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Investigating the co-evolution of node reputation and edge-strategy in prisoner's dilemma game

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ABSTRACT

In practice, there exists numerous examples indicating the phenomenon that individuals are inclined to employ different strategies when interacting with the others. This is incurred by the fact that individuals are able to adjust their strategies adaptively under different interacting environments. Scholars have devoted their efforts to this topic; nevertheless, the impact of diverse interactions on cooperation still needs to be further explored. In this manuscript, we propose a mechanism aiming to investigate the co-evolution of personal reputation and strategy under a general framework of interactive diversity (being referred to as edge-strategy for simplicity). Numerous simulations are conducted with sufficient analyses of the obtained results being provided. As illustrated by the evolutionary dynamics, we find that there exists an optimal reputation value which can promote the frequency of cooperation by a large extent. Furthermore, we can clearly conclude that the consideration of interaction diversity is able to ensure the maintenance of cooperation even if the temptation to adopt antisocial behavior is relatively large. Aiming to understand the phenomenon better, we also quantitatively analyze the results by investigating the statistics of interaction chain, cluster size and other microscopic information. Overall, we hope the findings here can provide some interesting insights in solving social dilemmas.

1. Introduction

For human-beings, they are inherently self-interested and the profits obtained by the defectors are usually higher than those of the cooperators, being indicated by the results of prisoner's dilemma (PD) game [1–3]. Thus, the desirable Nash equilibrium strategy is supposed to be mutual defection in the PD game. Furthermore, the evolutionary dynamics also indicate that cooperators are likely to be eliminated by natural selection in the survival of the fittest [4–7]. Nevertheless, in conflict with the natural selection, cooperative behaviors are widely observed either in biological systems or social ones. For instance, normal operation of our society is highly dependent on the cooperative behaviors, either the hunting/working

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together activities among tribes in hunter-gather societies, or the complex and orderly social process in our modern society [8–10]. Hence, numerous scholars have devoted their endless efforts into investigating the emergence and maintenance of cooperative behaviors in recent decades either theoretically or experimentally given the altruistic property of human-beings[11–16].

In order to understand this phenomenon better, some scholars introduced evolutionary game theory as a simple and effective framework [17–21]; for instance, Nowak and May introduced evolutionary game theory into complex network aiming to study the evolution of group cooperative behavior [22]. They find that network topology has contributed to the formation of compact clusters consisting of cooperators which play important roles in avoiding the invasion from defectors. This topic has attracted the interests of a bunch of researchers. Along this line, scholars have also tried to provide meaningful explorations from different perspectives; various mechanisms have been proposed, e.g., kin selection [23], direct reciprocity [24,25], group selection [26], indirect reciprocity [27]. Furthermore, game theory has been also considered to investigate the dynamics in the process of epidemic spreading [28–30]. Aiming to study the evolutionary dynamics of cooperation in complex networks in detail, various factors are considered, for instance, punishment, individual heterogeneity [31,32] and etc. Nevertheless, majority of the above-mentioned studies so far mainly focus on individual's competition for nodes, which is the basis for the formation of an evolutionary system (being referred to as node-strategy for short). Despite their insights, those works rely on the assumption that each player takes a fixed action to interact with the counterparts around them.

Some recently published works on network dynamics indicate that the dynamic properties defined at the edge-strategy seems to be quite different from those of node-strategy. Correspondingly, scholars have tried to solve the social dilemma problems from the perspective of various social relations, e.g., the study of migration, dynamic network and temporary network [33]. In fact, because of the narrow sense of altruism and egalitarian preference, individuals are inclined to make different behavioral decisions during different interactions and adjust their behaviors adaptively. Su *et al.* prove that edge-strategy is able to improve the frequency of cooperation to a relatively high level and such promotional effects are also remarkable on networks with high average degrees [34]. Su et al. find that interactive diversity elevates cooperation to a high level and this is strongly robust against variations of game metaphors, population types, payoff patterns and learning manners [35]. The authors in [36] propose two typical interaction patterns, i.e., interactive identity and interactive diversity, aiming to explore the evolution of cooperation. Hence, this incurs a diversity of edge-dynamics which are capable of capturing the inherent nature of many key biological systems; here, it is referred to as edge-strategy for short [35].

In previous studies, various co-evolutionary mechanisms have been investigated [37,38]; while individuals usually update their strategies by comparing benefits which are determined by the strategy pair. The benefits are supposed to be affected by various factors, for instance, a player's reputation. In order to understand the effect of node reputation on cooperation when considering edge-strategy, we devote our efforts to investigate the evolutionary dynamics of PD games in which a player's reputation directly affects his or her own revenue (benefit). A co-evolving mechanism of node reputation and edge strategy on the PD game is investigated thoroughly. For the proposed mechanism, if the benefit obtained by certain player decreases (increases), then the reputation of the strategy reference decreases (increases) according to the provided updating rules. After sufficient analyses of the obtained results, we come to the conclusion that our mechanism can promote the frequency of cooperation to a relatively high level. Furthermore, cooperation can still be sustained even under great temptation.

