SPG/SEG北京2016年国际地球物理会议

SPG/SEG 2016 International Geophysical Conference

Modified pseudo-spectral method for wave propagation in general anisotropic media

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Outline

- Earth's media & Seismic wave propagation
  - heterogeneity and anisotropy
  - purposes and approaches of seismic simulation
- Pseudo-spectral method and its challenges
  - artifacts suppressing
  - extension to general anisotropy
- Modified pseudo-spectral method
  - rotated staggered grid
- Numerical examples
  - two-layer VTI
  - Hess VTI
  - BP2007 TTI
  - 3D triclinic
- Conclusions

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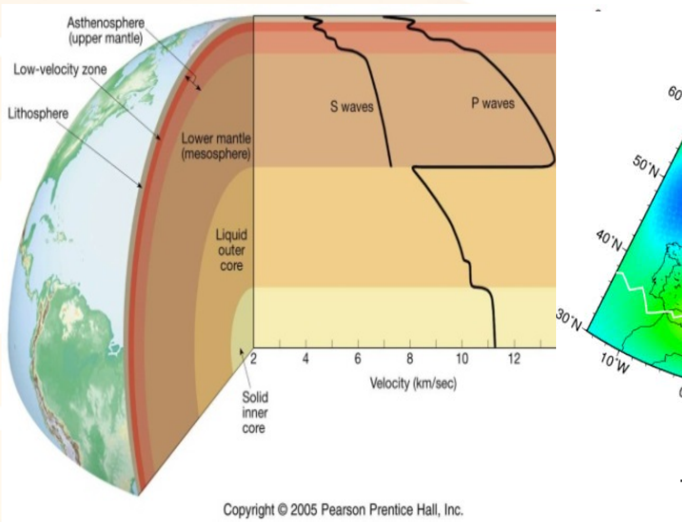
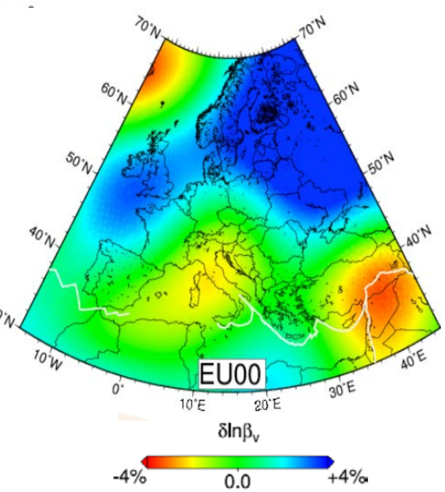
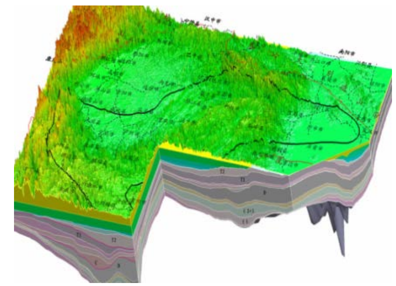
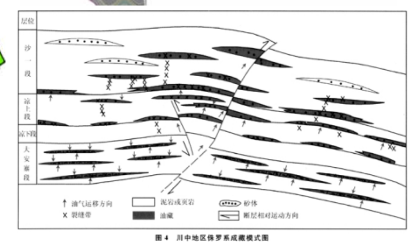
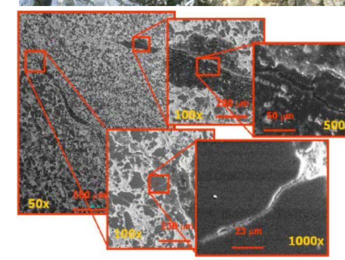
Multi-scale heterogeneities of the Earth







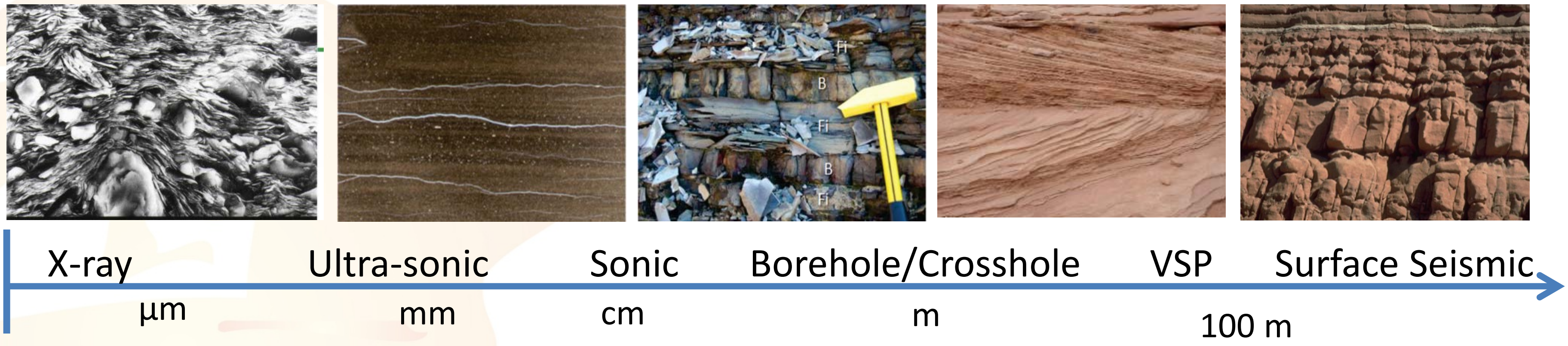




global **regional** **basin & sequence** **rock** **grain**

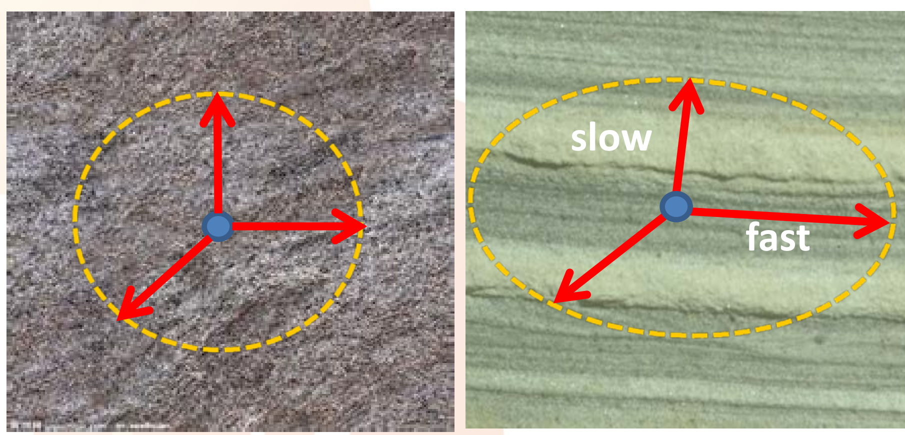
Aligned heterogeneities induced seismic anisotropy

Anisotropy arises from ordered heterogeneities much smaller than the wavelength

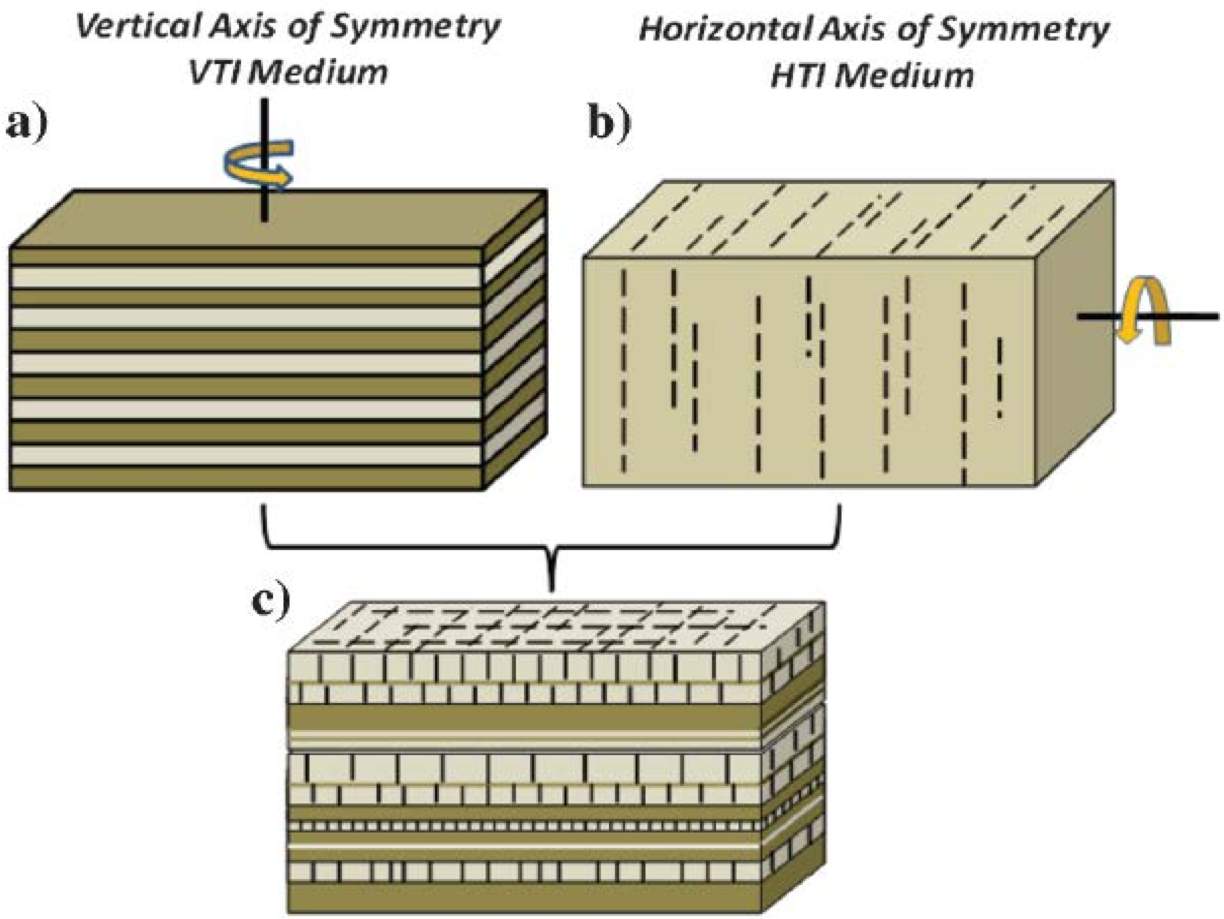


Seismic anisotropy includes:

