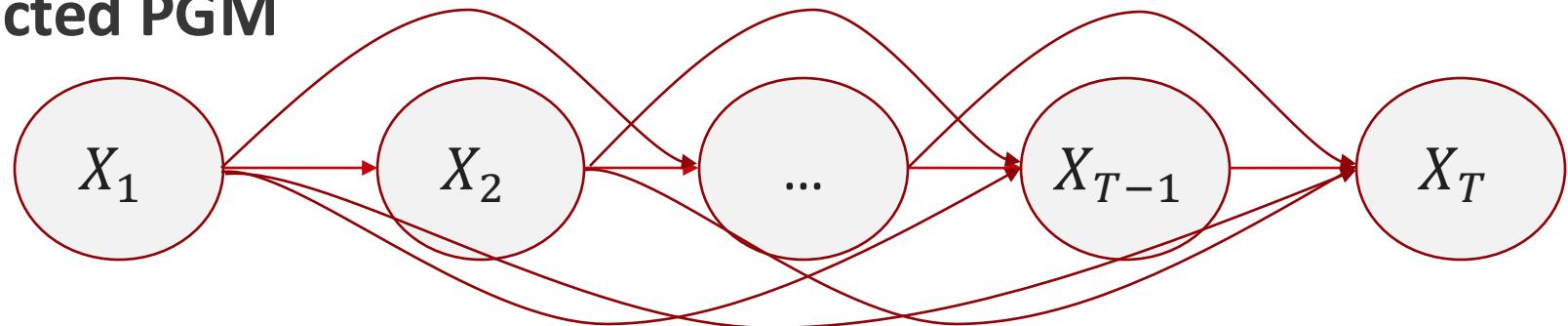
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Where we're going

GPT = Auto-regressive probabilistic model

- Directed PGM



$$P_\theta(X) = \prod_i \prod_t P_\theta(X_{i,t} \mid X_{i,<t})$$

- **Probabilistic objective:** Max log-likelihood of observed seqs

$$\max_{\theta} \sum_i \sum_t \log P_\theta(X_{i,t} \mid X_{i,<t})$$

[Radford et al., [Improving Language Understanding by Generative Pre-Training](#)]



Today

- **Different Ways to Model Text**
- Sequence Modeling with RNNs
- Different Types of Sequence Modeling Tasks
- Backpropagation Through Time
- Long-Short Term Memory (LSTM)
- Many-to-one Word RNNs

A classic approach: Bag-of-words

"Raw" training dataset

$x^{[1]} = \text{"The sun is shining"}$

$x^{[2]} = \text{"The weather is sweet"}$

$x^{[3]} = \text{"The sun is shining,}$
the weather is sweet, and
one and one is two"

$y = [0, 1, 0]$

class labels

```
vocabulary = {  
    'and': 0,  
    'is': 1  
    'one': 2,  
    'shining': 3,  
    'sun': 4,  
    'sweet': 5,  
    'the': 6,  
    'two': 7,  
    'weather': 8,  
}
```

Training set as design matrix

$$\mathbf{X} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 2 & 3 & 2 & 1 & 1 & 1 & 2 & 1 & 1 \end{bmatrix}$$

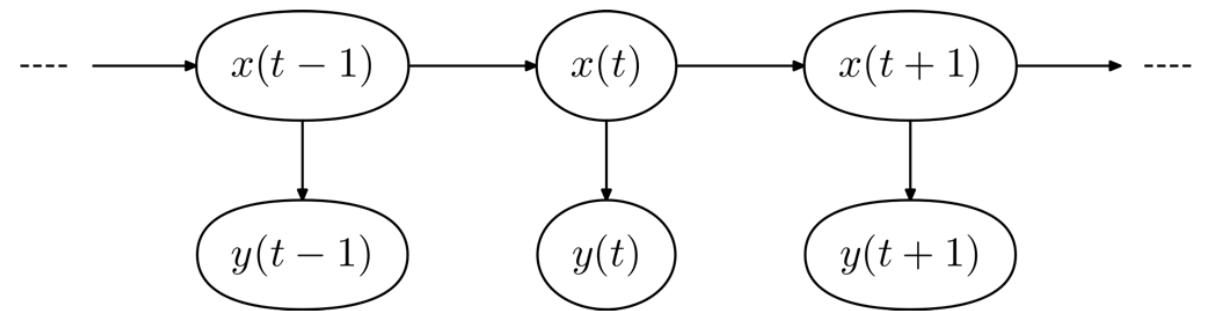
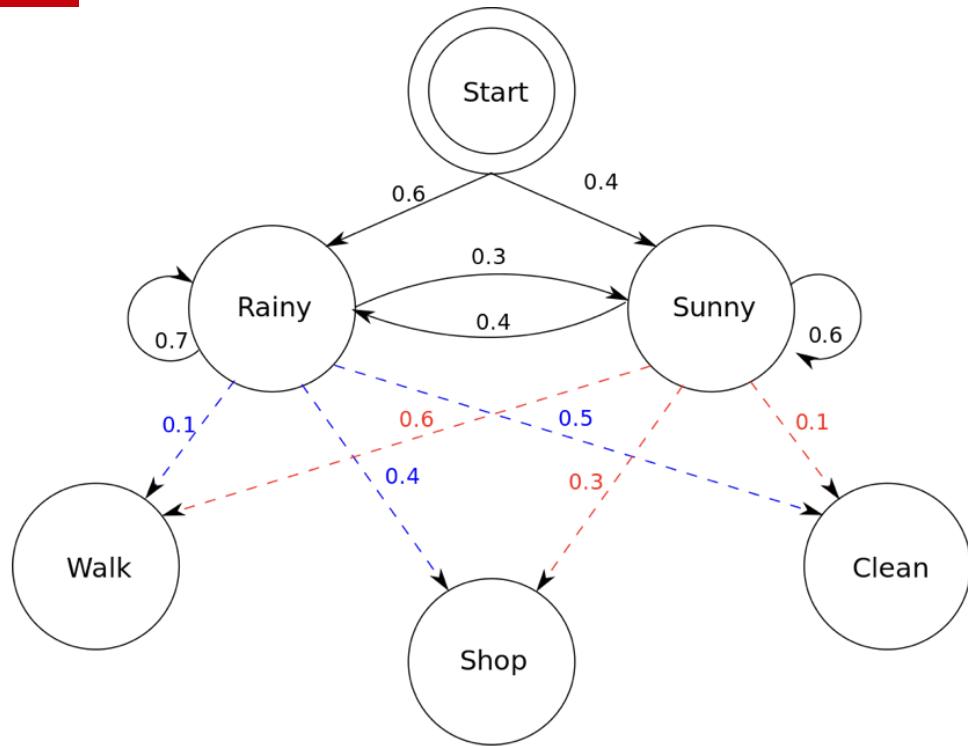
$$\mathbf{y} = [0, 1, 0]$$

class labels

training

Classifier

Another classic approach: Hidden Markov Model

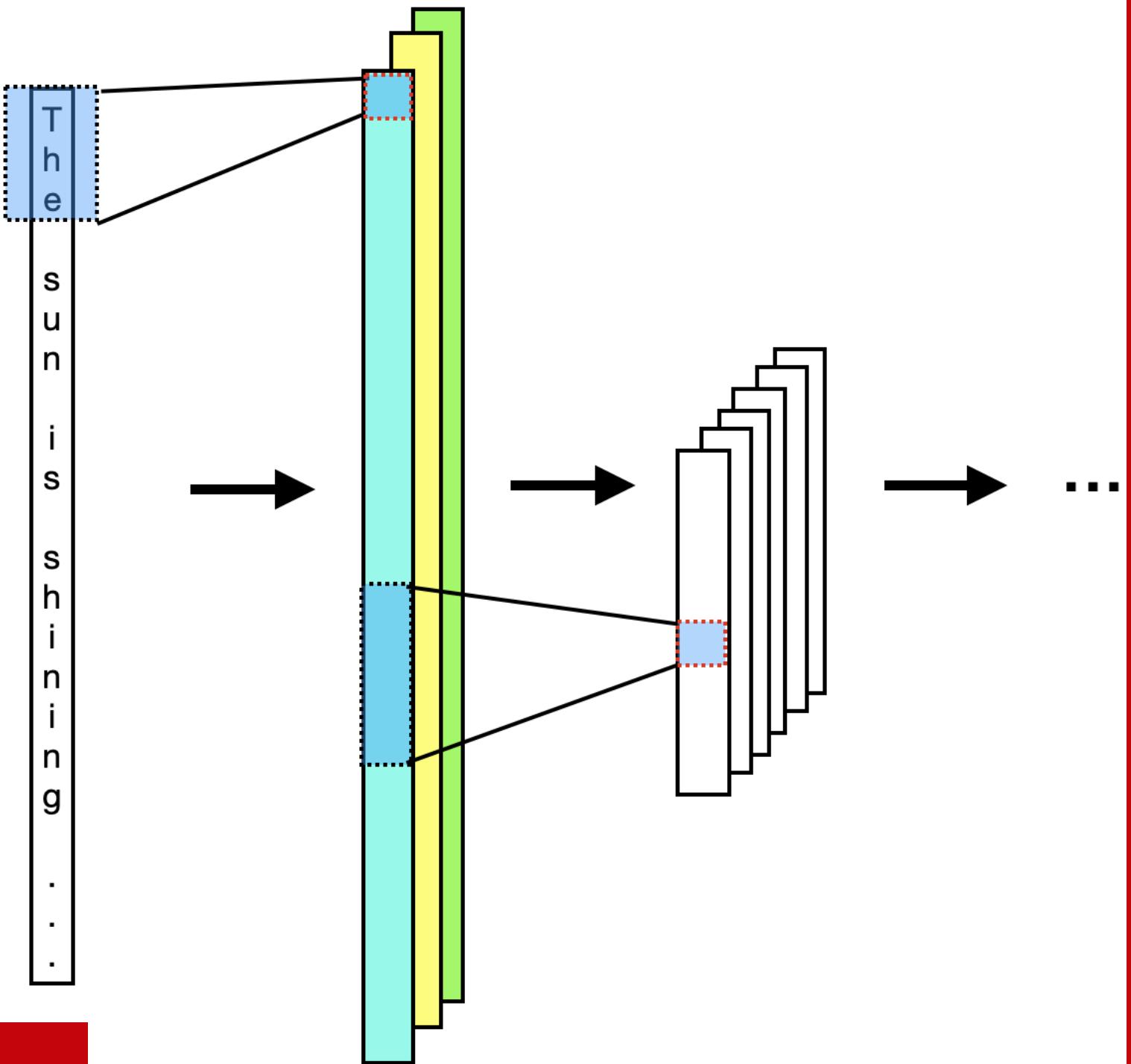


Wikipedia example: each day, weather follows a Markov chain, and activities are observables

$$\mathbb{P}(Y_n = y | X_1 = x_1, \dots, X_n = x_n) = \mathbb{P}(Y_n = y | X_n = x_n)$$

Another approach: CNNs

Can't handle variable length input,
→ need padding to max input length





Today

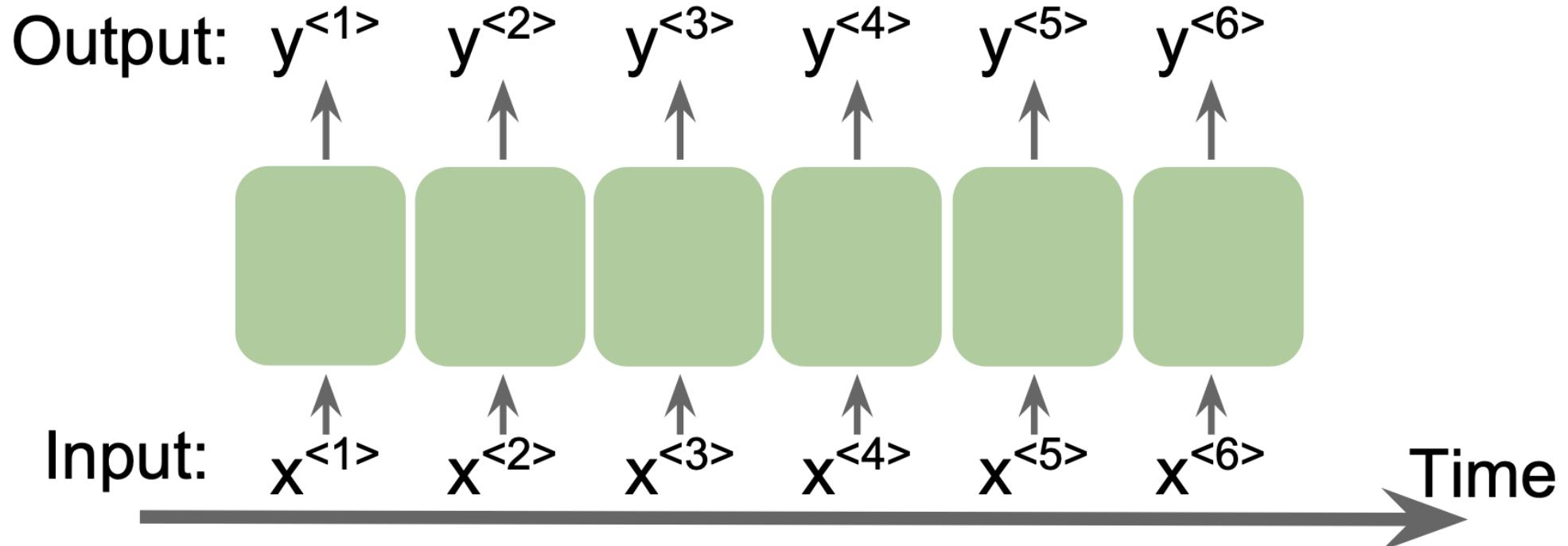
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Sequence data: order matters

