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Evaluating fitness by integrating the highest payoff within the neighborhood promotes cooperation in social dilemmas

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ABSTRACT

In this paper, we propose a modified fitness evaluation mechanism, which integrates the environmental factors into the focal player's fitness calculation, to investigate the evolution of cooperative behaviors in the prisoner's dilemma game. Here, the fitness of a player is computed by combining the individual raw payoff and the highest payoff within the neighborhood, which is regulated by a single parameter termed as trust level η. We show, compared to the traditional version ($\eta = 0$), that the cooperation level can be highly enhanced for $\eta > 0$. Meanwhile, we illustrate the dynamical evolution of cooperators on the square lattice, and for different defection parameters b the $F_C - K$ curves are utilized to investigate the impact of noise during the strategy updates. Likely, the role of pursuing the highest payoff within the neighborhood also favors the survival of cooperators in the spatial snowdrift game. In addition, the sensibility of knowing the external factors is often not identical for all individuals and we consider the distributed trust level in which η is a distributed parameter, and the results indicate that pursuing the highest payoff in the neighborhood is also inspiring as a consequence of its positive effect on cooperation. The current results are highly instructive for us to further understand the maintenance and emergence of cooperation under the framework of evolutionary game theory.

1. Introduction

Darwinian evolutionary theory has laid out the theoretical foundations for species development [1]. According to his theory, any behavior that brings benefits to others but not directly to oneself will soon disappear. However, extensive collective phenomena and cooperative behaviors can often be found in social and biological systems, which is not completely consistent with Darwinian classical theory. How to understand and analyze the widespread cooperative behaviors between individuals becomes an active topic, present in all scales of organization, from natural science to social science [2,3]. Among them, evolutionary game theory has proven to be one of the most fruitful approaches to investigate this problem [4,5], using evolutionary models based on so-called social dilemmas, such as the prisoner's dilemma game (PDG) [6–8], the snowdrift game (SDG) [9,10] and public goods game (PGG) [11,12].

Most notably, the prisoner's dilemma game has received particular renown and becomes the leading paradigm to explore the evolution of cooperation among selfish individuals [13–15]. As a metaphor, the prisoner's dilemma game is often

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selected to explore the evolution of cooperation between pairwise individuals. In its primitive version, two players must simultaneously decide to adopt one of two possible strategies: to cooperate (C) or to defect (D). Mutual cooperation leads to the reward (R), and mutual defection results in the punishment (P). However, any unilateral action will bring the highest payoff for the defector (i.e., the temptation to defect, T) and the lowest one for the cooperator (i.e., the Sucker's payoff, S). These payoffs strictly obey the rankings: $T > R > P > S$ and $2R > T + S$. In fact, it is often observed that the weak satisfying rule doesn't alter the qualitative result when we consider the following parameter setting: $T = b\,(1 \le b \le 2)$, $R = 1$, and $P = S = 0$ as described in Ref. [16] Thus, the corresponding payoff matrix can be simplified as,

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \end{array} \tag{1}$$

where b stands for the temptation to defection and is the only parameter in the game. In such a setting, obviously, to defect is the optimal choice because it brings the highest benefit irrespective of its opponent's strategy. Hence, the so-called social dilemma appears as a single individual tends to defect but the cooperation among players leads to the higher benefit from the perspective of the whole population.

Since the prisoner's dilemma represents the most adverse situation to cooperation, a plethora of extensions, which include spatial structure, complex topology, and various interaction mechanisms, have been put forward to further understand the origin of cooperation. In their pioneering work [16], Nowak and Robert seminally introduced spatial structure into the PGD, and found that the cooperation could be greatly enhanced by forming compact clusters on the square lattice. After that, various spatial extensions are integrated into the evolutionary PDG models and found to be beneficial to the cooperative behaviors [13]. At the same time, major progress has been made in the field of complex networks in which vertices denote the individuals or players and links mimic the interaction between them [17]. Many real systems display the characteristic small-world effect and scale-free properties which are found to highly promote the cooperation among individuals on networking communities [18,19]. In addition, various microscopic interaction mechanisms are also presented to promote the cooperative behaviors between individuals, such as kin selection [20], direct and indirect reciprocity [21], learning and heterogeneous teaching activities [22,23], environmental noise [24,25], asymmetric payoff [26], voluntary participation [27], reward and punishment [28], partner switching [29], memory [30], individual's mobility [31–33], bounded rationality [34], cooperation robustness [35], social diversity [36], vertex weighting mechanisms [37], as well as aspiring to the fittest [38,39]. All these works provide a variety of valuable clues to understand the maintenance and emergence of cooperation within the structured populations. For better reviews of the recent progress of evolutionary game theory, we recommend the readers to several latest reviews devoted to capturing succinctly recent advances in this field [13–15].