- velocity anisotropy
- attenuation anisotropy



Widely used (also simplest) anisotropic models



Orthorhombic (ORT) anisotropy

Wave propagation simulation

Purposes

- To study complex wave phenomena
- To provide a forward modeling engine for seismic imaging and inversion

Main approaches

- Finite-difference method (FDM)
- Pseudo-spectral method (PSM)
- Finite-element method (FEM)

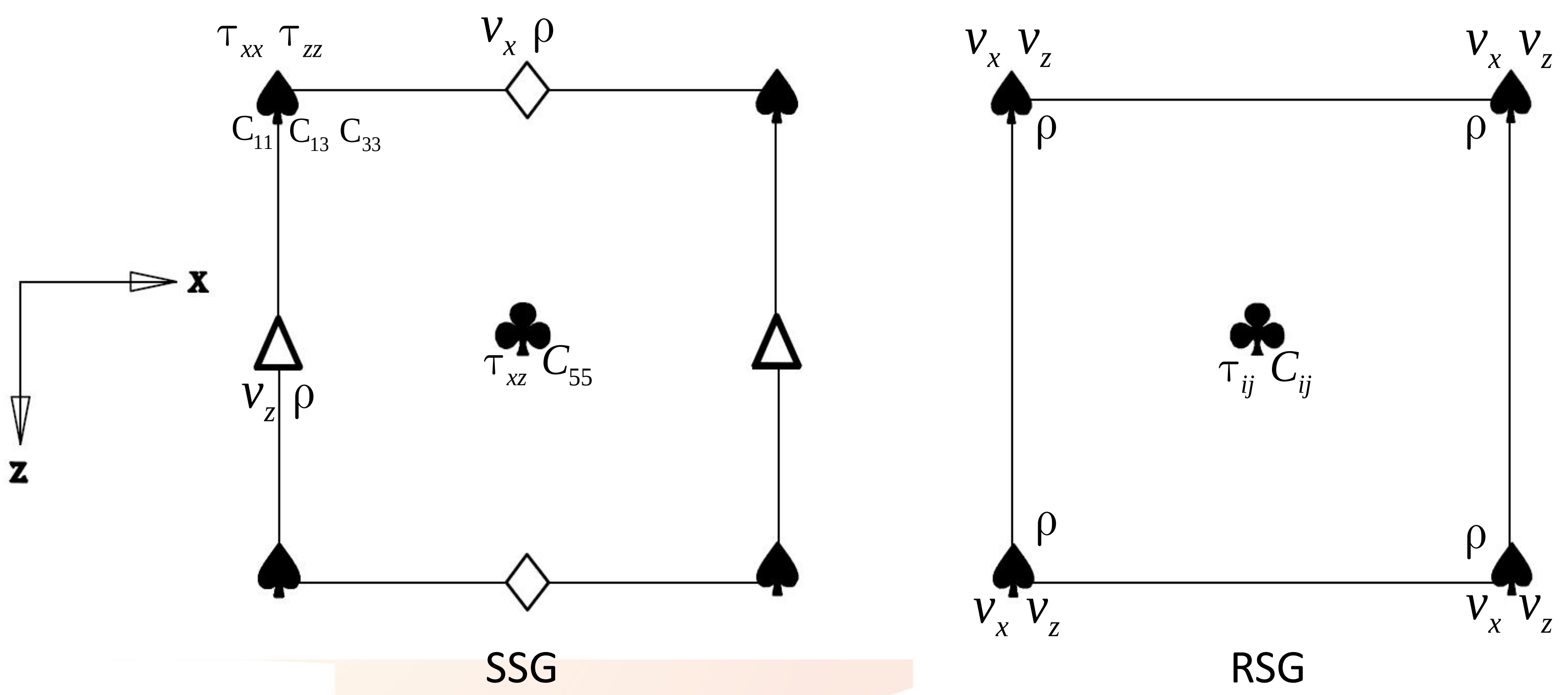
Notes:

-- FDM is the most widely used (simplest and efficient).

-- PSM has the highest accuracy (exact up to Nyquist frequency) and most memory economical (only two grids per wavelength), especially in 3D case.

Grid configuration for wave simulation

Widely used grid configuration for wave propagation simulation are standard staggered grid (SSG)[1] and rotated staggered grid (RSG)[2].



[1] Virieux, 1984
[2] Saenger et at, 2000

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PSM for 2D elastic wave equation

Taking the 2D elastic wave equation as an example:

$$\begin{aligned}
\rho\partial_t v_x &= \partial_x \tau_{xx} + \partial_z \tau_{xz} + f_x, \\
\rho\partial_t v_z &= \partial_x \tau_{xz} + \partial_z \tau_{zz} + f_z, \\
\partial_t \tau_{xx} &= C_{11}\partial_x v_x + C_{13}\partial_z v_z, \\
\partial_t \tau_{zz} &= C_{13}\partial_x v_x + C_{33}\partial_z v_z, \\
\partial_t \tau_{xz} &= C_{55}(\partial_x v_z + \partial_z v_x).
\end{aligned}$$

(for isotropic, VTI, HTI, orthorhombic media)

Spatial derivative approximation based on Fourier transform:[1]

$$D_x^{PS}\phi = \sum_{k_x=0}^{k_x(N)} ik_x\tilde{\phi}(k_x)\exp(ik_x x)$$

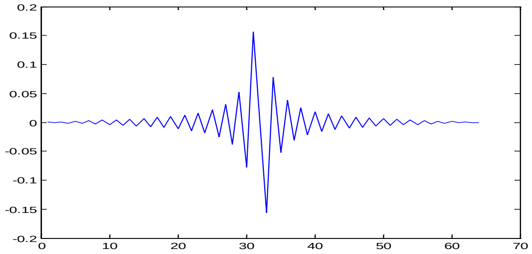
Finite-difference for temporal derivative (leap-frog):

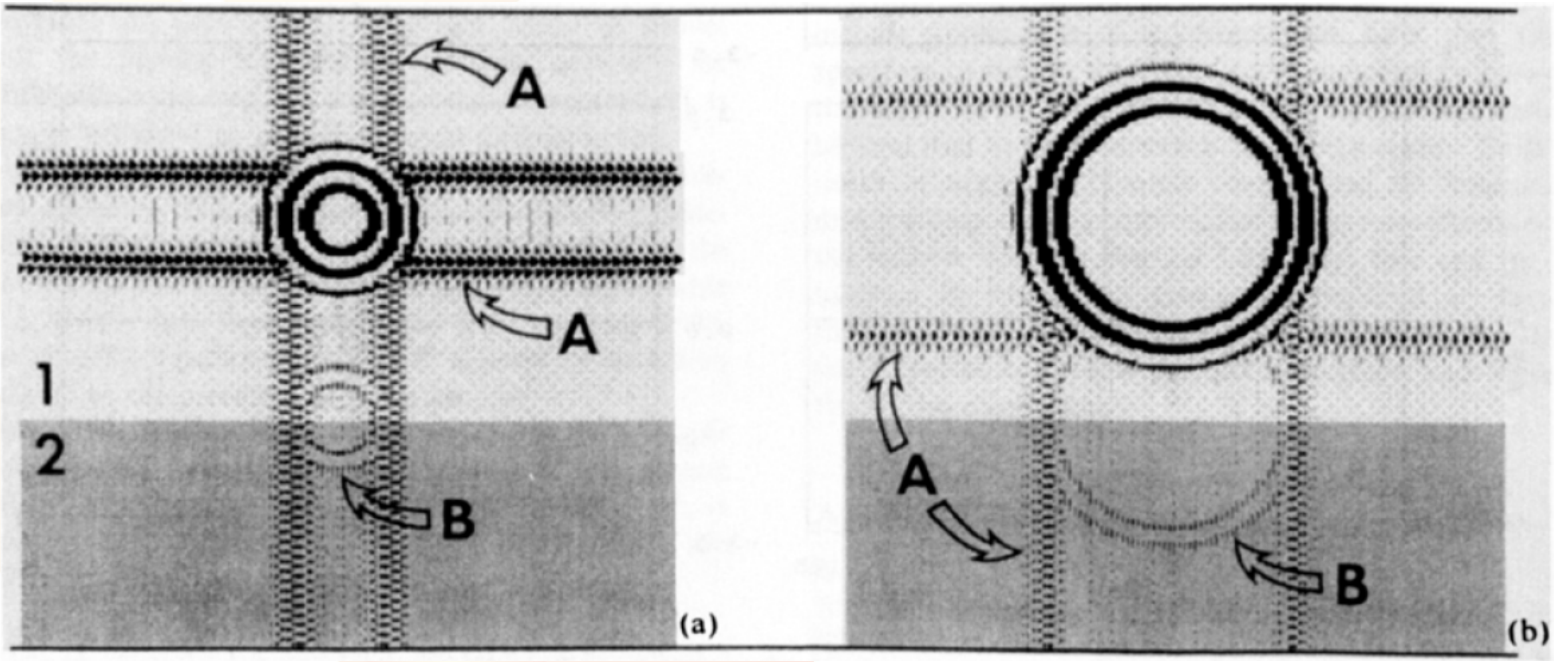
$$D_t v(t) = \frac{v(t+\Delta t/2) - v(t-\Delta t/2)}{\Delta t}$$

[1] Kosloff and Baysal, 1982

Challenge 1: non-causal ringing artifacts

1st spectral derivative operator will induce oscillation in spike derivative, which present as non-causal ringing in wavefield[1].





