Xuecheng_Zhang_HW2

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1 HW 2

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2 Question 1

2.0.1 Part a

A generative model learns the joint probability distribution p(x,y) while a discriminative model learns the conditional probability distribution p(y|x) “probability of y given x”. SO it is a discriminative approach.

2.0.2 Part b

Find the β of the MLE:

$$p(Y = k|X = x) = \frac{(\beta X)^k}{k!}e^{-\beta X}$$

next

$$L(\beta) = \prod_{i=1}^{n} P_{(y_i|x_i)}P_{(x_i)}$$

so

$$l(\beta) = lnL(\beta) = \sum_{i=1}^{n} ln[P_\beta(y_i|x_i)] + \sum_{i=1}^{n} P_\beta(x_i) = nln(\beta) + nln(t) - \beta t - ln(n!)$$

$$\text{where } n = \sum_{k=0}^{n} y_k, t = \sum_{i=0}^{n} x_i$$

$$\frac{d}{d\beta}[nln(\beta) + nln(t) - \beta t - ln(n!)] = 0$$

$$\frac{n}{\beta} - t = 0 \rightarrow \beta = \frac{n}{t}$$

Given n training samples $(x_1, y_1), ..., (x_n, y_n)$, the parameter $\beta = \frac{n}{t}$ via MLE.

2.0.3 part c

Given a new sample x, since we already have the parameter β so we put the new sample x in

