Math 241 Homework#2

due 9/12 Thursday in class

Heat equation

Read *Applied PDE by Haberman (5th edition) Chapter 1.4-1.5.

1. *Applied PDE by Haberman, chapter 1.4, exercise 1.4.1
2. *Applied PDE by Haberman, chapter 1.4, exercise 1.4.7
3. *Applied PDE by Haberman, chapter 1.4, exercise 1.4.10
4. Prove that there always exists an equilibrium solution to heat equation in 1D with constant prescribed temperature at the boundary points. Assume the equation is

$$u_t = u_{xx} + Q(x)$$

   with $u(0,t) = T_1$ and $u(L,t) = T_2$.

5. Let $u(x, y)$ be a function in $2D$. Prove the Laplacian of $u(x, y)$ has the following form under polar coordinates

$$\Delta u = \nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

6. Consider the Laplace equation on the region $D = \{(x, y)|1 \leq x^2 + y^2 \leq 9\}$.

$$\Delta u = 0$$

   with Neumann boundary conditions $\frac{\partial u}{\partial r} = 2$ when $r = 1$ and $\frac{\partial u}{\partial r} = \frac{2}{3}$ when $r = 3$.

   (a) Find one solution u_0.

   (b) Prove that any solution u has the form $u = u_0 + C$ with a constant C.

7. *Applied PDE by Haberman, chapter 1.5, exercise 1.5.8.
8. *Applied PDE by Haberman, chapter 1.5, exercise 1.5.9. (There is not heat sources inside the annulus.)

   (One more problem on the second page)

Calculus

1. Integrate by parts twice to prove the following identities for $n, m \in \mathbb{N}$:

$$\int_0^{2\pi} \sin nx \cos mx \ dx = 0$$

$$\int_0^{2\pi} \sin nx \sin mx \ dx = \begin{cases} 0 & n \neq m \\ \pi & n = m \end{cases}$$

$$\int_0^{2\pi} \cos nx \cos mx \ dx = \begin{cases} 0 & n \neq m \\ \pi & n = m \end{cases}$$