Statistical and Computational Tradeoffs in Biconvex Optimization

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Outline

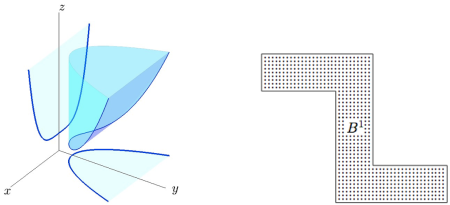
- Motivations
- General theory of biconvex optimization
- Application 1: bigraphical model
- Application 2: joint clustering and network estimation
- Future work

Background: Biconvex Optimization

- A function $g(x, y) : \mathcal{A} \times \mathcal{B} \to \mathbb{R}$ is biconvex if $g(x, y)$ is convex in x for fixed $y \in \mathcal{B}$, and convex in y for fixed $x \in \mathcal{A}$.
- Biconvex optimization:

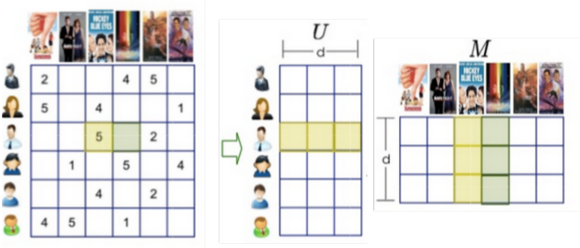
$$\min \; g(x, y)$$
$$s.t. \; x \in \mathcal{A}, y \in \mathcal{B}$$

Figure : Biconvex function and biconvex set



Resource: https://inst.eecs.berkeley.edu/ ee127a

Motivation: Non-negative matrix factorization



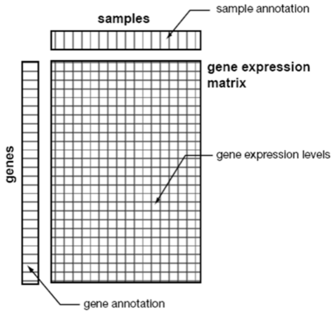
Source: A. Karatzoglou, ESSIR 2013 Recommender Systems tutorial

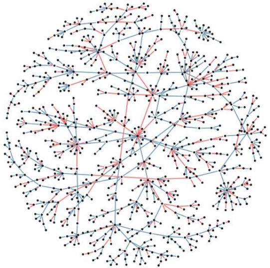
- Non-negative matrix factorization solves

$$\begin{aligned} \min_{U,M} \quad & \frac{1}{2}\|X - UM\|_F^2 \\ s.t. \quad & U_{ij} \geq 0, M_{ij} \geq 0. \end{aligned}$$

Motivation: Bigraphical model

Traditional Graphical Model:





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Motivation: Bigraphical model

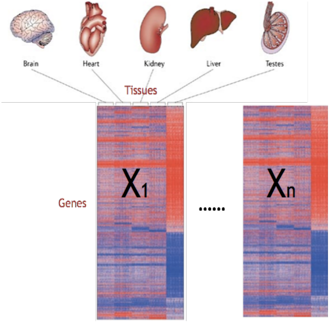


Figure : Matrix-variate data

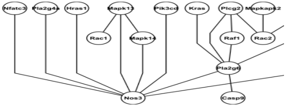


Figure : Gene network

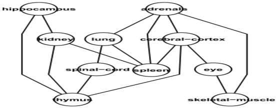


Figure : Tissue network

Source: Yin and Li (2012)

Biconvex Optimization

- Population objective function: $g(\mathbf{a}, \mathbf{b})$

$$
\begin{aligned}
(\mathbf{a}^*, \mathbf{b}^*) &= \arg\min \; g(\mathbf{a}, \mathbf{b}) \\
& \quad s.t. \; \mathbf{a} \in \mathcal{A}, \mathbf{b} \in \mathcal{B}
\end{aligned}
$$

- Sample objective function: $g_n(\mathbf{a}, \mathbf{b})$
- Goal: Find a minimizer $(\widehat{\mathbf{a}}, \widehat{\mathbf{b}})$ via $g_n(\mathbf{a}, \mathbf{b})$ s.t. $\|\widehat{\mathbf{a}} - \mathbf{a}^*\|_2$ and $\|\widehat{\mathbf{b}} - \mathbf{b}^*\|_2$ are small given limited computational resources.

