Bayesian Modeling of Complex-valued fMRI Signals

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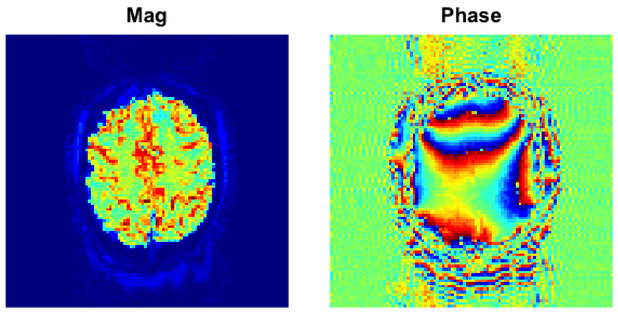
Motivation and Goal

- fMRI data are complex-valued. However, most fMRI studies use magnitude-only data and discard phase information.
- We propose a **Bayesian variable seletion method, both EM and MCMC algorithms, that takes real and imaginary parts of fMRI signals into account to detect which voxels are activated while a subject performs a task.**



Complex Normal and Complex-valued Linear Regression: Rowe (2005)

- A n-dimensional r.v. $\boldsymbol{Z} = \boldsymbol{X} + i\boldsymbol{Y}$ is complex normal $CN_n(\mu_Z, \Gamma, \boldsymbol{C})$ iff $\boldsymbol{Z}$ has real representation:

$$\begin{pmatrix} \boldsymbol{X} \\ \boldsymbol{Y} \end{pmatrix} \sim N_{2n}\left(\begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}, \Sigma = \begin{pmatrix} \Sigma_X & \Sigma_{XY} \\ \Sigma_{YX} & \Sigma_Y \end{pmatrix} \right),$$

  where $\Gamma := \mathrm{E}(\boldsymbol{Z}\boldsymbol{Z}^H)$, $\boldsymbol{C} := \mathrm{E}(\boldsymbol{Z}\boldsymbol{Z}')$ if $\mu_Z = \boldsymbol{0}$.
- Given time $t = 1, \ldots, T$ and at voxel $v = 1, \ldots, V$, we have

$$y_t^v = \rho_t^v \cos(\theta^v) + i\rho_t^v \sin(\theta^v) + \eta_t^v,$$
$$\rho_t^v = \beta_0^v + \beta_1^v x_{1,t} + \beta_2^v x_{2,t} + \cdots + \beta_p^v x_{p,t},$$

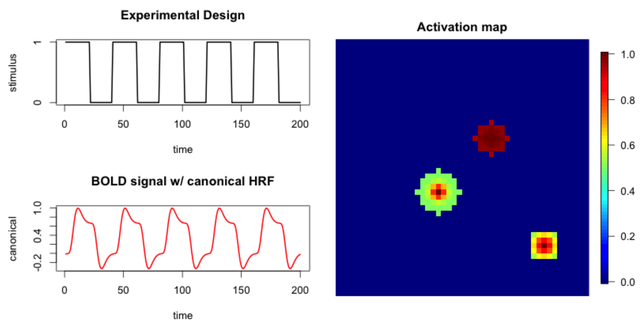
- $y_t^v = y_{t,re}^v + iy_{t,im}^v$ and $\eta_t^v = \eta_{t,re}^v + i\eta_{t,im}^v$.
- $\eta_t^v \sim CN_1(0, \Gamma_v, C_v)$, and usually circular, i.e., $C_v = 0$.
- ρ_t^v is the magnitude of y_t^v, and θ^v is the time-invariant phase of y_t^v
- $\mathbf{y}^v = \boldsymbol{X}(\beta^v \cos(\theta^v)) + i\boldsymbol{X}(\beta^v \sin(\theta^v)) + \boldsymbol{\eta}^v$, and hence $\mathbf{y}^v = \boldsymbol{X}\gamma^v + \boldsymbol{\eta}^v$,

Simulation Example I

- Simulated 48×48 fMRI data with a linear trend and one task-related covariate, $t = 1 : T$, $T = 200$.
- The task-related covariate is the blood oxygenation level dependent (BOLD) signal contrast, which is the convolution of the stimulus indicator function with a so called hemodynamic response function (HRF).
- The model is

$$y_{t,Re}^v = (\beta_0 + \beta_1(t/T) + \beta_2 f_v BOLD_t)\cos(\phi) + \eta_{t,Re}^v, \quad \eta_{t,Re}^v \sim N(0, \sigma^2)$$
$$y_{t,Im}^v = (\beta_0 + \beta_1(t/T) + \beta_2 f_v BOLD_t)\sin(\phi) + \eta_{t,Im}^v, \quad \eta_{t,Im}^v \sim N(0, \sigma^2)$$

- $\beta_1 = 0.00001$, $\sigma^2 = 0.25$, $\phi = \pi/6$
- Examine the performance of our method through different signal-noise ratio $SNR = \beta_0/\sigma$ and contrast-to-noise ratio $CNR = \beta_2/\sigma$.