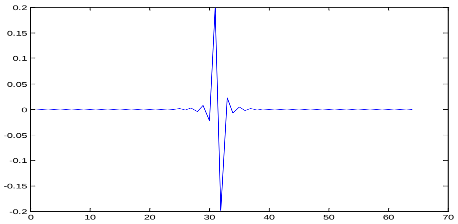
[1] Ozdenvar and McMechan, 1996

A solution to suppress the artifacts

A half-grid space-shift in wavenumber domain can suppress the oscillation[1][2]

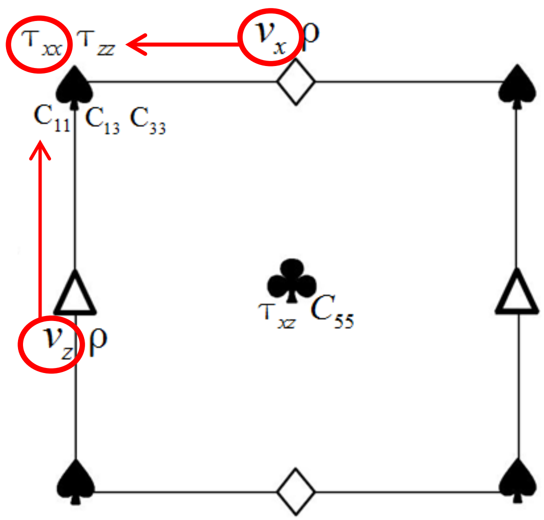
$$D_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} ik_x \boxed{\exp\left(\pm ik_x\Delta x/2\right)}\tilde{\phi}(k_x)\exp\left(ik_x x\right),$$

Shift term



However, this involves the elastic wave propagation simulation based upon standard staggered grid (SSG).

$$\begin{aligned}
\rho\partial_t v_x &= \partial_x\tau_{xx} + \partial_z\tau_{xz} + f_x,\\
\rho\partial_t v_z &= \partial_x\tau_{xz} + \partial_z\tau_{zz} + f_z,\\
\Aboxed{\partial_t\tau_{xx} &= C_{11}\partial_x v_x + C_{13}\partial_z v_z,}\\
\partial_t\tau_{zz} &= C_{13}\partial_x v_x + C_{33}\partial_z v_z,\\
\partial_t\tau_{xz} &= C_{55}(\partial_x v_z + \partial_z v_x).
\end{aligned}$$



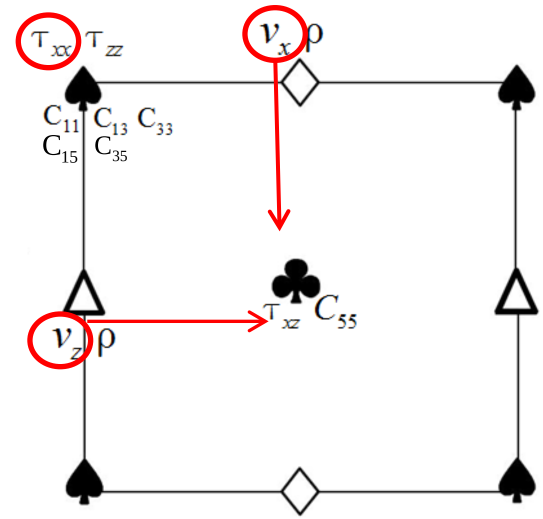
The SSG configuration is enough for isotropic case, as well as anisotropy with higher symmetry than orthorhombic media.

[1] Virieux, 1984
[2] Correa et al, 2002

Challenge 2: Extension for general anisotropy

When the anisotropy symmetry is lower than orthorhombic or the symmetry axis not align with coordinate, the elastic wave equation has more term.

$$
\begin{aligned}
\rho\partial_t v_x &= \partial_x\tau_{xx} + \partial_z\tau_{xz} + f_x, \\
\rho\partial_t v_z &= \partial_x\tau_{xz} + \partial_z\tau_{zz} + f_z, \\
\partial_t\tau_{xx} &= C_{11}\partial_x v_x + C_{13}\partial_z v_z + C_{15}(\partial_x v_z + \partial_z v_x), \\
\partial_t\tau_{zz} &= C_{13}\partial_x v_x + C_{33}\partial_z v_z + C_{35}(\partial_x v_z + \partial_z v_x), \\
\partial_t\tau_{xz} &= C_{55}(\partial_x v_z + \partial_z v_x) + C_{15}\partial_x v_x + C_{35}\partial_z v_z,
\end{aligned}
$$



The additional term can not be well staggered.
In 3D case the non-staggered term increase significantly.

A possible solution (interpolation)

A solution is interpolation the derivative of the particle velocity component to their corresponding position. The interpolation in space domain means shift in wavenumber domain.

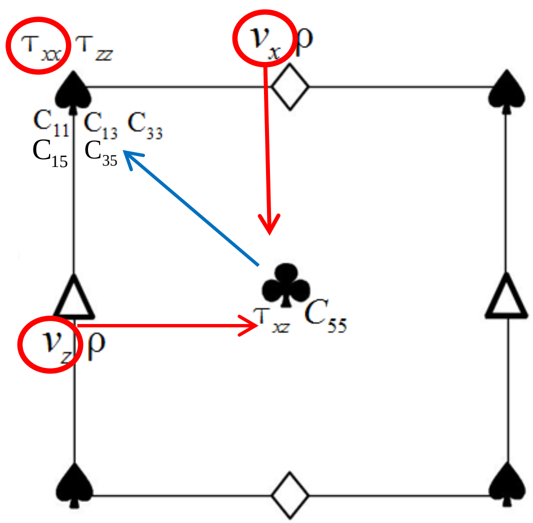
$$C = \begin{bmatrix} C_{11} & C_{13} & C_{15} S_x^- S_z^- \\ C_{13} & C_{33} & C_{35} S_x^- S_z^- \\ S_x^+ S_z^+ C_{15} & S_x^+ S_z^+ C_{35} & C_{55} \end{bmatrix}^{[1]}$$

shift operator

where S is the shift operator:

$$S_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} \exp(\pm i k_x \Delta x / 2)\tilde{\phi}(k_x)\exp(i k_x x)$$

$$\begin{aligned}
\rho \partial_t v_x &= \partial_x \tau_{xx} + \partial_z \tau_{xz} + f_x, \\
\rho \partial_t v_z &= \partial_x \tau_{xz} + \partial_z \tau_{zz} + f_z, \\
\partial_t \tau_{xx} &= C_{11} \partial_x v_x + C_{13} \partial_z v_z \boxed{+ C_{15}(\partial_x v_z + \partial_z v_x)}, \\
\partial_t \tau_{zz} &= C_{13} \partial_x v_x + C_{33} \partial_z v_z + C_{35}(\partial_x v_z + \partial_z v_x), \\
\partial_t \tau_{xz} &= C_{55}(\partial_x v_z + \partial_z v_x) + C_{15} \partial_x v_x + C_{35} \partial_z v_z.
\end{aligned}$$



[1] Bale, 2002

The staggered elastic tensor for 2D and 3D

After applying the shift operator, the elastic tensor for 2D:

$$C = \begin{bmatrix} C_{11} & C_{13} & C_{15} S_x^- S_z^- \\ C_{13} & C_{33} & C_{35} S_x^- S_z^- \\ S_x^+ S_z^+ C_{15} & S_x^+ S_z^+ C_{35} & C_{55} \end{bmatrix}$$

$$S_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} \exp(\pm i k_x \Delta x / 2)\tilde{\phi}(k_x)\exp(i k_x x)$$

and for 3D:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} S_y^- S_z^- & C_{15} S_x^- S_z^- & C_{16} S_x^- S_y^- \\ C_{12} & C_{22} & C_{23} & C_{24} S_y^- S_z^- & C_{25} S_x^- S_z^- & C_{26} S_x^- S_y^- \\ C_{13} & C_{23} & C_{33} & C_{34} S_y^- S_z^- & C_{35} S_x^- S_z^- & C_{36} S_x^- S_y^- \\ S_y^+ S_z^+ C_{14} & S_y^+ S_z^+ C_{24} & S_y^+ S_z^+ C_{34} & C_{44} & S_y^+ S_z^+ C_{45} S_x^- S_z^- & S_y^+ S_z^+ C_{46} S_x^- S_y^- \\ S_x^+ S_z^+ C_{15} & S_x^+ S_z^+ C_{25} & S_x^+ S_z^+ C_{35} & S_x^+ S_z^+ C_{45} S_y^- S_z^- & C_{55} & S_x^+ S_z^+ C_{56} S_x^- S_y^- \\ S_x^+ S_y^+ C_{16} & S_x^+ S_y^+ C_{26} & S_x^+ S_y^+ C_{36} & S_x^+ S_y^+ C_{46} S_y^- S_z^- & S_x^+ S_y^+ C_{56} S_x^- S_z^- & C_{66} \end{bmatrix}$$

However, shift operator S significantly increase the number of Fourier transform, particularly in 3D case.

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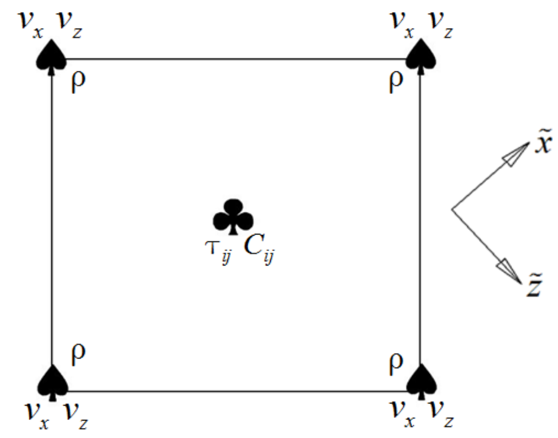
Our solution: rotated staggered grid (RSG)

In RSG configuration, all the stress components are defined at the same position and all the particle velocity components are defined at the other set of points[1] .

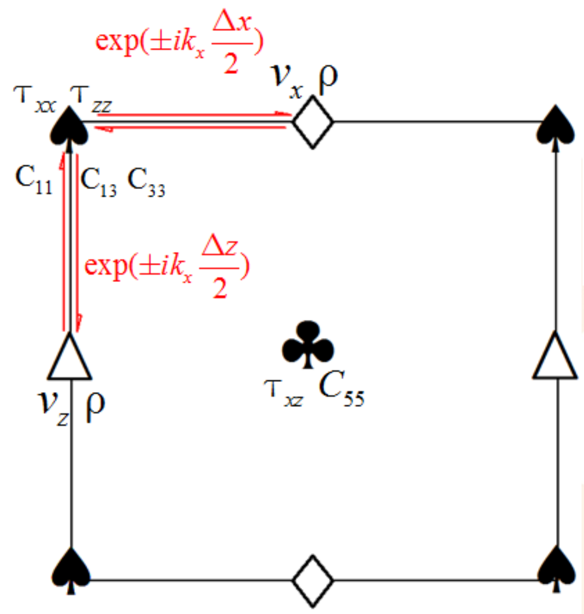


Saenger applied this grid configuration to FDM for simulating wave propagation in arbitrary anisotropic media.