Overall, this manuscript is organized as follows: we first present a detailed description of the mechanism in the model section which illustrates the co-evolution of node reputation and edge strategy. Then, we perform numerous simulations and corresponding results are presented with sufficient discussions. Finally, the conclusion of this manuscript is provided.

2. Model

For simplicity, the $L \times L$ square lattices are adopted in this manuscript for the evolutionary process while the lattices are anticipated to be with periodic boundary conditions. Hence, the size of the square lattice network equals to L^2 and each node represents an individual, they are allowed to interact with their four neighbors. For the simulations conducted in this manuscript, $L = 200$.

Initially, individuals with different strategies, i.e., cooperation and defection, are supposed to be distributed evenly in the network. For each interaction, the individuals play the PD game (either to be a cooperator (C) or defector (D)) while the individual can adopt different strategies when interacting with different neighbors. Two cooperators are anticipated to both receive a reward of R; while two defectors will obtain the punishment of P. However, if two players choose different strategies, the cooperator will bear the sucker's payoff S; whereas the defector earns the highest payoff T. Therefore, the matrix can be expressed as:

$$M = \begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array}.$$

As to this general 2D payoff matrix, dilemma strength is defined and corresponding effects on cooperation is thoroughly investigated in [39]. Two parameters, i.e., $D_g = T - R$ and $D_r = P - S$, are used as important classification indicators in quantifying the dilemma strength of an infinite mixed population game. Then, evolutionary dynamic and its internal equilibrium are completely determined by D_g and D_r. Here, we focus on investigating the PD game, where $T > R > P > S$, $2R > S + P$. The PD game incurs the social dilemma of collective and individual interests. In this manuscript, the above parameters are

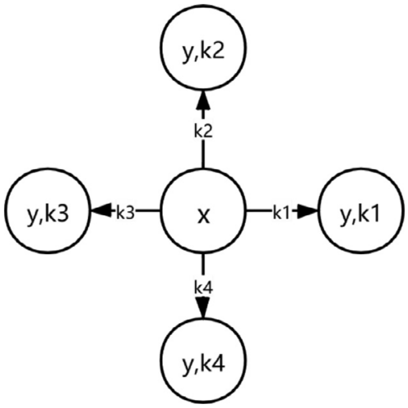

Fig. 1. Illustration of the reputation updating process. In the square lattices network, player x has four different neighbors in four directions being presented as k1, k2, k3 and k4, and the reputation of these four neighbors will be updated one by one.

characterized by setting $R = 1$, $S = -0.5$, $T = b$ and $P = 0$. To be consistent with previous investigations, b is supposed to be in the range $1 \le b < 2$ while relevant aspects of the PD game can be captured inherently.

For the PD game investigated here, corresponding payoff matrix is characterized as follows:

$$M_{PD} = \begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & -0.5 \\ b & 0 \end{pmatrix} \end{array}.$$

Then, a player x is randomly selected among all the players in the spatial network. Among the neighbors of x, an individual y is chosen and a profit P_{xy} can be obtained after the interaction between x and y. Then, player x will get an accumulated payoff, i.e., P_x, by interacting with his k neighbors and P_x can be obtained as follows:

$$P_x = \sum_{y \in \Phi_x}^{k} P_{xy} \tag{1}$$

where k represents the degree of player x (while $k = 4$ for any player in the square lattice network considered here) and φ_x denotes the neighbor set of player x. Therefore, the fitness of player x can be expressed as:

$$F_x = w_x * P_x \tag{2}$$

where w_x represents the reputation held by player x. We assume that all players are equivalent initially; then the initial reputations of all individuals are set to 1. With the proceeding of the evolutionary process, corresponding reputations will be updated subsequently. Considering the non-negativity of individual's reputation, the lower and upper bounds are assigned to be 0 and 2 respectively.

For the considered central individual x, two tasks are anticipated to be completed: (1) updating the strategies against the four neighbors which will be illustrated later; (2) updating the reputations of the neighbors by comparing the payoffs. As illustrated in Fig. 1, player x (which is considered as the central individual) has neighbors in four directions (i.e., k1, k2, k3 and k4); let $P_{y,k}$ represent the payoff of the neighbor for player x in the kth direction ($k \in \{$k1, k2, k3 and k4$\}$), while $w_{y,k}$ denotes the reputation of the neighbor for player x in the kth direction. By comparing P_x and $P_{y,k}$, corresponding reputation can be updated and the rules are provided as follows:

$$\begin{cases} w_{y,k} = w_{y,k} + \delta & P_x > P_{y,k} \\ w_{y,k} = w_{y,k} & P_x = P_{y,k} \\ w_{y,k} = w_{y,k} - \delta & P_x < P_{y,k} \end{cases} \tag{3}$$

where $\delta(\delta \in [0,1])$ represents the amount of the reputation adjustment. Overall, the reputation of certain individual is not determined by itself but by the neighbor. As the asynchronous updating mechanism is incorporated, the updated reputation is regarded as the latest reputation of the player. When the reputation of certain individual is decreased to 0 or increased to 2, then the reputations of individuals in the whole network stop evolving while only the strategy updating proceeds.