However, the impact about external and internal factors is usually separated in previous literatures, namely, some works focus on the effect of individual properties inherited from its ancestors, while the other works are based on the impact of external factors standing for the role of the environment in evolution. In fact, the personal characteristic ingredients and external factors can commonly influence their strategies during the decision processes, thus it is necessary to simultaneously take these two factors into account. For example, in Ref. [40,41], the authors integrate the average payoff of one player's neighbors into the PDG model to account for the role of the environmental factors within the cooperative behavior. In Ref. [42], spatial evolutionary games are studied with myopic players whose payoff interest, as a personal characteristic, is tuned from selfishness to other-regarding preference via fraternity. Nevertheless, most people try to mimic the best neighbor around them, and often establish the best neighbor as an example to imitate or go beyond. Although Yamauchi et al. consider that imitating the neighbor with the maximum payoff can bring higher cooperation than other stochastic updating rules, the individual factors will be totally ignored during evolution in this condition [43,44]. Nevertheless, it is more natural that most people try to mimic the best neighbor around them, and often establish the best neighbor as an example to imitate or go beyond. Here we try to present an approach which combines the individual benefit and the payoff of its best neighbor to make the assessment or decision for the game actions. As a matter of fact, many paradigmatic examples can also be found in social and biological systems to support this idea. By considering the traditional payoff accumulation which can be computed through playing with a random neighbor as something related to inheritance, and by pursuing the highest payoff among its neighbors as being representative for the environment, we introduce a modified fitness assessing method into the evolutionary game model, in which we can regulate the proportion of individual inheritance and exterior environment by a single tunable parameter. To some extent, the proposed fitness evaluating function implies a co-evolutionary ingredient since the impact of these two factors dynamically relies on their expected performance.

In what follows, the rest of this paper is composed of three sections. In Section 2 we give a detailed description of our evolutionary game model and the computing method of the fitness function. Section 3 presents extensive numerical simulation results. Finally, we end this paper with some concluding remarks in Section 4.

2. The model

As mentioned in Eq. (1), we consider a variant of the strict PDG model termed as so-called weak prisoner's dilemma, in which $P = S = 0$ is used rather than $P > S$. Although it is also termed as the boundary game between PDG and SDG, this version nearly captures all the relevant properties of the PDG model [43,44]. Furthermore, we also investigate the spatial SDG to verify our conclusions at the end of Section 3, in which the payoff matrix is normalized as follows,

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 1-r \\ 1+r & 0 \end{pmatrix} \end{array} \tag{2}$$

where $0 \le r \le 1$ denotes the cost-to-benefit ratio, and means these parameters strictly follow the ranking $T > R > S > P$. In reality, the SDG is frequently investigated as an alternative to the better known prisoner's dilemma, and even striking behaviors are found in the SDG where the spatial structure doesn't favor cooperation among agents [9].

As for the interaction network, we use a $L \times L$ square lattice with periodic boundary conditions as the underlying topology. Initially, each site is designated as a cooperator ($s_i = C$) or defector ($s_i = D$) with equal probability. Then, the game is played repeatedly according to the Monte Carlo (MC) simulation procedure containing the following elementary steps. First, a player i obtains its payoff P_i by playing the game with all its nearest neighbors. Next, the external factor or the environment is characterized by conquering the highest payoff within all its nearest neighbors as follows,

$$P_{\max} = \max_{j \in N_i} P_j \tag{3}$$

where N_i denotes the set of all the nearest neighbors of player i, P_j is the payoff of player j which is one of the neighbors of player i and $P_{\max}$ takes the highest payoff among all the nearest neighbors.

