Modified Pseudo-spectral Method for Wave Propagation Modelling in Arbitrary Anisotropic Media

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Summary

We propose a modified pseudo-spectral method (PSM) for numerical simulation of wave propagation in general anisotropic media. This approach uses rotated staggered grid configuration to implement spatial Fourier derivatives operators. It overcomes some shortages of the pseudo-spectral scheme based on the standard staggered grids, and thus can be efficiently used to simulate seismic wave propagation in anisotropic media with symmetries lower than orthorhombic. Synthetic examples demonstrate that this approach can obtain dispersion-free wave propagation modelling for those complex anisotropic media.

Introduction

PSM is a grid based modelling method similar to the FDM, with one key difference. Instead of using finite-difference operators, Fourier transforms are used to calculate the spatial derivatives. The main advantage is that it requires considerably fewer grid points per wavelength, to attain any desired accuracy. However, the PSM has its own difficulties, among which the Nyquist errors and the generation of non-causal ringing artifacts cause serious challenge. It is well established that modelling on staggered grids mitigates these effects. The standard staggered grid (SSG) schemes work well in isotropic media as well as in anisotropic media with relatively high symmetry. For anisotropy with symmetries of lower order than orthorhombic, or when the axis of anisotropy is not aligned with the grid, the straightforward application of the SSG scheme is difficult because of essentially different and complicated representations of Hooke's law. A solution to expand the PSM to arbitrary anisotropy case is interpolation of desired fields of propagation, which involves a lot of additional Fourier transforms. We present a modified PSM based upon the rotated configuration to tackle arbitrary anisotropy. This scheme has the same computational cost and similar formula with SSG PSM.

Methodology

For 2D general anisotropic media, the elastic wave equation can be written as:

$$
\begin{aligned}
\partial_t v_x &= \partial_x \tau_{xx} + \partial_z \tau_{xz} + f_x,\\
\rho\partial_t v_z &= \partial_x \tau_{xz} + \partial_z \tau_{zz} + f_z,\\
\partial_t \tau_{xx} &= C_{11}\partial_x v_x + C_{13}\partial_z v_z + C_{15}(\partial_x v_z + \partial_z v_x),\\
\partial_t \tau_{zz} &= C_{13}\partial_x v_x + C_{33}\partial_z v_z + C_{35}(\partial_x v_z + \partial_z v_x),\\
\partial_t \tau_{xz} &= C_{15}\partial_x v_x + C_{35}\partial_z v_z + C_{55}(\partial_x v_z + \partial_z v_x).
\end{aligned}
$$

The PSM is using Fourier transform operator to calculate the spatial derivatives,

$$D_x\phi = \sum_{k_x=0}^{k_x(N)} ik_x\tilde{\phi}(k_x)\exp(ik_x x).$$

But this standard spectral operator meets strong Nyquist error problems which lead oscillation in 1D spike derivative and non-causal ringing in 2D wavefields (Figure 1).

A good solution for suppressing the non-causal ringrings is simulating wave propagation on staggered grid (Ozdenvar and McMechan, 1996). However, the standard staggered grid (SSG) can't be applied to arbitrary anisotropic case directly (Igel et al, 1995). Following the idea of Igel et al.(1995), Bale (2002) using Fourier shift operator in wavenumber domain to stiffness matrix to assistance staggering. After shifting the stiffness matrix becomes as:

$$C = \begin{bmatrix} C_{11} & C_{13} & C_{15}S_x^- S_z^- \\ C_{13} & C_{33} & C_{35}S_x^- S_z^- \\ S_x^+ S_z^+ C_{15} & S_x^+ S_z^+ C_{35} & C_{55} \end{bmatrix},$$

where

$$S_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} \exp(\pm ik_x\Delta x/2)\tilde{\phi}(k_x)\exp(ik_x x).$$

Bale's solution involves a lot of additional Fourier transform, thus increases the computational cost. In this study, we introduce the RSG configuration (Figure 2) to PSM. Unlike the SSG-PSM, the half-grid shift should be along the diagonal direction. So we modify the spectral derivative operator as:

$$D_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} ik_x\exp(\pm i(k_x\Delta x/2 + k_z\Delta z/2))\tilde{\phi}(k_x)\exp(ik_x x).$$

Using this operator the elastic equation can be easily solved.