Alternative Update Algorithm

Input: function $g_n(\mathbf{a}, \mathbf{b})$, maximal number of iterations T.

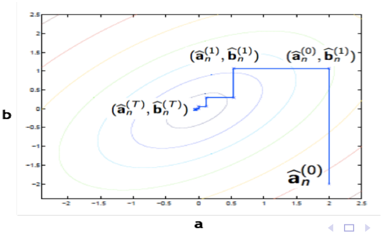
Initialize: $\widehat{\mathbf{a}}_n^{(0)}$.

For $t = 1$ **to** T:

- Fix $\widehat{\mathbf{a}}_n^{(t-1)}$, update $\widehat{\mathbf{b}}_n^{(t)} = \arg\min_{\mathbf{b}} g_n(\widehat{\mathbf{a}}_n^{(t-1)}, \mathbf{b})$;
- Fix $\widehat{\mathbf{b}}_n^{(t)}$, update $\widehat{\mathbf{a}}_n^{(t)} = \arg\min_{\mathbf{a}} g_n(\mathbf{a}, \widehat{\mathbf{b}}_n^{(t)})$;

End For

Output: $\widehat{\mathbf{a}} = \widehat{\mathbf{a}}_n^{(T)}$ and $\widehat{\mathbf{b}} = \widehat{\mathbf{b}}_n^{(T)}$.



Theory: Outline

$$\begin{tikzcd} \boxed{g_n(\mathbf{a}, \mathbf{b})} \arrow[d] & \boxed{g(\mathbf{a}, \mathbf{b})} \arrow[d] \\ (\widehat{\mathbf{a}}_n^{(t)}, \widehat{\mathbf{b}}_n^{(t)}) \arrow[d] & (\mathbf{a}^{(t)}, \mathbf{b}^{(t)}) \arrow[d] \\ {} & {} \end{tikzcd}$$

$$\left\|\widehat{\mathbf{a}}_n^{(t)} - \mathbf{a}^*\right\|_2 \leq \underbrace{\left\|\mathbf{a}^{(t)} - \mathbf{a}^*\right\|_2}_{\textit{Population}} + \underbrace{\left\|\widehat{\mathbf{a}}_n^{(t)} - \mathbf{a}^{(t)}\right\|_2}_{\textit{Sample}}$$

Theory: Population Version

- We focus on the Euclidean ball of radius $\alpha > 0$ for $\mathcal{A}$ and $\mathcal{B}$.

$$\begin{aligned}\mathcal{A} = \mathcal{B}(\alpha; \mathbf{a}^*) &:= \{\mathbf{a} \in \mathbb{R}^p : \|\mathbf{a} - \mathbf{a}^*\|_2 \leq \alpha\} \\ \mathcal{B} = \mathcal{B}(\alpha; \mathbf{b}^*) &:= \{\mathbf{b} \in \mathbb{R}^q : \|\mathbf{b} - \mathbf{b}^*\|_2 \leq \alpha\}\end{aligned}$$

- Denote $\nabla_1 g(\mathbf{a}, \mathbf{b})$ be the gradient w.r.t. $\mathbf{a}$ and $\nabla_2 g(\mathbf{a}, \mathbf{b})$ be the gradient w.r.t. $\mathbf{b}$.

