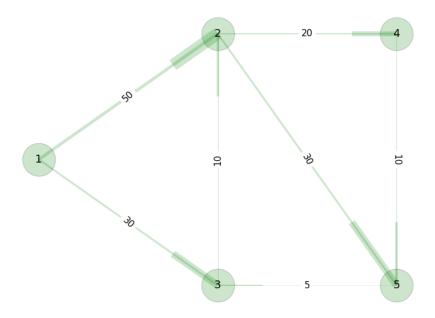
## flow\_network\_analysis

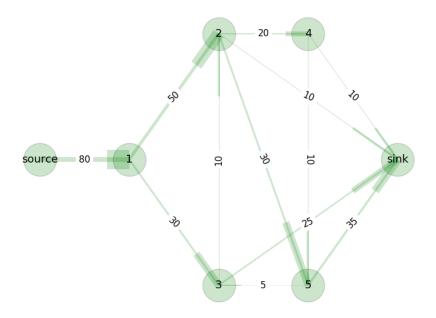
May 31, 2017

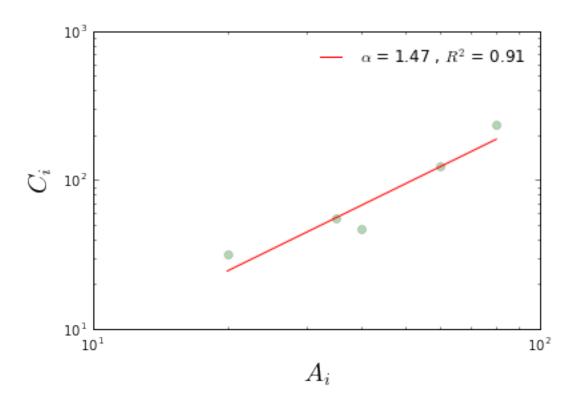
## 1 Example

```
In [11]: G = nx.DiGraph()
         G.add_edge(1, 2, weight = 50)
         G.add\_edge(1, 3, weight = 30)
         G.add\_edge(3, 2, weight = 10)
         G.add_edge(2, 4, weight = 20)
         G.add\_edge(2, 5, weight = 30)
         G.add_edge(5, 3, weight = 5)
         G.add\_edge(4, 5, weight = 10)
In [12]: fig = plt.figure(figsize=(16, 10),facecolor='white')
         pos={1: np.array([ 0.2, 0.5]),
          2: np.array([ 0.4, 0.8]),
          3: np.array([ 0.4, 0.2 ]),
          4: np.array([ 0.6, 0.8]),
          5: np.array([ 0.6, 0.2])}
         width=[float(d['weight']*.1) for (u,v,d) in G.edges(data=True)]
         edge_labels=dict([((u,v,),d['weight']) for u,v,d in G.edges(data=True)])
         nx.draw_networkx_edge_labels(G,pos,edge_labels=edge_labels, font_size = 15,alpha = .5)
         nx.draw(G, pos, node_size = 3000, node_color = 'green',
                 alpha = 0.2, width = width, edge_color='green',style='solid')
         nx.draw_networkx_labels(G,pos,font_size=18)
         plt.show()
```



```
In [13]: H = fn.flowBalancing(G)
In [24]: fig = plt.figure(figsize=(16, 10),facecolor='white')
        pos={1: np.array([ 0.2, 0.5]),
         2: np.array([ 0.4, 0.8]),
         3: np.array([ 0.4, 0.2 ]),
         4: np.array([ 0.6, 0.8]),
         5: np.array([ 0.6, 0.2]),
          'sink': np.array([ 0.8, 0.5]),
          'source': np.array([0, 0.5])}
         width=[float(d['weight']*.1) for (u,v,d) in H.edges(data=True)]
         edge_labels=dict([((u,v,),d['weight']) for u,v,d in H.edges(data=True)])
         nx.draw_networkx_edge_labels(H,pos,edge_labels=edge_labels, font_size = 15,alpha = .5)
        nx.draw(H, pos, node_size = 3000, node_color = 'green',
                alpha = 0.2, width = width, edge_color='green',style='solid')
         nx.draw_networkx_labels(H,pos,font_size=18)
        plt.show()
```