Then, we quantify the individual fitness (f_i) of focal player i according to the following utility function,

$$f_i = (1 - \eta) \times P_i + \eta \times P_{\max} \tag{4}$$

where η is a tunable parameter which represents the supporting level to a focal player, say i, from the external environments or its neighborhood, termed as the trust level in this paper. $\eta = 0$ indicates that an individual will not think of the exterior environmental factors at all during the process of fitness evaluation, and this case is equivalent to the traditional case. While for $\eta = 1$, the fitness of player i is fully dependent on the exterior environment, that is, all individuals find it possible to adopt the individual strategy with the highest evolutionary payoff. From the long-term and global evolution viewpoint, cooperation is better than defection, and thus the introduction of η will make the focal agents consider the neighboring information when they evaluate their fitness. Meanwhile, a focal agent tends to keep her own strategy, which leads to making agents' strategy updating proceed more slowly than in the standard case, and this works to support the collective cooperation in the other direction. In addition, an individual makes the fitness evaluation centered around itself and refers to the environmental information in part, we usually consider the case of $0 \le \eta \le 0.5$ in this paper. However, theoretically η can be an arbitrary value which lies between 0 and 1.

After the fitness of focal player i is determined, its strategy is updated in accordance with the Fermi rule [45], that is, player i will randomly select one neighbor, say j, from all its nearest neighbors and adopt the strategy of player j (whose fitness f_j is also calculated in the same way as f_i) with the probability

$$W(s_j \to s_i) = \frac{1}{1 + \exp[(f_i - f_j)/K]} \tag{5}$$

where K denotes the amplitude of noise or its inverse $(1/K)$ the so-called intensity of selection. Positive values of K mean that better performing players are probably imitated, but it is possible to adopt the strategy of a player who does worse. Such errors in judgment and decision can be attributed to some mistakes or external fluctuations that affect the evaluation of the opponent. After each player on average has one chance to perform the above-mentioned strategy update, the next MC simulation step or iteration begins.

In the following section, all numerical results of Monte Carlo simulation are performed on 200×200 to 400×400 lattices; other lattice sizes are also tested and the results are qualitatively the same. The stationary fraction of cooperators F_c is determined within 10^4 full MC steps after sufficiently long transients (here taking 10^4 MCS) are discarded. Moreover, the final results are averaged over up to 20 independent runs for each set of parameters to eliminate the influence of some uncertainties.

3. Monte Carlo simulation results

First, we depict the relationship between the fraction of cooperators F_c and the parameter b for different values of trust level η in Fig. 1. Here, η is varied from 0 to 0.5 and is identical for each player. It can be observed that, compared to the traditional version of the PDG (i.e. $\eta = 0$), the cooperation level is largely enhanced when η is more than 0. When the trust level η is slightly increased to 0.1, F_c will boost to a very high value and the extinction threshold of cooperation is extended to 1.107, whereas this threshold is only about 1.0375 in the traditional case. As the η increases, it is shown that the extinction threshold is largely augmented. When $\eta = 0.5$ is taken into account, the cooperators can dominate the whole population for most ranges of b, and cooperation tends to be extinct only after b is beyond 1.85. When $\eta > 0.5$, the promotion of cooperation becomes more pronounced and it will even display full cooperation when b lies between the whole range of

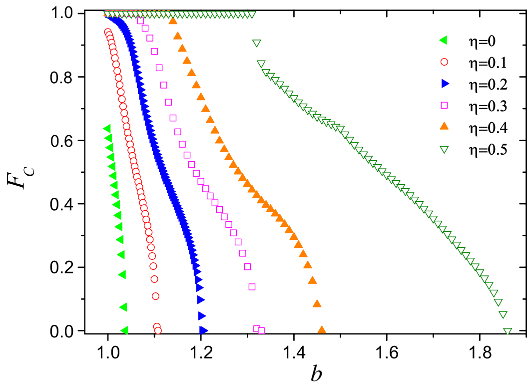

Fig. 1. (Color online) Relationship between F_C and b for different trust level (η) which is varied from 0 to 0.5. From left to right, prisoner's dilemma game is carried out for the traditional and five kinds of trust level. All these results are obtained for $K = 0.1$ and $L = 200$.