But no one use this grid configuration to PSM for suppressing the non-causal ringing.

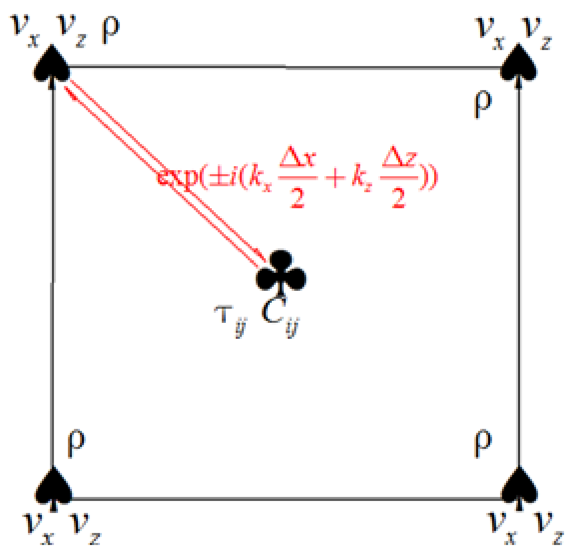
[1] Saenger et al, 2000

Our solution: RSG-PSM



In SSG PSM, shifting along x- or z-direction

$$D_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} ik_x \boxed{\exp\left(\pm ik_x\Delta x/2\right)} \tilde{\phi}(k_x)\exp\left(ik_x x\right),$$



In RSG PSM, shifting along the diagonal direction

$$D_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} ik_x \boxed{\exp\left(\pm i(k_x\Delta x/2 + k_z\Delta z/2)\right)} \tilde{\phi}(k_x)\exp\left(ik_x x\right).$$

The discrete formula for 2D and 3D arbitrary anisotropy

Using the RSG spectral derivative operator, the 2D and 3D arbitrary anisotropic discrete wave equation have uniform formula:

2D

$$
\begin{aligned}
\rho\partial_t v_x &= D_x^+\tau_{xx} + D_z^+\tau_{xz} + f_x,\\
\rho\partial_t v_z &= D_x^+\tau_{xz} + D_z^+\tau_{zz} + f_z,\\
\partial_t\tau_{xx} &= C_{11}D_x^- v_x + C_{13}D_z^- v_z + C_{15}(D_x^- v_z + D_z^- v_x),\\
\partial_t\tau_{zz} &= C_{13}D_x^- v_x + C_{13}D_z^- v_z + C_{35}(D_x^- v_z + D_z^- v_x),\\
\partial_t\tau_{xx} &= C_{15}D_x^- v_x + C_{35}D_z^- v_z + C_{55}(D_x^- v_z + D_z^- v_x).
\end{aligned}
$$

$$
\boxed{D_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} ik_x \exp\left(\pm i(k_x\Delta x/2 + k_z\Delta z/2)\right)\tilde{\phi}(k_x)\exp\left(ik_x x\right).}
$$

The motion equation: forward shift

The Hooke's law: backward shift

3D

$$
\begin{aligned}
\rho\partial_t v_x &= D_x^+\tau_{xx} + D_y^+\tau_{xy} + D_z^+\tau_{xz} + f_x,\\
\rho\partial_t v_y &= D_x^+\tau_{xy} + D_y^+\tau_{yy} + D_z^+\tau_{yz} + f_y,\\
\rho\partial_t v_z &= D_x^+\tau_{xz} + D_y^+\tau_{yz} + D_z^+\tau_{zz} + f_z,\\
\partial_t\tau_{xx} &= C_{11}D_x^- v_x + C_{12}D_y^- v_y + C_{13}D_z^- v_z + C_{14}(D_z^- v_y + D_y^- v_z)\\
&\quad + C_{15}(D_x^- v_z + D_z^- v_x) + C_{16}(D_y^- v_x + D_x^- v_y),\\
\partial_t\tau_{yy} &= C_{12}D_x^- v_x + C_{22}D_y^- v_y + C_{23}D_z^- v_z + C_{24}(D_z^- v_y + D_y^- v_z)\\
&\quad + C_{25}(D_x^- v_z + D_z^- v_x) + C_{26}(D_y^- v_x + D_x^- v_y),\\
\partial_t\tau_{zz} &= C_{13}D_x^- v_x + C_{23}D_y^- v_y + C_{33}D_z^- v_z + C_{34}(D_z^- v_y + D_y^- v_z)\\
&\quad + C_{35}(D_x^- v_z + D_z^- v_x) + C_{36}(D_y^- v_x + D_x^- v_y),\\
\partial_t\tau_{yz} &= C_{14}D_x^- v_x + C_{24}D_y^- v_y + C_{34}D_z^- v_z + C_{44}(D_z^- v_y + D_y^- v_z)\\
&\quad + C_{45}(D_x^- v_z + D_z^- v_x) + C_{46}(D_y^- v_x + D_x^- v_y),\\
\partial_t\tau_{xz} &= C_{15}D_x^- v_x + C_{25}D_y^- v_y + C_{35}D_z^- v_z + C_{45}(D_z^- v_y + D_y^- v_z)\\
&\quad + C_{55}(D_x^- v_z + D_z^- v_x) + C_{56}(D_y^- v_x + D_x^- v_y),\\
\partial_t\tau_{xy} &= C_{16}D_x^- v_x + C_{26}D_y^- v_y + C_{36}D_z^- v_z + C_{46}(D_z^- v_y + D_y^- v_z)\\
&\quad + C_{56}(D_x^- v_z + D_z^- v_x) + C_{66}(D_y^- v_x + D_x^- v_y).
\end{aligned}
$$

$$
\boxed{D_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} ik_x \exp\left(\pm i(k_x\Delta x/2 + k_y\Delta y/2 + k_z\Delta z/2)\right)\tilde{\phi}(k_x)\exp\left(ik_x x\right).}
$$

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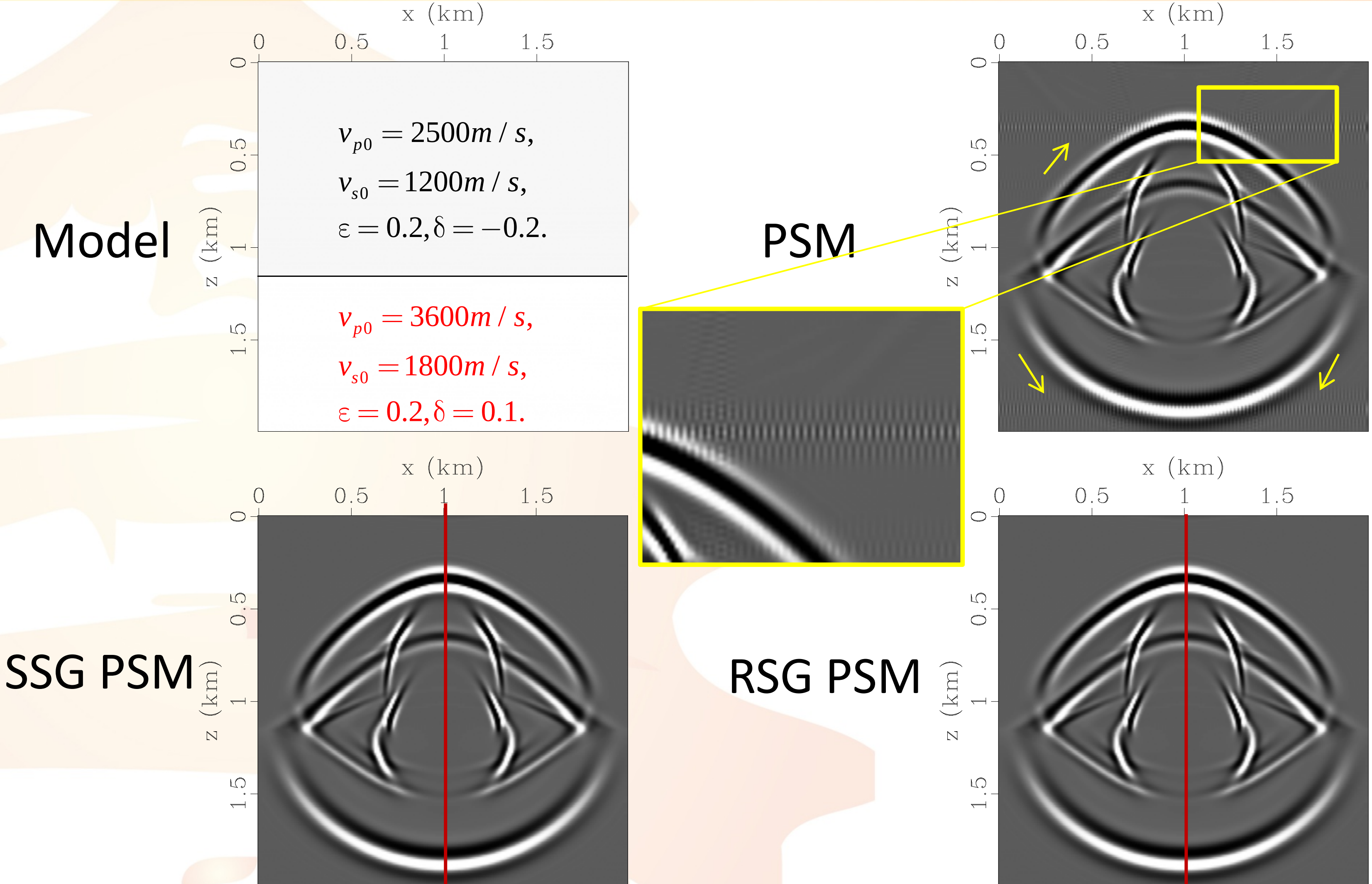
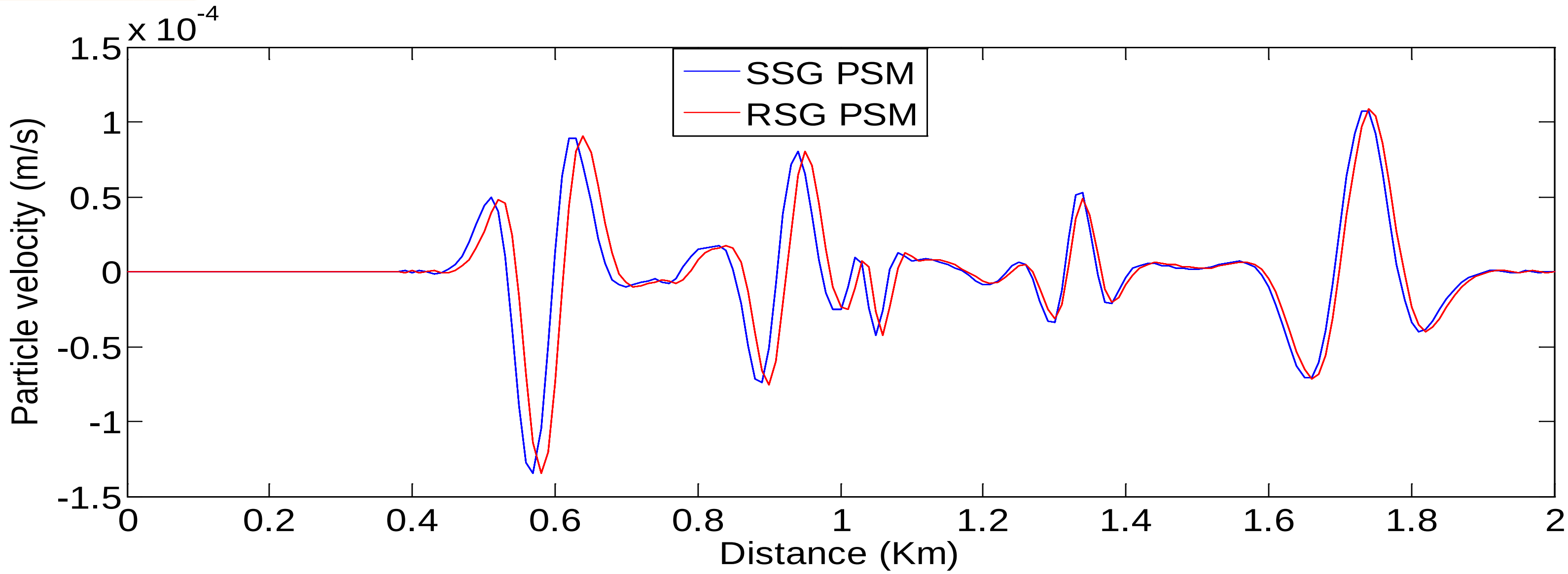
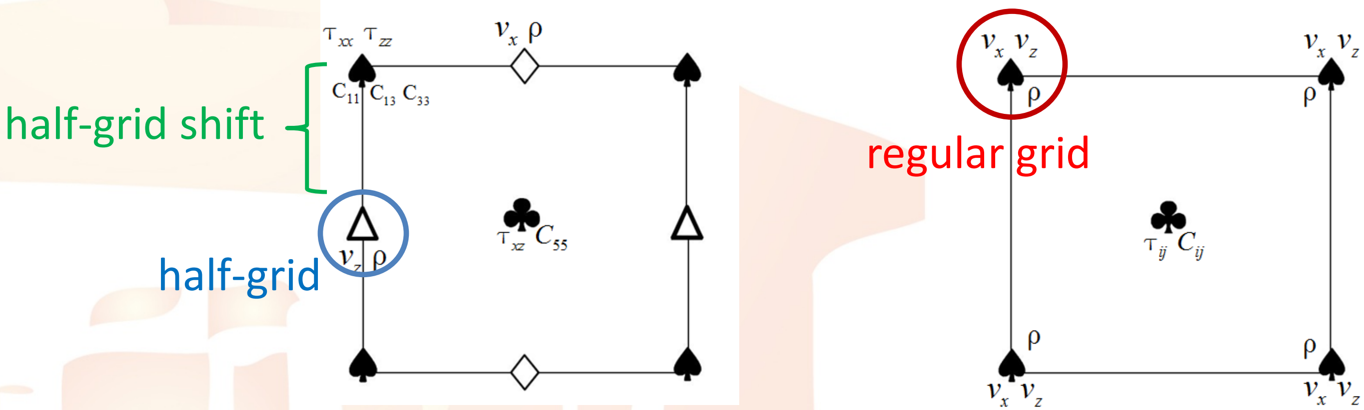
Two-layer VTI model



