COM 530500 **Network Science Homework #1**

DUE: Thursday, October 14, 2021

No late homework will be accepted.

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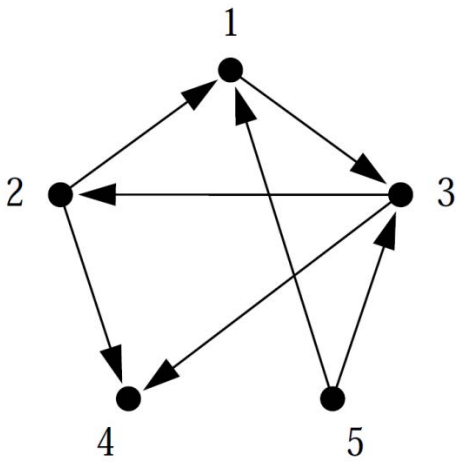

Figure 1: Network (a).

Problem 1.(10%)

(a) (5%) Write down the adjacency matrix of network (a).

(b) (5%) Write down the cocitation matrix of network (a).

Solution:

(a) $\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(b) Cocitation matrix $C = AA^T = \begin{bmatrix} 2 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ ◇

Problem 2.(10%)

(a) (5%) Write down the incidence matrix of network (b).

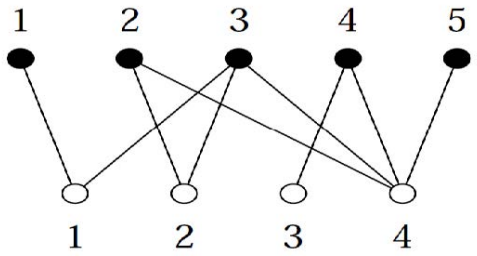

Figure 2: Network (b).

(b) (5%) Write down the projection matrix for the projection of network (b) onto its black vertices.

Solution:

(a) I regard white dots as different groups and black dots as vertexs.

$$\text{Incidence matrix} = B = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(b) Projection matrix $P = B^T B = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 1 \\ 1 & 2 & 3 & 1 & 1 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$

◇

Problem 3.(10%) Consider a bipartite network, with its two types of vertices. Suppose there are n_1 vertices of type 1 and n_2 vertices of type 2. Show that the mean degrees c_1 and c_2 of the two types are related by $c_2 = \frac{n_1}{n_2} c_1$.

Solution: If there are m edges between two types of vertices, The mean degree of type 1 vertices $c_1 = \frac{m}{n_1}$, and $c_2 = \frac{m}{n_2}$. Therefore, $\frac{c_1}{c_2} = \frac{\frac{m}{n_1}}{\frac{m}{n_2}} = \frac{n_2}{n_1}$, and we get $c_2 = \frac{n_1}{n_2} c_1$. ◇

Problem 4.(20%) Given

$$A = \begin{pmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{pmatrix},$$

(a) (10%) Find all eigenvalues of matrix A.

(b) (10%) Find an orthogonal matrix U that diagonalizes A.

Solution: Type your answer here. ◇

Problem 5.(15%) Read the tutorial from Ping-En Lu's GitHub repository to install Python3 and python-igraph (if you need).

Paste your screenshots of "Hello, World!" of both Python 3 (5%) and python-igraph (5%), and write a brief report (5%). (For example, you can write down some problems you encountered, and how you solved them.)

Solution: Type your answer here. ◇

Problem 6.(35%) Please download the **tvshow** dataset from Ping-En Lu's GitHub repository, and find the following information from this dataset.

- Number of nodes. (5%)
- Number of edges. (5%)
- Mean degree. (5%)
- Maximum degree. (5%)
- Diameter. (5%)

You need to upload your **python source code** to iLMS, and **write a brief report (10%) including screenshots, README file, and descriptions of your code** below the solution area. There will be no points for this problem if you do not upload your python source code to iLMS.

Solution: Type your answer here. ◇