UIUC IE 522 Statistical Methods in Finance Project

Due 11:59pm Tuesday 12/17/2024

1. You must upload your report in a single pdf file and supporting documents (code, tables, plots, data, etc.) in a single zip file on https://canvas.illinois.edu before 11:59pm on Tuesday 12/17/2024. Half or all points will be taken off from a late project.
2. Your files will go through plagiarism screening. Make sure you submit your own work, not report copied from someone else, or code written by someone else.
3. The project should be done in a team of four students. Only one submission is needed per team.
4. Programming language: Both C++ and R can be used. If you use R, you might use R Markdown so that you have code and report in the same file. C++ is much faster and can save your time significantly.
5. Grading criteria: (1) Are the results complete and correct? (2) Is the report well organized? (3) Are the results well explained using tables and plots? (4) Does the team make efforts on reducing computational time? Report and explain such efforts in your report. (5) To ensure that all students are fully engaged and contribute equally, each of you are required to provide an evaluation of your members. The average evaluation a student gets will be an important part of the student's final score.
6. In this project, we use Monte Carlo methods to price options.

- 1). (4 points) Consider a European vanilla put option with strike price K and maturity T (in years). Assume the Black-Scholes-Merton model for the underlying asset price:

$$S_t = S_0 \exp\Big((r - q - \frac{1}{2}\sigma^2)t + \sigma B_t\Big), \quad 0 \le t \le T.$$

  Here S_0 is the initial asset price, r is the risk free interest rate per year with continuous compounding, q is continuous yield of the asset, σ is the volatility per year for the underlying asset, B_t is a standard Brownian motion. The price of the European put is given by $p = e^{-rT}\mathbb{E}\Big[\max(0, K - S_T)\Big]$. Write a program to price the put option using Monte Carlo simulation. Your program should output the sample size, option price, estimated standard error, 95% confidence interval, the absolute pricing error (to compute this error, use Black-Scholes formula to compute the exact put option price), and the computational time in seconds. For $S_0 = K = 100, T = 0.5, r = 0.04, q = 0.02, \sigma = 0.2$, investigate the convergence of the Monte Carlo method when the sample size N increases. Your Monte Carlo estimate should be accurate up to cent.

  - 1a). (Optional, 2 extra points) Research and implement the antithetic approach and compare with the above standard approach.

- 2). (4 points) Consider an Asian call option in the Black-Scholes-Merton model with strike price K and maturity T (in years). The option payoff at maturity is given by $\max(0, \bar{S}_T - K)$, where $\bar{S}_T$ is the average asset price

$$\bar{S}_T = \frac{1}{m}\sum_{i=1}^{m} S_{t_i}, \qquad t_i = i\Delta t,\ \Delta t = T/m.$$

  The price of the Asian call is given by $c = e^{-rT}\mathbb{E}\Big[\max(0, \bar{S}_T - K)\Big]$. Write a program to price the Asian call option using Monte Carlo simulation. Your program should output the sample size, option price, estimated standard error, 95% confidence interval, and the computational time in seconds. For $S_0 = K = 100, T = 1, r = 10\%, q = 0, \sigma = 20\%, m = 50$, investigate the convergence of the Monte Carlo method when the sample size N increases. Your approximate option price should be accurate up to cent.

  - 2a). (Optional, 2 extra points) Research and implement the control variate approach. Use geometric Asian call as a control. Compare the approaches with and without control and investigate the effectiveness of the control variate technique.
  - 2b). (Optional, 2 extra points) Research and implement the moment matching approach for the above Asian call option.

- 3). (4 points) American style options can be exercised at any time at or before option maturity. The 2001 paper "Valuing American Options by Simulation: A Simple Least-Squares Approach" by Longstaff and Schwartz describes how to price American options using Monte Carlo simulation (https://people.math.ethz.ch/~hjfurrer/teaching/LongstaffSchwartzAmericanOptionsLeastSquareMonteCarlo.pdf). Research and implement this approach for an American vanilla put option. For $S_0 = K = 100, T = 1/12, r = 4\%, q = 2\%, \sigma = 20\%$, investigate how the sample size N (number of asset price paths generated), number of time steps M (time step size $\Delta t = T/M$), and number of regressors K affect the approximate option price.
  - 3a). (Optional, 2 extra points) Research and implement the Binomial Black-Scholes with Richardson Extrapolation (BBSR) approach for the above American put option (Broadie and Detemple, 1996, American Option Valuation: New Bounds, Approximations, and a Comparison of Existing Methods, The Review of Financial Studies, Vol 9, No. 4, 1211-1250). This allows you to compute a very accurate option price to use as benchmark in part 3).