HW 5. Problem 7.

Artin. 4.10 a) chapter 10.

	(1)	(4)	(5)	(3)	(5⁻)
	1	a	b	c	d
χ_1	1	1	1	1	1
χ_2	1	1	−1	−1	1
χ_3	1	1	$-i$	i	−1
χ_4	1	1	$-i$	i	−1
χ_5	a_1	a_2	a_3	a_4	a_5.

→ elements in group G.

χ_5?

$\rho(e) = \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix}$

$\chi(e) =$ dim

Missing row use second orthogonal relations.

First column $1^2 + 1^2 + 1^2 + 1^2 + a_1^2$

$= |G| = 1+4+3+8+4$

$= 20.$

$a_1^2 = 16, \quad a_1 = 4.$

outher column pair with first column. $= 0.$

some equation for $a_2, a_3, a_4, a_5.$

HW 5. Problem 2.

When dim = 1, character is a group homomorphism.

$\chi: G \to \mathbb{C}^* = GL(1)$

$\rho: G \to GL(1) = \mathbb{C}^*$ · group homo.

$\operatorname{tr}\rho = \chi$. when dim = 1

when dim ≠ 1. $\chi: G \to \mathbb{C}$.

① $\operatorname{Im}\chi$ may not be inside $\mathbb{C}^*$

$\chi(g)$ can be zero.

② $\chi(e) = \dim \rho \neq 1$ when dim ≠ 1.

HW 6 Problem 1.

$$[G, G] \subset \bigcap_{\dim \chi = 1} \ker \chi$$

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Find enough χ, $\dim \chi = 1$.

$G / [G, G]$ is abelian group.

has only 1-dim'l irreducible characters.

$\chi_1, \cdots \chi_s$. $s = |G / [G, G]|$

$\chi_1 \cdots \chi_s$ can be lifted to characters of G.

$\tilde{\chi}_i : G \xrightarrow{\pi} G / [G, G] \xrightarrow{\chi_i} \mathbb{C}$

$\ker \tilde{\chi}_i = \pi^{-1}(\ker \chi_i)$.