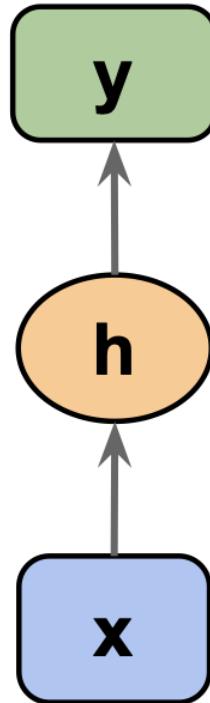
The movie my friend has not seen is good

The movie my friend has seen is not good



Recurrent Neural Networks (RNNs)

Networks we used previously: also called feedforward neural networks



Recurrent Neural Network (RNN)

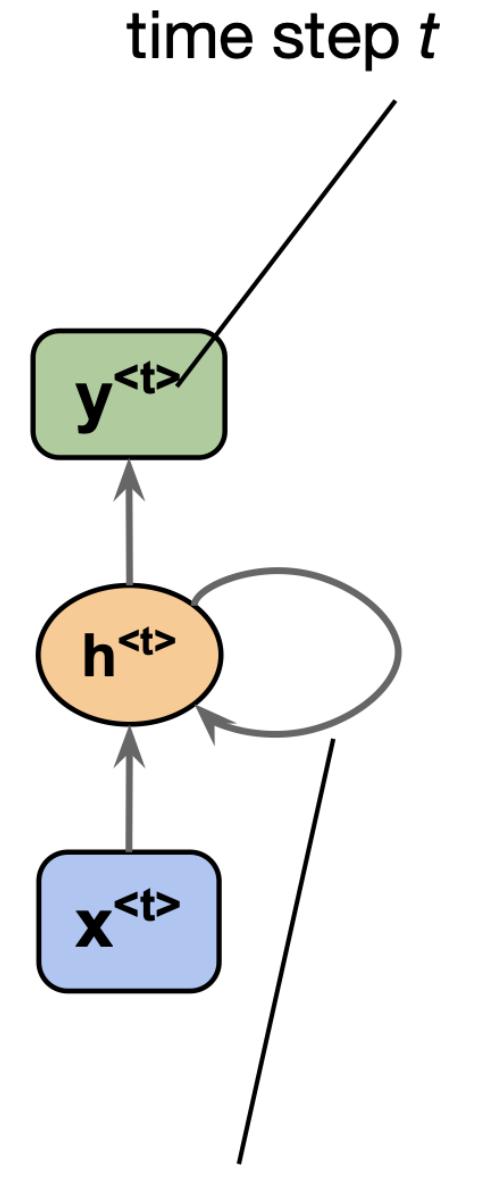


Image source: Sebastian Raschka, Vahid Mirjalili. Python Machine Learning. 3rd Edition. Packt, 2019

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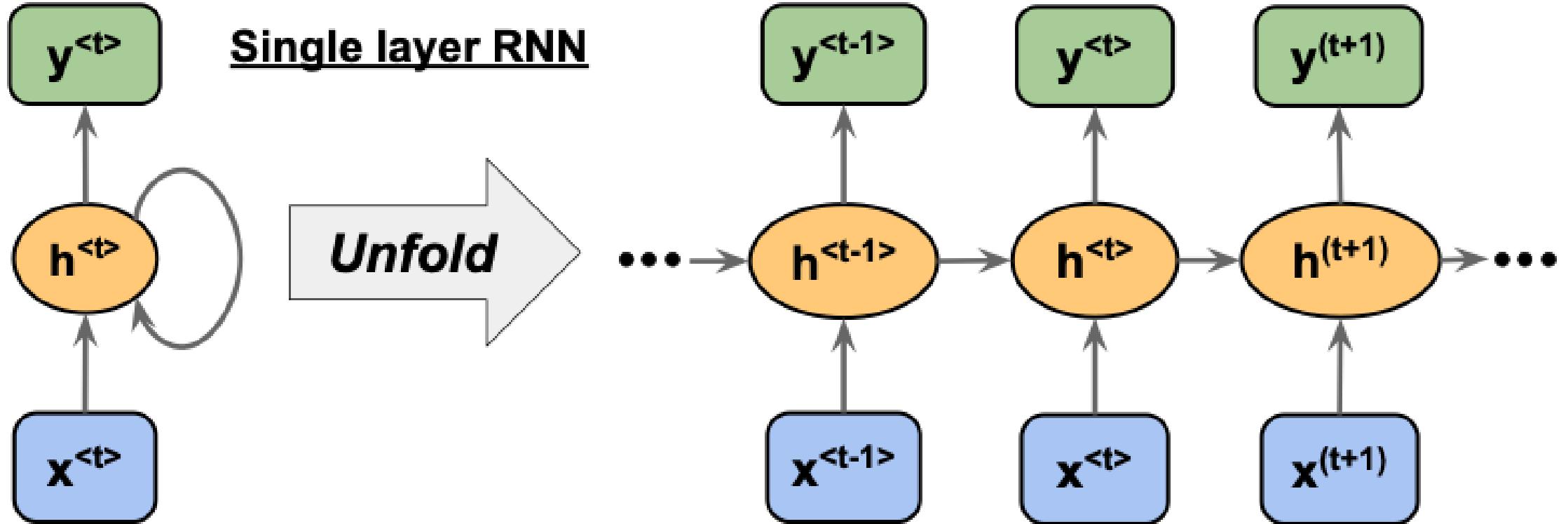


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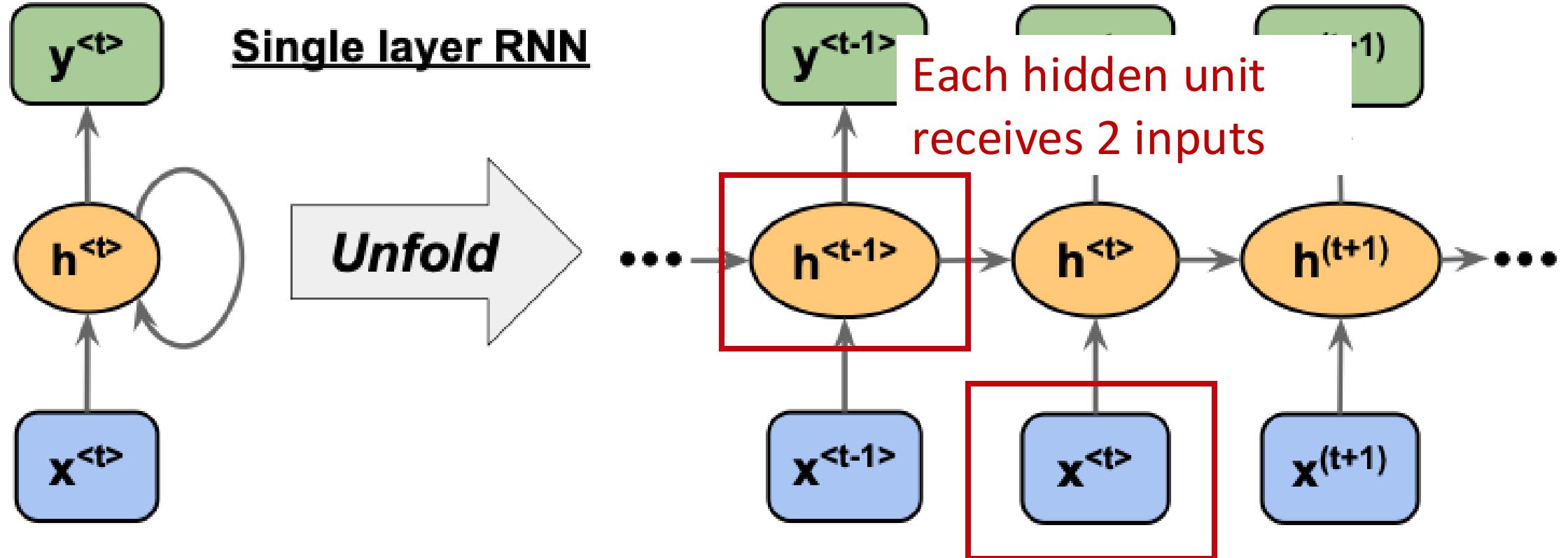


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Multilayer RNNs

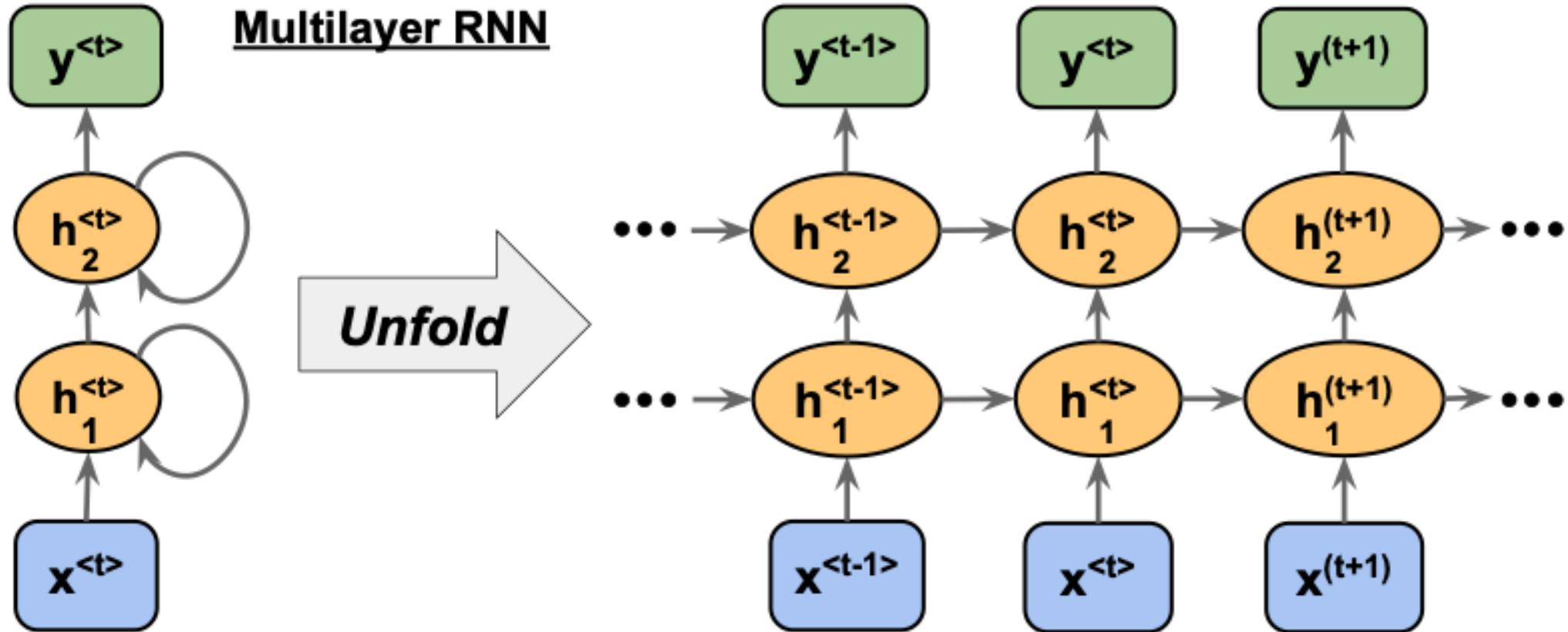


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Recurrence unlocks many types of sequence tasks