Condition ((λ, λ')-Strong-Convexity)

The function $g(\mathbf{a}^*, \cdot)$ is λ-strongly convex, and $g(\cdot, \mathbf{b}^*)$ is λ'-strongly convex. That is, for any $\mathbf{b}_1, \mathbf{b}_2 \in \mathcal{B}$ and $\mathbf{a}_1, \mathbf{a}_2 \in \mathcal{A}$,

$$\begin{aligned}g(\mathbf{a}^*, \mathbf{b}_1) - g(\mathbf{a}^*, \mathbf{b}_2) - \langle \nabla_2 g(\mathbf{a}^*, \mathbf{b}_2), \mathbf{b}_1 - \mathbf{b}_2 \rangle &\geq \frac{\lambda}{2} \cdot \|\mathbf{b}_1 - \mathbf{b}_2\|_2^2 \\ g(\mathbf{a}_1, \mathbf{b}^*) - g(\mathbf{a}_2, \mathbf{b}^*) - \langle \nabla_1 g(\mathbf{a}_2, \mathbf{b}^*), \mathbf{a}_1 - \mathbf{a}_2 \rangle &\geq \frac{\lambda'}{2} \cdot \|\mathbf{a}_1 - \mathbf{a}_2\|_2^2\end{aligned}$$

Theory: Population Version

- Denote population minimization functions:

$$M_1(\mathbf{a}) = \arg\min_{\mathbf{b}} g(\mathbf{a}, \mathbf{b}); \quad M_2(\mathbf{b}) = \arg\min_{\mathbf{a}} g(\mathbf{a}, \mathbf{b}).$$

Condition ((γ, γ')-Lipschitz-Gradient)

The function $\nabla_2 g(\mathbf{a}, \cdot)$ satisfies γ-Lipschitz gradient condition, and the function $\nabla_1 g(\cdot, \mathbf{b})$ satisfies γ'-Lipschitz gradient condition. That is, for any $\mathbf{a} \in \mathcal{A}$ and any $\mathbf{b} \in \mathcal{B}$,

$$\left\| \nabla_2 g(\mathbf{a}^*, M_1(\mathbf{a})) - \nabla_2 g(\mathbf{a}, M_1(\mathbf{a})) \right\|_2 \leq \gamma \cdot \|\mathbf{a}^* - \mathbf{a}\|_2$$

$$\left\| \nabla_1 g(M_2(\mathbf{b}), \mathbf{b}^*) - \nabla_1 g(M_2(\mathbf{b}), \mathbf{b}) \right\|_2 \leq \gamma' \cdot \|\mathbf{b}^* - \mathbf{b}\|_2.$$

Theory: Population Version

Theorem

Under $(\lambda, \lambda^{'})$-Strong-Convexity and $(\gamma, \gamma^{'})$-Lipschitz-Gradient conditions, we have

$$\|M_1(\mathbf{a}) - \mathbf{b}^*\|_2 \leq (\gamma/\lambda) \cdot \|\mathbf{a} - \mathbf{a}^*\|_2 \quad \textit{for any } \mathbf{a} \in \mathcal{A},$$
$$\|M_2(\mathbf{b}) - \mathbf{a}^*\|_2 \leq (\gamma^{'}/\lambda^{'}) \cdot \|\mathbf{b} - \mathbf{b}^*\|_2 \quad \textit{for any } \mathbf{b} \in \mathcal{B}.$$

Moreover, for any initialization $\mathbf{a}^{(0)}$, the solutions from the population alternative updates converge linearly,

$$\|\mathbf{b}^{(t)} - \mathbf{b}^*\|_2 \leq (\frac{\gamma}{\lambda})^t (\frac{\gamma^{'}}{\lambda^{'}})^{t-1} \cdot \|\mathbf{a}^{(0)} - \mathbf{a}^*\|_2$$
$$\|\mathbf{a}^{(t)} - \mathbf{a}^*\|_2 \leq (\frac{\gamma}{\lambda})^t (\frac{\gamma^{'}}{\lambda^{'}})^t \cdot \|\mathbf{a}^{(0)} - \mathbf{a}^*\|_2.$$

Theory: Sample Version

- Denote sample minimization functions:

$$M_{1n}(\mathbf{a}) = \arg\min_{\mathbf{b}} g_n(\mathbf{a}, \mathbf{b}); \quad M_{2n}(\mathbf{b}) = \arg\min_{\mathbf{a}} g_n(\mathbf{a}, \mathbf{b}).$$

Condition (Statistical-Error(ϵ_s, δ, n))

Uniformly over all $\mathbf{a} \in \mathcal{A}$ with $\mathbf{b} \in \mathcal{B}$, we have that

$$\max\left\{ \left\| M_{1n}(\mathbf{a}) - M_1(\mathbf{a}) \right\|_2, \left\| M_{2n}(\mathbf{b}) - M_2(\mathbf{b}) \right\|_2 \right\} \le \epsilon_s$$

with probability at least $1 - \delta$.