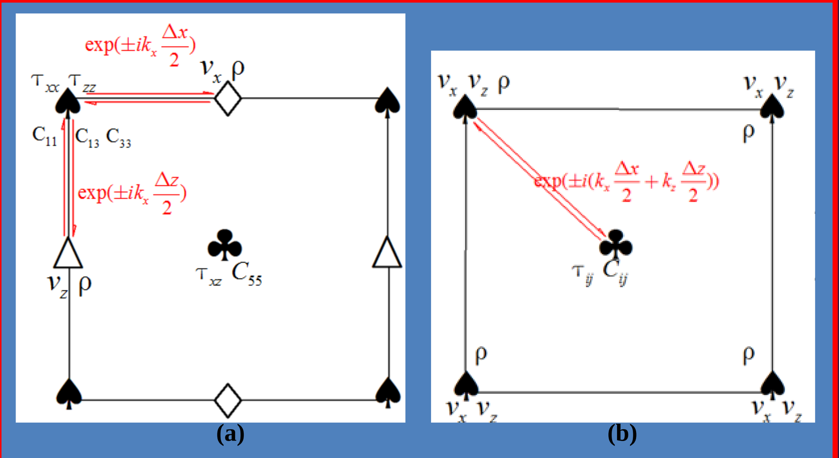

Figure 2. Elementary cells of SSG (a) and RSG (b). Locations where strains, displacements, velocities, and elastic parameters are defined. The red arrows denote the half-grid space shift.

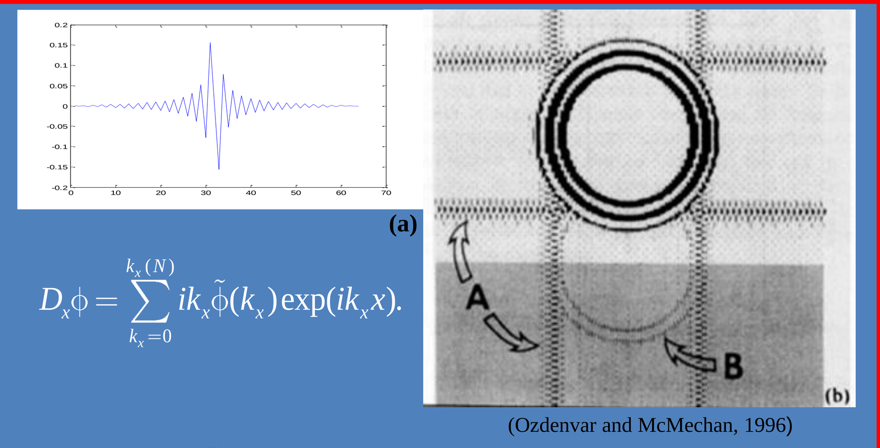

Figure 1. The oscillation (a) and non-causal ringring (b) caused by standard spectral derivative operator.

Examples

1. Using two-layer VTI model to validate our approach.
2. Test on tilt angel sharply variation case (BP2007 TTI model).
3. Strong anisotropy 3D case (triclinic media).

Main reference

Igel, H., Mora, P. and Riollet, B.,1995, *Geophysics*, **60**, 1203–1216.
Virieux, J., 984, *Geophysics*, **49**, 1933–1957.
Bale, R.A., 2003, *70th EAGE Conference and Exhibition 2008, Expanded Abstracts.*
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Ozdenvar, T. and McMechan, G.,1996, *Geophysical Journal International*, **126**, 819–829.

Conclusions

Based upon the RSG configuration, we have proposed a new PSM for elastic wave propagation modelling in heterogeneous anisotropic media. This approach has two main advantages: First, it can suppress the non-causal ringing artifacts; Second, it provides a more straightforward and accurate way to apply the PSM for anisotropic media with symmetries lower than orthorhombic. Numerical examples have demonstrated the validity of the proposed approach.

