IPP - Meta & THz (Sp-Su 2021) Notes

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Abstract

Metamaterials

Basics

1. Why metamaterials?

   Extreme Control of EM waves, e.x. Invisibility, Ideal/Perfect Lens, etc.

2. To be more specific, what is changed to achieve unusual control?

   The change of permitivity and permeativity, which leads to the change of refractive index (to be zero/negative), etc.

3. The theories/models?

   (a) Generalized Snell's Law

   The wavefront of light through normal optical components such as lenses rely on **gradual phase shift** accumulated along the optical path while an **abrupt phase shift** is introduced as a new freedom of control. Still, Fermat's principal (shortest path) governs the light's propagation.

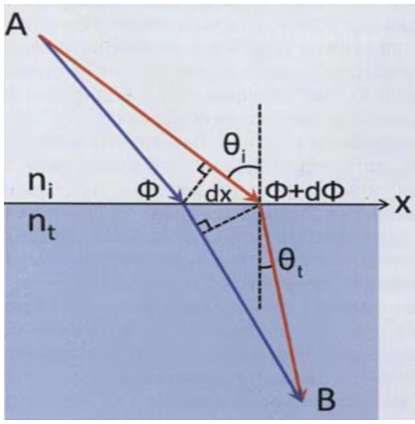
   The metasurface is extremely thin, even thinner than the wavelength. And thus, the gradual phase shift $\int_A^B d\phi(\vec{r})$ will be close to zero(or just zero) while the abrupt phase shift $\Phi(\vec{r_s})$ over the scale of wavelength can be introduced. It depends on coordinate $\vec{r_s}$ along the interface and the ultimate phase shift is calculated as $\Phi(\vec{r_s}) + \int_A^B d\phi(\vec{r})$ i)assume Φ being continuous,

$$
\begin{aligned}
sin(\theta_t)n_t - sin(\theta_i)n_i &= \frac{\lambda_o}{2\pi}\frac{d\Phi}{dx} \\
sin(\theta_r) - sin(\theta_i) &= \frac{\lambda_o}{2\pi n_r}\frac{d\Phi}{dx} \\
arcsin(\pm\frac{n_t}{n_i} - \frac{\lambda_o}{2\pi n_r}\frac{d\Phi}{dx}) &= \theta_c, \quad n_t < n_i \\
arcsin(1 - \frac{\lambda_o}{2\pi n_r}|\frac{d\Phi}{dx}|) &= \theta_c'
\end{aligned}
$$

   ii) Real-life: discreteness rather than continuity. Loss of energy rather than full transmission to reflection/refraction. So, we need numerical methods of Maxwell's equations. iii)Pictures about important parts:

   iii)How is this generalized snell's law used in papers?

   i. gradient phase shift $\longrightarrow$ reflection in obligue angle. (general confirmation with simulation) by TJCui, Coding metamaterials.
   ii. Find direction (Φ, θ) by assigning distributions of gradient phase responses. by L. Bao, Multi-Beam Forming and Controls.
   iii. Just mention it and calculate the angle shift by 14.5 ^{o} with phase discontinuity. by L. Zhang, Space-time-coding digital metamaterials



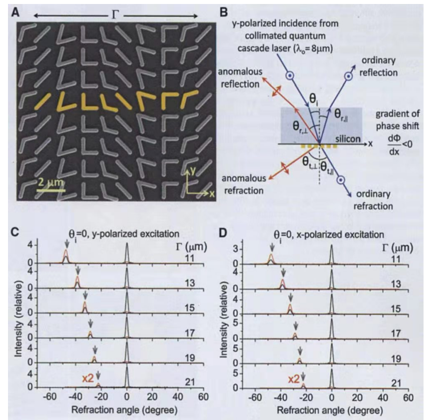
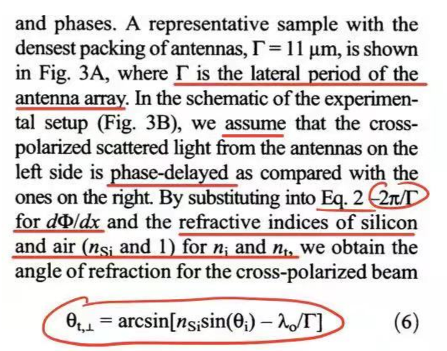