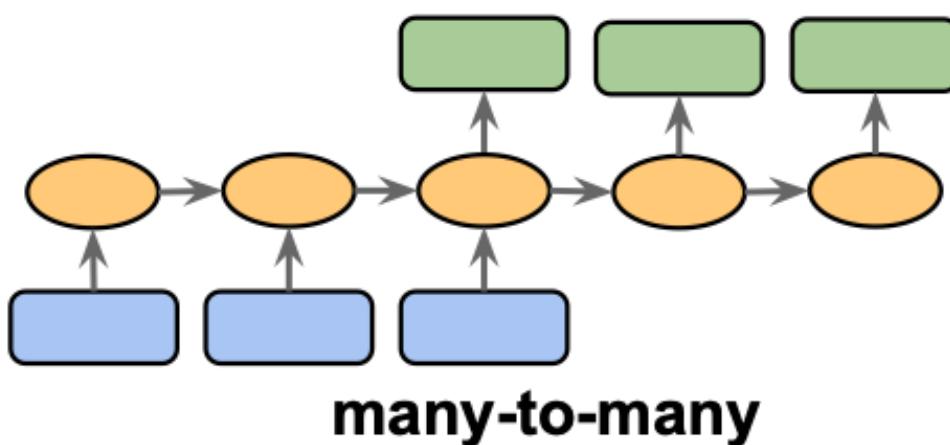
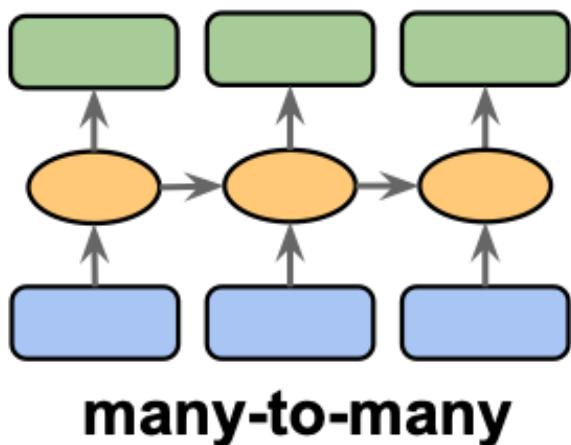
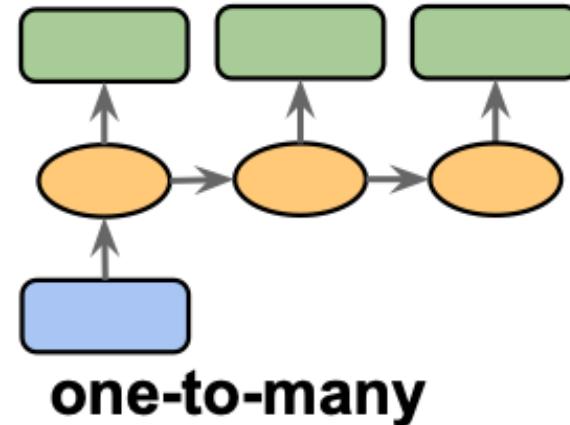
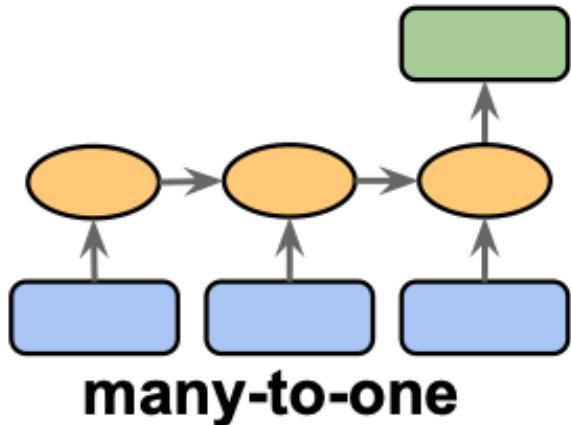


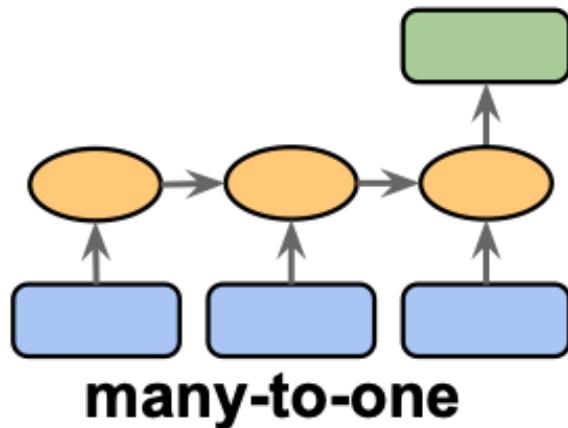
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Recurrence unlocks many types of sequence tasks

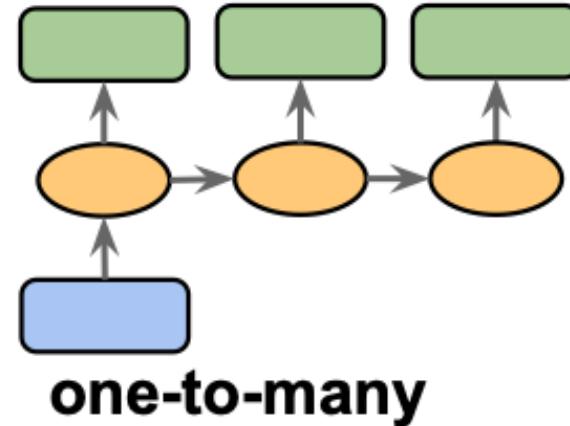


Many-to-one: The input data is a sequence, but the output is a fixed-size vector, not a sequence.

Example: sentiment analysis, the input is some text, and the output is a class label.

Image source: Sebastian Raschka, Vahid Mirjalili. Python Machine Learning. 3rd Edition. Packt, 2019

Recurrence unlocks many types of sequence tasks



One-to-many: Input data is in a standard format (not a sequence), the output is a sequence.

Example: Image captioning, where the input is an image, the output is a text description of that image

Image source: Sebastian Raschka, Vahid Mirjalili. Python Machine Learning. 3rd Edition. Packt, 2019

Recurrence unlocks many types of sequence tasks

Many-to-many: Both inputs and outputs are sequences. Can be direct or delayed.

Example: Video-captioning, i.e., describing a sequence of images via text (direct). Translation.

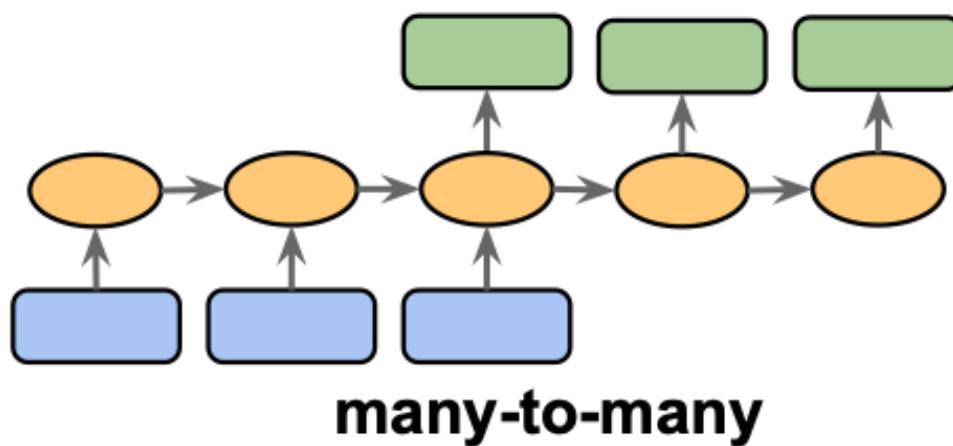
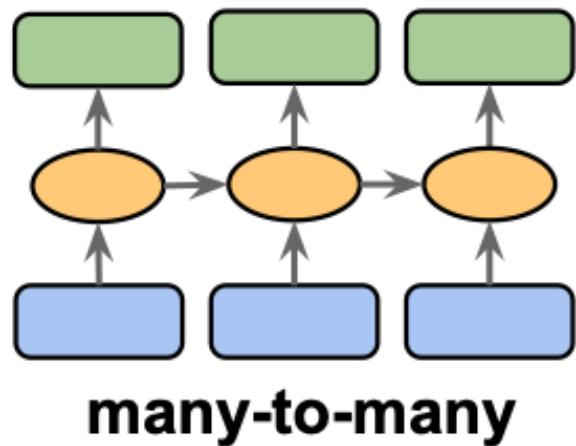


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Under the hood: weight matrices in an RNN

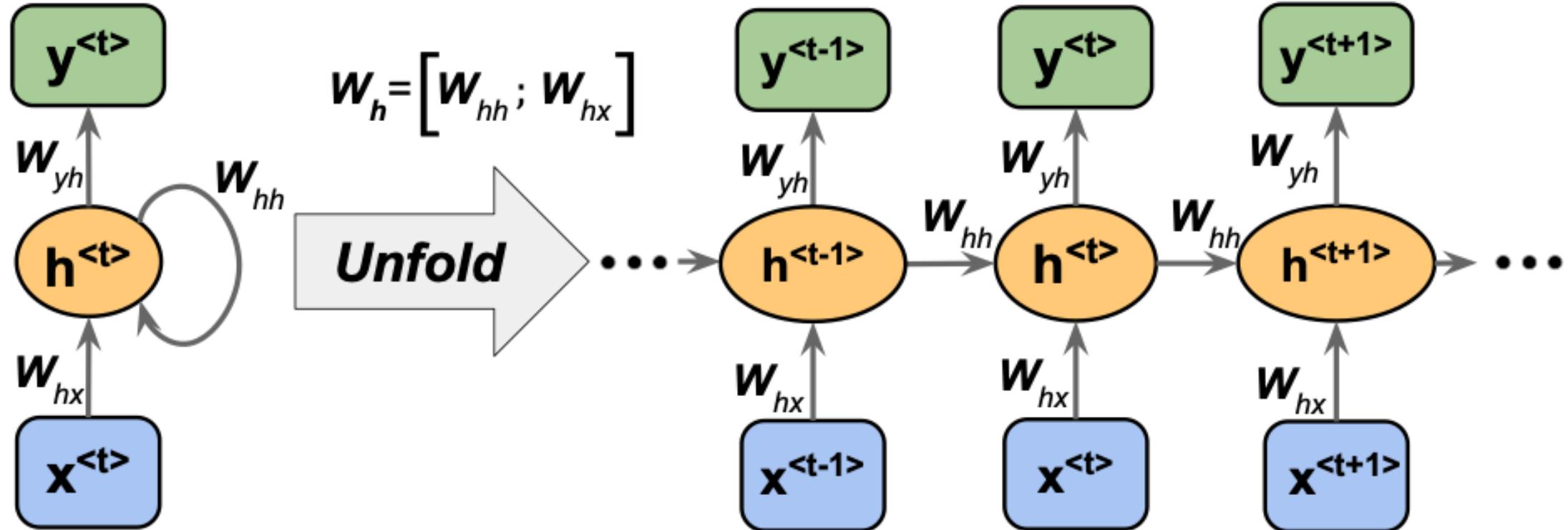
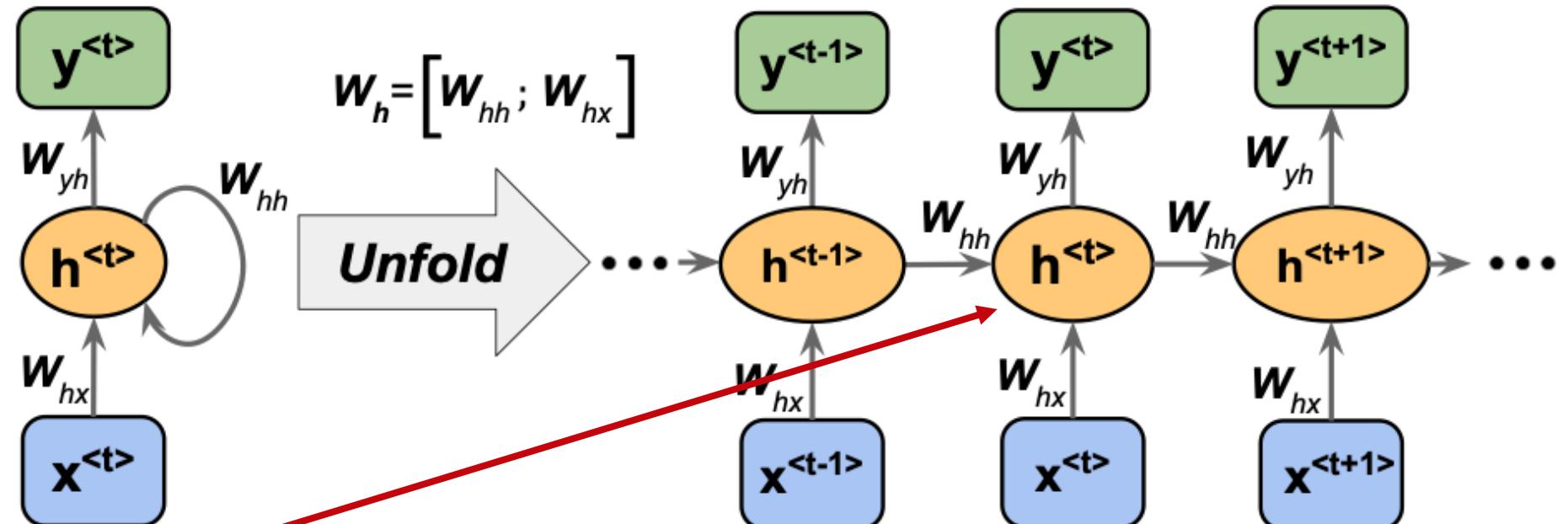


