問題 1 完整解（u[i,j]對應 x=0.1π*i, y=0.1π*j）：

```
[[ 1.      0.9511  0.809   0.5878  0.309   0.    ]
 [ 0.9511  0.7532  0.5646  0.3681  0.1728  0.    ]
 [ 0.809   0.5559  0.3476  0.1763  0.0531  0.    ]
 [ 0.5878  0.3332  0.1326 -0.005  -0.0589  0.    ]
 [ 0.309   0.0858 -0.0869 -0.1823 -0.1667  0.    ]
 [ 0.      -0.1732 -0.3056 -0.3539 -0.2699  0.    ]
 [-0.309   -0.4243 -0.5112 -0.5116 -0.3642  0.    ]
 [-0.5878 -0.6452 -0.6862 -0.642  -0.4413  0.    ]
 [-0.809   -0.8145 -0.8099 -0.7244 -0.4863  0.    ]
 [-0.9511 -0.9158 -0.8589 -0.7255 -0.4679  0.    ]
 [-1.      -0.9511 -0.809   -0.5878 -0.309   0.    ]]
```

問題 2：

前向差分法完整解

時間 t	r=0.5	r=0.6	r=0.7	r=0.8	r=0.9	r=1.0
0.0	0.0000	20.0000	40.0000	60.0000	80.0000	100.0000
0.5	12.5000	32.8125	53.1250	73.4375	93.7500	120.0000
1.0	23.4375	43.7500	64.0625	84.3750	104.6875	140.0000
1.5	33.2031	53.5156	73.8281	94.1406	114.4531	160.0000
2.0	41.9922	62.3047	82.6172	102.9297	123.2422	180.0000
2.5	49.9512	70.2637	90.5762	110.8887	131.2012	200.0000
3.0	57.2026	77.5151	97.8276	118.1401	138.4526	220.0000
3.5	63.8462	84.1587	104.4712	124.7837	145.0962	240.0000
4.0	69.9648	90.2773	110.5898	130.9023	151.2148	260.0000
4.5	75.6289	95.9414	116.2539	136.5664	156.8789	280.0000
5.0	80.8984	101.2109	121.5234	141.8359	162.1484	300.0000
5.5	85.8242	106.1367	126.4492	146.7617	167.0742	320.0000
6.0	90.4492	110.7617	131.0742	151.3867	171.6992	340.0000
6.5	94.8096	115.1221	135.4346	155.7471	176.0596	360.0000
7.0	98.9355	119.2480	139.5605	159.8730	180.1855	380.0000
7.5	102.8525	123.1650	143.4775	163.7900	184.1025	400.0000
8.0	106.5818	126.8943	147.2068	167.5193	187.8318	420.0000
8.5	110.1416	130.4541	150.7666	171.0791	191.3916	440.0000
9.0	113.5469	133.8594	154.1719	174.4844	194.7969	460.0000
9.5	116.8105	137.1230	157.4355	177.7480	198.0605	480.0000
10.0	119.9434	140.2559	160.5684	180.8809	201.1934	500.0000

後向差分法完整解

時間 t	r=0.5	r=0.6	r=0.7	r=0.8	r=0.9	r=1.0
0.0	0.0000	20.0000	40.0000	60.0000	80.0000	100.0000
0.5	13.2143	33.3929	53.5714	73.7500	93.9286	120.0000
1.0	25.0000	45.1786	65.3571	85.5357	105.7143	140.0000
1.5	35.7143	55.8929	76.0714	96.2500	116.4286	160.0000
2.0	45.5357	65.7143	85.8929	106.0714	126.2500	180.0000
2.5	54.6071	74.7857	94.9643	115.1429	135.3214	200.0000
3.0	63.0357	83.2143	103.3929	124.5714	144.7500	220.0000
3.5	70.9107	91.0893	111.2679	133.4464	153.6250	240.0000
4.0	78.3036	98.4821	118.6607	141.8393	162.0179	260.0000
4.5	85.2732	105.4518	125.6304	149.8089	169.9875	280.0000
5.0	91.8696	112.0482	132.2268	157.4054	177.5839	300.0000
5.5	98.1354	118.3140	138.4926	164.6712	184.8498	320.0000
6.0	104.1068	124.2854	144.4640	171.6426	191.8212	340.0000
6.5	109.8152	129.9938	150.1724	178.3510	198.5296	360.0000
7.0	115.2876	135.4662	155.6448	184.8234	205.0020	380.0000
7.5	120.5474	140.7260	160.9046	191.0832	211.2618	400.0000
8.0	125.6150	145.7936	165.9722	197.1508	217.3294	420.0000
8.5	130.5085	150.6871	170.8657	203.0443	223.2229	440.0000
9.0	135.2433	155.4219	175.6005	208.7791	228.9577	460.0000
9.5	139.8328	160.0114	180.1900	214.3686	234.5472	480.0000
10.0	144.2885	164.4671	184.6457	219.8243	240.0029	500.0000

