

## Homework 6

Deadline: November 13th.

**Problem 1.** Compute the subdifferentials of the following functions

(a)  $f(\mathbf{x}) = \|\mathbf{x}\|_2$

(b) Given a closed convex set  $\mathcal{C}$ , define

$$f(\mathbf{x}) = \begin{cases} 0 & \text{if } \mathbf{x} \in \mathcal{C} \\ +\infty & \text{otherwise.} \end{cases}$$

**Problem 2.** If function  $f$  is convex, Show that  $\partial f(\mathbf{x}) \neq \emptyset$  for all  $\mathbf{x} \in \text{dom } f$ .

**Problem 3.** If function  $f$  is  $\mu$ -strongly convex, and  $\mathbf{g}$  is a subgradient of  $f$  at  $\mathbf{x}$ . Show that for any  $\mathbf{y} \in \text{dom } f$ ,

$$f(\mathbf{y}) \geq f(\mathbf{x}) + \langle \mathbf{g}, \mathbf{y} - \mathbf{x} \rangle + \frac{\mu}{2} \|\mathbf{y} - \mathbf{x}\|_2^2.$$