Figure 1: Snell Foundamentals   Figure 2: Snell Sample Comparison


and phases. A representative sample with the densest packing of antennas, $\Gamma = 11$ μm, is shown in Fig. 3A, where Γ is the lateral period of the antenna array. In the schematic of the experimental setup (Fig. 3B), we assume that the cross-polarized scattered light from the antennas on the left side is phase-delayed as compared with the ones on the right. By substituting into Eq. 2 $-2\pi/\Gamma$ for $d\Phi/dx$ and the refractive indices of silicon and air (n_{Si} and 1) for n_i and n_t, we obtain the angle of refraction for the cross-polarized beam

$$\theta_{t,\perp} = \arcsin[n_{Si}\sin(\theta_i) - \lambda_o/\Gamma] \qquad (6)$$

Figure 3: Application of the snell's law

iv. Extract effective refractive index n_r^{eff} (conventional snell's law)

v. phase pattern determines scattering directions of a beam on the surface. by Q. Ma Information metamaterial: bridging...

iv)Conclusion for Snell's Law, not used widely, just taken as something for confirmation after the simulated outcome is present.

(b) Theory Background for Electromagnetic Principles:(not fully understandable)

i. General Maxwell Equations when viewing metamaterials as dielectrics/nonconductors:

$$\begin{aligned}
\bigtriangledown \cdot D &= 0 \\
\bigtriangledown \cdot B &= 0 \\
\bigtriangledown \times E + \frac{\partial B}{\partial t} &= 0 \\
\bigtriangledown \times H + \frac{\partial D}{\partial t} &= 0 \\
\text{and} \quad D &= \epsilon E \\
B &= \mu H
\end{aligned}$$

Note that the electric permitivity and magnetic permeativity are dispersive right now, i.e. function of the frequency(complex function). $\epsilon(\omega)$, the solution $\frac{\epsilon(\omega)}{\epsilon_0}$ is called Kramers-Kronig relations(covered in HW of Vv286, forgotten).
Anyway, the developed equations/solution travelling in x direction is in the form of

$$\begin{aligned}
(\bigtriangledown^2 + \mu\epsilon\omega^2)E/B &= 0 \\
E(x,t) &= E_0 e^{i(kx-\omega t)} \\
B(x,t) &= B_0 e^{i(kx-\omega t)} \\
k &= \sqrt{\epsilon\mu}\omega
\end{aligned}$$

k is the wave number, sometimes called as spatial modulation frequency β when the environment is changed, so is ω.
Next, phase velocity, refractive index and wave impedance are introduced as

$$\begin{aligned}
v &= \frac{\omega}{k} = \frac{1}{\epsilon\mu} = \frac{c}{n} \\
n &= \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} = \sqrt{\mu_r}\epsilon_r \\
Z &= \frac{E_0}{H_0} = \sqrt{\frac{\mu}{\epsilon}} = \frac{\mu_r}{\epsilon_r} Z_0 = \zeta Z_0
\end{aligned}$$

ii. Negative Index, how is that happening?
When taking square root, we are not taking negative because conventionally the materials are right-handed/positive($\epsilon > 0, \mu > 0$).
However, reconstruct complex $epsilon_r$ and μ_r and find $\epsilon_r = e^{i\Phi_\epsilon}, \mu_r = e^{i\Phi_\mu}$ where the phases Φ_ϵ and Φ_μ are between pi/2 and pi, finding n to be negative.

iii. Propagation

Reduce the equations to

$$\begin{aligned} k \times E &= \omega\mu H \\ k \times H &= -\omega\epsilon E \\ S &= E \times H \end{aligned}$$

where S is the poynting vector telling the energy flux, in anti-parallel with the wave vector k in left-handed materials.

iv. Single-negative ingredient propagation
It is decaying wave since k's imaginary part is decaying.