Theory: Main Result

Denote the initialization error as $\epsilon_0 := \|\widehat{\mathbf{a}}_n^{(0)} - \mathbf{a}^*\|_2$.

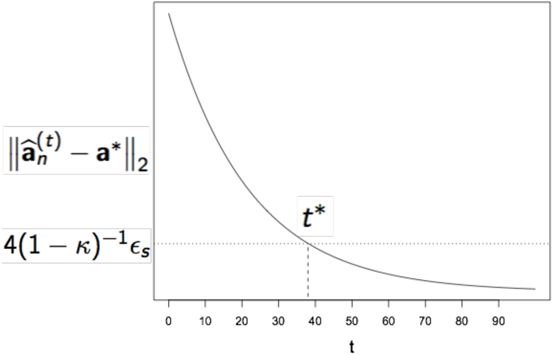
Theorem

Under above assumptions and assume $\gamma < \lambda$ and $\gamma' < \lambda'$ s.t. $\kappa = \gamma\gamma'/(\lambda\lambda') < 1$. Assume $\widehat{\mathbf{a}}_n^{(0)} \in \mathcal{B}(\alpha; \mathbf{a}^*)$, and n is sufficiently large such that $\epsilon_s \le \min\{(1-\gamma/\lambda)\alpha, (1-\gamma'/\lambda')\alpha\}$. Then

$$
\begin{aligned}
\left\|\widehat{\mathbf{b}}_n^{(t)} - \mathbf{b}^*\right\|_2 &\le 2(1-\kappa)^{-1}\epsilon_s + \kappa^{t-1}\epsilon_0, \\
\left\|\widehat{\mathbf{a}}_n^{(t)} - \mathbf{a}^*\right\|_2 &\le \underbrace{2(1-\kappa)^{-1}\epsilon_s}_{Statistical\ Error} + \underbrace{\kappa^t\epsilon_0}_{Optimization\ Error}.
\end{aligned}
$$

Theory: Main Result

$$\left\|\widehat{\mathbf{a}}_n^{(t)} - \mathbf{a}^*\right\|_2 \le \underbrace{2(1-\kappa)^{-1}\epsilon_s}_{\text{Statistical Error}} + \underbrace{\kappa^t \epsilon_0}_{\text{Optimization Error}}$$



Application 1: Bigraphical model

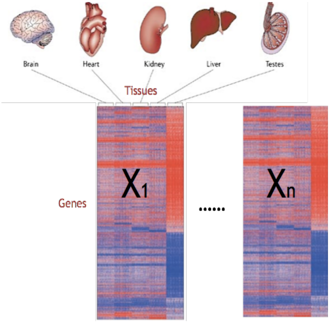


Figure : Matrix-variate data

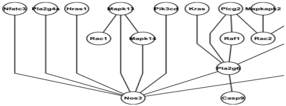


Figure : Gene network

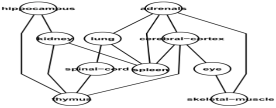


Figure : Tissue network

Application 1: Bigraphical Model

A random matrix $\mathbf{X} \in \mathbb{R}^{p \times q}$ follows a matrix-variate normal if

$$\mathrm{vec}(\mathbf{X}) \sim N_{pq}(\mathbf{0}, \Psi \otimes \Sigma).$$

- Denote the precision matrices $\Lambda = \Psi^{-1}$ and $\Omega = \Sigma^{-1}$
- Zeros in Λ (or Ω) define pairwise conditional independence of corresponding entries given all other entries.
- Goal: Estimate sparse Λ and Ω.