Extracted line at 1Km



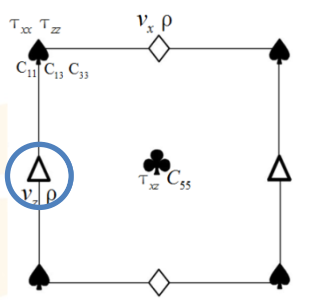
The extracted trance demonstrates that the SSG and RSG PSM have the same kinematics and dynamics, except a half-grid space-shift.



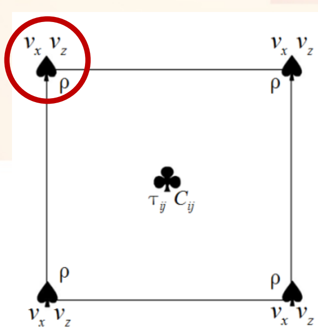
Elastic wave mode decoupling

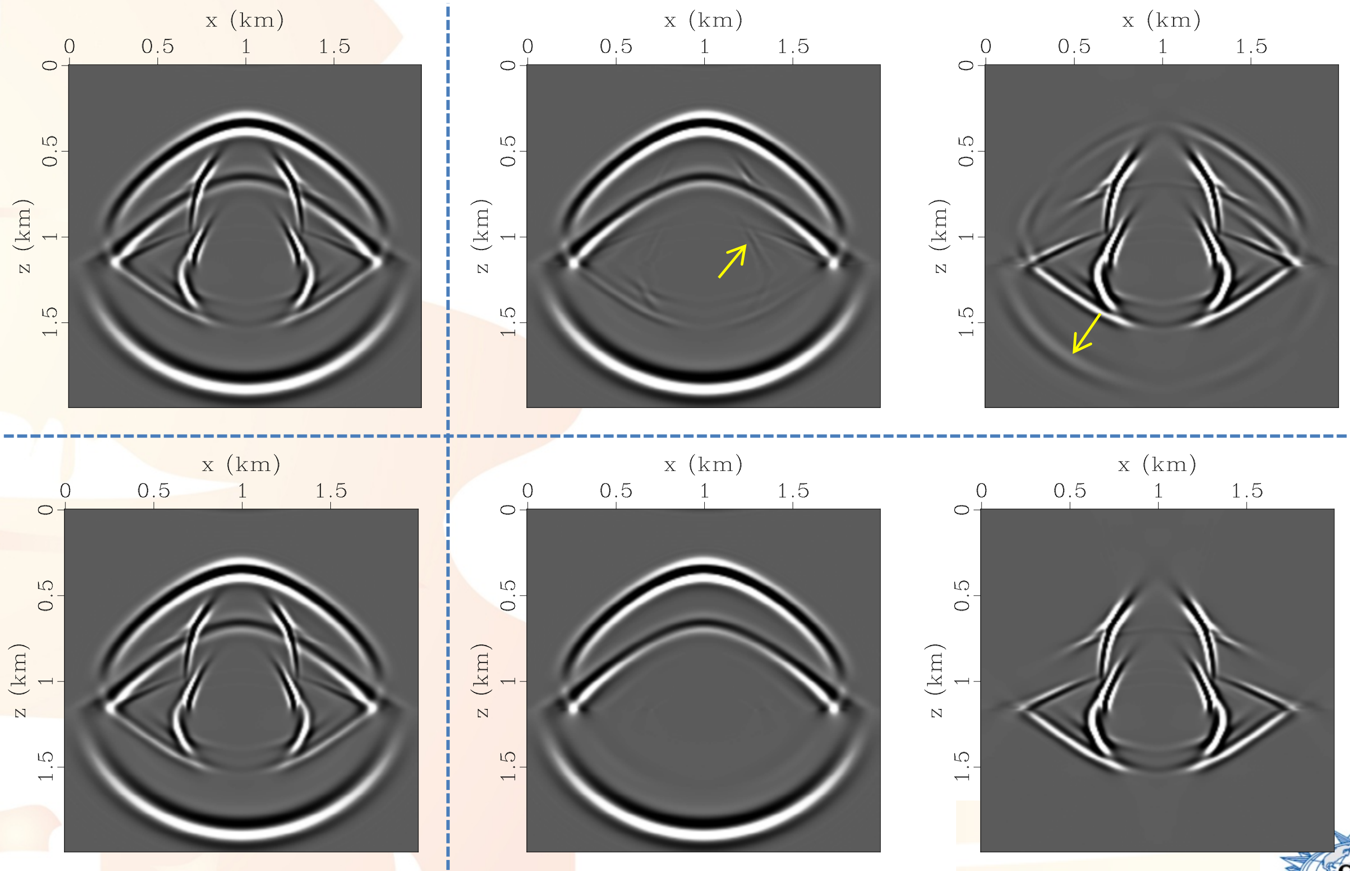
To obtain a physically interpretable imaging or inversion result, we usually need decompose the elastic wave modes. Wave mode decoupling needs the wavefeild define at the regular grid.

