# Tag Integrated spatial model equations

*Process dynamics model*

This model assumes an annual time-step cycle which applies the following dynamics

1. Recruitment and release tags if we tag in this year
2. Total mortality and ageing
3. Markovian movement
4. Tag-shedding

The untagged partition has four attributes, age (), region (), year (), and sex (). The recruitment, mortality and ageing process dynamics are applied following,

Where is the annual recruitment for region and

* is the sex and year specific fishery selectivity
* is the annual fishing mortality rate for fishery
* is the annual natural mortality rate.

Annual fishing mortality rates are calculating each year using a Newton Raphson iterative algorithm based on the known catch. This is analogous to Stock Synthesis hybrid fishing mortality process (**Add link to manual or technical appendix**)

Once recruitment, ageing and total mortality have taken place the model applies a Markovian movement dynamic as

Where is a vector of numbers at age for a sex across all regions, is a movement matrix, and denotes the numbers at age and sex after the movement process has been applied.

The recruitment dynamics does not assume a stock recruitment relationship and is parameterized as,

Where, is the estimable average recruitment for region , is an estimable regional annual recruitment deviation and is the assumed recruitment deviation variance. The model applies the following penalty to the objective function,

Then tagged fish are in the partition, they have the same ageing, mortality and growth assumptions as the untagged members of the partition. When tagged fish are released into the partition they are indexed by the release event index denoted by which is region and year specific (). Tag induced mortality and initial tag-loss is applied as an initial mortality rate denoted by ,

Ongoing annual tag-shedding denoted by is also applied as a mortality process whereby tagged fish are deleted from the partition as

It is assumed that tagged fish have known age and sex. In the sablefish application this is done by using the survey age-length key which is the main method for removals.

*Initialization*

An equilibrium age structure is derived by running the annual cycle times with i.e., no fishing mortality. This populates the numbers at age for all regions except the plus age cohort. The annual cycle is then run with one more time to calculate the number of individuals that moved into each sex and regions plus age cohort (), denoted by . This will be the result of ageing, mortality and movement. The equilibrium plus age cohort for region is calculated assuming an infinite geometric series with solution,

After the equilibrium, age-structure is calculated. If the model starts after known fishing exploitation has occurred. There is the option to also calculate an initial age-structure, which repeats the above but, will apply, a total mortality with some estimable that assumes the fixed gear fishery selectivity. Once the initial age-structure has been calculated, there is an option to estimate age specific deviation to allow the model to start with a non-equilibrium age-structure denoted by ,

To help with estimation there is a penalty on that assumes a central tendency of zero with an estimable variance parameter ()

*Growth*

Mean length at age () and mean weight at age () for each sex and year are assumed known without error. These are used to convert numbers into weight, such as in spawning stock, relative index of biomass and catch calculations. Growth is not assumed to vary among regions in this model. Age-length transition matrices for each sex and year denoted by is used to convert numbers at age to numbers at length is also assumed known without error.

Spawning stock biomass (SSB) calculations are a female only quantity and calculated as,

Where, is the proportion of mature females in age cohort . The total biomass is also outputted, which is the total weight of both female and male fish in a region and year

*Selectivity*

Both the fixed gear and the survey assume a logistic selectivity,

The trawl fishery was assumed to be a domed gamma selectivity,

*Observation model*

*Catch at age*

Fishery dependent catch at age observations are available for the fixed gear fishery, but are also needed to calculate catch at length observations for the trawl fishery. Catch at age for fishery  is denoted by  and model fitted values are calculated following

Fitted catch at age numbers are normalized for a given year and region to sum to one over sex and age following,

The likelihood assumed for catch-at-age observations is the Dirichlet-multinomial, where

Where, is an estimable over-dispersion parameter and is a vector of numbers over the sex and age dimensions, has the same dimension.

*Catch at length*

Catch at length are fishery dependent length frequencies denoted by . It uses the catch-at-age calculations, but converts them to numbers at length using the age-length transition matrix before normalizing.

Fitted catch at length numbers are normalized for a given year and region to sum to one over sex and age following,

The likelihood assumed for catch-at-length observations is the Dirichlet-multinomial, where

Where, is an estimable over-dispersion parameter and is a vector of numbers over the sex and length bin dimensions, has the same dimension.