Bigraphical Model

- Given i.i.d. data $\mathbf{X}_1 \ldots, \mathbf{X}_n$, the lasso penalized likelihood estimator minimizes

$$g_n(\Omega, \Lambda) = \frac{1}{npq} \sum_{i=1}^{n} \mathrm{tr}(\mathbf{X}_i \Lambda \mathbf{X}_i^\top \Omega) - \frac{1}{p} \mathrm{logdet}(\Omega) - \frac{1}{q} \mathrm{logdet}(\Lambda) + \lambda_1 \|\Omega\|_1 + \lambda_2 \|\Lambda\|_1.$$

- The population version likelihood function is

$$g(\Omega, \Lambda) = \frac{1}{pq} \mathbb{E}\big[\mathrm{tr}(\mathbf{X} \Lambda \mathbf{X}^\top \Omega)\big] - \frac{1}{p} \mathrm{logdet}(\Omega) - \frac{1}{q} \mathrm{logdet}(\Lambda).$$

- The objective function $g(\Omega, \Lambda)$ is biconvex in (Ω, Λ).

Bigraphical Model: Algorithm

Input: samples $\mathbf{X}_1 \ldots, \mathbf{X}_n$, maximal number of iterations T, tuning parameters λ_1, λ_2.
Initialize $\Omega^{(0)}$.
For $t = 1$ **to** T: Alternatively update $\Omega^{(t)}, \Lambda^{(t)}$ as

- Given $\Omega^{(t-1)}$, compute $\mathbf{S}_1 = (np)^{-1} \sum_{i=1}^n \mathbf{X}_i^\top \Omega^{(t-1)} \mathbf{X}_i$ and solve the glasso problem

$$\Lambda^{(t)} = \arg\min_{\Lambda} \left\{ \frac{1}{q} \mathrm{tr}(\mathbf{S}_1 \Lambda) - \frac{1}{q} \mathrm{logdet}(\Lambda) + \lambda_\Lambda \|\Lambda\|_1 \right\}$$

- Given $\Lambda^{(t)}$, compute $\mathbf{S}_2 = (nq)^{-1} \sum_{i=1}^n \mathbf{X}_i \Lambda^{(t)} \mathbf{X}_i^\top$ and solve the glasso problem

$$\Omega^{(t)} = \arg\min_{\Omega} \left\{ \frac{1}{p} \mathrm{tr}(\mathbf{S}_2 \Omega) - \frac{1}{p} \mathrm{logdet}(\Omega) + \lambda_\Omega \|\Omega\|_1 \right\}$$

End For
Output: $\widehat{\Omega} = \Omega^{(T)}$ and $\widehat{\Lambda} = \Lambda^{(T)}$.

Bigraphical Model: Convergence Analysis

- Step 1: Verify Strongly-Convexity of $g(\Omega^*, \cdot)$ and $g(\cdot, \Lambda^*)$, and Lipschitz-Gradient conditions of $\nabla_1 g(\cdot, \Lambda)$ and $\nabla_2 g(\Omega, \cdot)$.
- Define

$$\begin{aligned} \mathcal{B}(\alpha; \Omega^*) &:= \{\Omega \in \mathbb{R}^{p \times p} : \|\Omega - \Omega^*\|_F \le \alpha\} \\ \mathcal{B}(\alpha; \Lambda^*) &:= \{\Lambda \in \mathbb{R}^{q \times q} : \|\Lambda - \Lambda^*\|_F \le \alpha\}. \end{aligned}$$

Bigraphical Model: Convergence Analysis

The alternative update algorithm via the population objective function $g(\Omega, \Lambda)$ is locally contractive.

Corollary

For the population objective function $g(\Omega, \Lambda)$, we have that $g(\Omega^*, \cdot)$ and $g(\cdot, \Lambda^*)$ are strongly convex with parameters, respectively,

$$\lambda = p^{-1}[\|\Omega^*\|_2 + 3\alpha]^{-2} \ \mathit{and} \ \lambda' = q^{-1}[\|\Lambda^*\|_2 + 3\alpha]^{-2}.$$

Both $\nabla_1 g(\cdot, \Lambda)$ and $\nabla_2 g(\Omega, \cdot)$ satisfy the Lipschitz-Gradient conditions with

$$\gamma = \gamma' = (pq)^{-1}\|\Sigma^* \otimes \Psi^*\|_F.$$

If $\|\Sigma^* \otimes \Psi^*\|_F$ is bounded and if there exist constants $C_1, C_2 > 0$ such that $C_1 \leq \|\Omega^*\|_2, \|\Lambda^*\|_2 \leq C_2$, then

$$\gamma < \lambda, \gamma' < \lambda'.$$

- Step 2: Compute the statistical error.