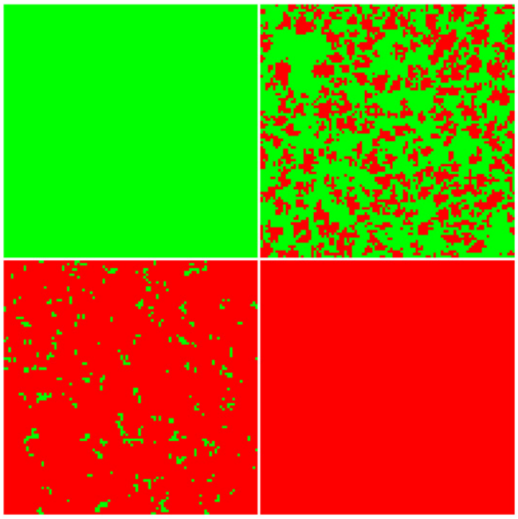

Fig. 2. (Color online) Characteristic snapshot of cooperators and defectors on the square lattice at the stationary state. In the upper two panels, η is set as 0 and 0.1, respectively. For the lower two panels, η corresponds to 0.3 and 0.5. All these results are obtained for $b = 1.08$, $L = 100$ and $K = 0.1$ after 10^4 MC time steps.

[1, 2] for $\eta \geq 0.7$, thus we don't present these results in Fig. 1. Altogether, the results indicate that the fitness assessment considering the external factors, here implemented by pursuing the highest payoff within the neighborhood, can drastically facilitate the cooperators among the population and lead to the emergence of cooperation between players.

Next, the stationary distribution of cooperators and defectors on the square lattice is visualized in Fig. 2. The red dots represent the cooperators and green dots stand for the defectors. (For clarity, we recommend the readers to refer to the web version of this paper.) For the traditional case, there is only the spatial reciprocity to support the cooperators which are easily exploited by the defectors. As a consequence, the whole population is taken over by the defectors. However, the introduction of external factors, e.g., referring to the neighbor with the highest payoff, can highly facilitate the evolution of cooperation. The cooperative clusters begin to form in the population as the trust level η is slightly increased, and even the cooperators will completely dominate the population when η is increased to over 0.3. That is, apart from the spatial structure, our modified fitness evaluation mechanism contributes to form the cooperative clusters in which the *C-type* player is supported by a large payoff originated from the bulk of the C domain, whereas the *D-type* players cannot gain such a benefit from its partners who weaken each other behind it. Thus, integrating the highest payoff within the neighborhood into the individual fitness assessment should be mainly responsible for the advancement in the level of cooperation within the population.

In Fig. 3, we illustrate the temporal evolution of the fraction of cooperators, $F_C(t)$, for different trust level η and a specific temptation to defection $b = 1.08$. In the beginning, defectors are more successful and the fraction of cooperators in the population decreases since the initial setup is randomly selected. This is a universal property of the PDG in spatially structured arrangements [13]. Nevertheless, different values of η will exhibit diverse behaviors within the subsequent process. The traditional case ($\eta = 0$) doesn't form compact clusters of cooperators, thus resulting in a quick extinction of

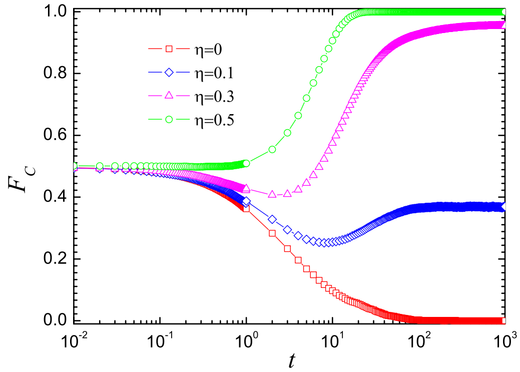

Fig. 3. (Color online) The dynamical evolution of the fraction of cooperators on the square lattice for different trust level η, which is 0, 0.1, 0.3 and 0.5 from bottom to top. All these results are obtained for $b = 1.08$, $L = 200$ and $K = 0.1$.