Crank-Nicolson 法完整解

時間 t	r=0.5	r=0.6	r=0.7	r=0.8	r=0.9	r=1.0
0.0	0.0000	20.0000	40.0000	60.0000	80.0000	100.0000
0.5	12.8571	33.0357	53.2143	73.3929	93.5714	120.0000
1.0	24.6429	44.8214	65.0000	85.1786	105.3571	140.0000
1.5	35.5357	55.7143	75.8929	96.0714	116.2500	160.0000
2.0	45.6786	65.8571	86.0357	106.2143	126.3929	180.0000
2.5	55.1786	75.3571	95.5357	115.7143	135.8929	200.0000
3.0	64.1190	84.2976	104.4762	124.6548	144.8333	220.0000
3.5	72.5685	92.7470	112.9256	133.1042	153.2827	240.0000
4.0	80.5845	100.7630	120.9415	141.1202	161.2986	260.0000
4.5	88.2159	108.3944	128.5729	148.7516	168.9300	280.0000
5.0	95.5046	115.6831	135.8616	156.0403	176.2187	300.0000
5.5	102.4868	122.6653	142.8438	163.0225	183.2009	320.0000
6.0	109.1940	129.3725	149.5510	169.7297	189.9081	340.0000
6.5	115.6536	135.8321	156.0106	176.1893	196.3677	360.0000
7.0	121.8893	142.0678	162.2463	182.4250	202.6034	380.0000
7.5	127.9222	148.1007	168.2792	188.4579	208.6363	400.0000
8.0	133.7708	153.9493	174.1278	194.3065	214.4849	420.0000
8.5	139.4516	159.6301	179.8086	199.9873	220.1657	440.0000
9.0	144.9793	165.1578	185.3363	205.5150	225.6934	460.0000
9.5	150.3666	170.5451	190.7236	210.9023	231.0807	480.0000
10.0	155.6249	175.8034	195.9819	216.1606	236.3390	500.0000

問題 3 完整解（T[i,j]對應 r=0.5+0.1*i, θ =j* π /30）：

```
[[  0.          50.          50.          50.          50.
   50.          50.          50.          50.          50.
    0.        ]
 [  0.          22.8586075   36.44333036  44.48079529  48.74299536
   50.07816604  48.74299592  44.48079624  36.44333141  22.85860825
    0.        ]
 [  0.          21.57683786  37.94279839  48.63795459  54.52330344
   56.38804926  54.52330415  48.63795581  37.94279973  21.57683881
    0.        ]
 [  0.          29.5169253   49.09215545  60.59514888  66.51643034
   68.33449835  66.51643091  60.59514985  49.09215652  29.51692607
    0.        ]
 [  0.          49.85485465  69.75482052  78.6307789   82.60107591
   83.75159852  82.6010762   78.63077939  69.75482106  49.85485503
    0.        ]
 [  0.         100.         100.         100.         100.
  100.         100.         100.         100.         100.
    0.        ]]
```

問題 4 完整解（p[j,i]對應 t=0.1*j, x=0.1*i）：

```
[[ 1.00000000e+00    8.09016994e-01    3.09016994e-01 -3.09016994e-01
   -8.09016994e-01 -1.00000000e+00 -8.09016994e-01 -3.09016994e-01
    3.09016994e-01    8.09016994e-01    1.00000000e+00]
 [ 1.00000000e+00    1.17833336e+00    9.06583427e-01    2.88549439e-01
   -4.39700628e-01 -1.00000000e+00 -1.17833336e+00 -9.06583427e-01
   -2.88549439e-01    4.39700628e-01    1.00000000e+00]
 [ 1.00000000e+00    1.09756643e+00    1.15786580e+00    7.75899793e-01
    9.75664329e-02 -6.18033989e-01 -1.09756643e+00 -1.15786580e+00
   -7.75899793e-01 -9.75664329e-02    2.00000000e+00]
 [ 1.00000000e+00    9.79532444e-01    9.66882799e-01    9.66882799e-01
    5.97566433e-01 -3.33066907e-16 -5.97566433e-01 -9.66882799e-01
   -9.66882799e-01    7.84399578e-01    2.00000000e+00]
 [ 1.00000000e+00    8.69316366e-01    7.88549439e-01    7.88549439e-01
    8.69316366e-01    6.18033989e-01    1.30683634e-01 -4.06583427e-01
    5.93416573e-01    1.13068363e+00    2.00000000e+00]
 [ 1.00000000e+00    8.09016994e-01    6.90983006e-01    6.90983006e-01
    8.09016994e-01    1.00000000e+00    8.09016994e-01    1.69098301e+00
    1.69098301e+00    1.80901699e+00    2.00000000e+00]
 [ 1.00000000e+00    8.21666640e-01    7.11450561e-01    7.11450561e-01
    8.21666640e-01    1.00000000e+00    2.56029937e+00    2.90658343e+00
    2.90658343e+00    2.56029937e+00    2.00000000e+00]
 [ 1.00000000e+00    9.02433567e-01    8.42134195e-01    8.42134195e-01
    9.02433567e-01    2.38196601e+00    3.09756643e+00    3.77589979e+00
```