Acknowledgements

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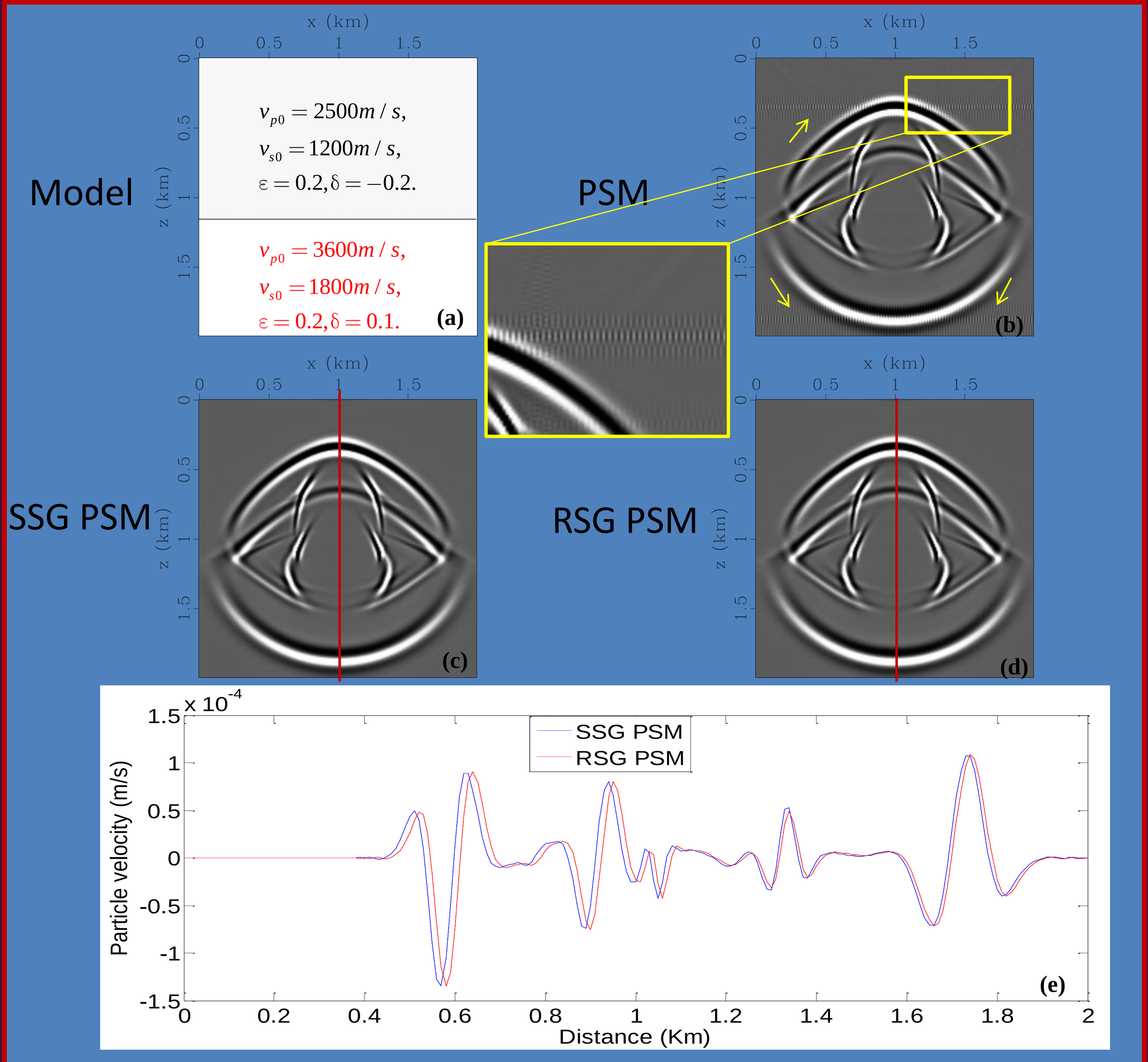

Figure 3. Snapshot of z-component elastic wavefields with model parameters(a), using PSM (b), SSG-PSM (c) and using RSG-PSM (d). The RSG-PSM can suppress the artifacts well as the SSG-PSM. The half-grid space shift between the RSG and SSG presented in (e) well.