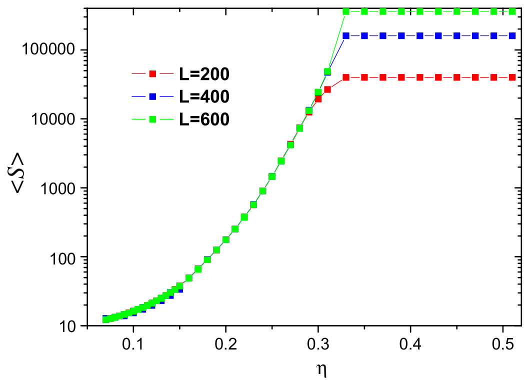

Fig. 4. (Color online) The average cluster size of cooperators $\langle S\rangle$ varies as a function of the trust level η. All these results are obtained for $b = 1.08$, $L = 200, 400, 600$ and $K = 0.1$. Here MC time steps take 50,000 and we begin to sample the cluster size at $t = 45,000$ and the sampling length $T = 5000$.

cooperators. When η is increased to over 0.1, surprisingly, the initial drop of $F_C(t)$ is halted, and turns to the fast spreading of cooperators who outperform defectors and even dominate the system. Moreover, the earlier the initial decrease of $F_C(t)$ is halted, the faster the recovery of cooperation will be, and the more obvious the final dominance of cooperation is. That is, considering the highest payoff within a neighborhood can provide the necessary incentive to promote the evolution of cooperation.

At the same time, we explore how the clusters of cooperators form under this modified mechanism, which is crucial for protecting cooperators from being exploited by defectors. At the stationary state, we can count the number of cooperative connected clusters (nc) and the number of players within each cooperative cluster (S_i) at each time step. Next, we can define the average cluster size of cooperators $\langle S\rangle$ as follows,

$$\langle S\rangle = \frac{1}{T}\sum_{t}^{t+T-1}\frac{\sum_{i=1}^{nc} S_i}{nc} \tag{6}$$

where S_i denotes the size of i_{th} cluster at each time step, t is the beginning of sampling steps and T stands for the sampling length. We depict the $\langle S\rangle$ as a function of η in Fig. 4 for different lattice sizes $L = 200$, 400 and 600. Here we set the MC steps to be 50,000 and start to sample at $t = 45,000$ and the sampling length is set to be $T = 5000$. It can be clearly observed that with the augmentation of η the average cluster size of cooperators $\langle S\rangle$ will quickly increase, and even reach the saturation status when η goes beyond an invasion threshold (denoted by η_c) which lies around 0.33. Over this threshold the cooperators will completely dominate the population and the clusters are composed of the players, which are now all cooperators, on the square lattice. In addition, the invasion threshold (η_c) is independent of the lattice size L, and the average size of cooperative clusters is nearly the same for different lattice size L when $\eta < \eta_c$. That is, the clusters formed by cooperators are absolutely impervious to defector attacks at high values of η because getting more benefit from the highest payoff neighbors can guarantee the better environment for cooperators. Hence, the consideration of pursuing the highest payoff within the whole neighborhood can create more beneficial conditions for cooperators, then these cooperators can organize into the extremely robust cluster of cooperators, which finally leads to the dominance of cooperators in the whole population.