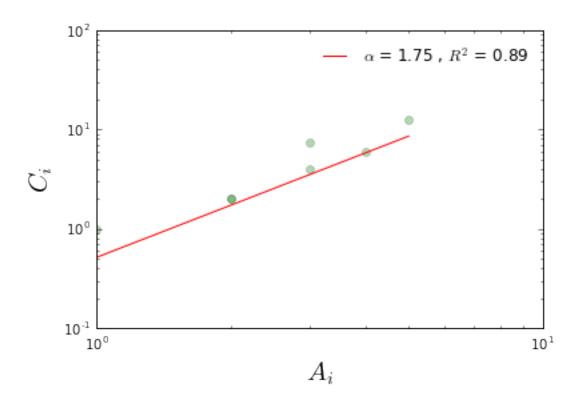
## 2 Demo Data

```
In [15]: demo = fn.attention_data
         demo
Out[15]: [['a', 0],
          ['a', 1],
          ['a', 2],
          ['b', 1],
          ['b', 2],
          ['c', 1],
          ['c', 2],
          ['c', 3],
          ['d', 2],
          ['d', 3],
          ['e', 0],
          ['e', 4],
          ['f', 0],
          ['f', 4],
          ['g', 0],
          ['g', 4],
          ['g', 5],
          ['h', 0],
          ['h', 5],
          ['i', 6]]
In [16]: gd = fn.constructFlowNetwork(demo)
In [17]: fig = plt.figure(figsize=(16, 10),facecolor='white')
         pos={0: np.array([ 0.6 , 0.8]),
          2: np.array([ 0.6, 0.2]),
          1: np.array([ 0.75, 0.6]),
          6: np.array([ 0.75, 0.4]),
          4: np.array([ 1, 0.8]),
          5: np.array([ 1, 0.5]),
          3: np.array([ 1, 0.2 ]),
          'sink': np.array([ 1.3, 0.5]),
          'source': np.array([.3, 0.5])}
         width=[float(d['weight']*1.2) for (u,v,d) in gd.edges(data=True)]
         edge_labels=dict([((u,v,),d['weight']) for u,v,d in gd.edges(data=True)])
         nx.draw_networkx_edge_labels(gd,pos,edge_labels=edge_labels, font_size = 15,alpha = .5)
         nx.draw(gd, pos, node_size = 3000, node_color = 'orange',
                 alpha = 0.2, width = width, edge_color='orange',style='solid')
         nx.draw_networkx_labels(gd,pos,font_size=18)
         for k, i in enumerate(demo):
             plt.text(-.1, .82-.035*k, '$%s \\rightarrow \%i$' \% (i[0], i[1]) , fontsize = 25)
         plt.arrow(.04, .5, .1, 0, color='orange', width=0.04, head_width = .07)
         plt.xlim(-.1, 1.5)
         plt.show()
```

```
a \rightarrow 0
a \rightarrow 1
a \rightarrow 2
b \rightarrow 1
b \rightarrow 2
c \rightarrow 1
c \rightarrow 2
c \rightarrow 3
d \rightarrow 2
d \rightarrow 3
                                                                                                                                                                                           sink
                                               source
e \rightarrow 0
e \rightarrow 4
f \rightarrow 0
f \rightarrow 4
q \rightarrow 0
g \rightarrow 4
g \rightarrow 5
h \rightarrow 0
h \rightarrow 5
i \rightarrow 6
```

```
In [6]: nx.info(gd)
Out[6]: 'Name: \nType: DiGraph\nNumber of nodes: 9\nNumber of edges: 15\nAverage in degree:
                                                                                            1.6667\nAv
In [7]: gh = fn.flowBalancing(gd)
       nx.info(gh)
Out[7]: 'Name: \nType: DiGraph\nNumber of nodes: 9\nNumber of edges: 15\nAverage in degree:
                                                                                            1.6667\nA
In [18]: m = fn.getFlowMatrix(gd)
Out[18]: matrix([[ 0., 1.,
                            0.,
                                 0., 3., 1., 0.,
                [ 0., 0.,
                            3.,
                                 0., 0.,
                                         0., 0.,
                                 2., 0.,
                                          0., 0.,
                            0.,
                       0.,
                            0.,
                                 0.,
                                     0.,
                                           0.,
                                               0.,
                                                    0.,
                            0.,
                                 0.,
                                     0.,
                                                    0.,
                                                         2.],
                [ 0.,
                       0.,
                                           1.,
                                               0.,
                       0.,
                                                    0.,
                [ 0.,
                            0.,
                                 0., 0.,
                                          0.,
                                               0.,
                                          0., 0.,
                            0.,
                                 0., 0.,
                            1., 0., 0., 0., 1.,
                                                    0.,
                                                         0.],
                            0.,
                                0., 0., 0., 0.,
In [19]: M = fn.getMarkovMatrix(m)
        М
Out[19]: array([[ 0.
                              0.2
                                           0.
                                                       0.
                                                                    0.6
                                                                 ],
                 0.2
                              0.
                                           0.
                                                       0.
               [ 0.
                              0.
                                           1.
                                                       Ο.
                                                                    0.
                 0.
                              0.
                                           0.
                                                       0.
```

```
[ 0.
                                 0.
                                               0.
                                                             0.5
                                                                        , 0.
                   0.
                                               0.
                                                             0.5
                                 0.
                                                                        ],
                 [ 0.
                                               0.
                                                             0.
                                                                           0.
                   0.
                                 0.
                                               0.
                                                             1.
                                                                        ],
                                               0.
                 [ 0.
                                 0.
                                                             0.
                                                                           0.
                                                             0.66666667],
                   0.33333333,
                                 0.
                                               0.
                 [ 0.
                                 0.
                                               0.
                                                             0.
                                                                           0.
                   0.
                                 0.
                                               0.
                                                             1.
                                                                        ],
                 [ 0.
                                 0.
                                               0.
                                                             0.
                                                                           0.
                   0.
                                 0.
                                               0.
                                                             1.
                                                                        ],
                                               0.11111111,
                 [ 0.5555556,
                                 0.2222222,
                                                             0.
                                                                           0.
                                                             0.
                   0.
                                 0.11111111,
                                               0.
                                                                        ],
                                                                        , 0.
                 [ 0.
                                 0.
                                               0.
                                                             0.
                                 0.
                                               0.
                   0.
                                                             0.
                                                                        ]])
In [21]: U = fn.getUmatrix(gh)
Out[21]: matrix([[ 1.
                                             , 0.2
                                                                         , 0.6
                                  0.2
                                                             0.1
                                             ],
                    0.4
                                  0.
                  [ 0.
                                  1.
                                                1.
                                                              0.5
                                                                            0.
                                             ],
                    0.
                                  0.
                  [ 0.
                                                              0.5
                                                                            0.
                                  0.
                                                1.
                    0.
                                  0.
                                             ],
                  [ 0.
                                  0.
                                                0.
                                                              1.
                                                                            0.
                    0.
                                  0.
                                             ],
                  [ 0.
                                  0.
                                                0.
                                                              0.
                                                                            1.
                    0.33333333,
                                  0.
                                             ],
                                                0.
                  [ 0.
                                  0.
                                                              0.
                                                                            0.
                                  0.
                                             ],
                    1.
                                                                          , 0.
                  [ 0.
                                  0.
                                                0.
                                                              0.
                    0.
                                  1.
                                             ]])
In [26]: ai, ci = fn.getAICI(gh).T
         fn.alloRegressPlot(ai,ci,'g','o','$A_i$','$C_i$', loglog=True)
```



```
In [27]: fn.networkDissipate(gh)
Out[27]: defaultdict(<function flownetwork.flownetwork.<lambda>>,
                     \{0: [0, 5, 5],
                      1: [0, 3, 2],
                      2: [2, 4, 1],
                      3: [2, 2, 0],
                      4: [2, 3, 0],
                      5: [2, 2, 0],
                      6: [1, 1, 1]})
In [28]: fn.flowDistanceFromSource(gd)
Out[28]: {0: 1.0,
          1: 1.33333333333333333333
          2: 2.0,
          3: 3.0,
          4: 2.0,
          5: 2.5,
          6: 1.0,
          'sink': 3.2222222222214}
In [29]: fn.averageFlowLength(gd)
Out[29]: 3.22222222222223
```