- Let $\mathcal{S}_1 := \{(i,j) : \Omega^*_{ij} \neq 0\}$ and $\mathcal{S}_2 := \{(i,j) : \Lambda^*_{ij} \neq 0\}$.
- Denote $s_1 = |\mathcal{S}_1| - p$ and $s_2 = |\mathcal{S}_2| - q$.
- Remind that

$$M_{1n}(\Omega) := \arg\min_{\Lambda} g_n(\Omega, \Lambda), \;\; M_{2n}(\Lambda) = \arg\min_{\Omega} g_n(\Omega, \Lambda),$$
$$M_{1}(\Omega) := \arg\min_{\Lambda} g(\Omega, \Lambda), \;\; M_{2}(\Lambda) = \arg\min_{\Omega} g(\Omega, \Lambda).$$

Condition (Bounded Eigenvalues)

There are positive constants C_1 and C_2 such that

$$0 < C_1 \leq \lambda_{\min}(\Sigma^*) \leq \lambda_{\max}(\Sigma^*) \leq 1/C_1 < \infty$$
$$0 < C_2 \leq \lambda_{\min}(\Psi^*) \leq \lambda_{\max}(\Psi^*) \leq 1/C_2 < \infty.$$

Condition (Tuning)

The tuning parameters satisfy

$$\lambda_\Omega = O\left(\sqrt{\frac{\log p}{np^2 q}}\right), \lambda_\Lambda = O\left(\sqrt{\frac{\log q}{npq^2}}\right).$$

Bigraphical Model: Convergence Analysis

Corollary

Under above two conditions, the statistical errors are

$$\sup_{\Omega \in \mathcal{B}(\alpha;\Omega^*)} \left\| M_{1n}(\Omega) - M_1(\Omega) \right\|_F = O_p\left(\sqrt{\frac{(q+s_2)\log q}{np}} \right),$$

$$\sup_{\Lambda \in \mathcal{B}(\alpha;\Lambda^*)} \left\| M_{2n}(\Lambda) - M_2(\Lambda) \right\|_F = O_p\left(\sqrt{\frac{(p+s_1)\log p}{nq}} \right).$$

- Step 1: Exploit the independence structure in $(\Omega^*)^{\frac{1}{2}}\mathbf{X}(\Lambda^*)^{\frac{1}{2}}$.
- Step 2: Use Talagrand inequality for the convergence rate of

$$(np)^{-1}\sum_{i=1}^{n} \mathbf{X}_i^\top \Omega \mathbf{X}_i - p^{-1}\mathbb{E}\left[\mathbf{X}^\top \Omega \mathbf{X}\right].$$

Bigraphical Model: Main Result

Combine above two Corollaries, we have

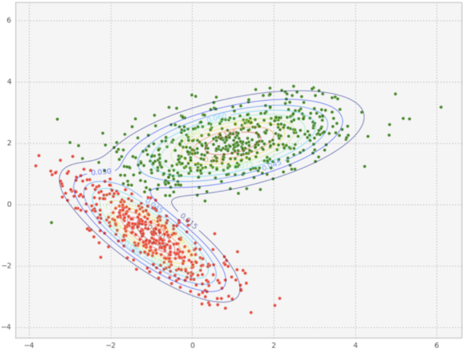
$$\|\widehat{\Lambda}^{(t)} - \Lambda^*\|_F \leq C\sqrt{\frac{(p+s_1)\log p}{nq}} + \kappa^{t-1}\epsilon_0,$$

$$\|\widehat{\Omega}^{(t)} - \Omega^*\|_F \leq C\sqrt{\frac{(q+s_2)\log q}{np}} + \kappa^{t}\epsilon_0.$$

- When $n = 1$, we can still consistently estimate Λ^* or Ω^*.
- Leng and Tang (2012) showed there existed a local minimizer which can obtain above statistical error.
- We prove that our algorithm can find such minimizer.
- The convergence rates showed in Yin and Li (2012), Tsiligkaridis et al. (2013) are slower than ours and they require at least $n > (p+q)(\log p + \log q)$.