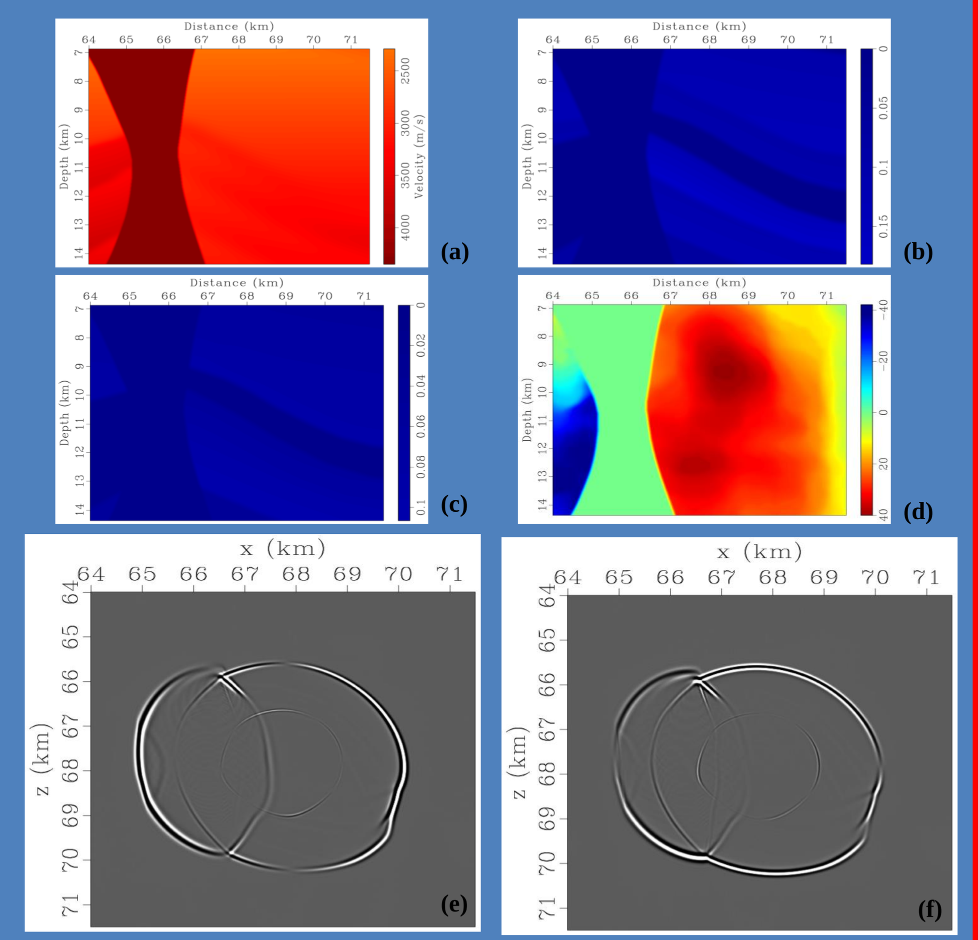

Figure 5. BP2007 TTI model (a), (b), (c) and (d) are Tomsen parameters Vp0, epsilon, delta and theta, respectively. (e) and (f) are the x- and z-component of the wavefields.

