

Homework 3

Deadline: October 17th.

Problem 1. Judge whether the following functions are smooth.

- (a) $f(x) = \sin x$.
- (b) $f(\mathbf{x}) = \|\mathbf{x}\|_1, \mathbf{x} \in \mathbb{R}^d$.

Problem 2. Judge whether the following functions are strongly convex.

- (a) $f(\mathbf{x}) = \sum_{i=1}^m (\mathbf{a}_i^\top \mathbf{x} - b_i)^2, \mathbf{a}_i, \mathbf{x} \in \mathbb{R}^d, m > d$.
- (b) $f(x_1, x_2) = 1/(x_1 x_2), x_1 > 0, x_2 > 0$.

Problem 3.

- (a) Suppose that $f : \mathbb{R}^d \rightarrow \mathbb{R}$ is β -smooth for some $\beta > \alpha$. Show that $h(\mathbf{x}) = f(\mathbf{x}) - \frac{\alpha}{2}\|\mathbf{x}\|^2$ is $(\beta - \alpha)$ -smooth.
- (b) Suppose that $f : \mathbb{R}^d \rightarrow \mathbb{R}$ is μ -strongly convex and L -smooth. Show that

$$\langle \nabla f(\mathbf{x}) - \nabla f(\mathbf{y}), \mathbf{x} - \mathbf{y} \rangle \geq \frac{\mu L}{\mu + L} \|\mathbf{x} - \mathbf{y}\|_2^2 + \frac{1}{\mu + L} \|\nabla f(\mathbf{x}) - \nabla f(\mathbf{y})\|_2^2.$$

(hint: by the conclusion of (a), $h(\mathbf{x}) = f(\mathbf{x}) - \frac{\mu}{2}\|\mathbf{x}\|^2$ is $(L - \mu)$ -smooth and convex.)