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Under the hood: weight matrices in an RNN

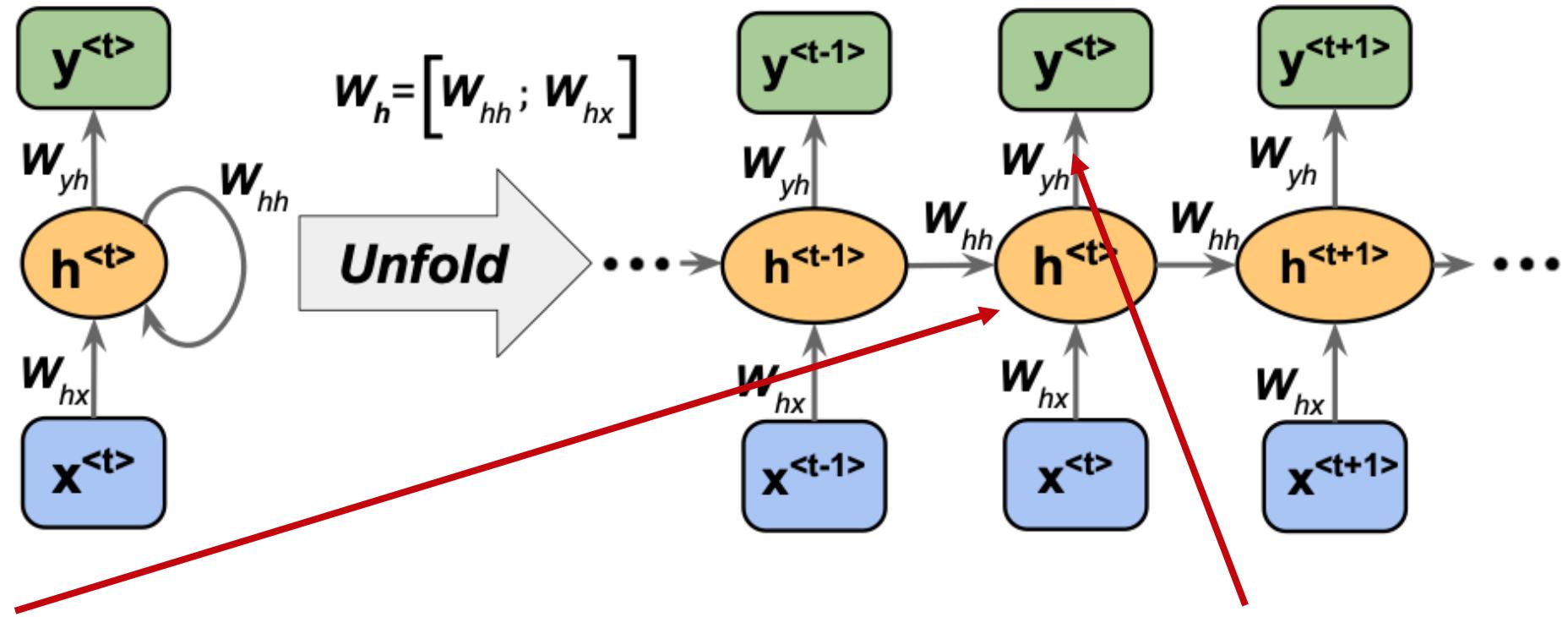


Net input: $\mathbf{z}_h^{(t)} = \mathbf{W}_{hx} \mathbf{x}^{(t)} + \mathbf{W}_{hh} \mathbf{h}^{(t-1)} + \mathbf{b}_h$

Activation: $\mathbf{h}^{(t)} = \sigma_h(\mathbf{z}_h^{(t)})$

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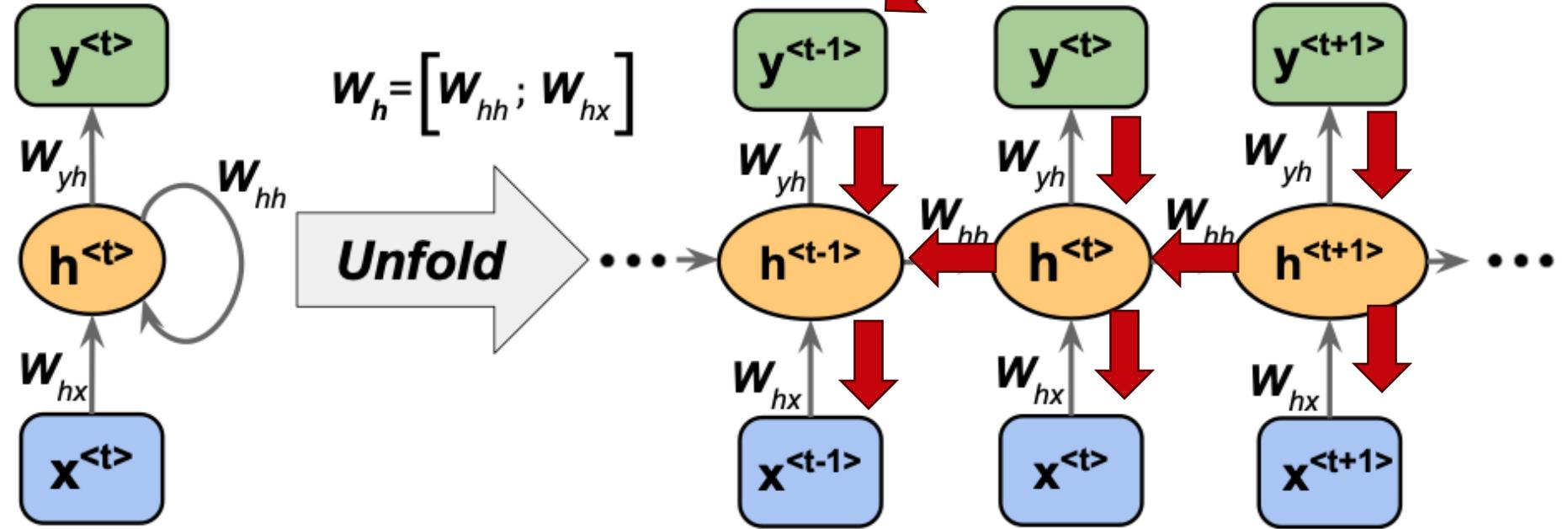
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Backpropagation through time



The overall loss can be computed as
the sum over all time steps

$$L = \sum_{t=1}^T L^{(t)}$$

Image source: Sebastian Raschka, Vahid Mirjalili. Python Machine Learning. 3rd Edition. Packt, 2019



Backpropagation through time

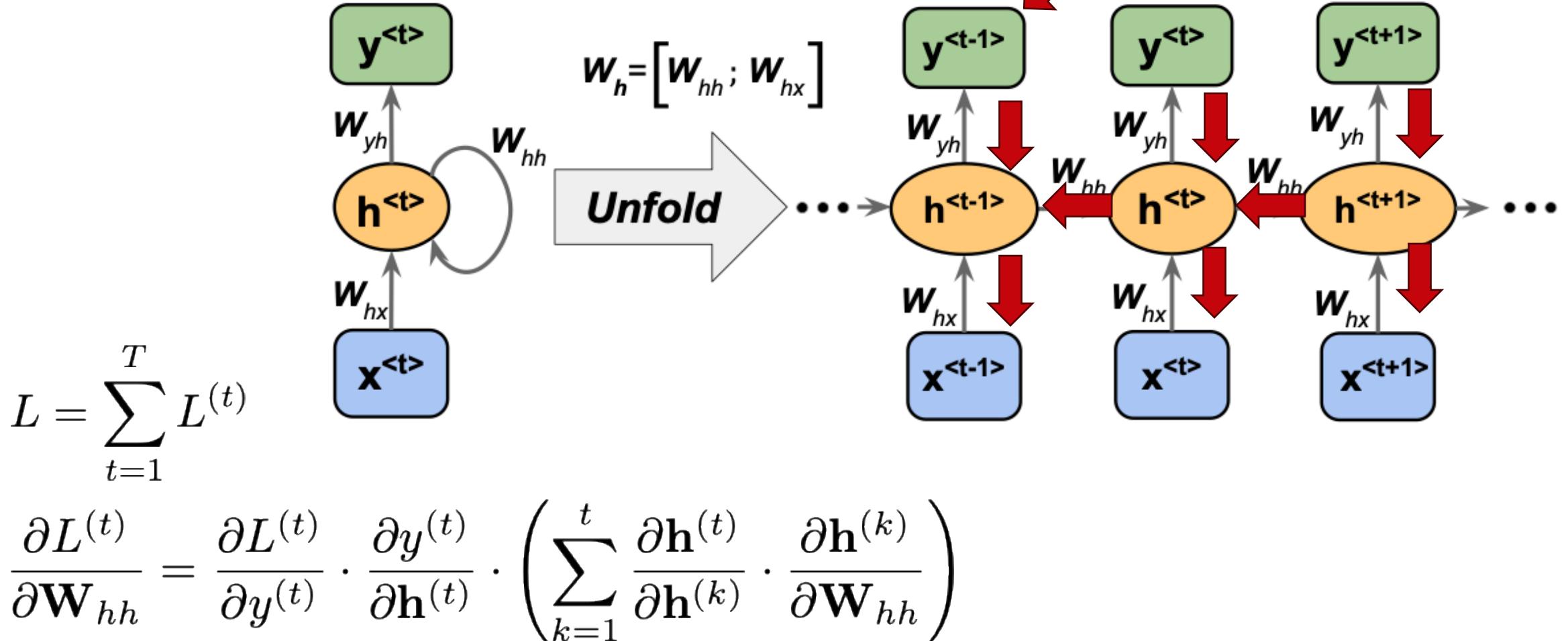
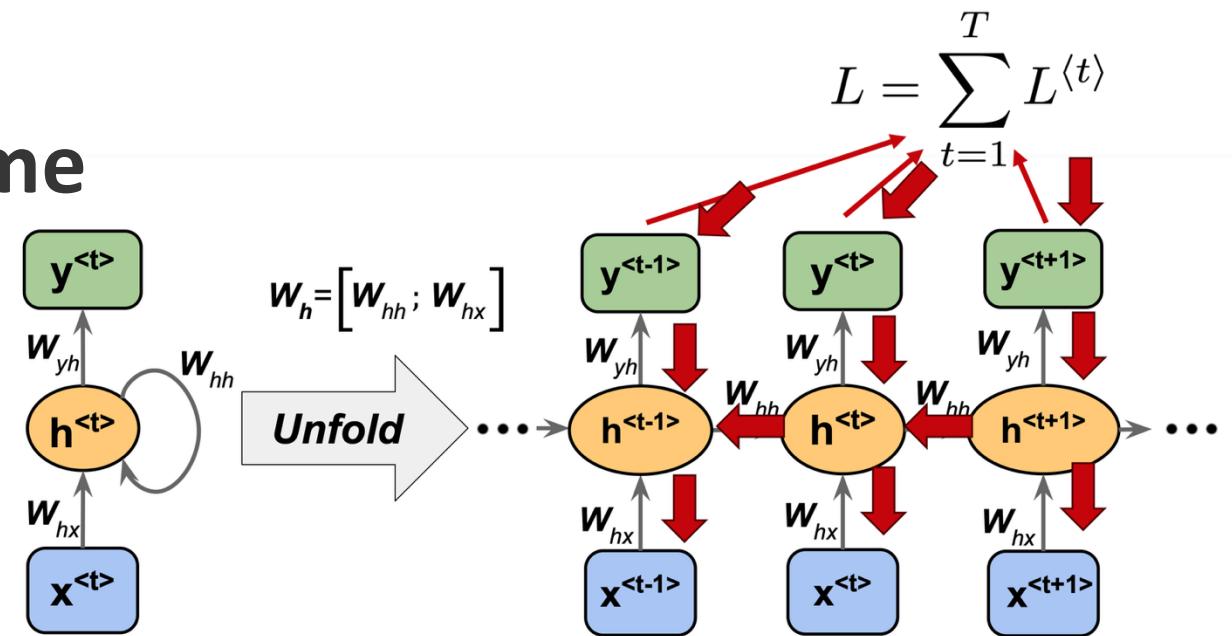