For the selected player x as presented in Fig. 2, it is supposed to update corresponding strategies against his four neighbors. When updating the strategy against certain neighbor (here, player y is discussed for an illustration), a reference player is necessary for the purpose of strategy learning, here, it is referred to as interactive diversity. Aiming to conduct the reference selecting process, we introduce another parameter p. The whole process is described as follows: when the strategy of player x against y is to be updated, then player y is anticipated to be chosen as the reference player with a probability of p; otherwise, one of the other neighbors (i.e., player z, w, or v) might be selected as the reference player witha probability of (1-p)/3. To be consistent with the updating rule in [35], the parameter p equals to 0. 919 throughout this manuscript.

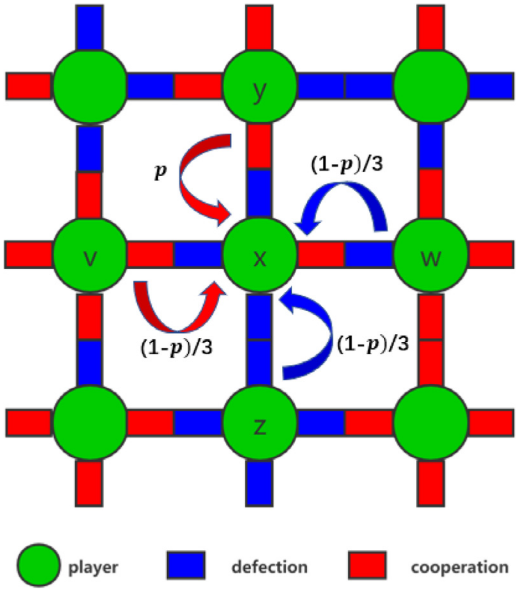

Fig. 2. Interactive diversity in square lattice networks. Here, green circles indicate players while rectangles denote strategies (here, red and blue represent cooperation and defection respectively). Each player is supposed to play games with its neighbors independently; then, the player accumulates his/her payoffs from all interactions. We assume when player *x* updates the strategy against the neighbor *y*, player *x* will select player *y* as a reference player with a probability *p*, and take *z, w*, or as reference player with a probability (1-)/3. We set $p = 0.919$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.) .

According to the above-described process, certain neighbor is chosen as the reference player for the incorporated player *x* in order to update the strategy against player *y*. Then, player *x* will update his/her strategy against player *y* accordingly by imitating the strategy of the selected reference (one of players *y, z, w*, and *v*) with the adoption of the well-known Femi updating rule [45]. Hence, when central individual is supposed to update the strategy against player *y*, he/she will imitate the strategy of the selected reference player. Here, the probability of local player *x* imitating strategy of the selected reference player is calculated as follows:

$$W = \begin{cases} p * \frac{1}{1+\exp((F_x - F_y)/K)} & if\ y\ is\ the\ reference\ player \\ \frac{(1-p)}{3} * \frac{1}{1+\exp((F_x - F_w)/K)} & if\ w\ is\ the\ reference\ player \\ \frac{(1-p)}{3} * \frac{1}{1+\exp((F_x - F_z)/K)} & if\ z\ is\ the\ reference\ player \\ \frac{(1-p)}{3} * \frac{1}{1+\exp((F_x - F_v)/K)} & if\ v\ is\ the\ reference\ player \end{cases} \tag{4}$$

where *K* indicates the selection intensity or the referred stochastic noises, representing the noise amplitude [40]. For simplicity, *K* is assigned to be 0.1 [41]. Similarly, the adopted strategies against the other neighbors can be determined accordingly. Over all, the key point of our proposed mechanism is that when a player is supposed to update the strategy against certain neighbor, then probability of this neighbor being selected as the reference player equals to *p*. Please refer to [35] for more information, the mechanism adopted in this manuscript is similar.

For the simulations conducted here, Monte Carlo (MC) steps are set to be 100,000. Aiming to ensure proper accuracy, the final result is averaged and each set of parameters was repeated 20 times independently.

3. Results and analysis

In order to investigate the effects of co-evolution of node reputation and edge strategy in the PD game, a number of simulations are conducted in this section with sufficient analyses of the obtained results being provided.