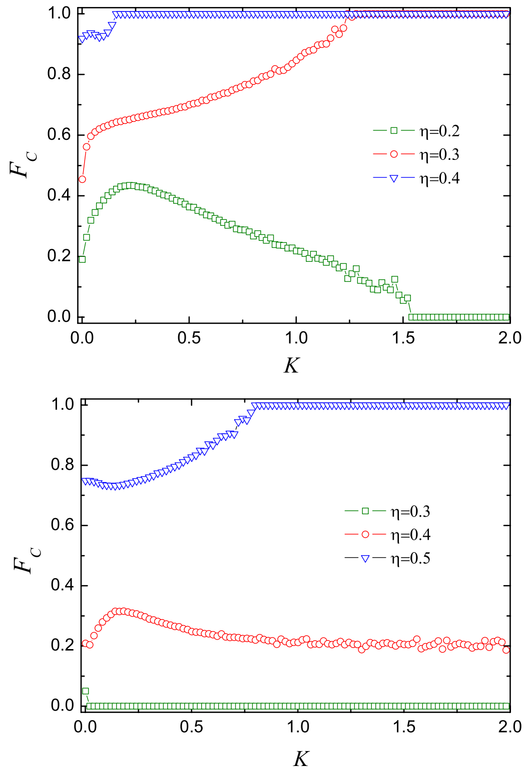

Fig. 5. (Color online) Fraction of cooperators F_C as a function of the noise K for different values of trust level η. (a) Top panel: $b = 1.15$; (b) Bottom panel: $b = 1.4$. All results are obtained for $L = 200$ and $MCS = 4 * 10^4$. Similar results are also obtained for larger lattice size and longer MC steps.

Till now, another important remaining problem is how robust the above mentioned results are under the effect of noise K during the strategy adoption (see (Eq. (5)). Fig. 5 features the frequency of cooperators F_C in dependency on K for different values of b (for clarity, the case of $F_C = 0$ or $F_C = 1$ for the whole interval of K is not included). For $b = 1.15$, when η is smaller than 0.2 cooperators tends to be extinct, irrespective of the values of K; while η is over 0.4 (for example, $\eta = 0.5$), all individuals tend to cooperate in the whole interval of K. Interestingly, when η is 0.2, there exists the non-monotonous phenomenon between F_C and K, that is, $K \approx 0.25$ allows the cooperation to be the most prosperous for the whole range of $K \in [0, 2]$. With the continuous augmentation of η, this trend transfers to the monotonously increasing one, where cooperation is largely boosted. Meanwhile, when η is equal to 0.4, the cooperation fluctuates a little for $K < 0.25$ but maintains a very high level, and then the cooperation dominates in the population for the rest of the range of K. When the larger value of b (e.g., $b = 1.4$) is taken into account, we can see that a similar phenomenon can also be obtained, which only needs higher values of η when compared to the case in Fig. 5(a), especially for $\eta < 0.3$ the full defection continuously emerges and doesn't change as K adapts and we don't illustrate this case in Fig. 5(b). Although Fig. 5 is obtained under the condition of $L = 200$ and $MCS = 4 * 10^4$, we also perform the simulations with larger lattice sizes and longer MC steps (e.g., $L = 600$ and $MCS = 10^5$) and find that the results are qualitatively the same for K between 0 and 2.0. Therefore, we argue that the effect of noise is robust for different b and it illuminates various behaviors under different values of η.

In order to further validate the generality of pursuing the maximum payoff within the neighborhood, we introduce this modified fitness assessment mechanism into the SDG model as well. Fig. 6 depicts how the cooperation evolves as a function of cost-to-benefit r for different values of η under the SDG model. It can be observed that the role of integrating the highest payoff within the neighborhood into the individual fitness evaluation is also positive and beneficial for the promotion of cooperation in the spatial SDG model. Thus, the current results are surprisingly inspiring for us to understand the cooperative behaviors inside the population since the spatial structure of the regular lattice is usually thought to be detrimental to the SDG game behavior in the population [9].