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Backpropagation through time



Computed as a multiplication of adjacent time steps:

$$L = \sum_{t=1}^T L^{(t)}$$

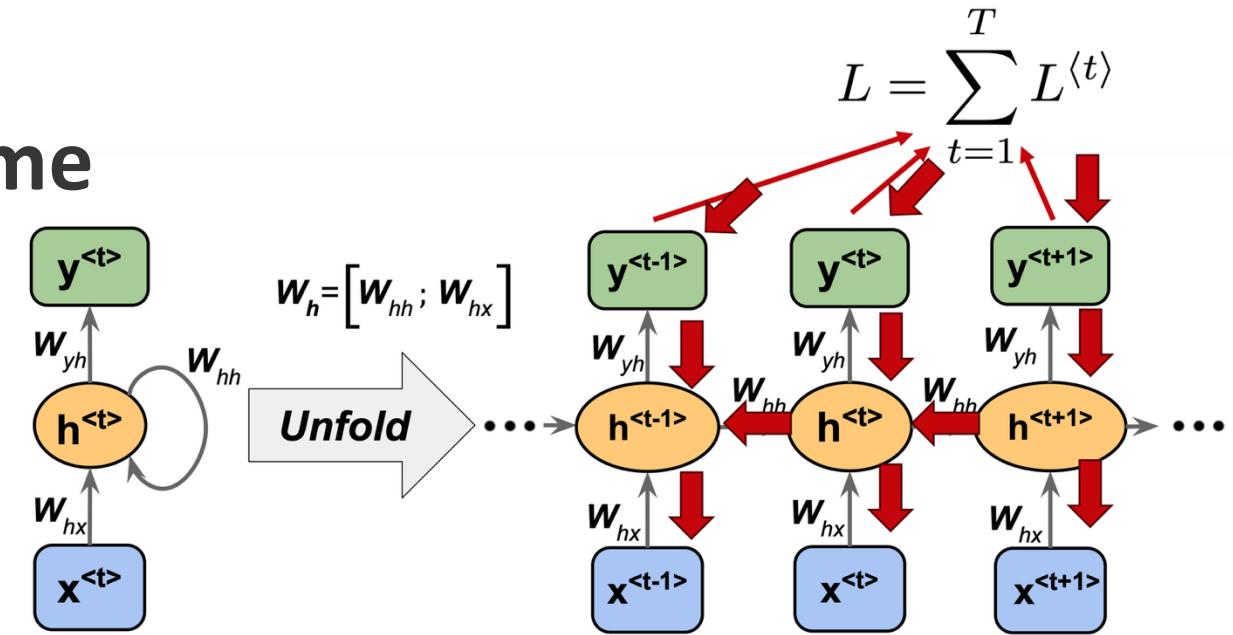
$$\frac{\partial L^{(t)}}{\partial \mathbf{W}_{hh}} = \frac{\partial L^{(t)}}{\partial y^{(t)}} \cdot \frac{\partial y^{(t)}}{\partial \mathbf{h}^{(t)}} \cdot \left(\sum_{k=1}^t \frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(k)}} \cdot \frac{\partial \mathbf{h}^{(k)}}{\partial \mathbf{W}_{hh}} \right)$$

$$\frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(k)}} = \prod_{i=k+1}^t \frac{\partial \mathbf{h}^{(i)}}{\partial \mathbf{h}^{(i-1)}}$$

Image source: Sebastian Raschka, Vahid Mirjalili. Python Machine Learning. 3rd Edition. Packt, 2019

Backpropagation through time

Straightforward, but problematic:
vanishing / exploding gradients!



Computed as a multiplication of adjacent time steps:

$$L = \sum_{t=1}^T L^{(t)}$$

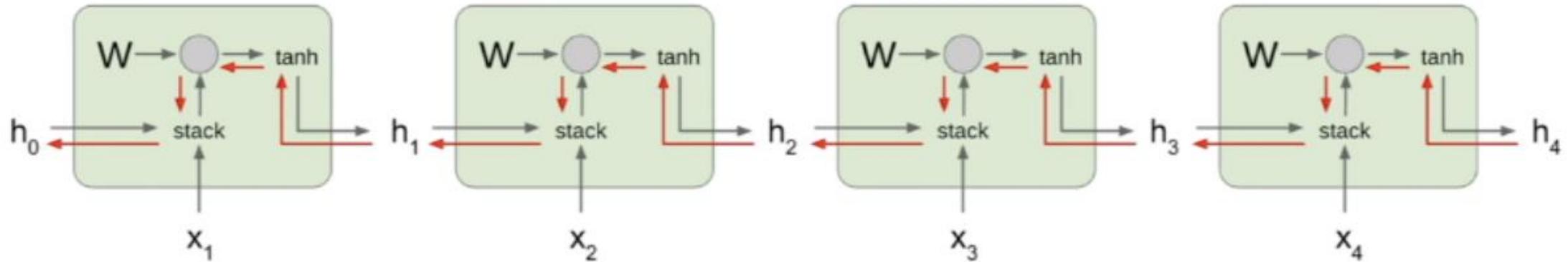
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A challenge: Vanishing / exploding gradients

$$\mathbf{h}_t = \tanh(W^{hh}\mathbf{h}_{t-1} + W^{hx}\mathbf{x}_t)$$



Computing gradient of \mathbf{h}_0 involves many factors of \mathbf{W} (and repeated \tanh)