$$p(Y = k|X = x) = \frac{(\frac{n}{t}X)^k}{k!}e^{-\frac{n}{t}X}$$

3 Question 2

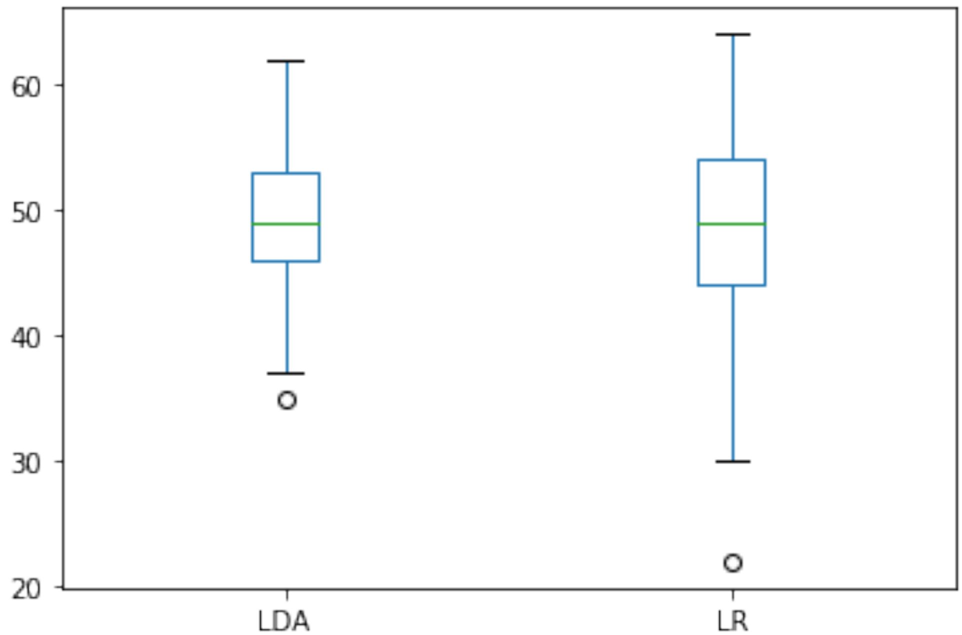
3.0.1 part a

[3]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import sklearn.linear_model as skl_lm
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
import matplotlib.pyplot
from sklearn import svm
```

[4]:
```
np.random.RandomState(1) # Set the random seed
def generata_data(n,p):
    n1=np.random.binomial(n,0.5)
    n2=100-n1
    x1=np.random.standard_t(1,n1)+p
    x2=np.random.standard_t(1,n2)
    y1=np.repeat(1,n1)
    y2=np.repeat(-1,n2)
    x=np.concatenate((x1,x2))# put x1 and x2 in same x
    y=np.concatenate((y1,y2))# put y1 and y2 in same y
    x=x.reshape(-1,1)#reshape x from 2D to 1D
    return x,y
err1=[]#define an empty list for LDA
err2=[]#define an empty list for logistic regression
lr = skl_lm.LogisticRegression(solver='newton-cg')
lda = LinearDiscriminantAnalysis(solver='svd')
for i in range(100):
    X_train,Y_train = generata_data(100,1)
    X_test,Y_test = generata_data(100,1)
    #LDA
    lda.fit(X_train,Y_train)#use training data to fit the lda model
    test_error1=sum(lda.predict(X_test)!=Y_test)#get the test error
    err1.append(test_error1)#put the test error in err1 list
    #logistic regression
    lr.fit(X_train,Y_train)#use the training dara to fit the logsitic␣
 ↪regression model
    test_error2=sum(lr.predict(X_test)!=Y_test)#get the test error
    err2.append(test_error2)#put the test error in err2 list
df= {'LDA':err1, 'LR':err2}
df = pd.DataFrame(data=df)#save them in data frame
#find the mean and variance
mean = df.mean()
var = df.var()
print("The mean of test error is",mean)
print("The variance of test error is",mean)
```

```
box_plot = df.plot.box()
```

```
The mean of test error is LDA    49.62
LR     48.43
dtype: float64
The variance of test error is LDA    49.62
LR     48.43
dtype: float64
```

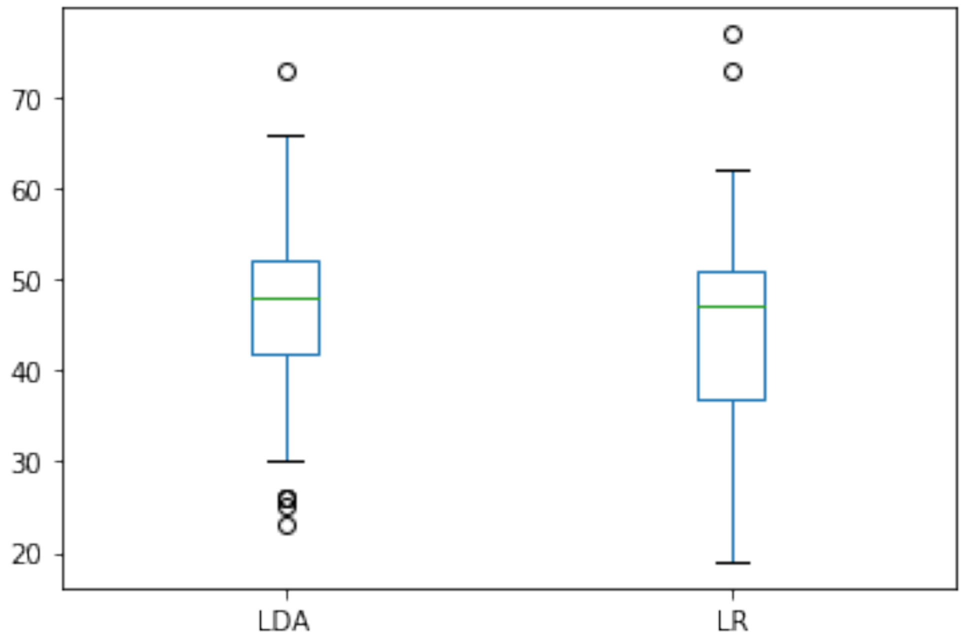


3.0.2 part b

[5]:
```python
np.random.RandomState(1) # Set the random seed
def generata_data(n,p):
    n1=np.random.binomial(n,0.5)
    n2=100-n1
    x1=np.random.standard_t(1,n1)+p
    x2=np.random.standard_t(1,n2)
    y1=np.repeat(1,n1)
    y2=np.repeat(-1,n2)
    x=np.concatenate((x1,x2))# put x1 and x2 in same x
    y=np.concatenate((y1,y2))# put y1 and y2 in same y
    x=x.reshape(-1,1)#reshape x from 2D to 1D
    return x,y
err1=[]#define an empty list for LDA
err2=[]#define an empty list for logistic regression
```

