

Math 241 Homework#4
due 10/03 Thursday in class

Heat equation

Read Applied PDE by Haberman (5th edition) Chapter 3.1-3.3.

1. Applied PDE by Haberman, chapter 3.2, exercise 3.2.1 (f).
2. Applied PDE by Haberman, chapter 3.2, exercise 3.2.2 (e).
3. Applied PDE by Haberman, chapter 3.3, exercise 3.3.1 (a).
4. Consider the function $f(x) = x^2$ on $[-L, L]$.
 - (a) Write down the Fourier series of $f(x)$.
 - (b) Use this series at $x = L$ and Fourier's Theorem to prove the following expression

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$$

This is called Basel's problem and first solved by Euler.