3.77589979e+00 3.09756643e+00 2.00000000e+00]

[1.00000000e+00 1.02046756e+00 1.03311720e+00 1.03311720e+00

2.40243357e+00 3.00000000e+00 3.59756643e+00 3.96688280e+00

3.96688280e+00 3.21560042e+00 2.00000000e+00]

[1.00000000e+00 1.13068363e+00 1.21145056e+00 2.59341657e+00

3.13068363e+00 3.61803399e+00 3.86931637e+00 3.78854944e+00

3.40658343e+00 2.86931637e+00 2.00000000e+00]

[1.00000000e+00 1.19098301e+00 2.69098301e+00 3.30901699e+00

3.80901699e+00 4.00000000e+00 3.80901699e+00 3.30901699e+00

2.69098301e+00 2.19098301e+00 2.00000000e+00]

[1.00000000e+00 2.56029937e+00 3.28854944e+00 3.90658343e+00

4.17833336e+00 4.00000000e+00 3.43970063e+00 2.71145056e+00

2.09341657e+00 1.82166664e+00 2.00000000e+00]

[1.00000000e+00 3.09756643e+00 3.77589979e+00 4.15786580e+00

4.09756643e+00 3.61803399e+00 2.90243357e+00 2.22410021e+00

1.84213420e+00 1.90243357e+00 2.00000000e+00]

[1.00000000e+00 2.21560042e+00 3.96688280e+00 3.96688280e+00

3.59756643e+00 3.00000000e+00 2.40243357e+00 2.03311720e+00

2.03311720e+00 2.02046756e+00 2.00000000e+00]

[1.00000000e+00 1.86931637e+00 2.40658343e+00 3.40658343e+00

2.86931637e+00 2.38196601e+00 2.13068363e+00 2.21145056e+00

2.21145056e+00 2.13068363e+00 2.00000000e+00]

[1.00000000e+00 1.19098301e+00 1.30901699e+00 1.30901699e+00

2.19098301e+00 2.00000000e+00 2.19098301e+00 2.30901699e+00

2.30901699e+00 2.19098301e+00 2.00000000e+00]

[1.00000000e+00 4.39700628e-01 9.34165727e-02 9.34165727e-02

4.39700628e-01 2.00000000e+00 2.17833336e+00 2.28854944e+00

2.28854944e+00 2.17833336e+00 2.00000000e+00]

[1.00000000e+00 -9.75664329e-02 -7.75899793e-01 -7.75899793e-01

-9.75664329e-02 6.18033989e-01 2.09756643e+00 2.15786580e+00

2.15786580e+00 2.09756643e+00 2.00000000e+00]

[1.00000000e+00 -2.15600422e-01 -9.66882799e-01 -9.66882799e-01

-5.97566433e-01 8.88178420e-16 5.97566433e-01 1.96688280e+00

1.96688280e+00 1.97953244e+00 2.00000000e+00]

[1.00000000e+00 1.30683634e-01 -4.06583427e-01 -7.88549439e-01

-8.69316366e-01 -6.18033989e-01 -1.30683634e-01 4.06583427e-01

1.78854944e+00 1.86931637e+00 2.00000000e+00]

[1.00000000e+00 8.09016994e-01 3.09016994e-01 -3.09016994e-01

-8.09016994e-01 -1.00000000e+00 -8.09016994e-01 -3.09016994e-01

3.09016994e-01 1.80901699e+00 2.00000000e+00]]