SSG PSM



RSG PSM





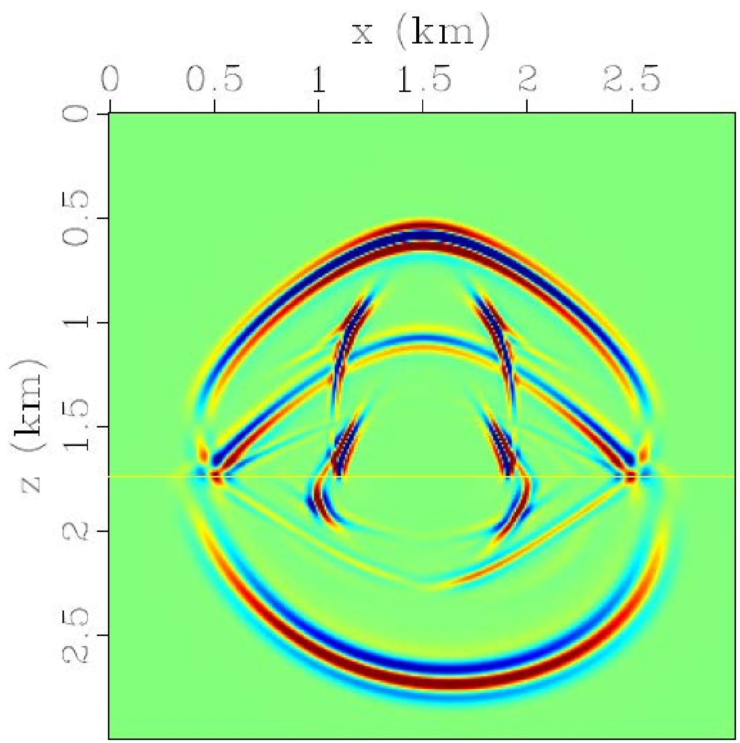
Total wavefield

P-wave

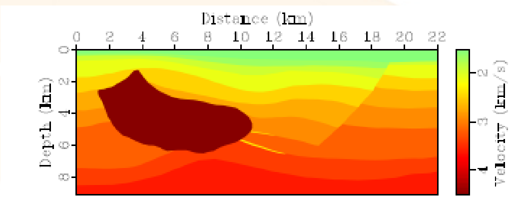
S-wave

Efficiency comparison

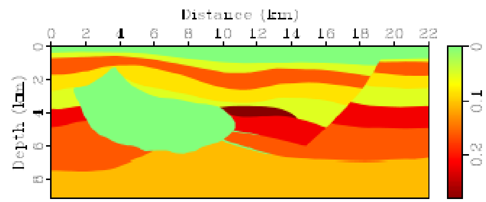
scheme	PSM	RSG-PSM	SSG+interpolation
CPU time	33.2s	33.4s	46.2s



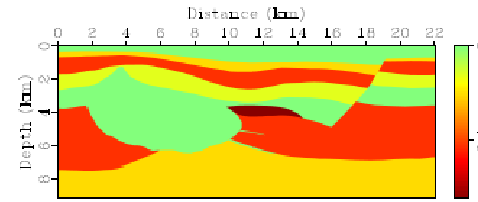
2D Hess VTI model
(RSG-PSM)



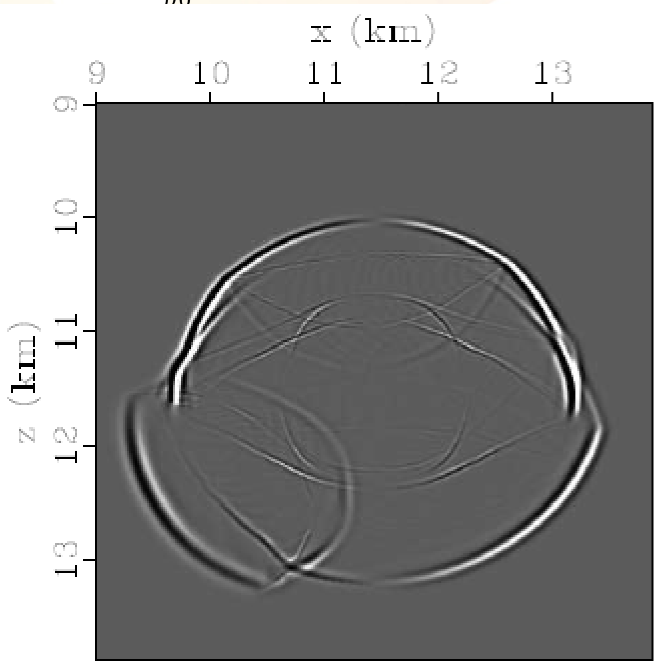
v_{p0}



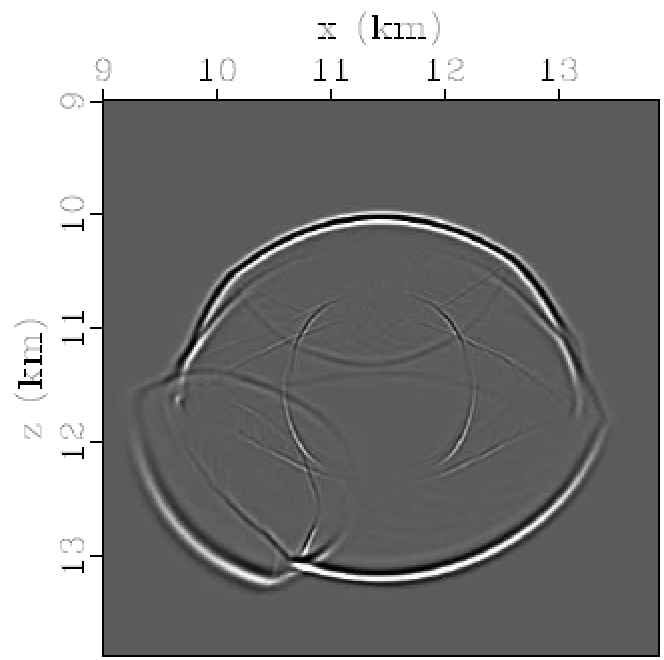
ε



δ

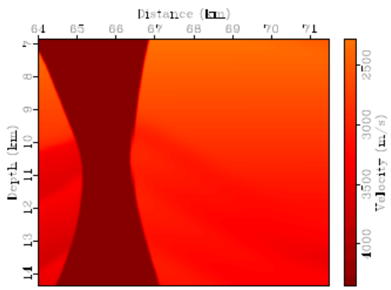


x-component

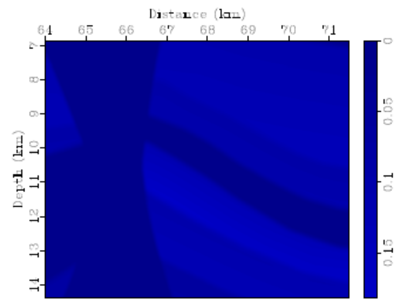


z-component

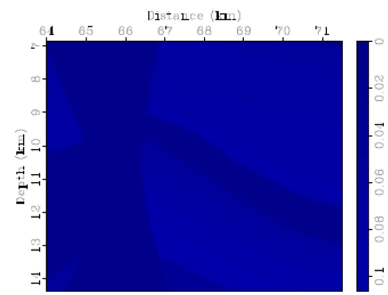
BP2007 TTI model
(RSG-PSM)



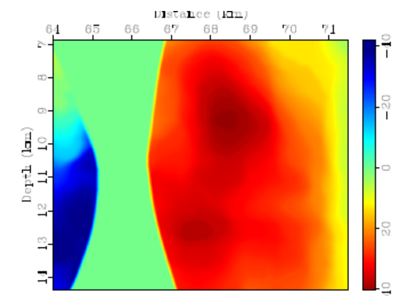
v_{p0}



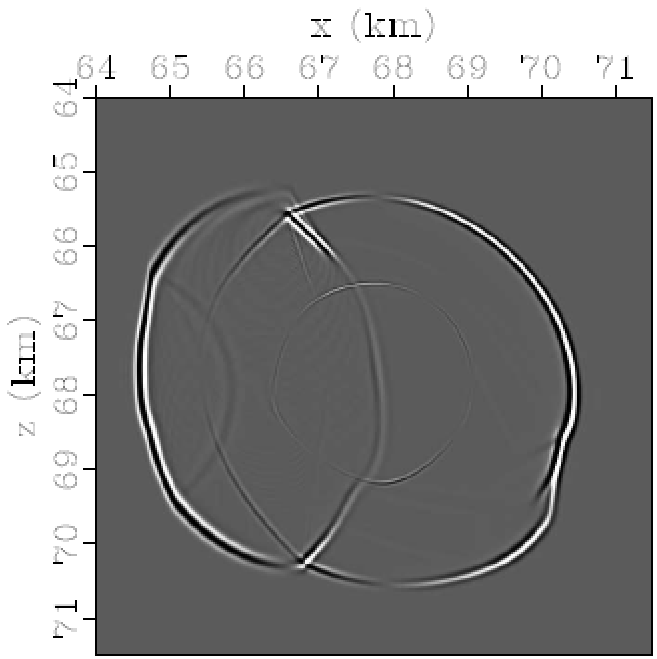
ε



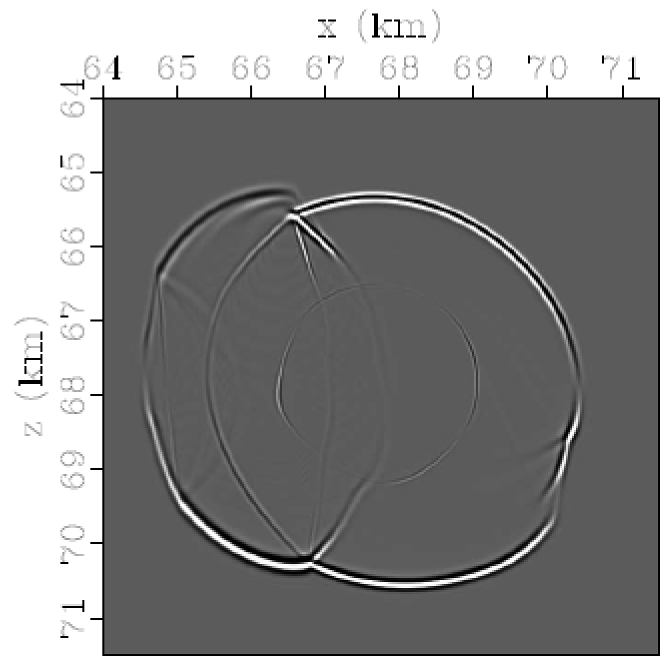
δ



θ



x-component



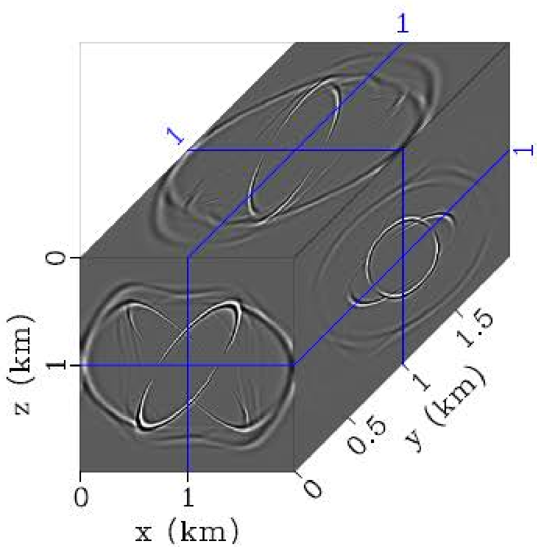
z-component

No smoothing in the tilt angle model.