```
lr = skl_lm.LogisticRegression(solver='newton-cg')
lda = LinearDiscriminantAnalysis(solver='svd')
for i in range(100):
    X_train,Y_train = generata_data(100,2)
    X_test,Y_test = generata_data(100,2)
    #LDA
    lda.fit(X_train,Y_train)#use training data to fit the lda model
    test_error1=sum(lda.predict(X_test)!=Y_test)#get the test error
    err1.append(test_error1)#put the test error in err1 list
    #logistic regression
    lr.fit(X_train,Y_train)#use the training dara to fit the logsitic␣
 ↪regression model
    test_error2=sum(lr.predict(X_test)!=Y_test)#get the test error
    err2.append(test_error2)#put the test error in err2 list
df= {'LDA':err1, 'LR':err2}
df = pd.DataFrame(data=df)#save them in data frame
#find the mean and variance
mean = df.mean()
var = df.var()
print("The mean of test error is",mean)
print("The variance of test error is",mean)
box_plot = df.plot.box()
```

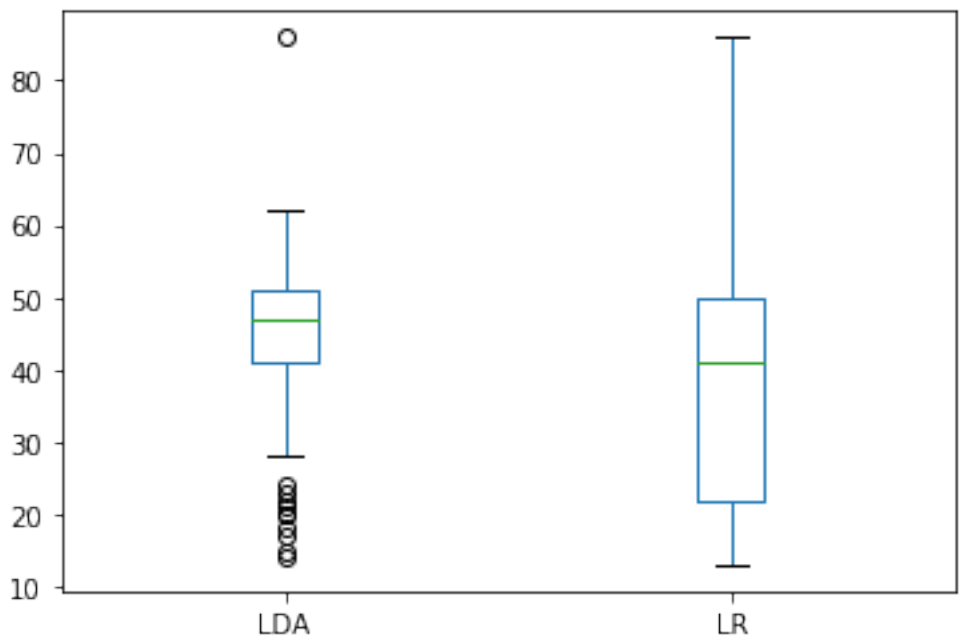
```
The mean of test error is LDA    46.42
LR     44.18
dtype: float64
The variance of test error is LDA    46.42
LR     44.18
dtype: float64
```



[6]:
```
np.random.RandomState(1) # Set the random seed
def generata_data(n,p):
    n1=np.random.binomial(n,0.5)
    n2=100-n1
    x1=np.random.standard_t(1,n1)+p
    x2=np.random.standard_t(1,n2)
    y1=np.repeat(1,n1)
    y2=np.repeat(-1,n2)
    x=np.concatenate((x1,x2))# put x1 and x2 in same x
    y=np.concatenate((y1,y2))# put y1 and y2 in same y
    x=x.reshape(-1,1)#reshape x from 2D to 1D
    return x,y
err1=[]#define an empty list for LDA
err2=[]#define an empty list for logistic regression
lr = skl_lm.LogisticRegression(solver='newton-cg')
lda = LinearDiscriminantAnalysis(solver='svd')
for i in range(100):
    X_train,Y_train = generata_data(100,3)
    X_test,Y_test = generata_data(100,3)
    #LDA
    lda.fit(X_train,Y_train)#use training data to fit the lda model
    test_error1=sum(lda.predict(X_test)!=Y_test)#get the test error
    err1.append(test_error1)#put the test error in err1 list
    #logistic regression
    lr.fit(X_train,Y_train)#use the training dara to fit the logsitic␣
↪regression model
```

```
    test_error2=sum(lr.predict(X_test)!=Y_test)#get the test error
    err2.append(test_error2)#put the test error in err2 list
df= {'LDA':err1, 'LR':err2}
df = pd.DataFrame(data=df)#save them in data frame
#find the mean and variance
mean = df.mean()
var = df.var()
print("The mean of test error is",mean)
print("The variance of test error is",mean)
box_plot = df.plot.box()
```