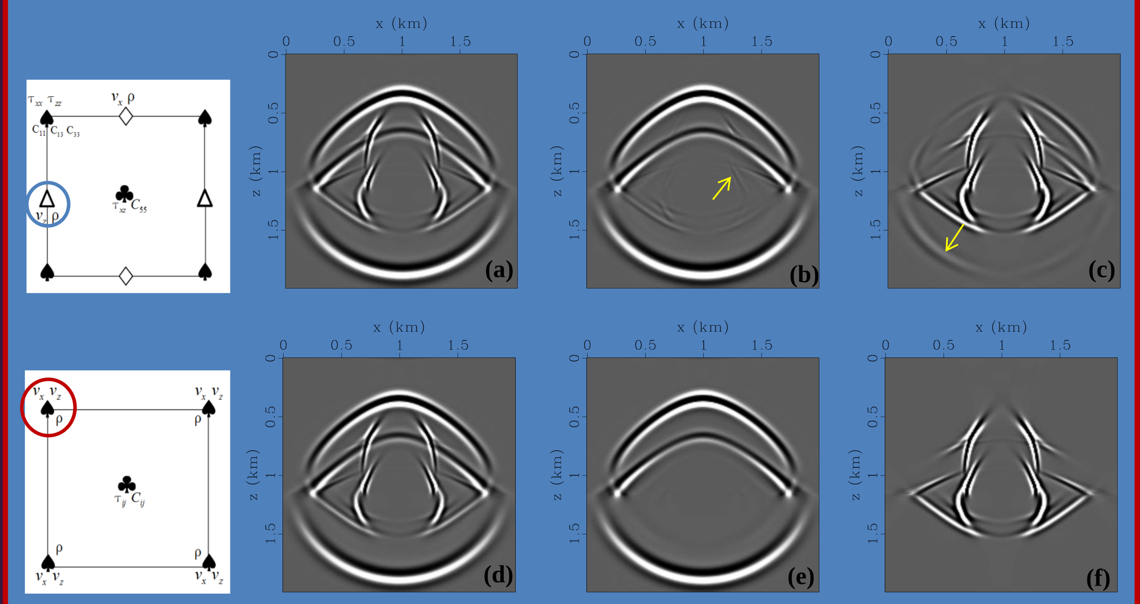

Figure 4. Elastic wave model decoupling based on SSG configuration (a), (b) and (c), on RSG configuration (d), (e) and (f). Total wavefield (a), (d) and P-wave (b), (e) and S-wave (c), (f).

$$C_{tri} = \begin{bmatrix} 10 & 3.5 & 2.5 & -5 & 0.1 & 0.3 \\ 3.5 & 8 & 1.5 & 0.2 & -0.1 & -0.15 \\ 2.5 & 1.5 & 6 & 1 & 0.4 & 0.24 \\ -5 & 0.2 & 1 & 5 & 0.35 & 0.525 \\ 0.1 & -0.1 & 0.4 & 0.35 & 4 & -1 \\ 0.3 & -0.15 & 0.24 & 0.525 & -1 & 3 \end{bmatrix}$$

(Igel et al., 1995)

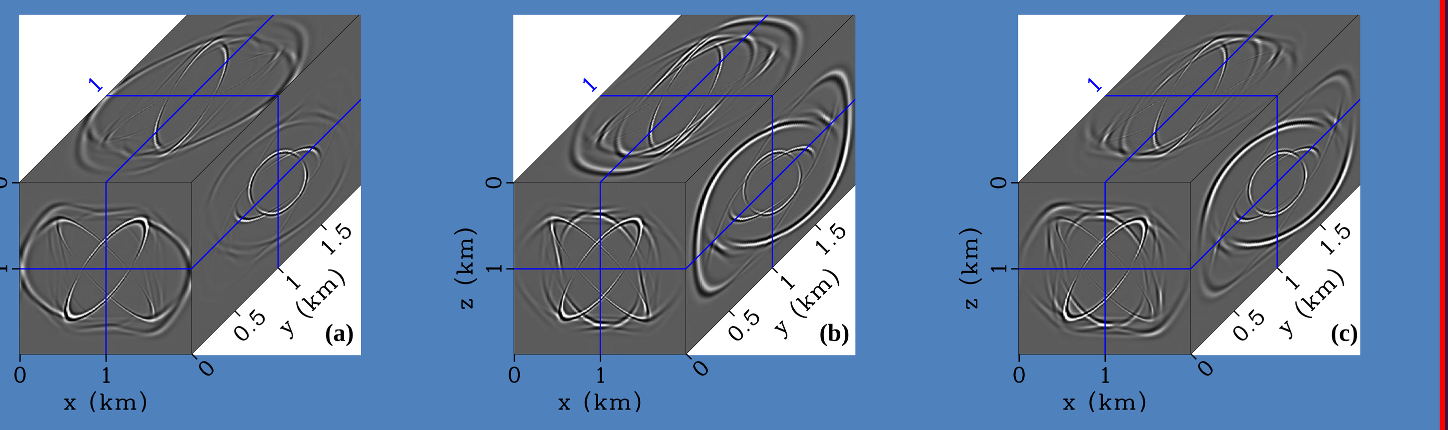

Figure 6. The complex snapshots of the triclinic model (a), (b) and (c) are x-, y- and z-component, respectively.