Firstly, we devote our efforts into considering the effects of varying parameters δ and *b* on the frequency of cooperation, i.e., F_c, as presented in Fig. 3. Here, if the parameter $\delta = 0$, then the scenario is simplified to the traditional PD game while the obtained results are the same as those of the traditional scenario and the cooperation frequency is not particularly evident. When $\delta = 1$, then the investigated scenario is similar as the traditional PD game. This is incurred by the assumption that the reputation evolving process of the network stops if the reputation of any player is reduced to 0 or increased to 2. Hence, the system meets this stopping condition only after one iteration; thus, the obtained results are consistent with those of the traditional scenario. For a fixed positive δ (i.e., $\delta > 0$), if δ is varying between 0.001 and 0.01, the effect of *b* on cooperation attenuates and individuals are able to maintain an acceptable of cooperation level even if *b* is larger than the value of the traditional scenario. While there exists an optimal parameter combination, i.e., $b = 1.7$ and $\delta = 0.01$, at which

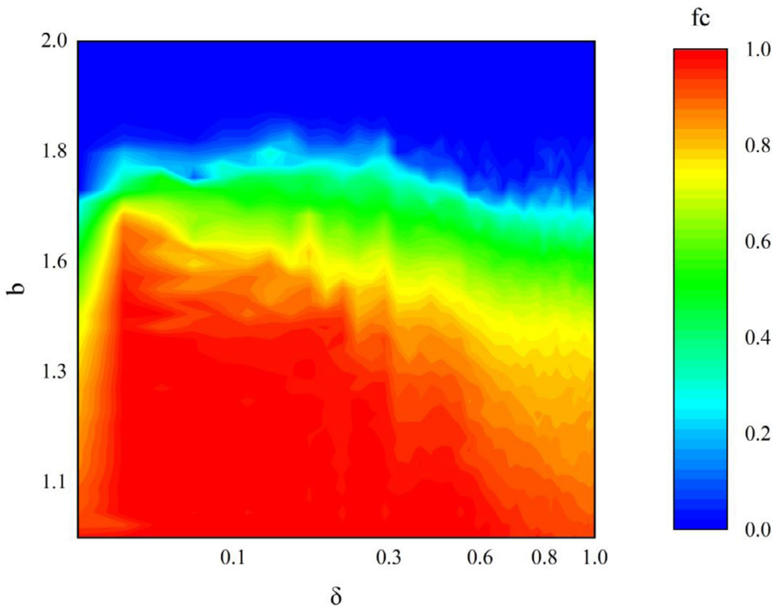

Fig. 3. Illustration of the fraction of cooperators for different parameter combination of b and δ, the color band on the right indicates the frequency of cooperation; experiments are conducted on square lattice networks here. As presented, the scenario is simplified to the traditional PD game for $\delta = 0$. Nevertheless, the level of cooperation begins to increase first and then decrease gradually with the increase of δ for a fixed b ($b < 1.7$) for the scenario considered.

the cooperation frequency is still capable of maintaining a desirable level compared with scenarios of other parameter combinations. For a fixed b ($b< 1.7$), we can clearly see that the level of cooperation begins to increase first and then decrease with the increase of δ. If $b > 1.8$, cooperators seem to be eliminated due to the large temptation to defect. Hence, we can conclude that the proposed mechanism in this manuscript is capable of attenuating the effects of b on cooperation for δ in certain range.

Then, let us turn our attention to the evolution of strategies over different time steps for scenarios with different δ. For $\delta = 0.045$, the level of cooperation can be improved significantly as presented in Fig. 4(a) compared with the other two scenarios as presented in Fig. 4(b) and (c) respectively. For $\delta = 0.8$, similar to the case of $\delta = 1$, the system will quickly stop the reputation evolving process and return to the traditional case, and only strategy evolution will be conducted. By contrast, the final cooperation frequency at the stationary state just increase slightly, but not obvious if $\delta = 0.5$ compared with the results presented in Fig. 4(c). From these results, we can conclude that, in our proposed mechanism, appropriate values of δ can increase the overall cooperation frequency to a higher level compared with the traditional scenario. Furthermore, in PD games, the network reciprocity mechanism increases the viscosity of the society, resulting in a cooperative equilibrium; the mechanism of this enhancement is illustrated by discussing the enduring period (END) and the expanding period (EXP) as in [42,43]. With the increase of δ, the minimal frequency of cooperators (i.e., the bottom cooperation fraction observed at the end of END period) increases while final (maximal) frequency of cooperator (i.e., equilibrium value observed at the end of EXP period) decreases. Inversely speaking, presuming an appropriately smaller δ, lower bottom value of the frequency of cooperators for END and higher celling value for EXP are realized. It implies that the number of C-Clusters surviving in END period becomes small, thus those surviving C-Clusters are able to swell smoothly to neighboring defective 'sea' relatively easily pushing the frequency of cooperator up to a higher value. This discussion can be fully consistent with the concept of END and EXP period [44,45]. In order to investigate the fractions of strategies with different reputation, we further divide the individuals into four categories according to players' reputations, they are listed as high-reputation cooperators (HC), low-reputation cooperators (LC), high-reputation defectors (HD) and low-reputation defectors (LD) respectively. In particular, at the beginning of each time step, we will calculate the average reputation of all individuals in the network. By comparing the individual's reputation value with the average one, we will divide the individuals into two types, players with high reputation and players with low reputation. Here, high reputation individual refers to the player's reputation is higher than the average value, while low reputation individual indicates the player's reputation is lower than the average reputation. As indicated by the obtained results, for small δ, the individuals are inclined to be high reputation players; while the players with low reputation will dominate absolutely for large δ. This is incurred by the inherent fact of our mechanics that when a player's reputation reaches 0 or 2, the reputations of individuals in the entire network stops evolving. Thus, if δ is relatively large, then it is highly likely for the reputation of certain player to be either zero or two shortly incurring the end of reputation evolving process. Hence, there is only a small number of players evolving into high-reputation individu-