```
The mean of test error is LDA    44.37
LR     37.28
dtype: float64
The variance of test error is LDA    44.37
LR     37.28
dtype: float64
```



4 Question 3

4.0.1 part a

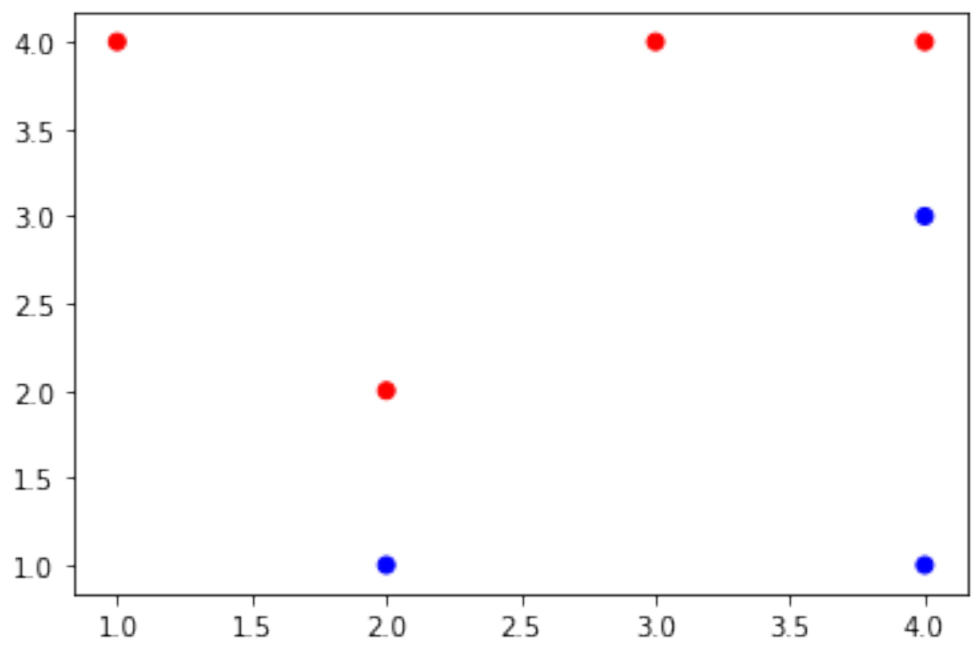
In this question my code based on stackoverflow and scikittlearn.

https://scikit-learn.org/stable/auto_examples/svm/plot_svm_margin.html https://scikit-learn.org/stable/auto_examples/svm/plot_separating_hyperplane.html#sphx-glr-auto-examples-svm-plot-separating-hyperplane-py

[7]:
```
import numpy as np
from matplotlib import pyplot as plt
import matplotlib.pyplot
import matplotlib.lines as mlines
```

[8]:
```
vector_x1 = np.array([3, 2, 4, 1, 2, 4, 4]);
vector_x2 = np.array([4, 2, 4, 4, 1, 3, 1]);
matplotlib.pyplot.scatter(vector_x1,vector_x2,color=["r", "r", "r", "r", "b",␣
↪"b", "b"])
```

[8]:
```
<matplotlib.collections.PathCollection at 0x1a25d072d0>
```

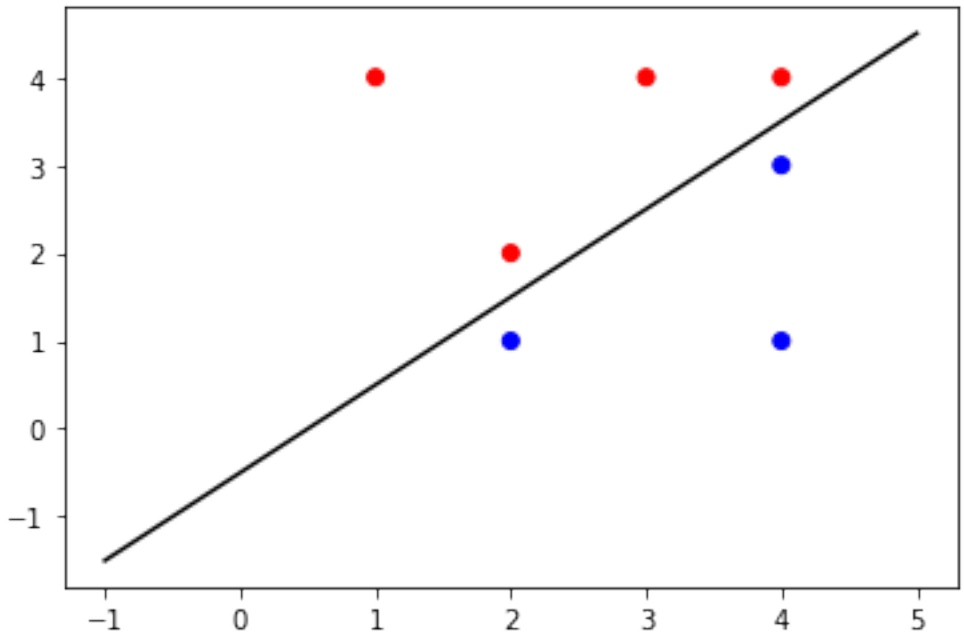


4.0.2 part b

[11]:
```
X = np.vstack((vector_x1,vector_x2)).T#add vector_x1 and vector_x2 as column␣
↪vector
Y = np.array([1]*4 + [0]*3)#"1" is red, and repect 4 times,"0"is blue which␣
↪repect 3 times
clf = svm.SVC(kernel='linear',C=9999)#since optimal separating hyperplane, so␣
↪we set Regularization parameter become a big number
clf.fit(X, Y)#fir X and Y in clf model
w = clf.coef_[0]# get the coefficients
a = -w[0] / w[1]# find the slope by negative beta1 divided beta 2
xx = np.linspace(-1, 5)
```