(c) Space-Time Modulation

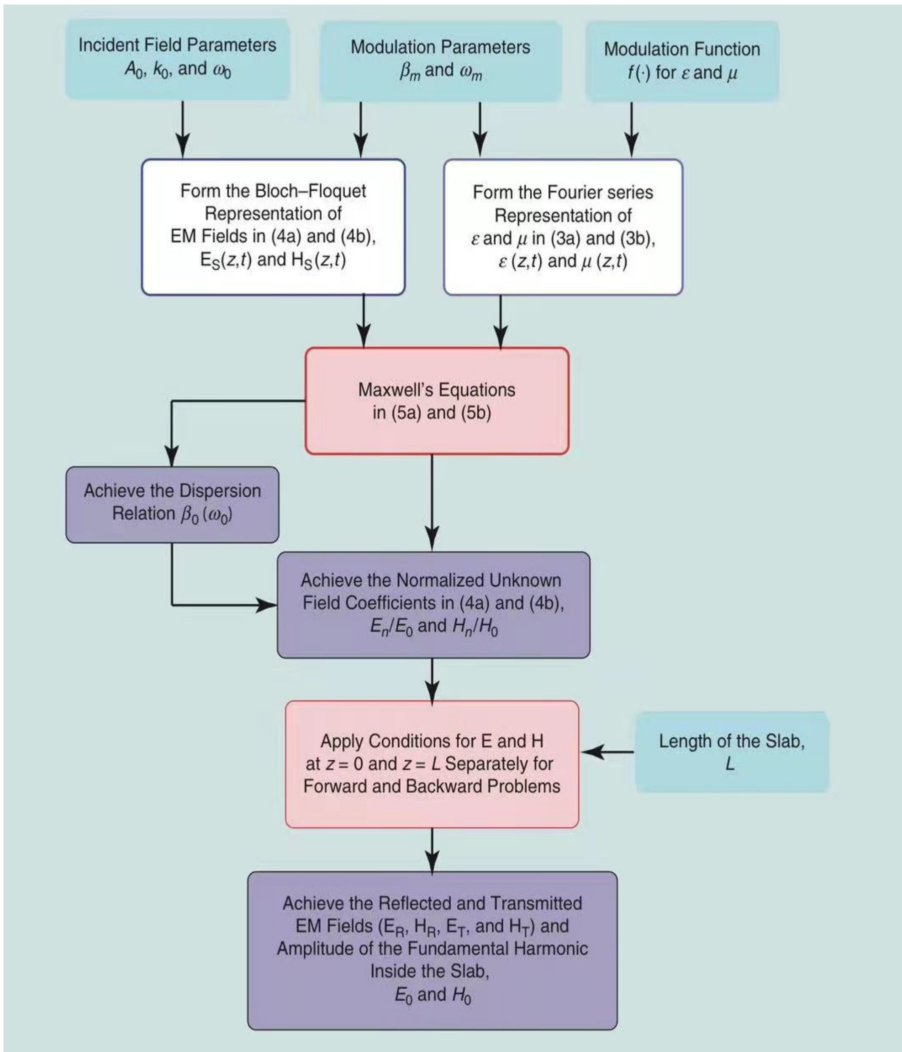


Figure 4: Find the scattered solution for STM slab

Equations mentioned:

$$\beta_m = \frac{\omega_m}{\nu_m} = \frac{\omega_m}{\gamma\nu_b}$$

$$\epsilon(z,t) = \sum_{k=-\infty}^{\infty} \tilde{\epsilon_k} e^{-jk(\beta_m z - \omega_m t)} \quad (3a)$$

$$\mu(z,t) = \sum_{k=-\infty}^{\infty} \tilde{\mu_k} e^{-jk(\beta_m z - \omega_m t)} \quad (3a)$$

$$E_s(z,t) = \hat{x} e^{-i(\beta_0 z - \omega_0 t)} \sum_{n=-\infty}^{\infty} E_n e^{in(\beta_m z - \omega_m t)} \quad (4a)$$

$$H_s(z,t) = \hat{y} e^{-i(\beta_0 z - \omega_0 t)} \sum_{n=-\infty}^{\infty} H_n e^{in(\beta_m z - \omega_m t)} (4b)$$

$$\frac{\partial E_x(z,t)}{\partial z} = -\frac{\partial[\mu(z,t)H_y(z,t)]}{\partial t} \quad (5a)$$

$$\frac{\partial H_y(z,t)}{\partial z} = -\frac{\partial[\epsilon(z,t)E_x(z,t)]}{\partial t} \quad (5b)$$