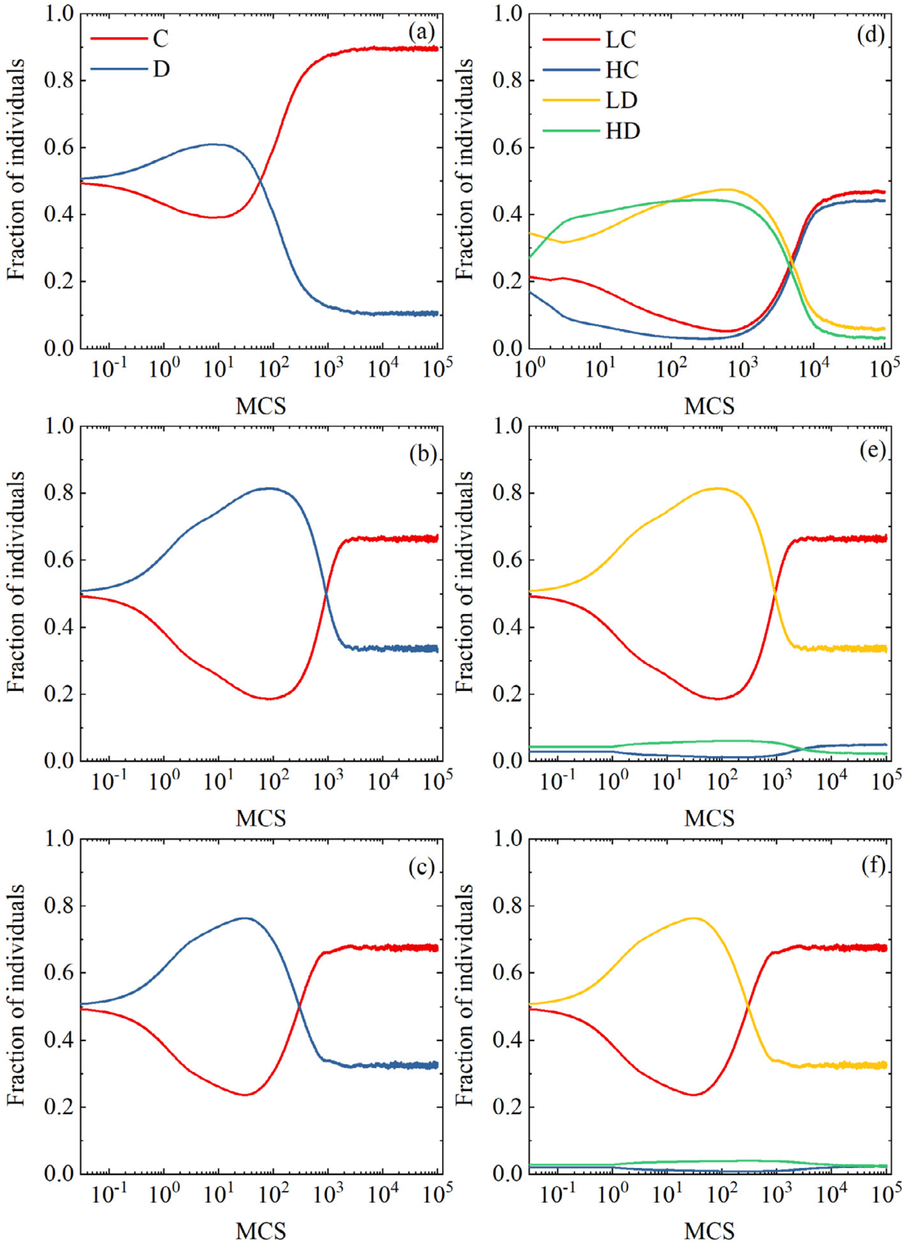

Fig. 4. The evolution of strategies for different parameter combination. Here we fix b= 1.5. Panels (a)–(c) correspond to the obtained results of scenarios δ = 0.045, 0.5, and 0.8 respectively; while panels (e)–(f) represent the evolution of different types of strategies with corresponding parameters. The frequency of cooperation with high-reputation (HC) and low-reputation (LC) are indicated by blue and red respectively; while the frequency of defection with high-reputation (HD) and low-reputation (LD) are denoted by green and yellow, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

als. Whereas the reputation evolution process for the scenario of $\delta = 0.045$ can be extensively performed incurring a large number of high-reputation individuals.

Aiming to understand the origin of observed phenomena extensively, characteristic snapshots of four types of strategies on the regular lattice network are further illustrated in Fig. 5 for different values.