```
yy = a * xx - (clf.intercept_[0]) / w[1]#create a relationship
matplotlib.pyplot.scatter(vector_x1,vector_x2,color=["r", "r", "r", "r", "b",␣
 ↪"b", "b"])
plt.plot(xx, yy, 'k-')
```

[11]: [<matplotlib.lines.Line2D at 0x1a25dffcd0>]



4.0.3 part c

The

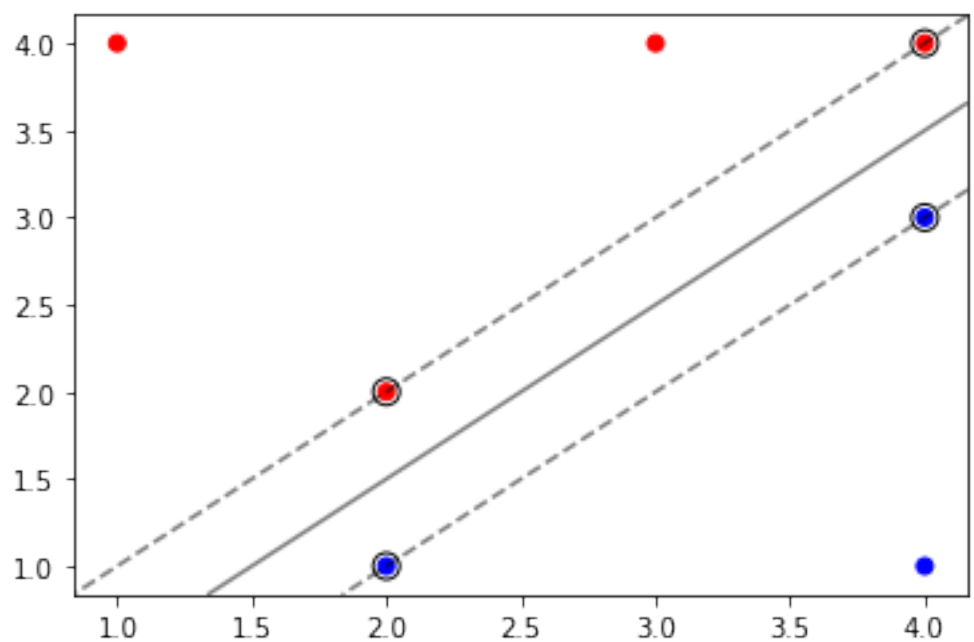
$$\beta_1 = -0.61538462, \beta_2 = 1.07692308, \beta_0 = -0.8461538461538456$$

if $-0.8461538461538456 - 0.61538462X_1 + 1.07692308X_2 > 0$ is red, otherwise it will be blue

4.0.4 part d

[12]:
```
# plot the line, the points, and the nearest vectors to the plane
X = np.vstack((vector_x1,vector_x2)).T#add vector_x1 and vector_x2 as column␣
 ↪vector
Y = np.array([1]*4 + [0]*3)#"1" is red, and repect 4 times,"0"is blue which␣
 ↪repect 3 times
clf = svm.SVC(kernel='linear',C=9999)#since optimal separating hyperplane, so␣
 ↪we set Regularization parameter become a big number
clf.fit(X, Y)#fir X and Y in clf model
w = clf.coef_[0]# get the coefficients
a = -w[0] / w[1]# find the slope by negative beta1 divided beta 2
```

