Part 2:

1. a) First, we compute the closure of each FD:

$L^+ = LNQSOPRM$

$MNR^+ = MNRO$

$O^+ = OM$

$NQ^+ = NQLSOPRM$

$S^+ = SOPRM$

It shows that $MNR \rightarrow O$, $O \rightarrow M$ and $S \rightarrow OPR$ has violated BCNF because these three FDs are not superkeys.

b) We first divide relation A into two parts using the $S \rightarrow OPR$:

$MNR \rightarrow O$

$A_1 = SLNQ$

$A_2 = SOPRM$

Now we project D onto A_1 :

S	**L**	**N**	**Q**	**Closure**	**FDs**
✓				$S^+ = SOPRM$	------------
	✓			$L^+ =LMNOPQRS$	$L \rightarrow NQS$
		✓		$N^+ = N$	------------
			✓	$Q^+ = Q$	------------
✓	✓			Don't need to check because it is a superset of L.	------------
✓		✓		$SN^+ = SNOPRM$	------------
✓			✓	$SQ^+ = SQOPRM$	------------
	✓	✓		It is a superset of L	------------
	✓		✓	It is a superset of L	------------
		✓	✓	$NQ^+ = NQLSMOPR$	$NQ \rightarrow LS$
✓	✓	✓		It is a superset of L	------------
✓	✓		✓	It is a superset of L	------------
✓		✓	✓	It is a superset of NQ	------------

	✓	✓	✓	It is a superset of L	------------
✓	✓	✓	✓	It is a set of all attributes	------------

The projected FD is {L → NQS, NQ → LS}, and all the FDs are superkeys. Therefore, A_1 is in BCNF form.

Now we project D onto A_2. Since attribute L, N, and Q are not in A_2, the only two FDs that will be considered are {O → M, S → OPR}. As we computed at the beginning of this questions, S → OPR is a superkey in A_2 while O → M is not, so we divide A_2 into two parts using O → M:

A_3 = SOPR

A_4 = OM

In this level, we can clearly see that A_3 only corresponds to one FD S → OPR, and A_4 only corresponds to O → M, where in both cases the FDs are superkeys.

Therefore, A_3 and A_4 are both in BCNF.

The final relations are: A_1, LNQS, A_3, MO, and A_4, OPRS.

Projected FDs onto each corresponding relations: {L → NQS, NQ → LS}, {O → M} and {S → OPR}.

2. (a)

Step 1: Split the RHSs to get the initial set of FDs, T1:

(a) AB → C

(b) C → A

(c) C → B

(d) C → D

(e) CFD → E

(f) E → B

(g) BF → E

(h) BF → C

(i) B → D

(j) $B \rightarrow A$

Step 2: For each FD, try to reduce the LHS:

(a) $A^+ = A$. $B^+ = BDAC$. So we can reduce the LHS, yielding the new FD: $B \rightarrow C$.

(b) Since the FD has only one attribute on the LHS, we cannot reduce the LHS of this FD.

(c) Since the FD has only one attribute on the LHS, we cannot reduce the LHS of this FD.

(d) Since the FD has only one attribute on the LHS, we cannot reduce the LHS of this FD.

(e) $C^+ = CABD$. So we can reduced this LHS, yielding the new FD: $CF \rightarrow E$.

(f) Since the FD has only one attribute on the LHS, we cannot reduce the LHS of this FD.

(g) $B^+ = BCDA$. $F^+ = F$. Since no singleton LHS yields anything, we cannot reduce the LHS of this FD.

(h) Since no singleton LHS yields anything, we cannot reduce the LHS of this FD.

(i) Since the FD has only one attribute on the LHS, we cannot reduce the LHS of this FD.

(j) Since the FD has only one attribute on the LHS, we cannot reduce the LHS of this FD.

Thus, the new set of FDs, T2, is:

(a) $B \rightarrow C$

(b) $C \rightarrow A$

(c) $C \rightarrow B$

(d) $C \rightarrow D$

(e) $CF \rightarrow E$

(f) $E \rightarrow B$

(g) $BF \rightarrow E$

(h) $BF \rightarrow C$

(i) $B \rightarrow D$

(j) $B \rightarrow A$

Step 3: Try to eliminate each FD.

