STA142B, S20
Prof. W. Polonik

Project 1

due Tue, April 21, 2020

A. Data analysis: Perform the data analysis projects as instructed in the file Project1.ipynb that you can find in 'files' on Canvas.

B. Methodology: Answer the following questions, and submit your answers through canvas (either a picture or a scan - clearly readable).

1. (a) Show that for a given m-dimensional vector a, the map

$$f_a(x) = \langle a, x \rangle$$

is linear.

(b) Now let A be a $(n \times m)$-matrix. Show that the map

$$f_A(x) = Ax$$

is linear.

2. Consider the symmetric matrix

$$A = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}.$$

(a) Verify that A is PSD.

(b) i. Draw a figure that shows the points $x_1 = \binom{1}{0}$ and $x_2 = \binom{1}{1}$ and also their images y_1, y_2 under the linear map induced by A, i.e.

$$y_1 = Ax_1 \qquad \text{and} \qquad y_2 = Ax_2.$$

ii. Find the unit eigenvectors of A and draw them into the same figure.

iii. Show how to obtain y_1 and y_2 in three steps by using the eigendecomposition of A :

- projection onto unit eigenvectors
- scaling by unit eigenvalues and
- adding up the two scaled projections.

Do so by indicating the three steps in one figure.

iv. Find the image of the line passing through x_1 and x_2 under A and draw the line and its image in a figure. Also include y_1 and y_2 in this figure.