As presented in Fig. 5(a1)–(a4) which correspond to the scenario of $\delta = 0.\ 045$, we can find that evolution of a player's reputation in the network seems to last for rather a long time; this is incurred by the fact that reputation evolves more slowly being incurred by a small varying amount which takes a long time to reach the threshold. This indicates that we are anticipated to have more cooperators with high reputation, incurring the formation of clusters consisting of cooperators with high reputations. These clusters can efficiently avoid the invasion from defectors. The expansion of such clusters can

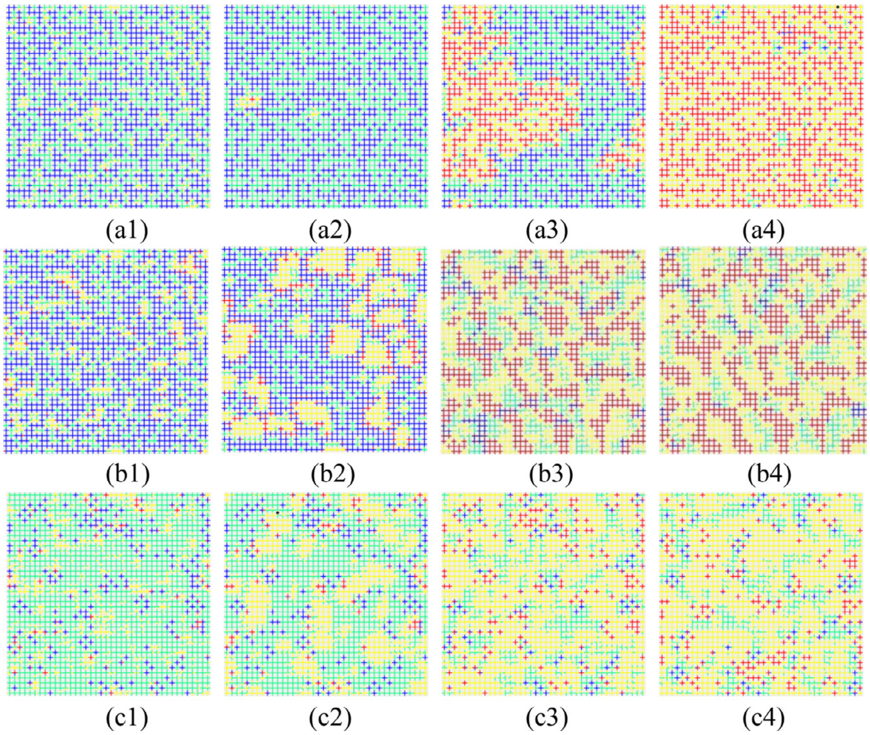

Fig. 5. The characteristic snapshots of individuals with different strategies on the regular lattice network. Here, cooperators with high reputation and low reputation are denoted by red and yellow, respectively; whereas defectors with high and low reputation are represented by blue and cyan, respectively. From top to bottom, δ equal to 0.045, 0.5 and 0.8, respectively. From left to right, the snapshots are selected at the following MC steps as 100, 2000, 10000 and 99999 respectively. The results are obtained for $b = 1.5$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

play important roles in promoting cooperation. Hence, the overall frequency of cooperation can be improved. With the continuation of evolving process, there will be more high-reputation cooperators and low-reputation cooperators. This is incurred by the fact that when a player brings positive benefits to his neighbors, his reputation will increase which results in the increase of corresponding revenue accordingly. Hence, clusters of high-reputation individuals will form gradually. On the contrary, the defector will get less and less benefit and eventually be eliminated.

Nevertheless, if δ equals to 0.5, we can find that the reputation evolution process is supposed to stop very soon (as indicated by Fig. 5(b1)–(b4)). Under such scenario, it is unlikely for the cooperative players with high reputations to increase to a high level, while majority of the cooperators possess low reputations. Compared with the traditional case, individuals with low reputations cannot provide a better benefit to the group; thus, the obtained cooperation frequency is similar as that of the traditional situation. Furthermore, we can get similar conclusion for the scenario with $\delta = 0.8$ (as in Fig. 5(c1)–(c4)).

In order to further explain the principle of this phenomenon, we also consider the variation of chains, i.e., the interaction strategy pairs between players, like CC chain, DD chain, CD chain and DC chain, respectively. The above four types of chain represent possible adopted strategy cases of two players; CC chain indicates two players cooperate with each other, DD chain means two players defect with each other, while CD chain shows that two players interacting differently, i.e., one cooperates and the other defects. Here, the proportions of various chains in the evolution process are provided in the form of time evolution diagrams as in Fig. 6(a)–(c). In the initialization stage, random distribution is adopted to control the player's strategy determination. As the evolution process proceeds, the frequency of CC chain in Fig. 6(a) seems to be higher than those of the other two scenarios. This seems to be the reason of higher frequency of cooperators for $\delta = 0.045$. As illustrated by Fig. 6(d), we can find that CD and DC chains varies gradually and eventually maintain a relatively low level regardless of the value of δ, this is incurred by the fact that these two types of links are unbalanced and easily to be broken. In other words, the CD and DC chains are temporary states due to the transitions between CC chain and DD chain. Therefore, during the whole evolution process, the fractions of CC and DD chains seem to be relatively high; the superiority of the profits on CC chain is obvious and CC chain dominates rapidly especially compared with DD chain. As in Fig. 6(d), the results are consistent with above discussions. Specially, if δ is relatively small, the frequency of the CC chain increases rapidly and reaches the summit; then with the increase of δ, corresponding frequency of CC chain decreases gradually. Hence, we can come to the conclusion that a small δ plays an important role in promoting cooperation.