Papers & Concepts

1. 2003 Kuester "generalized sheet transition conditions (GSTCs)':

    (a) How to derive conditions: replace discrete distribution of scatters with continuous one. Similar to Clausius–Mossotti–Lorenz–Lorentz procedure for determining the dielectric constant of a volume distribution of small scatterers.

    (b) GSTCs: an equivalent transition (boundary) condition for the specular interaction of electromagnetic waves with a surface of electrically small scatterers.

    (c) Scatters: characterized completely by their electric and magnetic polarizabilities and their density of distribution in the surface.

    (d) metafilm $\approx$ 2-D metamaterial: have certain desired reflection and transmission properties (e.g., total reflection or total transmission).

    (e) Importance: Generalized functional dependence of the polarizability densities to the reflection and transmission properties of the surface, realizing controllable surfaces.

    (f) Applicable to: metafilms located in a homogeneous(unifromly same) medium.

    (g) Conclude: Concept generation, the output condition is not easy for calculation. Also, it is just mentioned as background, not as actual calculation steps. So, now should focus on generalized snell's law and other practical formulas that have been widely adopted.

Structure and theroretical background

Simplest 1-bit case

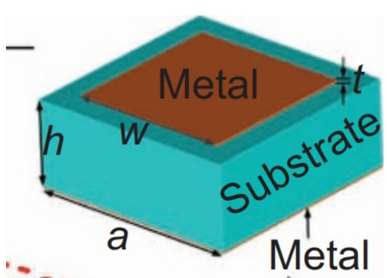


Figure 5: The Structure

And we have the following function of scattering field in a spherical coordinate system.

$$f(\theta,\varphi) = f_e(\theta,\varphi)$$

$$\sum_{m=1}^{N}\sum_{n=1}^{N} \exp\{-i\{\varphi(m,n) + kD\sin\theta[(m-1/2)\cos\varphi + (n-1/2)\sin\varphi]\}\} \quad (1)$$

