班級: 資應所

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Problem 1.(40%) Consider a k-regular undirected network (i.e., a network in which every vertex has degree k) with number of nodes n.

(a) (10%) Show that the n-vector $\mathbf{1}_n = [1, 1, \ldots, 1]$ is an eigenvector of the adjacency matrix with corresponding eigenvalue k.

(b) (10%) By making use of the fact that eigenvectors are orthogonal (or otherwise), show that there is no other eigenvector that has all elements positive. [*Note: The Perron-Frobenius theorem says that the eigenvector with all elements positive has the largest eigenvalue, and hence the eigenvector $\mathbf{1}_n$ gives, by definition, the eigenvector centrality of our k-regular network and the centralities are the same for every vertex.*]

(c) (10%) Find the **Katz centralities** of all vertices in a k-regular network.

(d) (10%) Find a centrality measure that can give different centralities for different vertices in a regular network. Please provide a specific example.

Solution: Type your answer here. $\diamond$

Problem 2.(60%) Please find a real dataset from the Internet. (**Note: You need to cite the dataset in the reference.**) Note that this dataset should be an undirected network, and the total number of nodes should be greater than 500. **Please do not use the same dataset in Homework #1.**

(a) (10%) Briefly introduce this dataset, and list some basic statistical information, such as the number of nodes, number of edges, average clustering coefficient, diameter, average degree, maximum degree, etc.

(b) (10%) Please visualize the dataset by plotting it.

(c) (20%) Please implement the Katz centrality measure (textbook chapter 7.3 [1]) **without** using the katz_centrality function and the katz_centrality_numpy function provided by NetworkX, and find the top 10 nodes ranked by the Katz centrality measure you've written.

(d) (10%) Please find the top 10 nodes by two other different centrality measures (you can use any packages and functions).

(e) (5%) Are the top 10 nodes ranked by different centrality measures in (c) and (d) the same? Explain why?

(f) (5%) Is there a best centrality measure for ranking this dataset? Explain why?

Solution: Type your answer here.   ◇

References

[1] Newman, Mark. *Networks: An Introduction.* Oxford University Press, 2010.