(a) $B^+_{T2\text{-}(a)} = BDA$. We need this FD.

(b) $C^+_{T2\text{-}(b)}$ = CBD<u>A</u>. We can remove this FD.

(c) $C^+_{T2\text{-}(b)\text{-}(c)}$ = CD. We need this FD.

(d) $C^+_{T2\text{-}(b)\text{-}(d)}$ = CB<u>D</u>A. We can remove this FD.

(e) $CF^+_{T2\text{-}(b)\text{-}(d)\text{-}(e)}$ = CFB<u>E</u>DA. We can remove this FD.

(f) $E^+_{T2\text{-}(b)\text{-}(d)\text{-}(e)\text{-}(f)}$ = E. We need this FD.

(g) $BF^+_{T2\text{-}(b)\text{-}(d)\text{-}(e)\text{-}(g)}$ = BFCDA. We need this FD.

(h) $BF^+_{T2\text{-}(b)\text{-}(d)\text{-}(e)\text{-}(h)}$ = BFE<u>C</u>DA. We can remove this FD.

(i) $B^+_{T2\text{-}(b)\text{-}(d)\text{-}(e)\text{-}(h)\text{-}(i)}$ = BCA. We need this FD.

(j) $B^+_{T2\text{-}(b)\text{-}(d)\text{-}(e)\text{-}(h)\text{-}(j)}$ = BCD. We need this FD.

Thus, the final set of FDs is:

$B \rightarrow A$

$B \rightarrow C$

$B \rightarrow D$

$BF \rightarrow E$

$C \rightarrow B$

$E \rightarrow B$

(b)

First, we try whether the FDs contain any key.

B^+ = BACD. no EFGH.

BF^+ = BFEACD. no GH.

C^+ = CBAD. no EFGH.

E^+ = EBACD. no FGH.

For each attribute, we want to get whether it appears in the LHS, RHS, both or neither.

LHS	RHS	Attributes
×	×	GH
✓	×	F
×	✓	AD
✓	✓	BCE

Thus, F,G,H is in every key, and we need to check B,C,E.

B	C	E	closure	
✓			$FGHB^+ = FGHBACDE$	is a key
	✓		$FGHC^+ = FGHCBADE$	is a key
		✓	$FGHE^+ = FGHEBACD$	is a key
✓	✓		no need to check, since FGHB is a key	
✓		✓	no need to check, since FGHB is a key	
	✓	✓	no need to check, since FGHC is a key	
✓	✓	✓	no need to check, since FGHB is a key	

Thus, all keys for P are: BFGH, CFGH, and EFGH.

(c)

By (a), the minimal basis M for T is $\{B \rightarrow A, B \rightarrow C, B \rightarrow D, BF \rightarrow E, C \rightarrow B, E \rightarrow B\}$.

Merge the right-hand sides to get a smaller FDs. Let's call the revised FDS M1:

$\{B \rightarrow ACD, BF \rightarrow E, C \rightarrow B, E \rightarrow B\}$.

The set of relations that would result would have these attributes:

R1(A, B, C, D), R2(B, F, E), R3(B, C), R4(B, E)

Since the attributes BC occur in R1, we don't need to keep the relation R3. Similarly, since the attributes BE occur in R2, we don't need to keep the relation R4.

By (b), we know all keys for P are BFGH, CFGH, and EFGH. Since there is no relation that includes a key, we need to add a relation whose schema is some key.

$ABCD^+ = ABCD$. no EFGH

$BFE^+ = BFEACD$. no GH

Thus, we can add a relation R5(B, F, G, H).

So, the final set of relations is:

R1(A, B, C, D), R2(B, F, E), R5(B, F, G, H).

(d)

We need to project the FDs onto each relation to check whether it violates BCNF and therefore allow redundancy.

We can quickly find a relation that violates BCNF without doing all the full projections. $E \rightarrow B$ projects onto the relation R2. $E^+ = EB$, which is not a superkey of the relation R2.

Thus, these schema allows redundancy.