## 3 Visualization

10°

```
In [30]: from random import random
    x = np.array(range(1, 100))
    y = (x+random()*x)**3

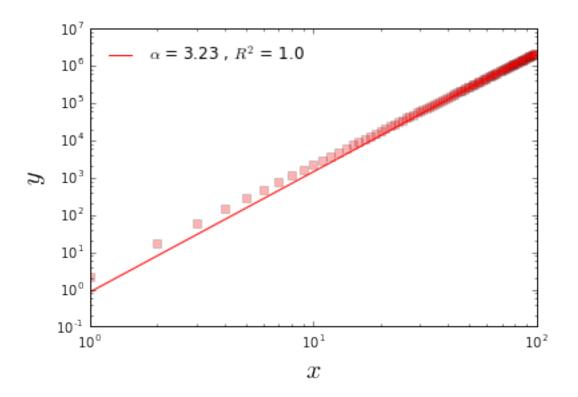
plt.plot(x, y, 'ro')
    plt.xscale('log');plt.yscale('log')
    plt.show()

107
100
103
104
100
100
```

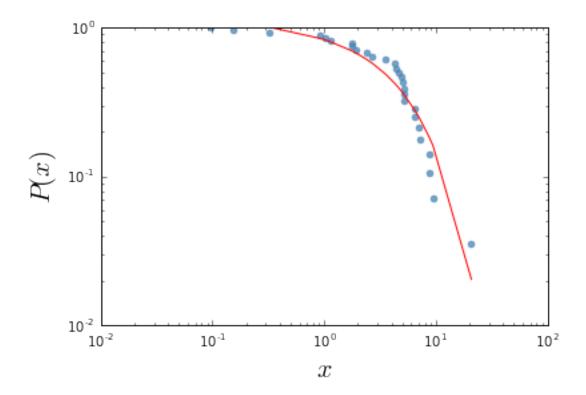
```
In [31]: fn.alloRegressPlot(x,y,'r','s','$x$','$y$', loglog=True)
```