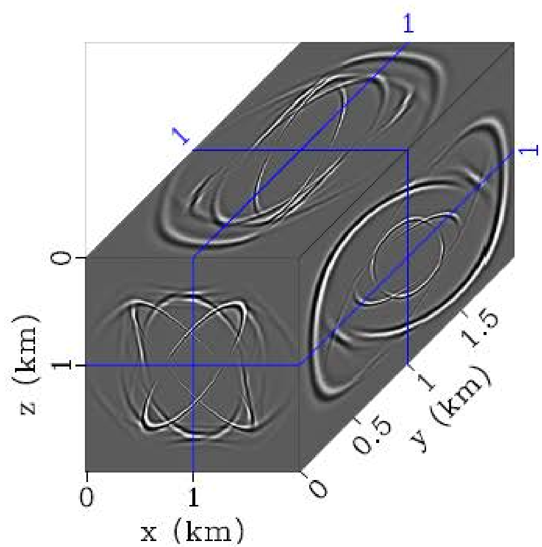
3D triclinic model
(RSG-PSM)

$$C_{tri} = \begin{bmatrix} 10 & 3.5 & 2.5 & -5 & 0.1 & 0.3 \\ 3.5 & 8 & 1.5 & 0.2 & -0.1 & -0.15 \\ 2.5 & 1.5 & 6 & 1 & 0.4 & 0.24 \\ -5 & 0.2 & 1 & 5 & 0.35 & 0.525 \\ 0.1 & -0.1 & 0.4 & 0.35 & 4 & -1 \\ 0.3 & -0.15 & 0.24 & 0.525 & -1 & 3 \end{bmatrix}$$ [1]

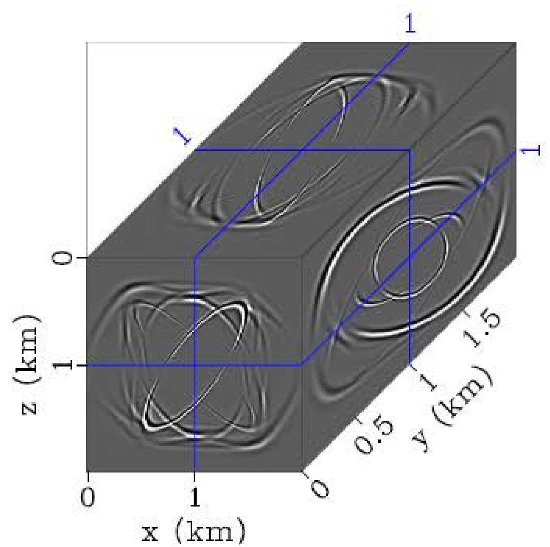
We obtained dispersion-free wavefields for this complex triclinic model.



x-component



y-component



z-component

[1] Igel et al, 1995

Outline

- Earth's media & Seismic wave propagation
  - heterogeneity and anisotropy
  - purposes and approaches of seismic simulation
- Pseudo-spectral method and its challenges
  - artifacts suppressing
  - extension to general anisotropy
- Modified pseudo-spectral method
  - rotated staggered grid
- Numerical examples
  - two-layer VTI
  - Hess VTI
  - BP2007 TTI
  - 3D triclinic
- Conclusions

Conclusions

We have proposed a rotated-staggered-grid based PSM to simulate wave propagation in arbitrary anisotropic media.

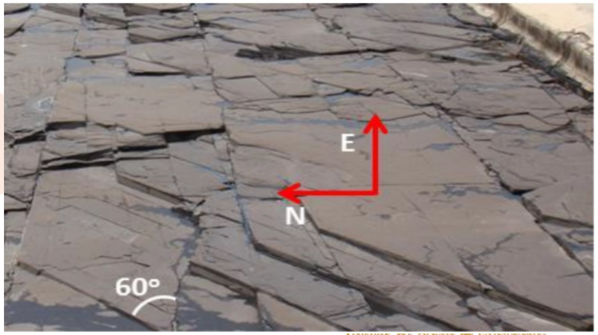
This new scheme has two main advantages:

-- *suppress the non-causal artifacts successfully*

-- *no additional interpolation to assist staggering for anisotropic media with lower symmetry*(high efficiency, saving memory).

The possible application is to provide an efficient forward modeling engine for seismic imaging and waveform inversion in unconventional reservoirs , especially for 3D fractured VTI rocks (e.g., tilted orthorhombic, monoclinic anisotropy)

Fractured tight-sand rock



Fractured shale rock

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- Madagascar free software
- Hess, BP (SEG VTI, TTI models)

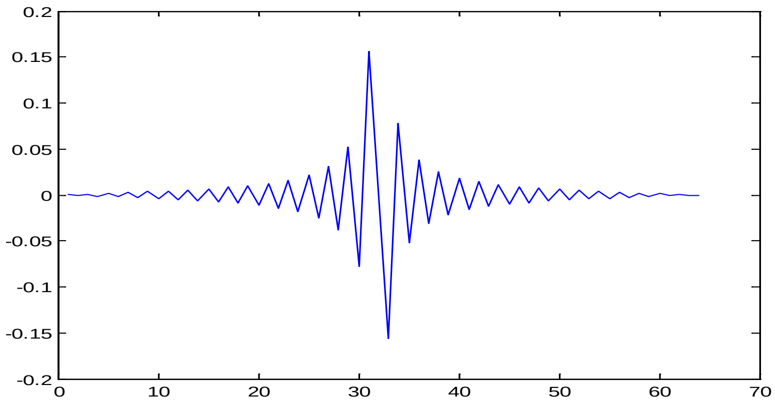
Thanks for your attention!

$$
{}_{SSG}D_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} ik_x \exp(\pm ik_x\Delta x/2)\tilde{\phi}(k_x)\exp(ik_x x)
$$

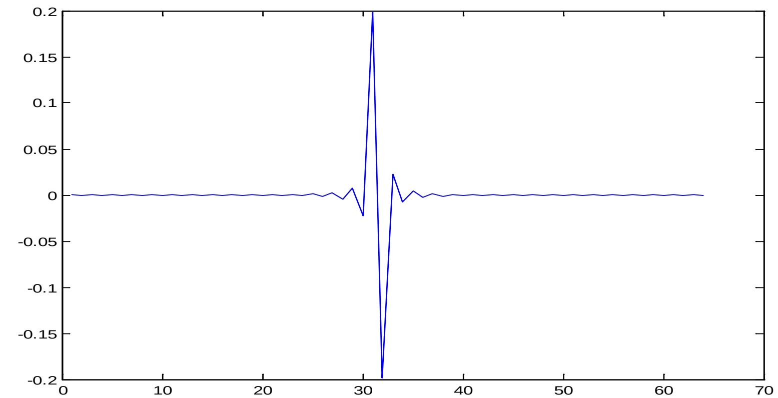
$$
{}_{RSG}D_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} ik_x \exp(\pm i(k_x\Delta x/2\boxed{+k_z\Delta z/2}))\tilde{\phi}(k_x)\exp(ik_x x)
$$

$$
\Downarrow
$$

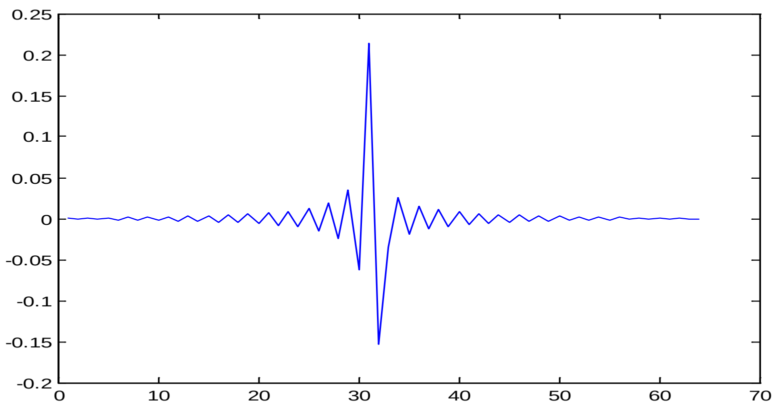
$$
{}_{SSG}D_x^{\pm}\phi = \sum_{k_x=0}^{k_x(N)} ik_x \exp(\pm ik_x\boxed{a\Delta x/2})\tilde{\phi}(k_x)\exp(ik_x x)
$$