Application 2: Joint Clustering and Network Estimation

- Gaussian mixture model (GMM) $\pi_k N(\boldsymbol{\mu}_k, \Sigma_k)$, $k = 1, \ldots, K$.



Source: http://www.nehalemlabs.net/

Application 2: Background

- Samples $\mathbf{x}_i, \ldots, \mathbf{x}_n$ follows a GMM with $\pi_k f_k(x; \boldsymbol{\mu}_k, \Sigma_k)$.
- If assume $\Sigma_k = \sigma_k \mathbb{1}_p$ (Pan and Shen, 2007, Sun et al., 2012), the clustering can be solved by minimizing

$$\sum_{i=1}^{n} \log \left(\sum_{k=1}^{K} \pi_k f_k(\mathbf{x}_i; \boldsymbol{\mu}_k, \Sigma_k) \right) - P(\boldsymbol{\mu}).$$

- If assume clustering assignment is given, the networks are estimated jointly (Guo et al., 2011, Danaher et al., 2014) via

$$\begin{aligned} \max_{\Omega_1,\ldots,\Omega_K} \quad & \sum_{k=1}^{K} n_k [\log\det(\Omega_k) - \mathrm{tr}(S_k \Omega_k)] - P(\Omega) \\ \text{s.t.} \quad & \Omega_1, \ldots, \Omega_K \text{ are positive definite.} \end{aligned}$$

Application 2: Joint Clustering and Network Estimation

- Denote the set of parameters as $\Theta := \{(\boldsymbol{\mu}_k, \Omega_k), k \in [K]\}$. Our optimization is formulated as

$$\max_{\pi_k, \boldsymbol{\mu}_k, \Omega_k} \sum_{i=1}^{n} \log \left(\sum_{k=1}^{K} \pi_k f_k \Big(\mathbf{x}_i; \boldsymbol{\mu}_k, \Omega_k \Big) \right) - P(\Theta).$$

- We focus on the l_1 penalty on $\boldsymbol{\mu}_k$ and fussed graphical lasso penalty (Danaher et al., 2014) on $\Omega_k = (\omega_{kij})$,

$$P(\Theta) = \lambda_1 \sum_{k=1}^{K} \sum_{j=1}^{p} |\mu_{kj}| + \lambda_2 \sum_{k=1}^{K} \sum_{i \neq j} |\omega_{kij}| + \lambda_3 \sum_{k<k'} \sum_{i,j} |\omega_{kij} - \omega_{k'ij}|.$$

Application 2: EM Algorithm

- Denote the K clusters as $\mathcal{A}_1, \cdots, \mathcal{A}_K$, denote cluster assignment matrix L with entry $L_{ik} = \mathbf{1}(X_i \in \mathcal{A}_k)$.
- The regularized complete log-likelihood function is

$$\log L_c(\Theta) := \sum_{i=1}^{n} \sum_{k=1}^{K} L_{ik} [\log \pi_k + \log f_k(x_i; \Theta_k)] - P(\Theta).$$

- E-step: compute the conditional expectation

$$Q(\Theta | \widehat{\Theta}^{(t)}) := \sum_{i=1}^{n} \sum_{k=1}^{K} \widehat{L}_{ik}^{(t)} [\log \pi_k + \log f_k(x_i; \Theta_k)] - P(\Theta).$$

- M-step: maximize $Q(\Theta | \widehat{\Theta}^{(t)})$ w.r.t. π_k, $\boldsymbol{\mu}_k$, Ω_k.

Application 2: EM Algorithm

M-step is a tri-convex optimization.

