COM 530500 **Network Science Homework #1**

Due: Thursday, October 14, 2021

No late homework will be accepted.

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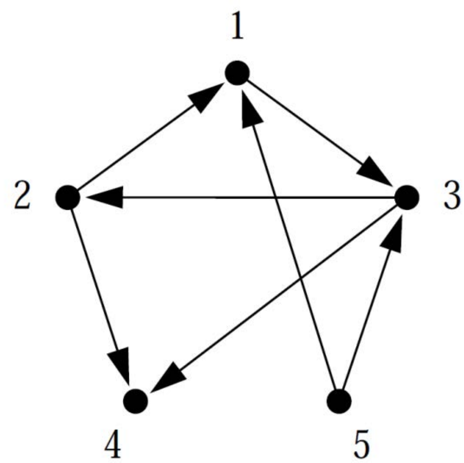

Figure 1: Network (a).

Problem 1.(10%)

(a) (5%) Write down the adjacency matrix of network (a).

(b) (5%) Write down the cocitation matrix of network (a).

Solution:

(a) $\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(b) Cocitation matrix $C = AA^T = \begin{bmatrix} 2 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ ◇

Problem 2.(10%)

(a) (5%) Write down the incidence matrix of network (b).

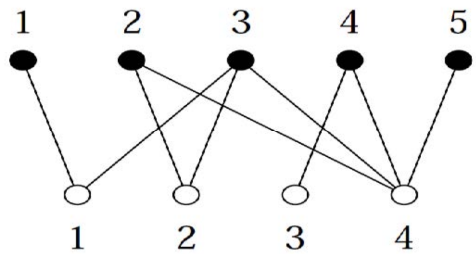

Figure 2: Network (b).

(b) (5%) Write down the projection matrix for the projection of network (b) onto its black vertices.

Solution:

(a) I regard white dots as different groups and black dots as vertexs.

$$\text{Incidence matrix} = B = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\text{(b) Projection matrix } P = B^T B = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 1 \\ 1 & 2 & 3 & 1 & 1 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

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Problem 3.(10%) Consider a bipartite network, with its two types of vertices. Suppose there are n_1 vertices of type 1 and n_2 vertices of type 2. Show that the mean degrees c_1 and c_2 of the two types are related by $c_2 = \frac{n_1}{n_2} c_1$.

Solution: If there are m edges between two types of vertices, The mean degree of type 1 vertices $c_1 = \frac{m}{n_1}$, and $c_2 = \frac{m}{n_2}$. Therefore, $\frac{c_1}{c_2} = \frac{\frac{m}{n_1}}{\frac{m}{n_2}} = \frac{n_2}{n_1}$, and we get $c_2 = \frac{n_1}{n_2} c_1$. ◇

Problem 4.(20%) Given

$$A = \begin{pmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{pmatrix},$$

(a) (10%) Find all eigenvalues of matrix A.

(b) (10%) Find an orthogonal matrix U that diagonalizes A.

Solution:

(a) Eigenvalues can be obtained by the equation: $Ax = \lambda x$, and $(A-\lambda I)$ is a singular matrix. Therefore, $det(A - \lambda I) = 0$.

$$det(A-\lambda I) = det\begin{pmatrix} 0-\lambda & 2 & -1 \\ 2 & 3-\lambda & -2 \\ -1 & -2 & 0-\lambda \end{pmatrix} = 0$$

$$-\lambda^3 + 3\lambda^2 + 9\lambda + 5 = 0$$

$$Eigenvalues(\lambda) = -1, -1, 5$$

(b) Orthogonal matrix is composed by eigenvectors of A.

1. When $\lambda = -1, A - (-1)I = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. The eigenvector is the form: $\begin{pmatrix} s+2t \\ -t \\ s \end{pmatrix} = s\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$. So a normalized eigenvector for $\lambda_1 = -1$ is $v_1 = (\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}), v_2 = (\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, 0)$.

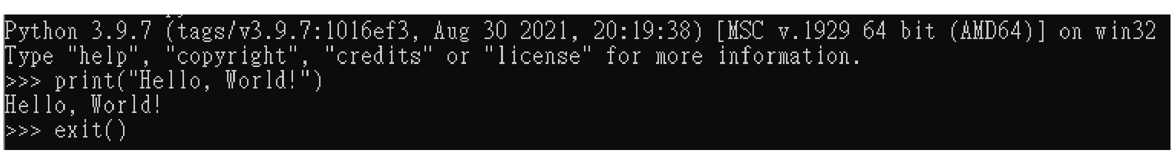
2. When $\lambda = 5, A - (5)I = \begin{pmatrix} -5 & 2 & -1 \\ 2 & -2 & -2 \\ -1 & -2 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 \\ -5 & 2 & -1 \\ -1 & -2 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$. So a normalized eigenvector for $\lambda_1 = -1$ is $v_1 = (\frac{-1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}})$.

Finally, orthogonal matrix $U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{6}} \\ 0 & \frac{-1}{\sqrt{5}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{6}} \end{pmatrix}$. ◇

Problem 5.(15%) Read the tutorial from Ping-En Lu's GitHub repository to install Python3 and python-igraph (if you need).

Paste your screenshots of "Hello, World!" of both Python 3 (5%) and python-igraph (5%), and write a brief report (5%). (For example, you can write down some problems you encountered, and how you solved them.)

Solution: I didn't encounter any problem during the installation of both Python and Python-iGraph. Additionally, I didn't meet the issue the tutorial has mentioned about installing Python-iGraph on Windows. I installed it directly by typing pip command in command line.



```
Python 3.9.7 (tags/v3.9.7:1016ef3, Aug 30 2021, 20:19:38) [MSC v.1929 64 bit (AMD64)] on win32
Type "help", "copyright", "credits" or "license" for more information.
>>> print("Hello, World!")
Hello, World!
>>> exit()
```

Figure 3: Python screenshot.

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```
Python 3.9.7 (tags/v3.9.7:1016ef3, Aug 30 2021, 20:19:38) [MSC v.1929 64 bit (AMD64)] on win32
Type "help", "copyright", "credits" or "license" for more information.
>>> from igraph import *
>>> g = Graph()
>>> print(g)
IGRAPH U--- 0 0 --
```

Figure 4: Python-iGraph screenshot.

Problem 6.(35%) Please download the **tvshow** dataset from Ping-En Lu's GitHub repository, and find the following information from this dataset.

- Number of nodes. (5%)
- Number of edges. (5%)
- Mean degree. (5%)
- Maximum degree. (5%)
- Diameter. (5%)

You need to upload your **python source code** to iLMS, and **write a brief report (10%) including screenshots, README file, and descriptions of your code** below the solution area. There will be no points for this problem if you do not upload your python source code to iLMS.

Solution:

- Number of nodes: 3892
- Number of edges: 17262
- Mean degree: 8.870503597122303
- Maximum degree: 126
- Diameter: 20(undirected network); 14(directed network)

My code is divided into two major part. The first part is to create whole graph by the dataset. I used a loop to read each row and add vertices and edges in the graph. The second part is to output the reslut. Because iGraph is powerful and very convenience, I simply called built-in methods to get the information I need. ◇

```
IGRAPH U--- 3892 17262 --
Number of nodes: 3892
Number of edges: 17262
Mean degree: 8.870503597122303
Maximum degree: 126
Diameter: 20
```

Figure 5: Result screenshot.