```
xx = np.linspace(-1, 5)
yy = a * xx - (clf.intercept_[0]) / w[1]#create a relationship
matplotlib.pyplot.scatter(vector_x1,vector_x2,color=["r", "r", "r", "r", "b",␣
↪"b", "b"])
# plot the decision function
ax = plt.gca()
xlim = ax.get_xlim()#the decision function of x
ylim = ax.get_ylim()#the decision function of y

# create grid to evaluate model
xx = np.linspace(xlim[0], xlim[1], 30)
yy = np.linspace(ylim[0], ylim[1], 30)
YY, XX = np.meshgrid(yy, xx)
xy = np.vstack([XX.ravel(), YY.ravel()]).T#add XX.ravel() and YY.ravel()as␣
↪column vector
Z = clf.decision_function(xy).reshape(XX.shape)
ax.contour(XX, YY, Z, colors='k', levels=[-1, 0, 1], alpha=0.5,
           linestyles=['--', '-', '--'])# plot decision boundary and margins
ax.scatter(clf.support_vectors_[:, 0], clf.support_vectors_[:, 1], s=100,
           linewidth=1, facecolors='none', edgecolors='k')
plt.show()# plot support vectors
```



4.0.5 part e

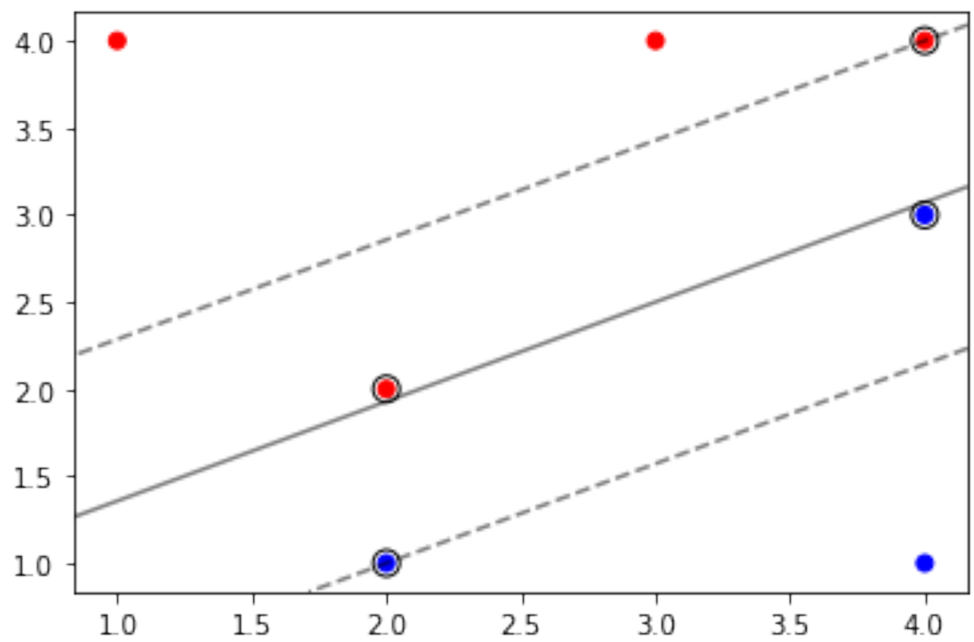
From part d), the plot shows that the support vectors are the points (2,1), (2,2), (4,3) and (4,4).

4.0.6 part f

Since (4,1),(1,4),(3,4)are not the support vectors, so if we moved those observations, we would not change the maximal margin hyperplane.

4.0.7 part g

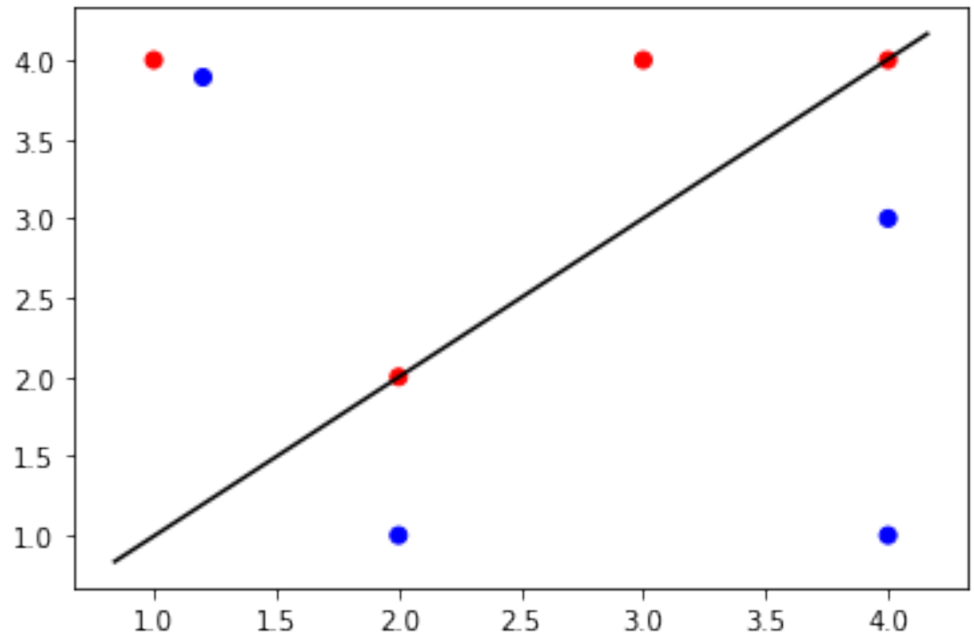
[13]:
```
# plot the line, the points, and the nearest vectors to the plane
X = np.vstack((vector_x1,vector_x2)).T#add vector_x1 and vector_x2 as column␣
↪vector
Y = np.array([1]*4 + [0]*3)#"1" is red, and repect 4 times,"0"is blue which␣
↪repect 3 times
clf = svm.SVC(kernel='linear')#since not optimal separating hyperplane, so we␣
↪do not set Regularization parameter
clf.fit(X, Y)#fir X and Y in clf model
w = clf.coef_[0]# get the coefficients
a = -w[0] / w[1]# find the slope by negative beta1 divided beta 2
xx = np.linspace(-1, 5)
yy = a * xx - (clf.intercept_[0]) / w[1]#create a relationship
matplotlib.pyplot.scatter(vector_x1,vector_x2,color=["r", "r", "r", "r", "b",␣
↪"b", "b"])
# plot the decision function
ax = plt.gca()
xlim = ax.get_xlim()#the decision function of x
ylim = ax.get_ylim()#the decision function of y