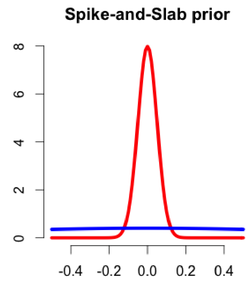


Model and Algorithm: C-EMVS and C-SSVS

- The basic model is

$$\boldsymbol{y}^v = \boldsymbol{X}\gamma^v + \boldsymbol{\eta}^v, \boldsymbol{\eta}^v \sim CN_T(\boldsymbol{0}, 2\sigma_v^2\boldsymbol{I}, \boldsymbol{0}),$$
$$\gamma_j^v|\psi_j^v \sim (1-\psi_j^v)CN_1(0, 2v_0\sigma_v^2, 0) + \psi_j^v CN_1(0, 2v_1\sigma_v^2, 0),$$
$$p(\sigma_v^2) \propto 1/\sigma_v^2, \quad \psi_j^v|\theta \sim Bernoulli(\theta), \quad \theta \sim Beta(a_\theta, b_\theta),$$

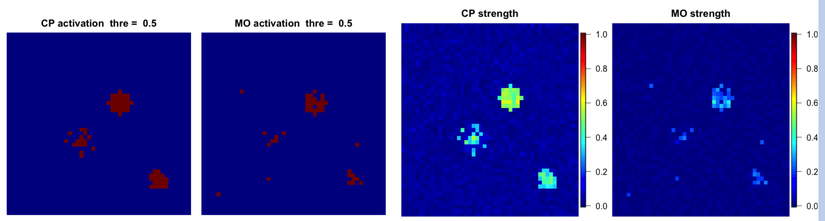
- dodge controling errors (e.g. Bonferroni correction)
- voxels are related through $Pr(\psi_j^v = 1|\theta) = \theta$ for all voxels $v = 1, \ldots, V$
- We generalize the SSVS and EMVS algorithm proposed by George and McCulloch (1993) and Rockova and George (2014) to the **complex-valued** domain.
- An voxel is active if $Pr(\psi_2^v = 1|y, \gamma, \sigma) > 0.5$



Simulation Results: Example I

		CNR = (.38, .53, .75)			CNR=(.625, .88, 1.25)		
Measure	Model	SNR = .5	SNR = 1	SNR = 2	SNR = .5	SNR = 1	SNR = 2
sensitivity	CP	0.321	0.335	0.340	0.829	0.842	0.852
	LA	0	0	0.083	0.266	0.664	0.824
	ALA	0	0.087	0.402	0.279	0.735	0.818
specificity	CP	1	1	1	1	0.999	0.999
	LA	1	1	1	1	0.999	0.999
	ALA	1	0.999	0.998	1	0.998	0.998
precision	CP	0.973	0.994	0.997	0.994	0.985	0.983
	LA	0	0	0.300	0.690	0.966	0.985
	ALA	0	0.274	0.937	0.900	0.955	0.959
accuracy	CP	0.969	0.970	0.970	0.992	0.992	0.993
	LA	0.955	0.955	0.959	0.966	0.984	0.992
	ALA	0.955	0.958	0.971	0.968	0.986	0.990

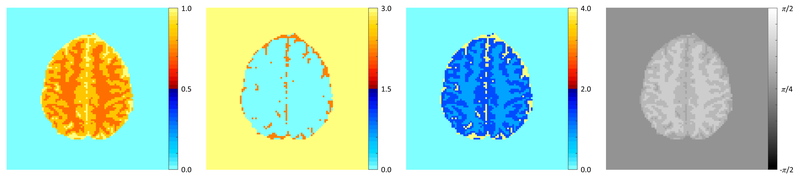
Table : CP: complex-valued EM, LA: Lasso, and ALA: Adaptive Lasso.