*Survey age composition*

Numbers at age for the survey () are calculated as,

Where, is the proportion of the year when the survey occurs and represents the survey selectivity.

Fitted catch at age numbers are normalized for a given year and region to sum to one over sex and age following,

The likelihood assumed for catch-at-age observations is the Dirichlet-multinomial, where

Where, is an estimable over-dispersion parameter and is a vector of numbers over the sex and age dimensions, has the same dimension.

*Survey index of abundance*

A relative index of biomass is calculated using the survey age calculations. The index in year and region is denoted by . Fitted values are calculated as,

Where, is the survey catchability. This is assumed to be distributed according to the lognormal distribution,

Where, is the standard deviation which is derived from standard errors from a design-based estimator.

*Catch biomass*

Observed fishery catch is also treated an observation even though annual fishing mortality rates are calculated assuming catch is known without error using a Newton Raphson minimization routine. Model fitted catch biomass is calculated as,

This is also treated as a lognormal random variable,

*Tag recovery observations*

All tag-recoveries were assumed to be from the fixed gear (hook and line and pot) fishery. For each tag-release group denoted by , there were possible recovery events, where is the number of regions in the model and is the number of years that we track the tagged cohorts in the model. We didn’t consider tag-recoveries in the first year of release to allow mixing assumptions to be satisfied. Each potential recovery event was indexed by which has an implied year and region of recovery (). Model fitted tag-recoveries for tag-release group in recovery event were calculated as,

Where, is the fixed gear tag reporting rate and is the fishing mortality for the fixed gear fishery. Tag-recoveries were assumed to be distributed according to the negative binomial likelihood as,

Where is the estimable over dispersion parameter for all tag recovery events.

*Index definitions*

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| Symbol | Definition | Estimable (Y/N) |
| **Model index’s** | | |
|  | Age index for real ages | N |
|  | Minimum age in the model, can be different from 1 | N |
|  | Maximum age in the model, which is an accumulating age cohort | N |
|  | Number of ages. Length of | N |
|  | Region index | N |
|  | Number of regions | N |
|  | Year index | N |
|  | Number of years | N |
|  | Sex index is males and is females | N |
|  | Length bin index | N |
|  | Number of length bins | N |
|  | Number of years tagged cohorts are tracked before going into an accumulating cohort where only region of release is known, not year of release | N |
| **Model Quantities** | | |
|  | Numbers at age by sex, year, and region |  |
|  | Numbers at age by sex, year, region for release event |  |
|  | Mean weight at age | N |
|  | Mean length at age | N |
|  | Age-length transition matrix | N |
|  | Proportion female mature | N |
|  | Selectivity for fishery | Y |
|  | Selectivity for survey | Y |
|  | Annual recruitment |  |
|  | Annual natural mortality | N |
|  | Annual fishing mortality |  |
|  | Annual total mortality | N |
|  | Annual movement transition matrix | Y |
|  | Survey year proportion (timing) | N |
| **Observations** | | |
|  | Observed numbers over age and sex from the fishery | N |
|  | Observed numbers over length and sex from the fishery | N |
|  | Observed numbers over age and sex from the survey | N |
|  | Observed survey biomass | N |
|  | Observed fishery catch biomass | N |
|  | Observed tag recoveries from release event and recovery event | N |
| **Estimated Parameters** | | |
|  | Mean recruitment | Y |
|  | Initial age-deviations | Y |
|  | Recruitment annual deviations | Y |
|  | Tag-reporting rate | Y |
|  | Survey catchability | Y |
|  | Initial fishing mortality | Y |
|  | Negative binomial over-dispersion parameter for all tag-recoveries | Y |
|  | Negative binomial over-dispersion parameter for survey numbers at age | Y |
|  | Negative binomial over-dispersion parameter for fishery numbers at age | Y |
|  | Negative binomial over-dispersion parameter for fishery numbers at length | Y |
|  | Recruitment variance | Y |
|  | Initial age deviation variance | Y |
| **Input Parameters** | | |
|  | Natural mortality | 0.1048 |
|  | Initial tag mortality | 0.1 |
|  | Annual tag-shedding | 0.02 |
|  | Standard deviation for survey biomass index |  |
|  | Standard deviation for fishery catch |  |
|  | Recruitment variance | 1.2 |
|  | Initial age deviation variance | 1.2 |

# 5A Base operating model (OM) parameter values

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