Homework 1

Total 40 points

Problem 1. (10 points) Judge the properties of the following sets (openness, closeness, boundedness, compactness) and give their interiors, closures, and boundaries:

(a) $\mathcal{C}_1 = \emptyset$.

(b) $\mathcal{C}_2 = \mathbb{R}^n$.

(c) $\mathcal{C}_3 = \{(x, y)^\top | x \geq 0, y > 0\}$.

(d) $\mathcal{C}_4 = \{k | k \in \mathbb{Z}\}$.

(e) $\mathcal{C}_5 = \{k^{-1} | k \in \mathbb{Z}\}$.

Problem 2. (12 points) For each of the following sequence, determine the rate of convergence and the rate constant:

(a) $x_k = 1 + 5 \times 10^{-2k}$.

(b) $x_k = 2^{-2^k}$.

(c) $x_k = 3^{-k^2}$.

(d) $x_{k+1} = x_k/2 + 2/x_k$, $x_1 = 4$.

Problem 3. (18 points) Compute the **gradient** and the **Hessian** of the following functions (write in vector or matrix form, rather than entrywise), give details:

(a) $f(\mathbf{x}) = (\mathbf{a}^\top \mathbf{x})(\mathbf{b}^\top \mathbf{x})$.

(b) $f(\mathbf{x}) = \frac{1}{2}||\mathbf{A}\mathbf{x} - \mathbf{b}||_2^2$.

(c) $f(\mathbf{x}) = \log \sum_{i=1}^m \exp(\mathbf{a}_i^\top \mathbf{x} + b_i)$.