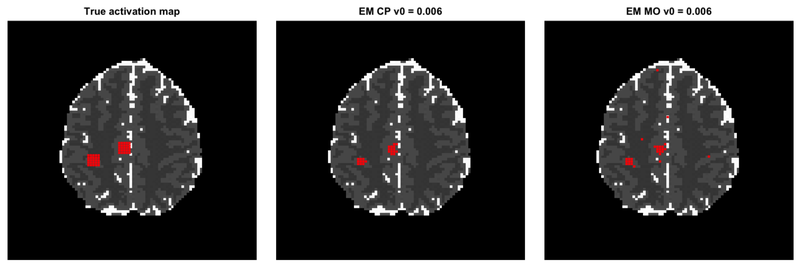
(a) Activation maps (b) Strength maps

Simulation Example II

- A simulated data with dimension $96 \times 96 \times 490$ was generated using **MR signal equation**.

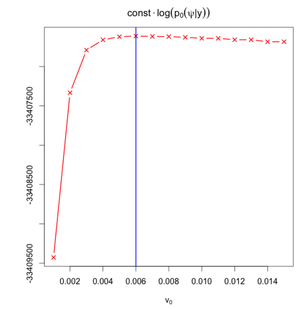


(c) PSD, $\rho(x, y)$ (d) TRR, $T_2^*(x, y)$ (e) LRR, $T_1(x, y)$ (f) Phase baseline



How to choose the tuning parameter v_0 and v_1?

- Wang et al (2015) suggests $v_1 = 1$.
- Rockova and George (2014) suggests to choose v_0^* with the highest marginal posterior probability of $p_0(\psi|\boldsymbol{y})$ that evaluates according to the submodel that contains only those variables for which $\psi_j^v = 1$.
- For fMRI, $1/\sqrt{100Tp} < v_0^* < 1/\sqrt{10Tp}$ usually.
- Suggestion: create a grid for v_0 between $1/\sqrt{100Tp}$ and $1/\sqrt{10Tp}$



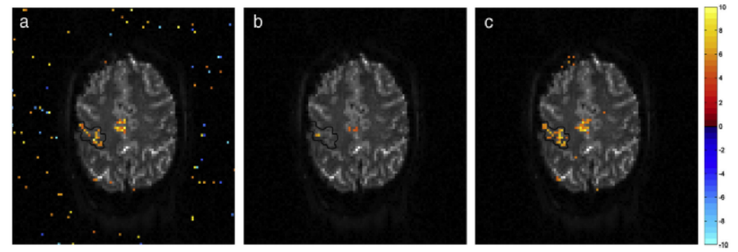
Data Analysis

A human subject complex data set from Karaman, Bruce and Rowe (2014) is considered.

- A bilateral finger-tapping task was performed.
- A block design with an initial 30 s rest followed by 16 epochs of 15 s on and 15 s off.
- Data dimension is $96 \times 96 \times 510$.

Results from Karaman, Bruce and Rowe (2014)

- a) Complex-Valued; b) Magnitude-Only; c) Nonlinear experiment-related.
- Likelihood ratio test statistics $-2\log\lambda$ and activation is thresholded at a 5% Bonferroni family-wise error rate.
- **a): many false positives outside the brain**; **b): few activations**; **c): a few false positives in the left-upper side of the brain.**



C-EMVS with AR(1) noise

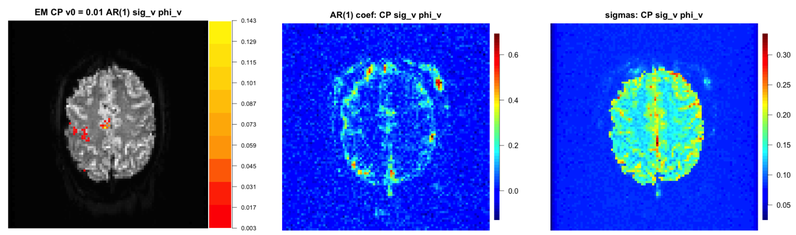


Figure : Human data: Activation map (left); Estimated values of ϕ_v^2 for a models with voxel-specific σ_v^2 (center plot); estimated values of the σ_v^2s

Conclusions

- The C-EMVS approach is able to detect **more true positives** and **less false positives** than magnitude-only models, especially when the SNR is small.
- The C-EMVS algorithms are computationally fast and have run times comparable to those needed by Lasso or Adaptive Lasso.
- The models considered here do not use any sophisticated spatio-temporal or nonlinear structure that more appropriately model fMRI data (we only considered an AR(1) temporal structure), but would also lead to more computationally-intensive models that may be not be feasible for detecting brain activation at the voxel-specific level.
- MCMC approaches for full posterior inference are also provided (results not shown).
- This algorithm is quite general, and can be applied to any complex-valued data, and for both circular or noncircular normal case.

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