More information

1. Digital control

   0-0 width of the metal: 3.75mm

   1-π width of the metal: 4.8mm

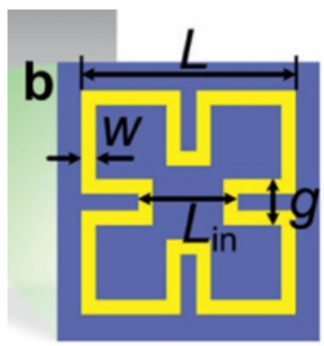
2. Frequency

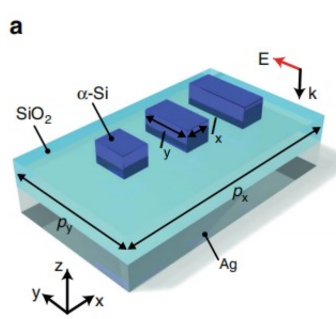
   8.1 - 12.7 GHz

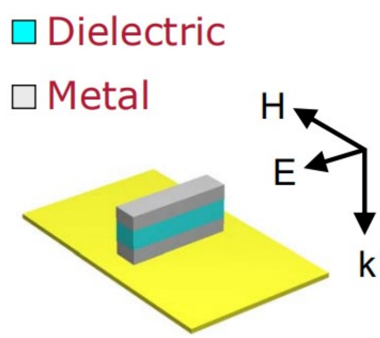
3. Control of the amplitude and phase

   Can't achieve separate control

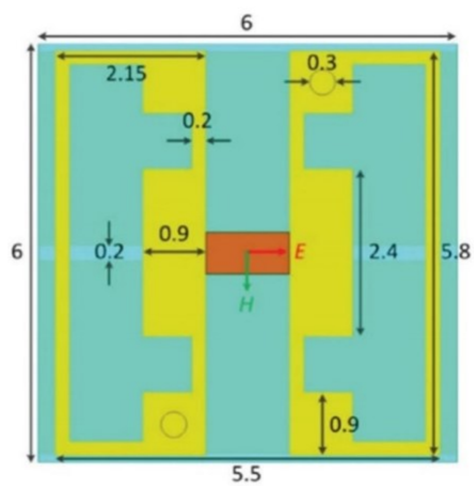
Other Passive Structures







From passive to active



Simulation

The simulation result from the paper:

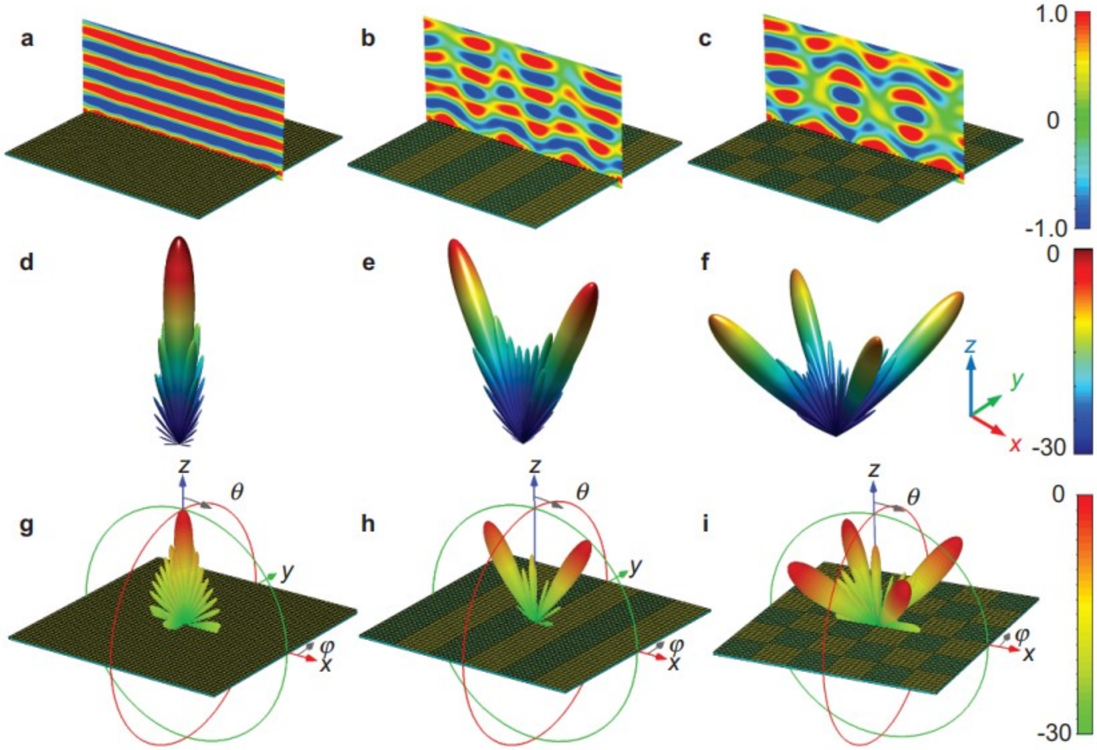


Figure 6: Result

Our simlation results using Matlab.