# create grid to evaluate model
xx = np.linspace(xlim[0], xlim[1], 30)
yy = np.linspace(ylim[0], ylim[1], 30)
YY, XX = np.meshgrid(yy, xx)
xy = np.vstack([XX.ravel(), YY.ravel()]).T#add XX.ravel() and YY.ravel()as␣
↪column vector
Z = clf.decision_function(xy).reshape(XX.shape)
ax.contour(XX, YY, Z, colors='k', levels=[-1, 0, 1], alpha=0.5,
           linestyles=['--', '-', '--'])# plot decision boundary and margins
ax.scatter(clf.support_vectors_[:, 0], clf.support_vectors_[:, 1], s=100,
           linewidth=1, facecolors='none', edgecolors='k')
plt.show()# plot support vectors
```



4.0.8 part h

[53]:
```
matplotlib.pyplot.scatter(vector_x1,vector_x2,color=["r", "r", "r", "r", "b",␣
↪"b", "b"])
plt.plot(xx, yy, 'k-')
plt.scatter(1.2,3.9,color="b")#this number should be red, but I set it to be␣
↪blue
```

[53]: <matplotlib.collections.PathCollection at 0x1a25620c50>



(1.2,3.9) this point should be red, but I set it to be blue, so the two classes are no longer separable by a hyperplane.