Math 241 Homework#7

due 10/24 Thursday in class

Heat equation

Read *A*pplied PDE by Haberman (5th edition) Chapter 5, 5.1-5.6. Main theorem on page 157 and examples 5.3.3.

1. *A*pplied PDE by Haberman, chapter 5.3, exercise 5.3.3.
2. *A*pplied PDE by Haberman, chapter 5.3, exercise 5.3.8.
3. *A*pplied PDE by Haberman, chapter 5.3, exercise 5.3.9.

   Hint: Euler's formula gives the following identity

   $$x^{\sqrt{-\lambda}} = e^{\sqrt{-1}(\sqrt{\lambda}\log x)} = \cos(\sqrt{\lambda}\log x) + \sqrt{-1}\sin(\sqrt{\lambda}\log x)$$

4. *A*pplied PDE by Haberman, chapter 5.4, exercise 5.4.3.
5. *A*pplied PDE by Haberman, chapter 5.4, exercise 5.4.4.
6. Consider the eigenvalue problem

   $$\phi''(x) - 2x\phi'(x) + \lambda\left(x^2+1\right)\phi(x) = 0, \quad x \in [0,2]$$

   $$\phi(0) = 0, \quad \phi'(2) = 0.$$

   Let $\lambda_1 < \lambda_2 < \lambda_3 < \cdots$ be all the eigenvalues of the above equation. Suppose that for $n \geq 1$ the eigenfunction $\phi_n(x)$ that corresponds to the eigenvalue λ_n is chosen so that

   $$\int_0^2 \phi_n^2(x)e^{-x^2}\left(x^2+1\right)dx = 2$$

   a) Is this a regular Sturm-Liouville problem? Justify your answer.
   b) If $a_n = \int_0^2 \left(x^3+2x^2-1\right)\phi_n(x)e^{-x^2}\left(x^2+1\right)dx$, calculate $\sum_{n=1}^{\infty} a_n\phi_n(1)$. Carefully explain your answer.

7. Fall 2015 final exam, problem 4.