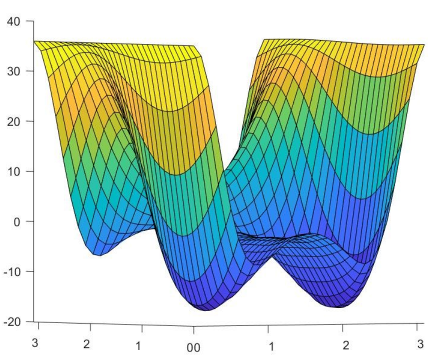


Figure 7: 000000/000000

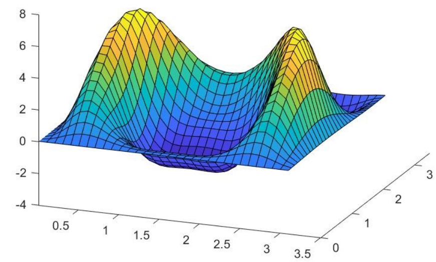


Figure 8: 010101/010101

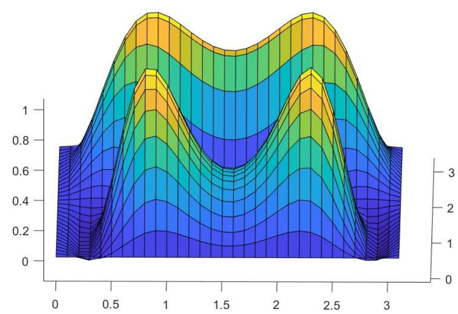


Figure 9: 010101/101010

Radar Cross Section

$$\text{RCS reduction} = \frac{\lambda^2}{4\pi N^2 D^2} \underset{\theta,\varphi}{\mathrm{Max}}[\mathrm{Dir}(\theta,\varphi)]$$

$$\mathrm{Dir}(\theta,\varphi) = 4\pi|f(\theta,\varphi)|^2 \Big/ \int_0^{2\pi}\int_0^{\pi/2} |f(\theta,\varphi)|^2 \sin\theta \mathrm{d}\theta \mathrm{d}\varphi$$

One of the aim is minimize the RCS.

C-shape

From "Multi-Beam Forming and Controls by Metasurface With Phase and Amplitude Modulations (002)"

Based on the above conclusion, the regulation mechanism of amplitude and phase are further improved.

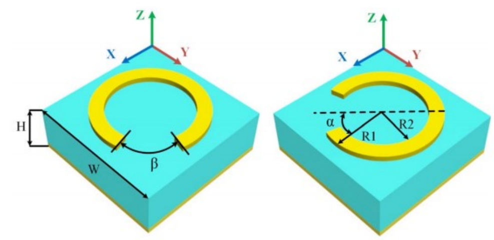


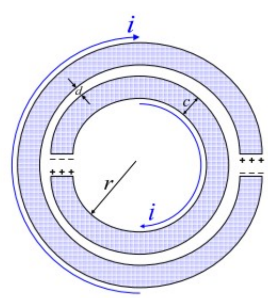
Figure 10: C-shape from 002

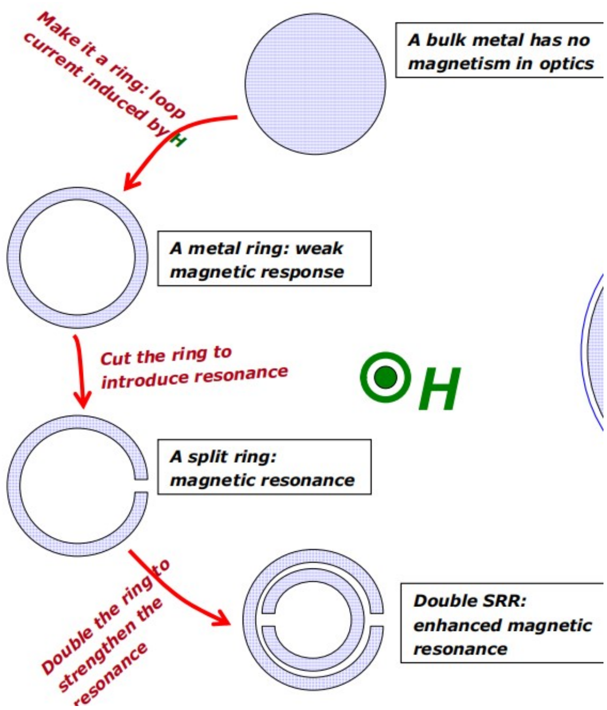
β for phase, α for amplitude.