Largest singular value > 1 :
Exploding gradients

Largest singular value < 1 :
Vanishing gradients

Gradient clipping: Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
    grad *= (threshold / grad_norm)
```

Bengio et al., 1994 “Learning long-term dependencies with gradient descent is difficult”
Pascanu et al., 2013 “On the difficulty of training recurrent neural networks”

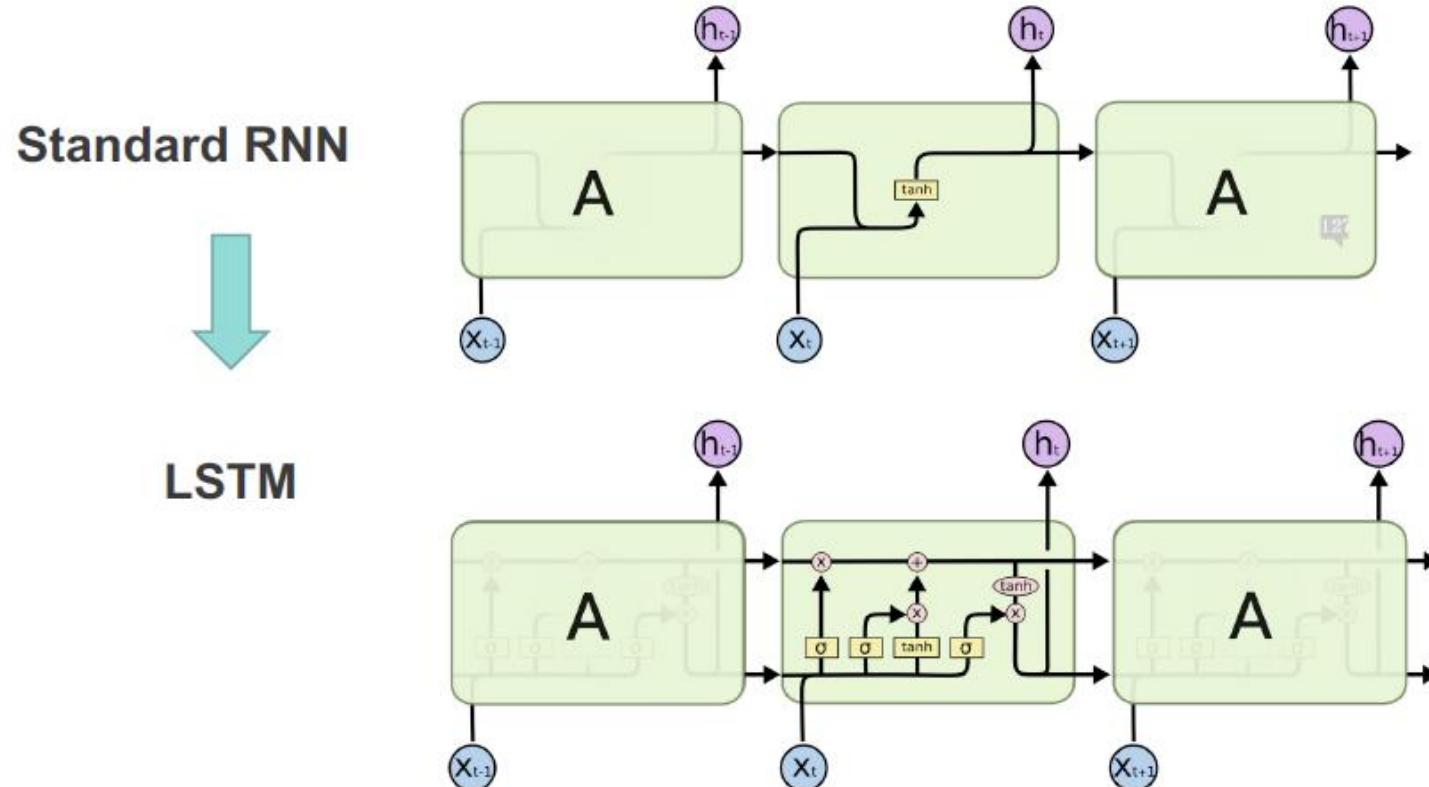


Solutions to Vanishing / Exploding Gradients

- **Gradient Clipping:** set a max value for gradients if they grow to large (solves only exploding gradient problem)
- **Truncated backpropagation through time (TBPTT):** limit the number of time steps the signal can backpropagate after each forward pass. E.g., even if the sequence has 100 elements/steps, we may only backpropagate through 20 or so.

Solutions to Vanishing / Exploding Gradients

Long short-term memory (LSTM): uses a *memory cell* for modeling long-range dependencies and avoid vanishing gradient problems





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Long-short term memory (LSTM)

- Not an oxymoron: **2 paths of memory**

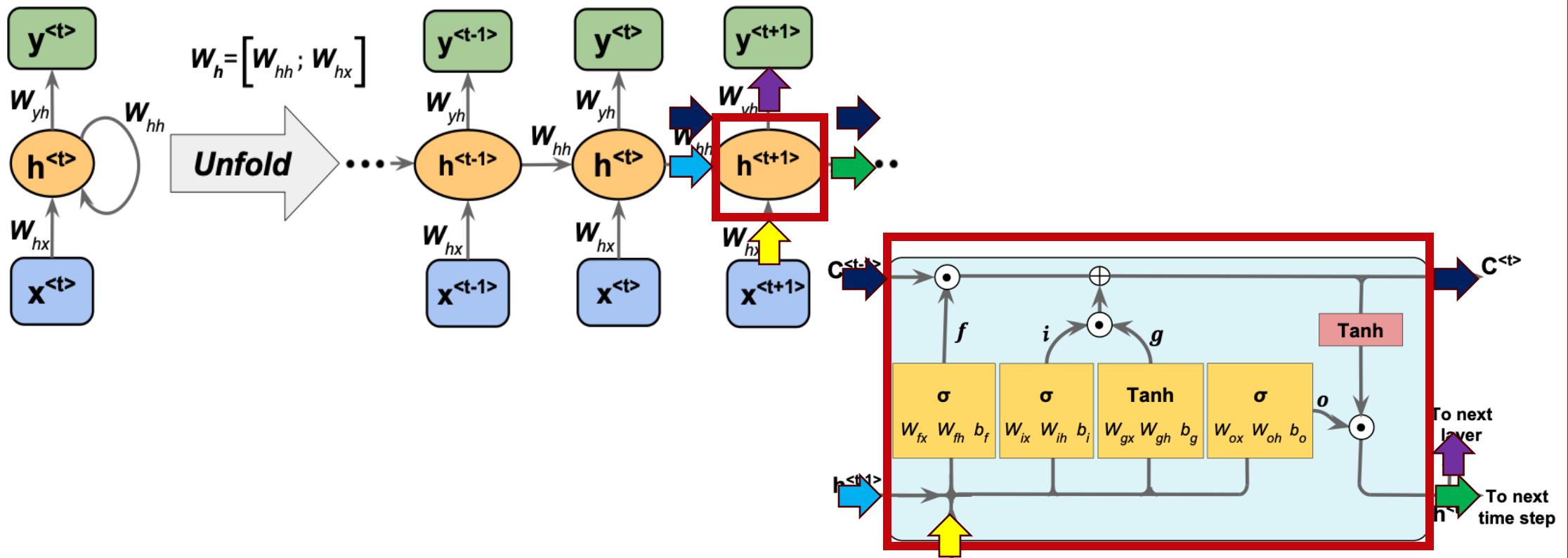


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Long-short term memory (LSTM)

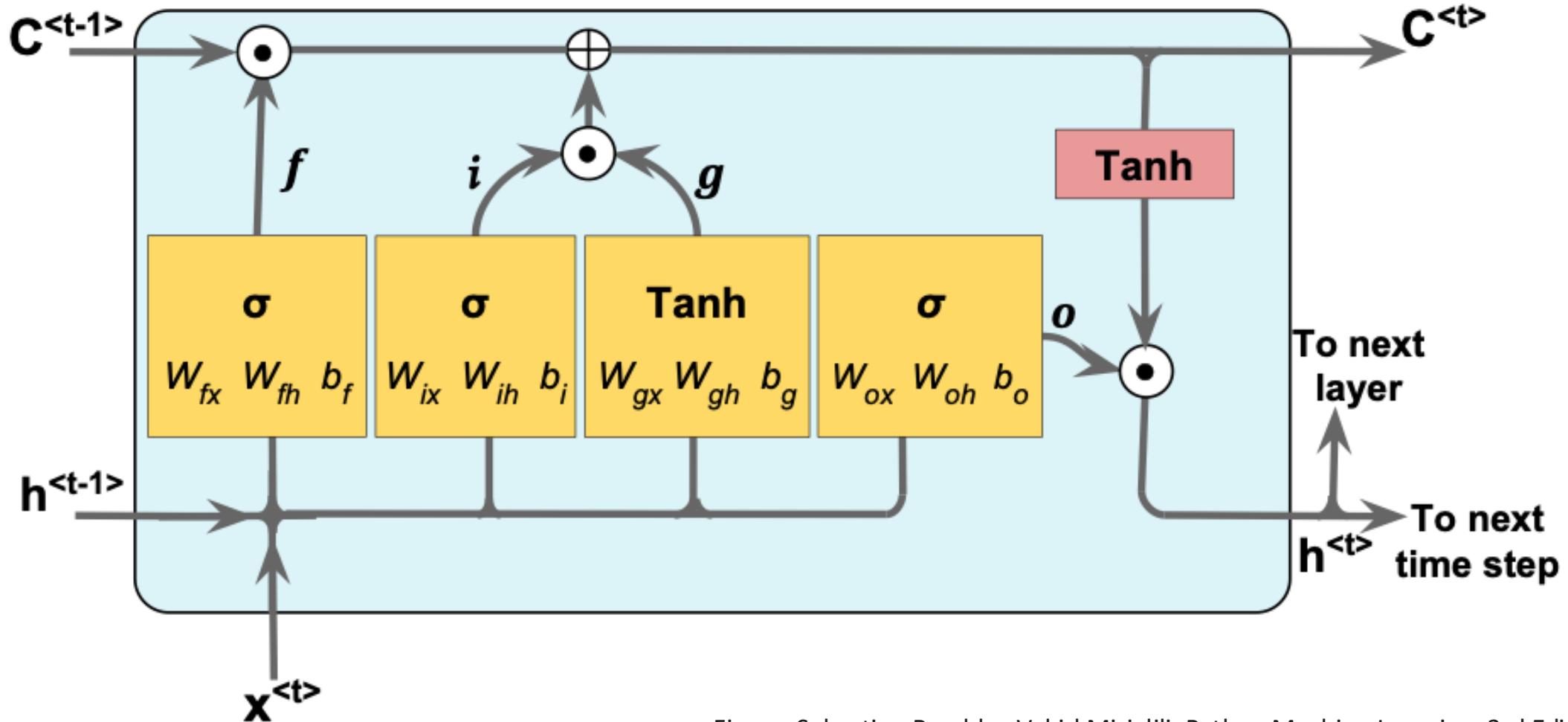
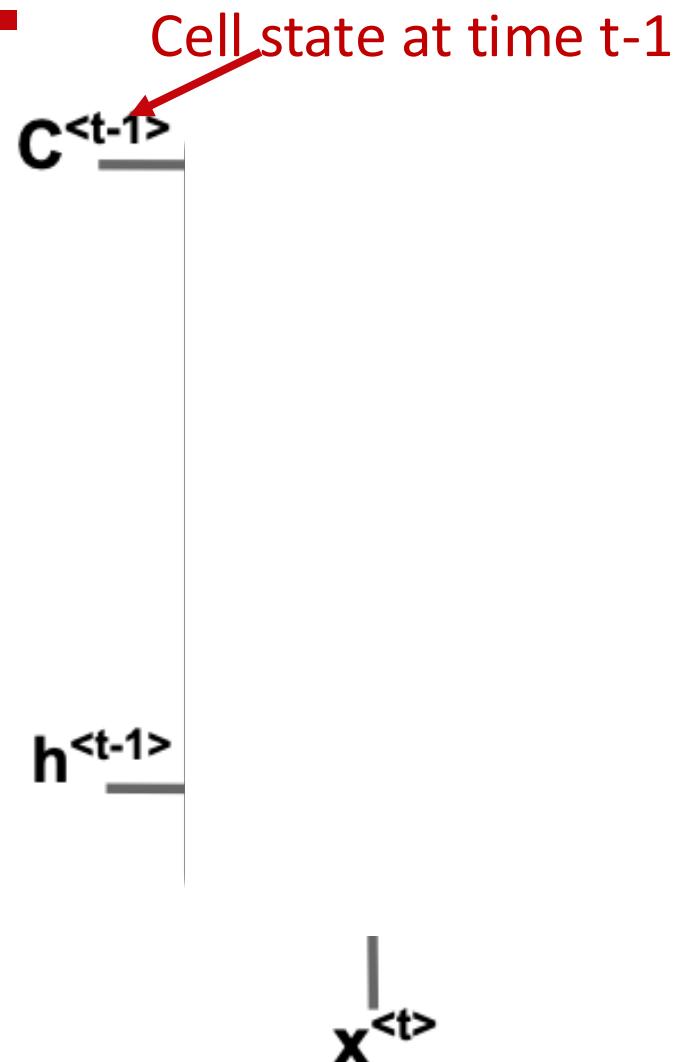


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Inside LSTM



Cell state at time t

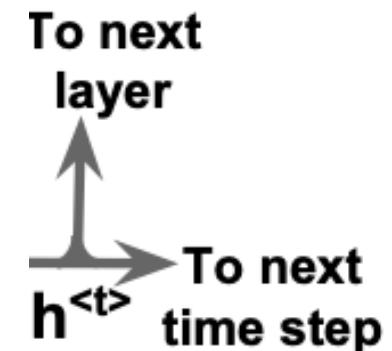


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Inside LSTM



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“Forget gate”: controls which information is remembered and which is forgotten

$$f_t = \sigma \left(\mathbf{W}_{fx} \mathbf{x}^{(t)} + \mathbf{W}_{fh} \mathbf{h}^{(t-1)} + \mathbf{b}_f \right)$$

Inside LSTM



Figure: Sebastian Raschka, Vahid Mirjalili. Python Machine Learning. 3rd Edition. Birmingham, UK: Packt Publishing, 2019

Inside LSTM

“Input gate”: $i_t = \sigma(\mathbf{W}_{ix}\mathbf{x}^{(t)} + \mathbf{W}_{ih}\mathbf{h}^{(t-1)} + \mathbf{b}_i)$

“Input node”: $g_t = \tanh(\mathbf{W}_{gx}\mathbf{x}^{(t)} + \mathbf{W}_{gh}\mathbf{h}^{(t-1)} + \mathbf{b}_g)$

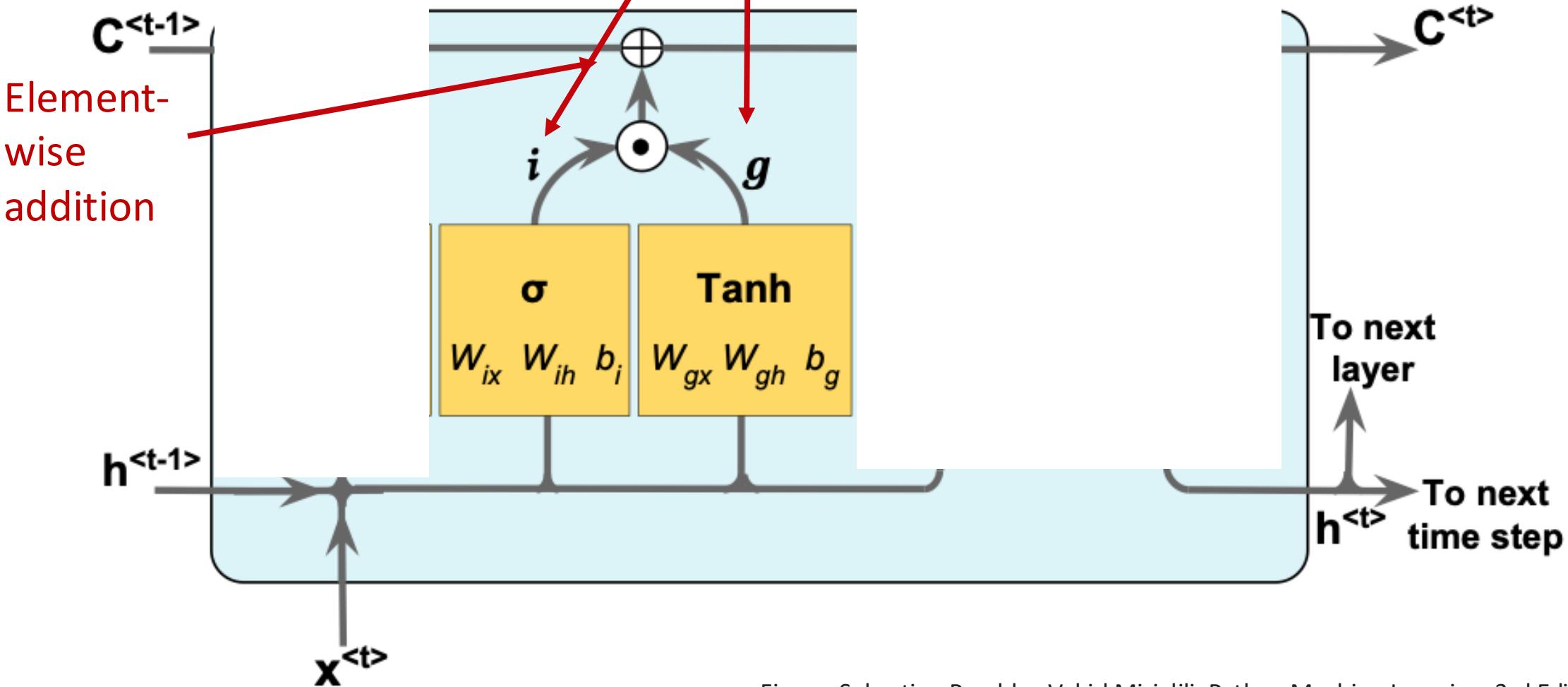


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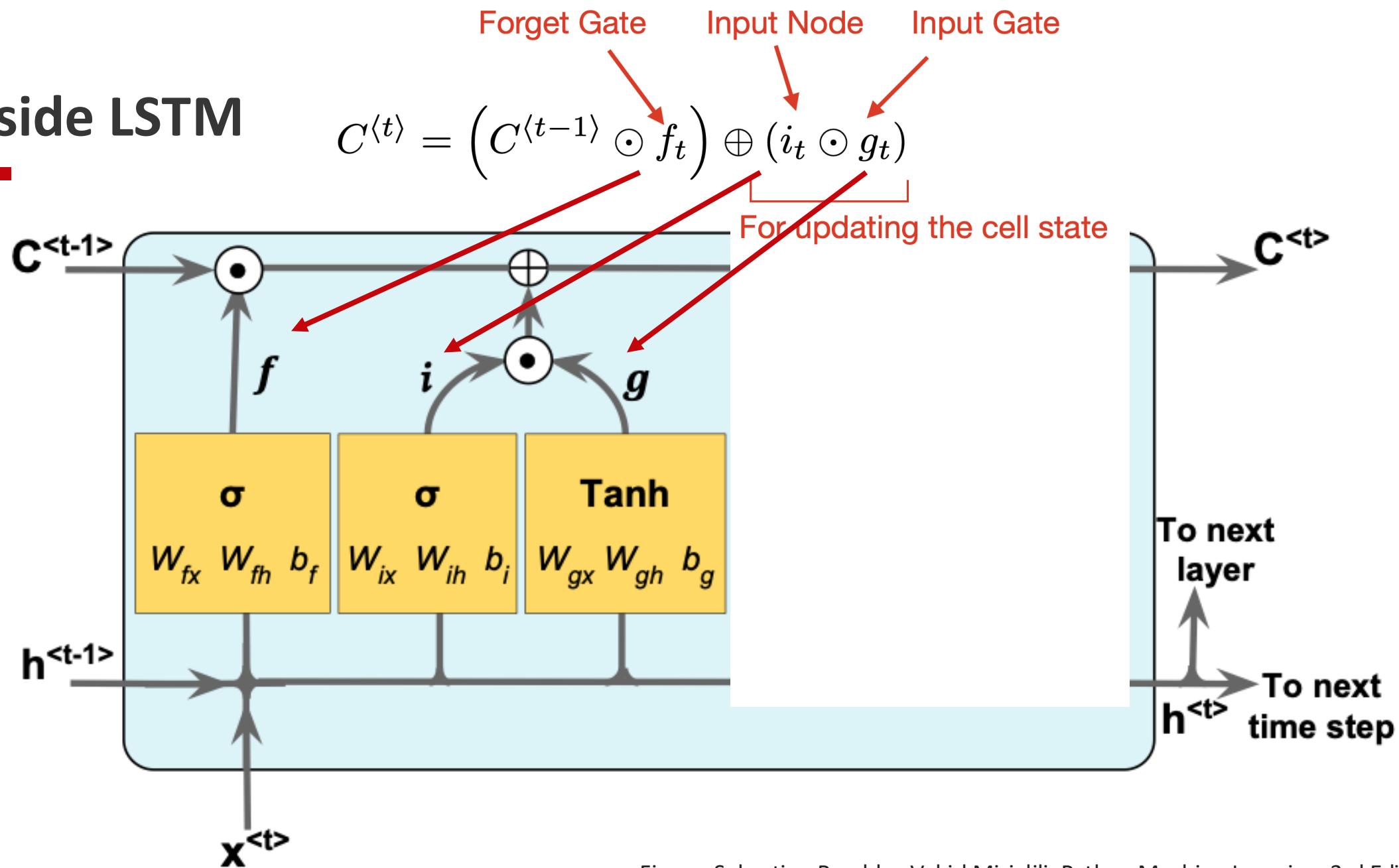


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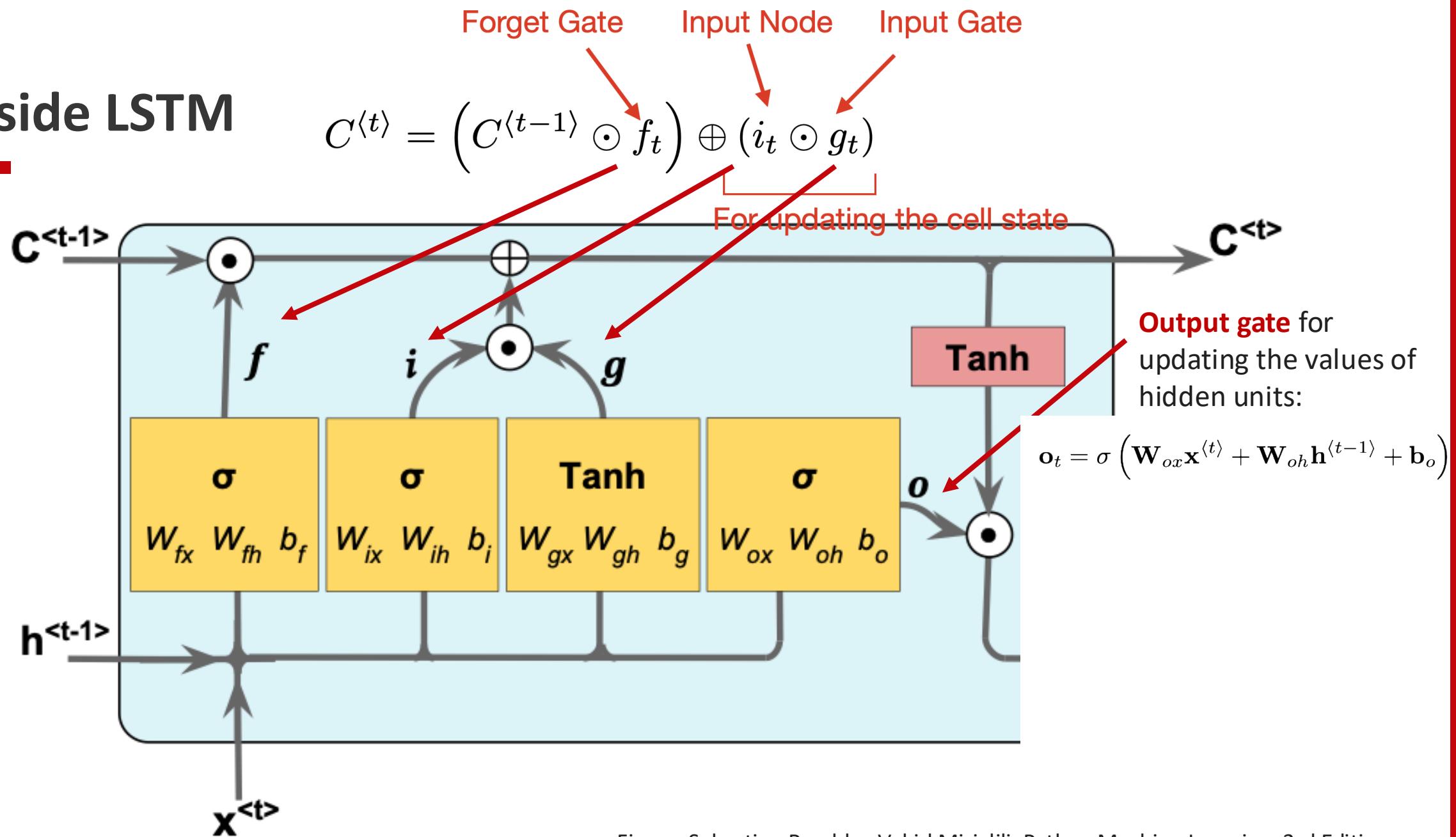


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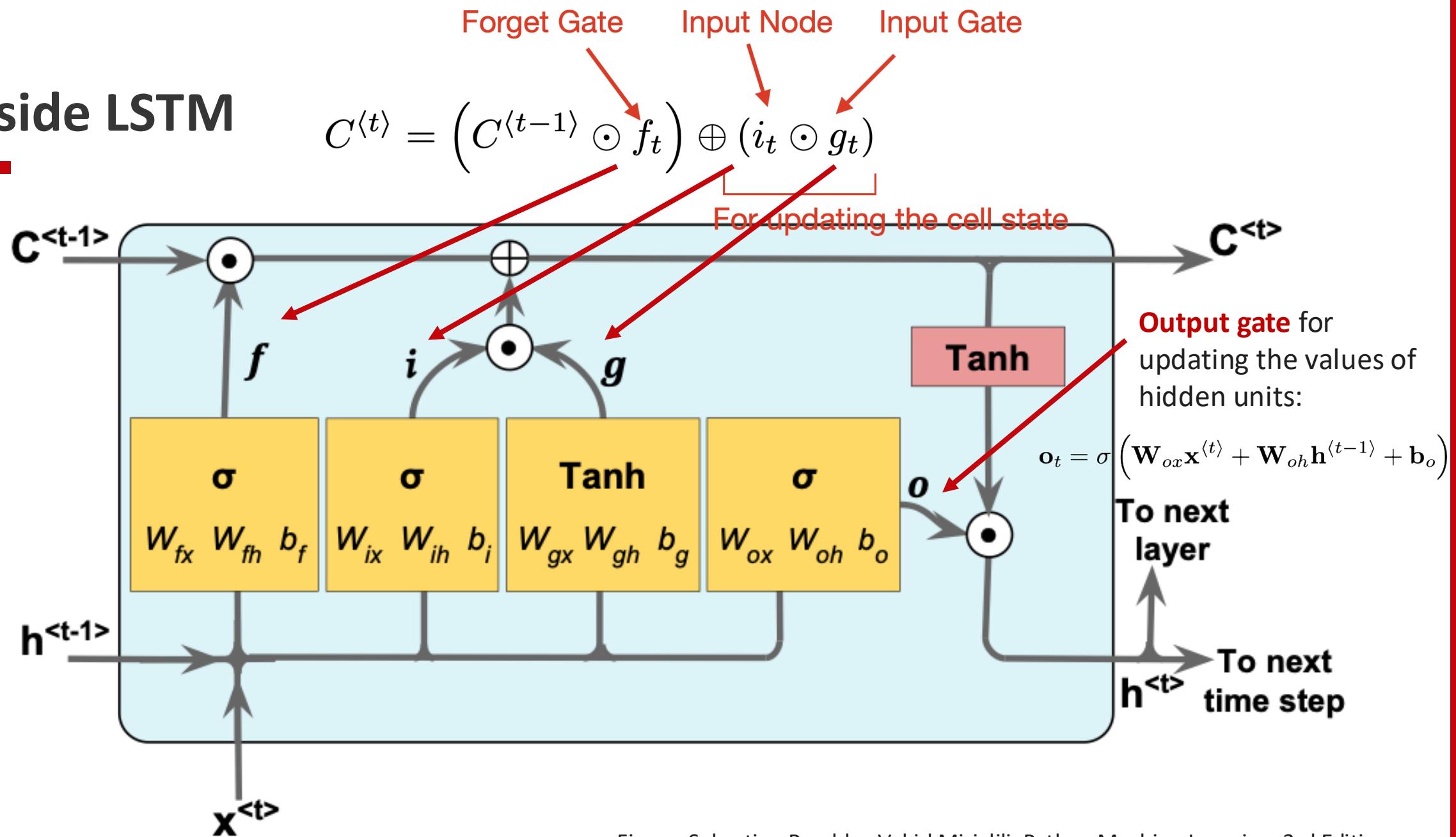


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LSTM Back Together

$$\mathbf{h}^{(t)} = \mathbf{o}_t \odot \tanh(\mathbf{C}^{(t)})$$

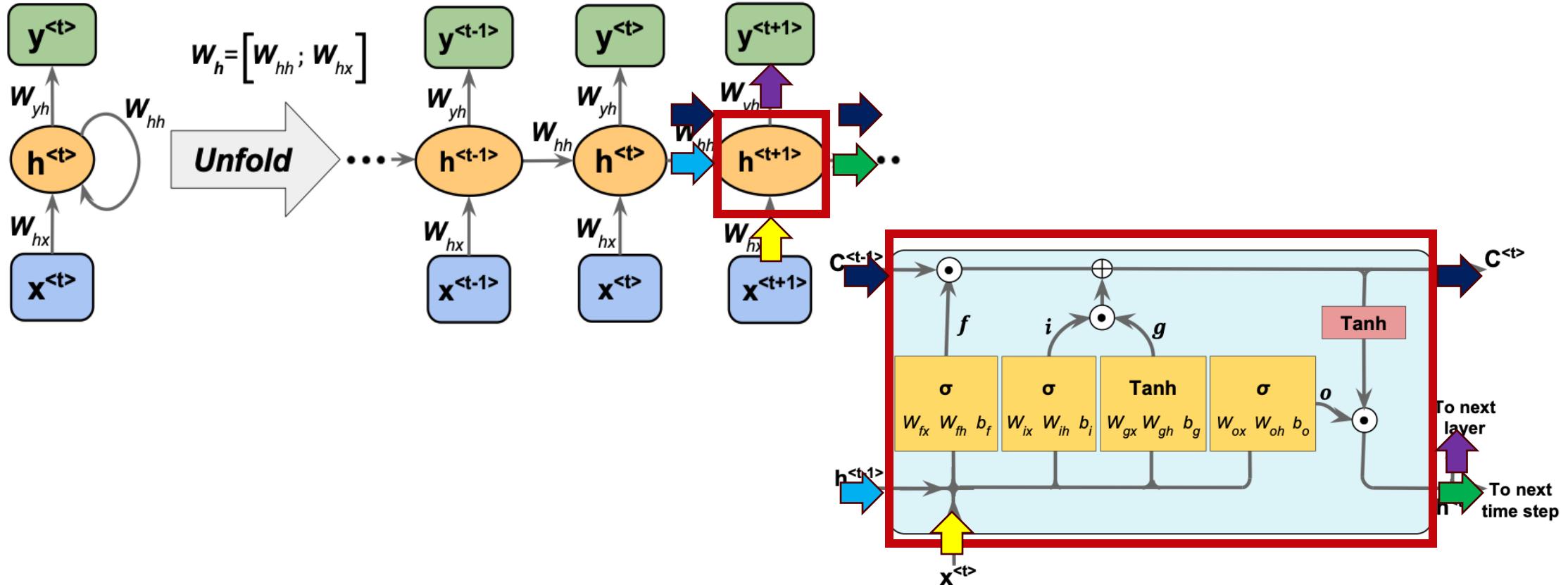


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RNN Step 1: Build Vocabulary

"Raw" training dataset

$x^{[1]}$ = "The sun is shining"

$x^{[2]}$ = "The weather is sweet"

$x^{[3]}$ = "The sun is shining,
the weather is sweet, and
one and one is two"

$y = [0, 1, 0]$

class labels



```
vocabulary = {  
    '<unk>': 0,  
    'and': 1,  
    'is': 2  
    'one': 3,  
    'shining': 4,  
    'sun': 5,  
    'sweet': 6,  
    'the': 7,  
    'two': 8,  
    'weather': 9,  
    '<pad>': 10 }
```



RNN Step 2: Convert text to indices

"Raw" training dataset

$x^{[1]} = \text{"The sun is shining"}$

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    'sun': 5,  
    'sweet': 6,  
    'the': 7,  
    'two': 8,  
    'weather': 9,  
    '<pad>': 10  
}
```

$x^{[1]} = \text{"The sun is shining"}$

[7 5 2 4 ... 10 10 10]

$x^{[2]} = \text{"The weather is sweet"}$

[7 9 2 6 ... 10 10 10]

$x^{[3]} = \text{"The sun is shining,}$
the weather is sweet, and
one and one is two"

[7 5 2 4 ... 3 2 8]

RNN Step 3: Convert indices to one-hot representation

"Raw" training dataset

$x^{[1]} = \text{"The sun is shining"}$

$x^{[2]} = \text{"The weather is sweet"}$

$x^{[3]} = \text{"The sun is shining,}$
 $\text{the weather is sweet, and}$
 $\text{one and one is two"}$

$x^{[1]} = \text{"The sun is shining"}$

```
vocabulary = {  
    '<unk>': 0,  
    'and': 1,  
    'is': 2  
    'one': 3,  
    'shining': 4,  
    'sun': 5,  
    'sweet': 6,  
    'the': 7,  
    'two': 8,  
    'weather': 9,  
    '<pad>': 10  
}
```

[7

5

2

4

...

10

10

10]

[0 0 0 0 0 0 0 1 0 0 0 0]

[0 0 0 0 1 0 0 0 0 0 0]

[0 0 1 0 0 0 0 0 0 0 0]

[0 0 0 1 0 0 0 0 0 0 0]

...

[0 0 0 1 0 0 0 0 0 0 0 1]

[0 0 0 1 0 0 0 0 0 0 0 1]

[0 0 0 1 0 0 0 0 0 0 0 1]



RNN Step 4: Convert one-hot to embeddings

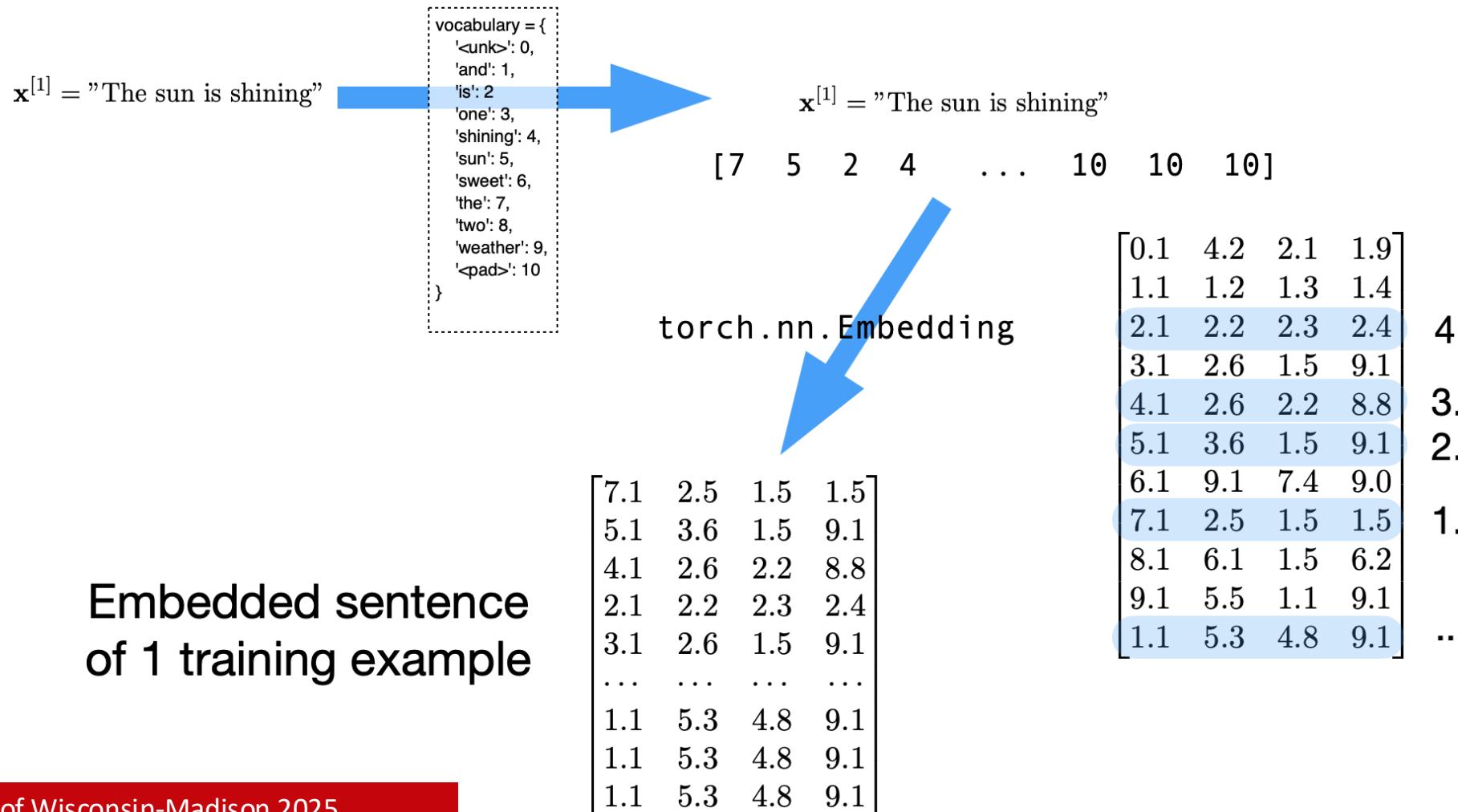
Embedding matrix

One-hot vector \times Embedding matrix = Hidden layer output

<p>One-hot vector</p> $[0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0]$	\times	<table border="1" style="margin-left: auto; margin-right: auto;"><tr><td>0.1</td><td>4.2</td><td>2.1</td><td>1.9</td></tr><tr><td>1.1</td><td>1.2</td><td>1.3</td><td>1.4</td></tr><tr><td>2.1</td><td>2.2</td><td>2.3</td><td>2.4</td></tr><tr><td>3.1</td><td>2.6</td><td>1.5</td><td>9.1</td></tr><tr><td>4.1</td><td>2.6</td><td>2.2</td><td>8.8</td></tr><tr><td>5.1</td><td>3.6</td><td>1.5</td><td>9.1</td></tr><tr><td>6.1</td><td>9.1</td><td>7.4</td><td>9.0</td></tr><tr><td>7.1</td><td>2.5</td><td>1.5</td><td>1.5</td></tr><tr><td>8.1</td><td>6.1</td><td>1.5</td><td>6.2</td></tr><tr><td>9.1</td><td>5.5</td><td>1.1</td><td>9.1</td></tr><tr><td>1.1</td><td>5.3</td><td>4.8</td><td>9.1</td></tr></table>	0.1	4.2	2.1	1.9	1.1	1.2	1.3	1.4	2.1	2.2	2.3	2.4	3.1	2.6	1.5	9.1	4.1	2.6	2.2	8.8	5.1	3.6	1.5	9.1	6.1	9.1	7.4	9.0	7.1	2.5	1.5	1.5	8.1	6.1	1.5	6.2	9.1	5.5	1.1	9.1	1.1	5.3	4.8	9.1	$= [7.1 \ 2.5 \ 1.5 \ 1.5]$
0.1	4.2	2.1	1.9																																												
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3.1	2.6	1.5	9.1																																												
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8.1	6.1	1.5	6.2																																												
9.1	5.5	1.1	9.1																																												
1.1	5.3	4.8	9.1																																												

PyTorch: Skip steps 3 and 4. Instead...

use a lookup function (`torch.nn.Embedding`)





LSTMs in PyTorch

<https://pytorch.org/docs/stable/generated/torch.nn.LSTM.html>

Parameters

- **input_size** – The number of expected features in the input x
- **hidden_size** – The number of features in the hidden state h
- **num_layers** – Number of recurrent layers. E.g., setting `num_layers=2` would mean stacking two LSTMs together to form a *stacked LSTM*, with the second LSTM taking in outputs of the first LSTM and computing the final results. Default: 1
- **bias** – If `False`, then the layer does not use bias weights b_{ih} and b_{hh} . Default: `True`
- **batch_first** – If `True`, then the input and output tensors are provided as (batch, seq, feature). Default: `False`
- **dropout** – If non-zero, introduces a *Dropout* layer on the outputs of each LSTM layer except the last layer, with dropout probability equal to `dropout`. Default: 0
- **bidirectional** – If `True`, becomes a bidirectional LSTM. Default: `False`
- **proj_size** – If > 0 , will use LSTM with projections of corresponding size. Default: 0

Examples:

```
>>> rnn = nn.LSTM(10, 20, 2)
>>> input = torch.randn(5, 3, 10)
>>> h0 = torch.randn(2, 3, 20)
>>> c0 = torch.randn(2, 3, 20)
>>> output, (hn, cn) = rnn(input, (h0, c0))
```



Good reading

- The Unreasonable Effectiveness of Recurrent Neural Networks by Andrej Karpathy
- On the difficulty of training recurrent neural networks by Razvan Pascanu, Tomas Mikolov, Yoshua Bengio

Questions?

