CS 146 - HW2

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// Problem #3 //
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(a)

Because of linear separability, there exists a hyperplane $\overline{v}^T\overline{x}+b$ such that

$[min(\overline{v}^T\overline{x}+b)\ when\ y\ =\ 1]\ \geq\ 0\ > [max(\overline{v}^T\overline{x}+b)\ when\ y\ =\ -1]$

Let $\overline{x_p}$ be the positive data point closest to the hyperplane and $\overline{x_n}$ be the negative data point closest to the hyperplane. Because of linear separability, $p^+ \geq 0 > p^-$. These two points are

$p^+ = [min(\overline{v}^T\overline{x}+b)\ when\ y\ =\ 1]\ = \overline{v}^T\overline{x_p}+b$

$p^- = [max(\overline{v}^T\overline{x}+b)\ when\ y\ =\ -1]\ = \overline{v}^T\overline{x_n}+b$

We can shift the hyperplane by a value c, so that the new hyperplane is equidistant from $\overline{x_p}$ and $\overline{x_n}$. We can find the value of c.

$\frac{|\overline{v}^T\overline{x}_p+b-c|}{||\overline{v}||} = \frac{|\overline{v}^T\overline{x_n}+b-c|}{||\overline{v}||}$

$\overline{v}^T\overline{x_p}+b-c =- (\overline{v}^T\overline{x_n}+b-c)$

$p^+ - c =- p^- + c$

$c = \frac{p^+ + p^-}{2}$

The new hyperplane $\overline{v}^T\overline{x}+b-c=0$ separates the data set D.

$[min(\overline{v}^T\overline{x}+b-c)\ when\ y\ =\ 1] = \frac{p^+ - p^-}{2}$

$[max(\overline{v}^T\overline{x}+b-c)\ when\ y\ =\ -1]\ = \frac{p^- - p^+}{2}$

Hence, $y(\overline{v}^T\overline{x}+b-c)\ \geq \frac{p^+ - p^-}{2}$

Since $p^+ > p^-$ and $c = \frac{p^+ + p^-}{2}$. We can set $\overline{w} = \frac{\overline{v}}{c}$ and $\theta = \frac{b-c}{c}$ and $\delta = 0$.

Now plugging in $\delta = 0$, we can see that

$\overline{w}^T\overline{x}+\theta \geq 1 \geq 0$ when y = 1, and

$\overline{w}^T\overline{x}+\theta \leq -1 < 0$ when y = -1

(b)

When $\delta > 0$, we can split this into a few cases:

When $1 > \delta > 0$, the data set is linearly separable because of the proof in part (a).

When $\delta \geq 1$, the data set can be separable or inseparable.

When min $\delta \geq 1$, the data set is not sesparable.

(c)

To satisfy the LP formulation given, the optimal solution would be when $\overline{w} = 0$, $\theta = 0$, and $\delta = 0$

However, this does not result in a hyperplane, so we cannot use this formulation.