Math 241 Homework#6

due 10/15 Tuesday in class

Heat equation

Read *A*pplied PDE by Haberman (5th edition) Chapter 4.

1. *A*pplied PDE by Haberman, chapter 4.4, exercise 4.4.9.
2. *A*pplied PDE by Haberman, chapter 4.4, exercise 4.4.11.
3. Using separation of variables, solve the wave equation $u_{tt} = c^2 u_{xx}$ for $0 < x < L$ with the following boundary conditions and initial conditions:

$$u(0,t) = 0, u(L,t) = 0, u(x,0) = \sin\frac{2\pi x}{L} + 7\sin\frac{5\pi x}{L}, u_t(x,0) = 2\sin\frac{3\pi x}{L} + 4\sin\frac{6\pi x}{L}.$$

4. Using the d'Alembert solution, solve the wave equation $u_{tt} = c^2 u_{xx}$ for $-\infty < x < +\infty$ with the following initial conditions:
   (a) $u(x,0) = \sin x, u_t(x,0) = \cos x$.
   (b) $u(x,0) = x, u_t(x,0) = x^2$.
5. [Causality Principle/Domain of dependence] Say $u_{tt} = 100u_{xx}$ for all $-\infty < x < \infty$ with $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$. Find the largest interval $J = \{a \leq x \leq b\}$ where modifying f or g inside this interval can change the value of $u(3,6)$.
6. Consider wave equation $u_{tt} = c^2 u_{xx}$, $0 \leq x \leq L$ with boundary conditions $u(0,t) = u(L,t) = 0$.
   (a) Show that the energy $E(t)$ is constant. (See Haberman, chapter 4.4, exercise 4.4.9. for definition)
   (b) If $u(x,0) = 0$ and $u_t(x,0) = 0$, what can you conclude? (This is related to uniqueness of solutions to wave equation)
7. Fall 2016 final exam, problem 7.