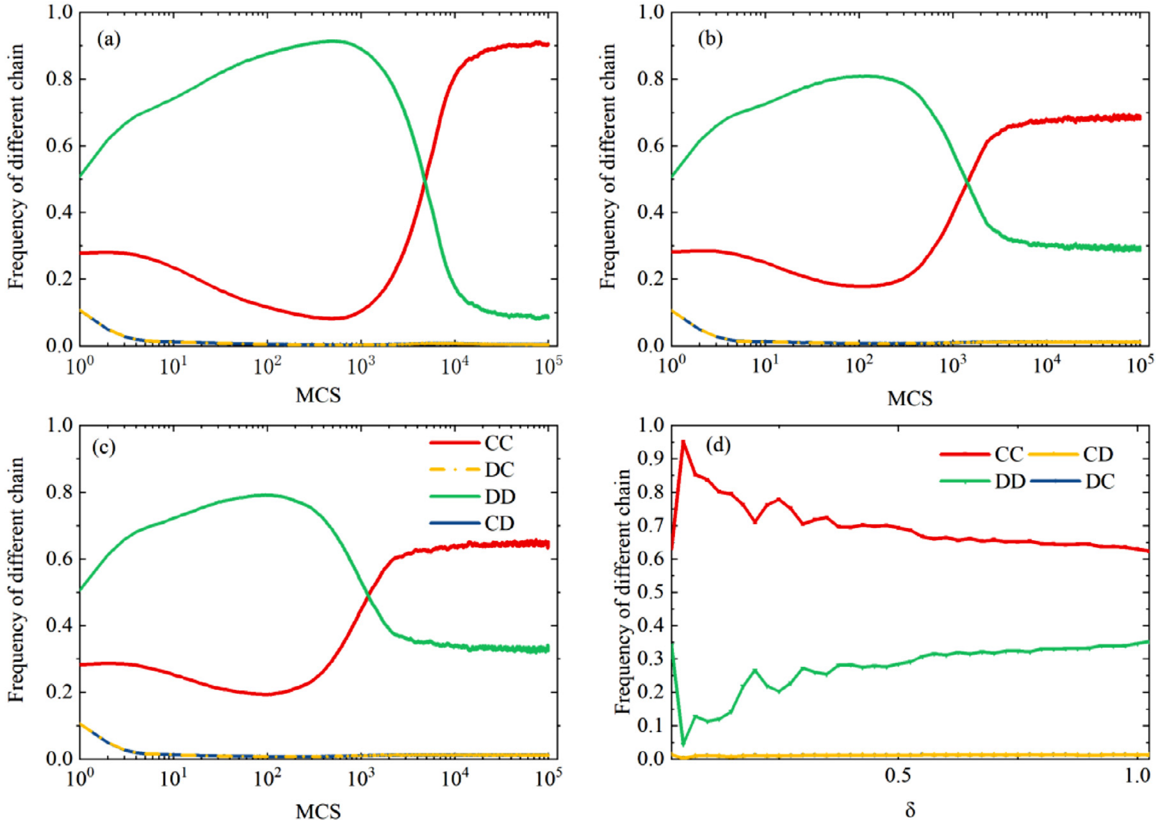

Fig. 6. Panels (a)–(c) show the evolution of fractions of four types of links for different δ (δ equal to 0.045, 0.5 and 0.8, respectively). Connection types in the network are divided into four types, namely CC chain, CD chain, DC chain and DD chain being represented by red, blue, yellow and green respectively. Panel (d) indicates the fractions of four types of link in steady state as a function of δ. Here, all results are obtained for $b = 1.5$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