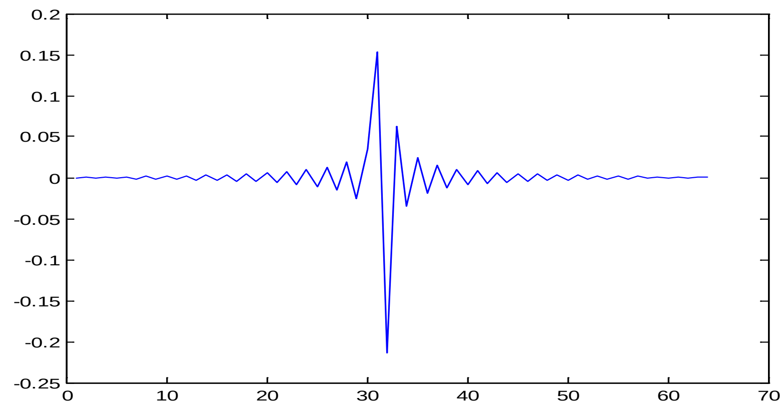
ik_x



$ik_x \exp(ik_x \Delta x / 2)$



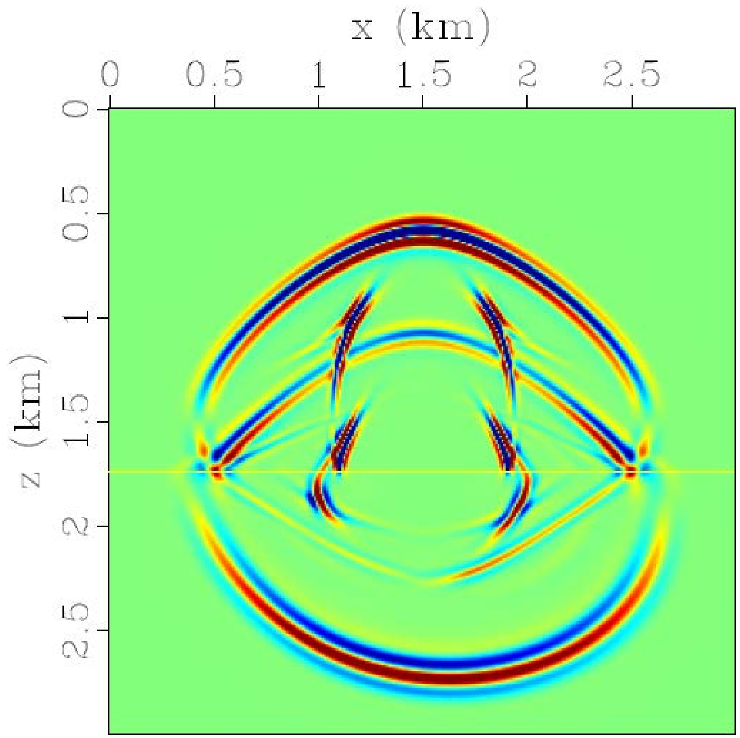
$ik_x \exp(ik_x \Delta x / 3)$

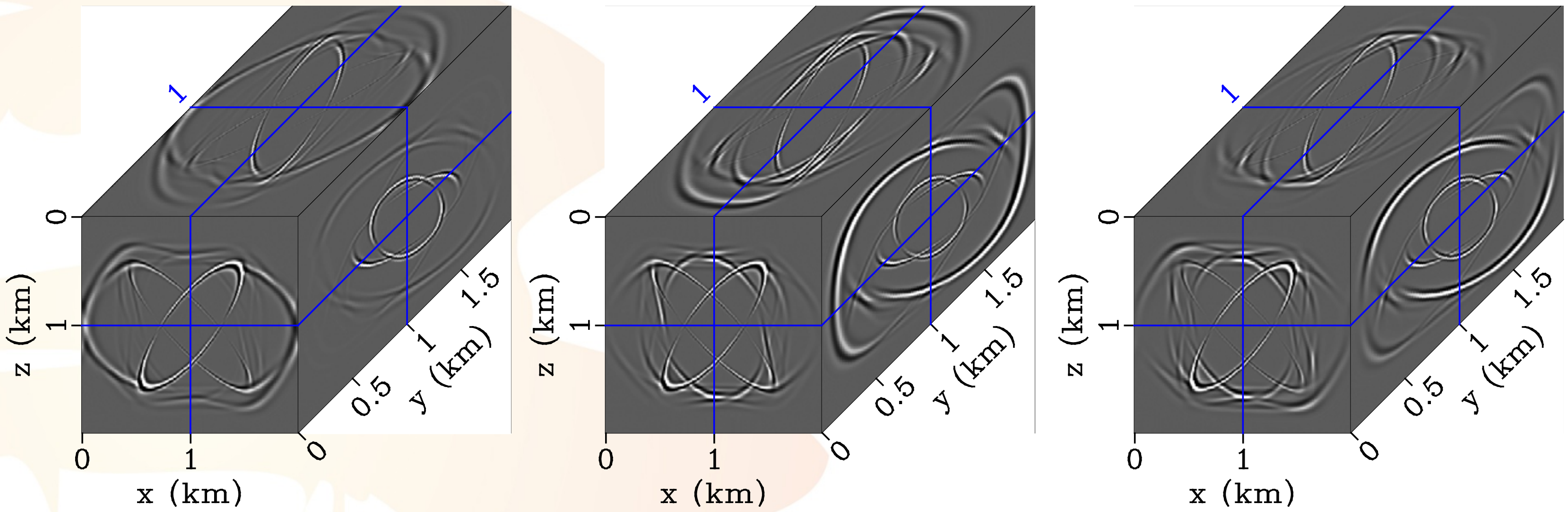


$ik_x \exp(ik_x 3\Delta x / 2)$

Efficiency comparison

scheme	PSM	RSG-PSM	interpolation	LG-PSM
CPU time	33.2s	33.4s	46.2s	56.1s





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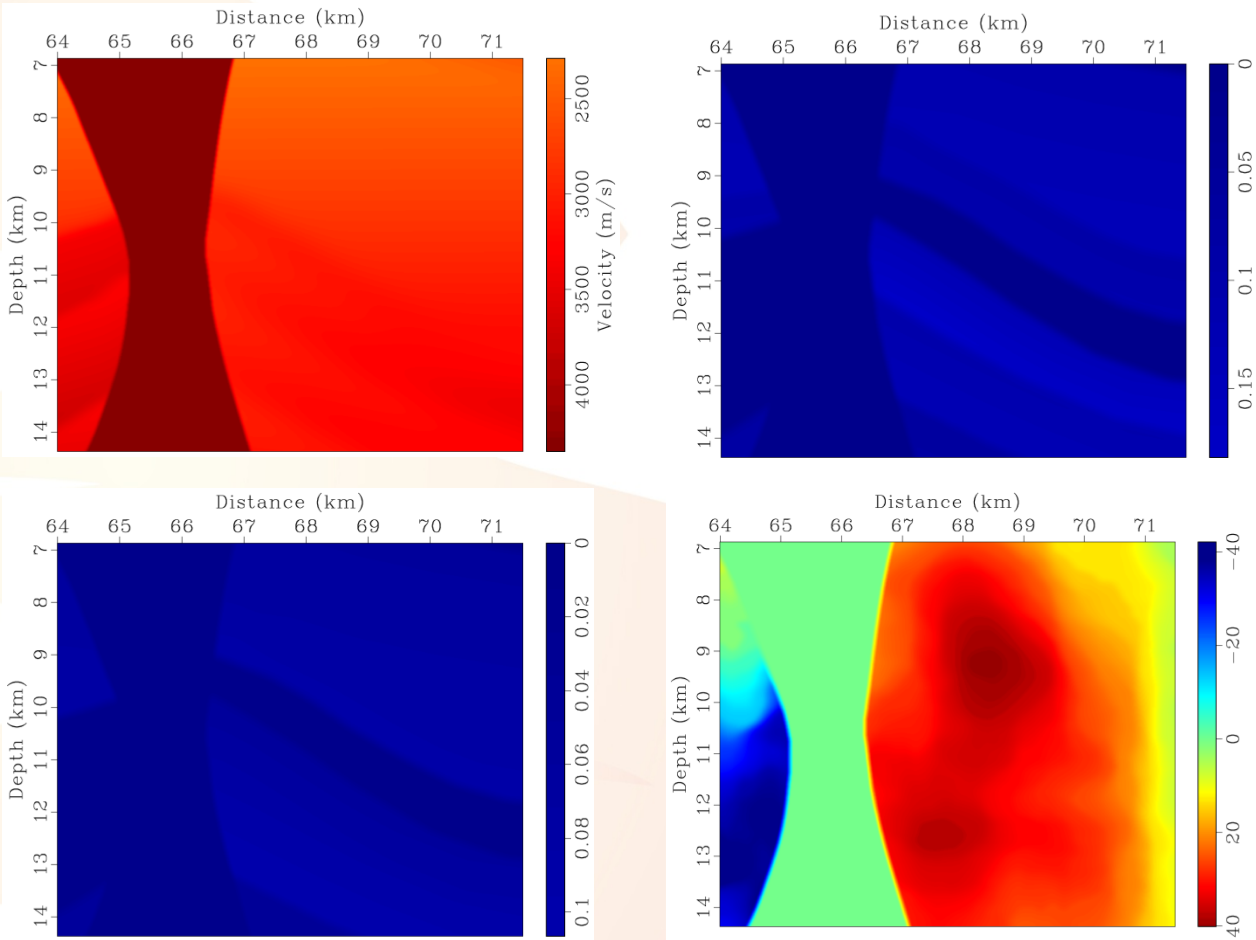
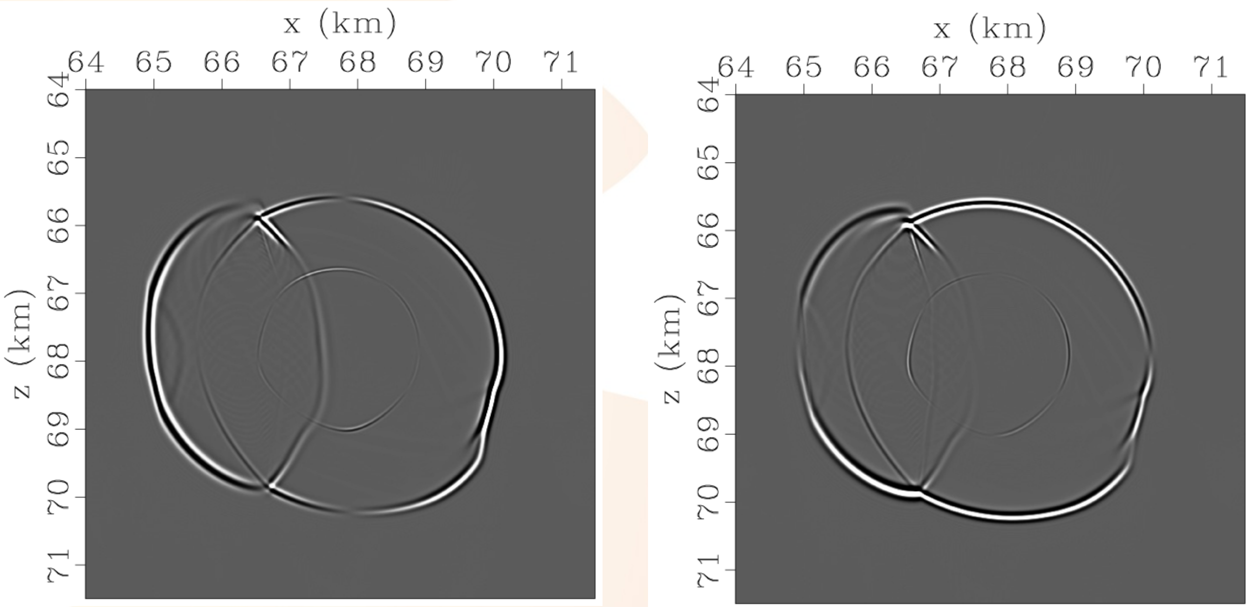

Distance (km)
64 65 66 67 68 69 70 71
Depth (km)
7 8 9 10 11 12 13 14
Velocity (m/s)
2500 3000 3500 4000
0 0.05 0.1 0.15
0 0.02 0.04 0.06 0.08 0.1
−40 −20 0 20 40


x (km)
64 65 66 67 68 69 70 71
z (km)
64 65 66 67 68 69 70 71
x (km)
64 65 66 67 68 69 70 71
z (km)
64 65 66 67 68 69 70 71