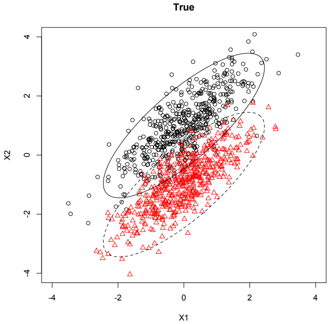
- Update π_k: $\widehat{\pi}_k^{(t+1)} = n^{-1} \sum_{i=1}^{n} \widehat{L}_{ik}^{(t)}$.
- Update $\boldsymbol{\mu}_k$: solve sparse mean via KKT condition.
- Update Ω_k: solve sparse networks via existing joint graphical lasso algorithms,

$$\max_{\Omega_1,\ldots,\Omega_K} \sum_{k=1}^{K} n_k [\log\det(\Omega_k) - \mathrm{trace}(\widetilde{S}_k \Omega_k)] - P(\Omega).$$

Application 2: Illustration

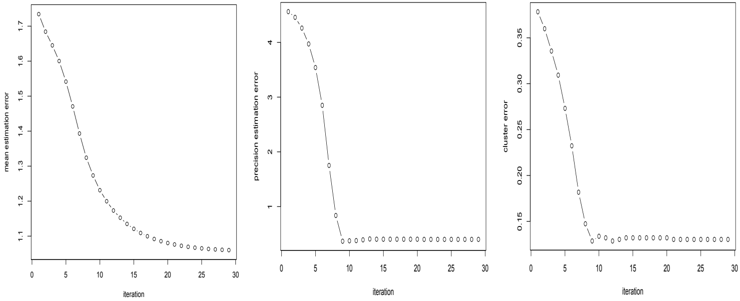
- $n = 1000$ with 500 from $N(\boldsymbol{\mu}_1, \Sigma)$ and 500 from $N(\boldsymbol{\mu}_2, \Sigma)$,

$$\boldsymbol{\mu}_1 = (0,1)^T, \boldsymbol{\mu}_2 = (0,-1)^T, \Sigma = \begin{pmatrix} 1 & 0.8 \\ 0.8 & 1 \end{pmatrix}.$$



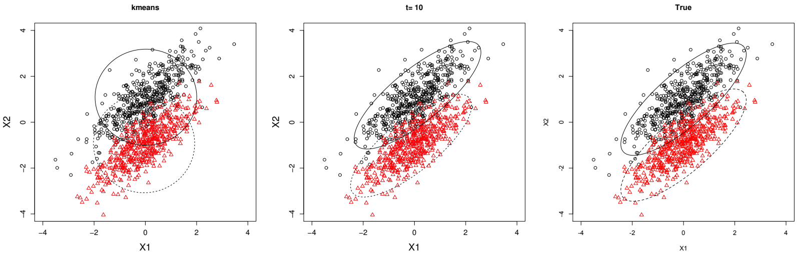
Application 2: Illustration

Figure : Mean vector estimation errors, precision matrix estimation errors, and cluster errors versus # of iterations.



Application 2: Illustration

Figure : Kmeans, Iteration $t = 10$ of our algorithm, and the truth.

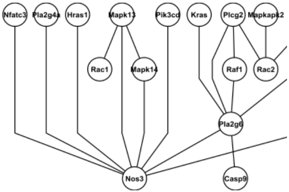


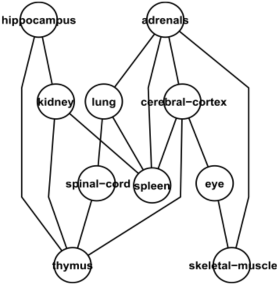
Summary

- A general convergence study of bi-convex problems.
- It reveals statistical and computational tradeoffs.
- Our theory is widely applicable to many models:
  - bigraphical model,
  - joint clustering and network estimation,
  - non-negative matrix factorization,
  - sparse tensor decomposition...

Future Work: Statistical Inference

- From parameter estimation to statistical inference.
- In the bigraphical model, test $H_0 : \omega_{ij} = 0$ v.s. $H_1 : \omega_{ij} \neq 0$
- Tools: Desparsify Lasso (van de Geer et al., 2014), De-correlated Score Test (Ning and Liu, 2014).





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