In all the above numerical results, the trust level η is identical for each player in the population and it also implies that each player has the same sensibility to the neighborhood information. In fact, the player usually has different knowledge or information about its local neighbors. Recently, it is also frequently observed that the behavior of individuals exhibits non-homogeneity. Henceforth, it is highly relevant for us to investigate the impact of distributional trust level η_i, and the role of inhomogeneity in η should not be ignored when we talk about the collective cooperation. In the initial setup, we assume that each player i takes a random strategy and a different trust level η_i, which is uniformly distributed and taken from the interval [0, 0.5], and then η_i stays fixed during the evolution of cooperation. Fig. 7 illustrates the impact of uniformly distributed trust level among the population, the cooperation level is indisputably boosted when compared to the traditional

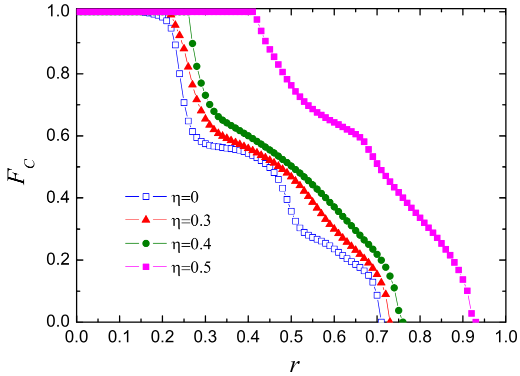

Fig. 6. (Color online) The fraction of cooperators F_C as a function of cost-to-benefit r in the SDG model. The results are also obtained for $L = 200$ and $K = 0.1$. Likely, the traditional case $\eta = 0$ is also described for the comparison here.

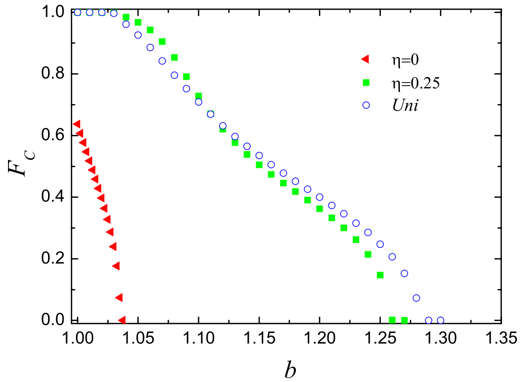

Fig. 7. (Color online) The fraction of cooperators F_C as a function of defection parameter b under the distributed trust level $\eta \in [0, 0.5]$. The results are also obtained for $K = 0.1$ and $L = 200$. For comparison, the traditional case and the case with $\eta = 0.25$ for all players are also displayed here.

case (i.e. $\eta = 0$). Even if referred to the case in which all players have the same trust level $\eta = 0.25$, cooperation can be enhanced a little when the defection parameter b is over 1.1, and the extinction threshold is further increased into around 1.3 for the distributed trust case. To some extent, the current results also coincide with the evidence that the heterogeneity supports the cooperation among selfish and unrelated individuals [18,36].

4. Conclusions

Many constraints or conditions, which include not only internal factors, such as individual ability, personality and characteristics, but also some external factors, for instance, natural resources, social norms and even cultural ambience, jointly play an important role in the evolution of cooperation. However, individuals can only obtain limited information around themselves, thus it becomes a challenging problem to make the decision based on the local information. An intuitive fact is that people usually make a comparison or evaluation between themselves and their neighbors, and try to perform the decision by utilizing related information among the neighborhood. Here, we propose a modified fitness evaluation mechanism, which commonly integrates their own payoffs and the highest payoff among the neighborhood into the fitness assessment, to investigate the evolution of cooperation. The results have indicated that pursuing the highest payoff within the neighborhood can largely promote the cooperation level. While this enhancement in the cooperation level is highly influenced by the tunable parameter (termed here as the trust level η). As η increases, the compact and strong cooperators' clusters are more easily created, and the resistance to defection is also greatly strengthened, which even leads to the complete dominance of cooperators. Numerical simulations validate the generality of this mechanism in the PDG model as well as the SDG model where the spatial structure is often found to hinder the cooperation among individuals. In addition, we still consider the impact of distributed trust level on the cooperative behaviors, in which the facilitating effect can also be observed and the extinction threshold of cooperators is further boosted.

To sum up, the current results help in further understanding the role of external environments in the maintenance or emergence of cooperation based on the framework of evolutionary game theory. Since pursuing the best neighbor is a widespread phenomenon, it is of high relevance to be integrated into the evolutionary game or decision model, and further studies deserve to be explored in the future.

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