Eventually, we try to explain this phenomenon from the perspective of cluster evolution with corresponding results being provided in Fig. 7. Here, we mainly investigate the scenario of $\delta = 0.045$ while b varies from 1 to 2. Firstly, we focus on individual strategies. As presented in Fig. 7(a) and (b), though the number of C clusters (i.e., cluster of cooperators) is small, the number of individuals in the largest cluster of C strategy is really big, which means cooperation prevails in the network due to the small value of b. With the increase of b, we can see the number of C clusters increases, and the number of individuals in the largest cluster of C strategy decreases gradually, which imply that the large C clusters are separated by D strategy players, forming a large number of smaller clusters. As b increases further, the number of C clusters is increasingly explored by strategy D players, and D clusters gradually increase and merge, finally dominate the whole network. Figs.7(c)-(d) discuss this phenomenon from the perspective of a more microscopic sense; besides strategy, we also consider the player's reputation. For a small b, the number of clusters is relatively small; nevertheless, HC dominates the largest clusters. For instance, the number of clusters is small for $b = 1$ as indicated by Fig. 7(c), whereas the size of largest cluster is almost 160,000, this indicates HC dominates the largest cluster. Then, with the increase of b, the number of clusters consisting of the four types of strategies increases gradually; due to the emergence of defection, the giant cooperation cluster will be exploited slowly while the dominance of cooperation with high reputation will be threatened and attenuated. As presented in Fig. 7(d), with the decrease of cooperation with high reputation (i.e., HC) then the number of cooperation with low reputation (i.e., LC) will increase temporarily as in Fig. 7(d); nevertheless, due to the large temptation to defect, defectors with high reputation start to increase. Overall, cooperation with high reputation plays an important role in avoiding the exploration from defection; even if = 1.6, there exists a bunch of clusters consisting of cooperators.

4. Conclusion

In this paper, aiming to investigate the evolution of edge-strategy, we considered a general framework which incorporates interactive diversity. This indicates that each player adopts separated strategies against different neighbors independently and takes adaptive adjustment according to the opponents' strategies. Furthermore, we also introduce the definition of reputation for a player in order to investigate the coevolution of reputation and edge-strategy thoroughly. Then, a mechanism is proposed in this manuscript aiming to investigate the co-evolution of personal reputation and strategy under a general

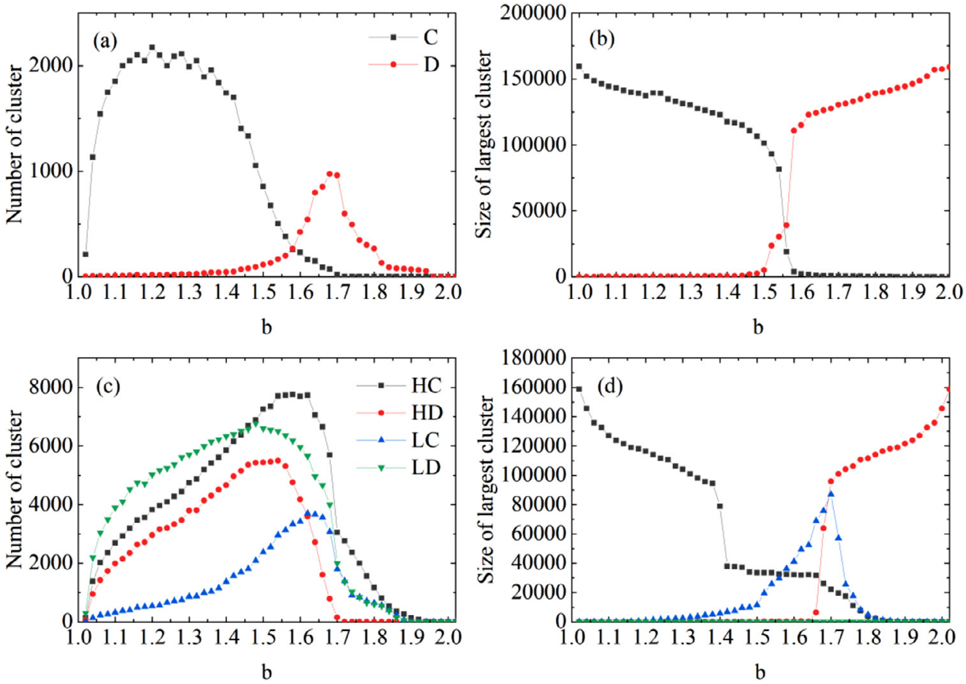

Fig. 7. Cluster evolution for $\delta = 0.045$ where b varies from 1 to 2. Panels (a) and (b) show the number of cluster and size of the largest cluster for two types of strategies, cooperation (black) and defection (red). Considering individuals' reputation, panels (c) and (d) show the number of cluster and size of the largest cluster for four types of strategies, cooperation with high-reputation (black), cooperation with low-reputation (blue), defection with high-reputation (red), and defection with low-reputation (green). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

framework of interactive diversity (i.e., edge-strategy here). After performing numerical simulations, we find that the level of cooperation can be effectively promoted under the proposed mechanism. A relatively small δ plays an important role in providing a high level of social fault tolerance which is able to maximize profits in a hostile environment and continue to affect others. Hence, a stable social structure consisting of cooperative behaviors can be formed and survived. The obtained results in this manuscript are capable of providing us with comprehensive understandings regarding the influence of nodal reputation on the evolution of cooperative behaviors. Furthermore, the number of neighbors might has a significant effect on the evolution of cooperation, and scholars have done meaningful works regarding how the average degree affects cooperative behavior [46]; however, most of such works focus on the node dynamics. Therefore, in future work, we will try to investigate the effects of network topology characteristics, such as network type and network average degree, on evolutionary dynamics under the consideration of edge dynamics.

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