When amplitude can be modified, the introduced new coefficient is calculated by

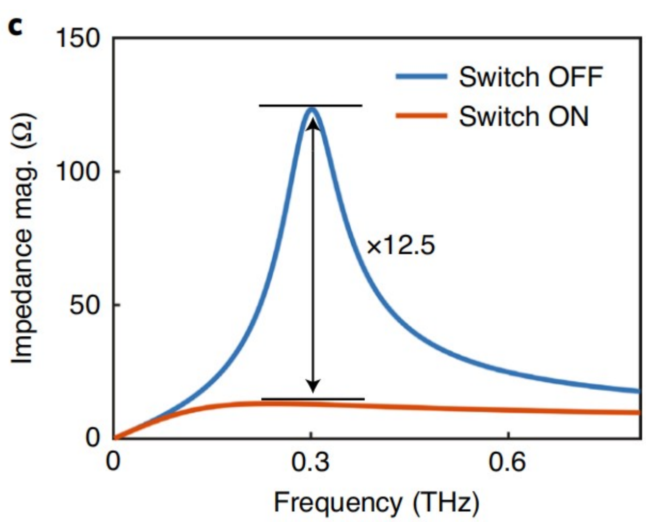
$$C_{r1} = \begin{bmatrix} A_1(1,1)e^{-j\varphi_1(1,1)} & \cdots & A_1(1,N)e^{-j\varphi_1(1,N)} \\ \vdots & \ddots & \vdots \\ A_1(M,1)e^{-j\varphi_1(M,1)} & \cdots & A_1(M,N)e^{-j\varphi_1(M,N)} \end{bmatrix}$$

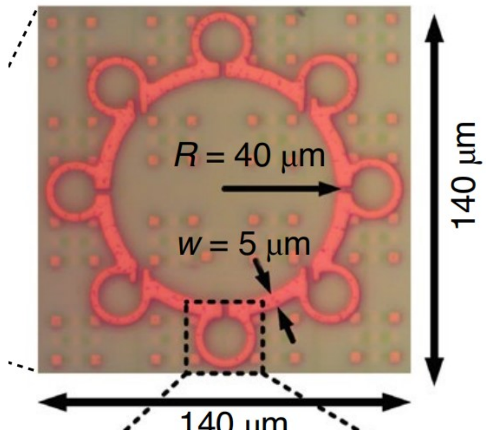
More on C-shape



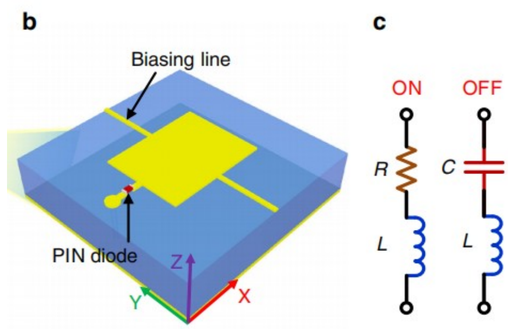


Resonance can improve the effect of switch on/off.

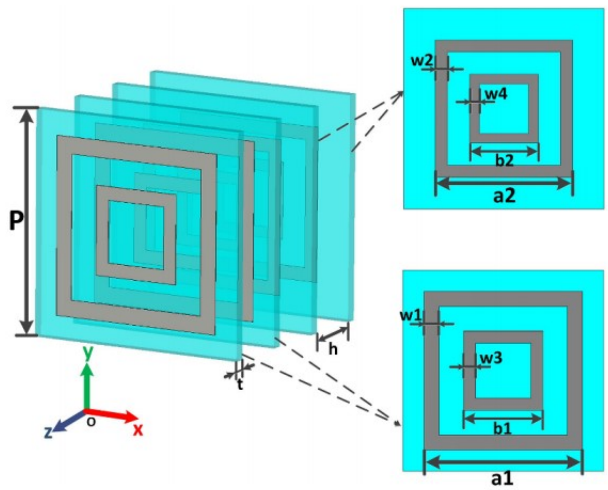




Space-time-coding digital metasurfaces



Multi-layer



Information

10GHz

00	0,0.1	0
01	0.3,0.4	-90
10	0.6,0.7	-180
11	0.85,0.95	-270