10¹

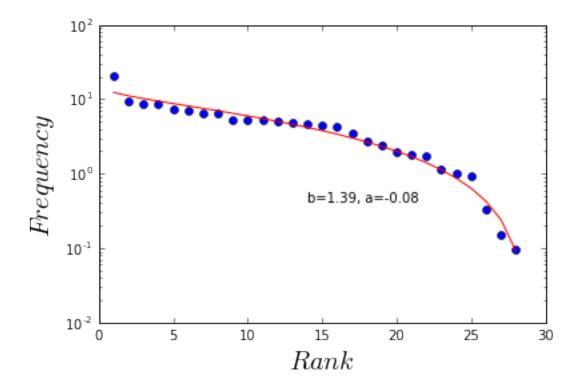
10<sup>2</sup>



```
In [32]: rg = np.array([ 20.7863444 ,  9.40547933,  8.70934714,  8.62690145,
             7.16978087,
                          7.02575052, 6.45280959,
                                                       6.44755478,
             5.16630287,
                           5.16092884,
                                         5.15618737,
                                                       5.05610068,
             4.87023561,
                           4.66753197,
                                         4.41807645,
                                                       4.2635671 ,
             3.54454372,
                           2.7087178 ,
                                         2.39016885,
                                                       1.9483156,
                           1.75432688,
             1.78393238,
                                         1.12789787,
                                                       1.02098332,
             0.92653501,
                           0.32586582,
                                         0.1514813 ,
                                                       0.09722761])
        fn.powerLawExponentialCutOffPlot(rg, '$x$', '$P(x)$')
Out[32]: [-0.0099301962503268171,
         -0.064764460567964449,
         -0.17705123513352666,
         0.89999847894045781]
```



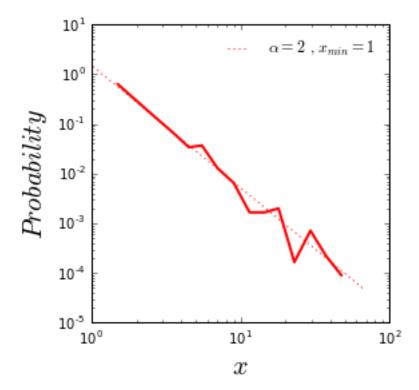
In [33]: fn.DGBDPlot(rg)



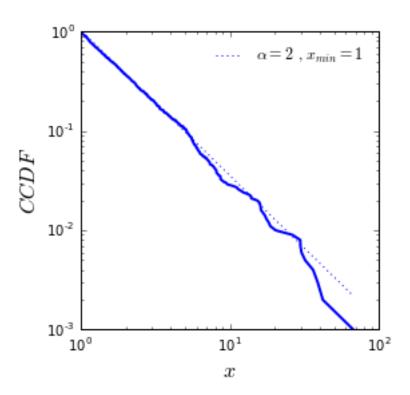
```
In [34]: from networkx.utils import powerlaw_sequence
     pl_sequence = powerlaw_sequence(1000,exponent=2.5)

fig = plt.figure(figsize=(4, 4),facecolor='white')
    ax = fig.add_subplot(111)
    fn.plotPowerlaw(pl_sequence,ax,'r','$x$')
```

Calculating best minimal value for power law fit



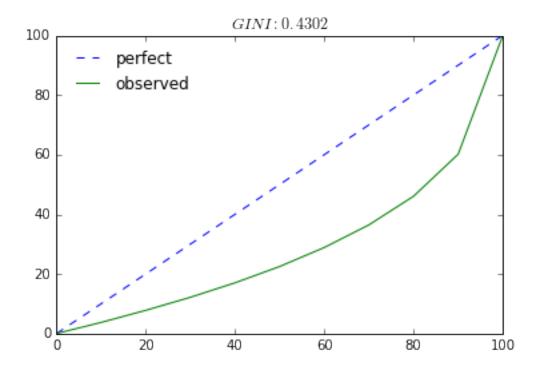
Calculating best minimal value for power law fit



```
In [36]: bins, result, gini_val = fn.gini_coefficient(np.array(pl_sequence))

plt.plot(bins, bins, '--', label="perfect")
   plt.plot(bins, result, label="observed")
   plt.title("$GINI: %.4f$" %(gini_val))

plt.legend(loc = 0, frameon = False)
   plt.show()
```



In []: