深蓝学院
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第三讲作业思路分享

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思路提示

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- Q4：SE(3)映射推导

$$\xi = \begin{bmatrix} \rho \\ \phi \end{bmatrix}$$

$$\xi^\wedge = \begin{bmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{bmatrix}$$

$$\begin{bmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{bmatrix} \cdot \begin{bmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{bmatrix} = \begin{bmatrix} (\phi^\wedge)^2 & \phi^\wedge \rho \\ 0^T & 0 \end{bmatrix}$$

$$\begin{bmatrix} (\phi^\wedge)^2 & \phi^\wedge \rho \\ 0^T & 0 \end{bmatrix} \cdot \begin{bmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{bmatrix} = \begin{bmatrix} (\phi^\wedge)^3 & (\phi^\wedge)^2 \rho \\ 0^T & 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{bmatrix}\right)^n = \left(\begin{bmatrix} (\phi^\wedge)^n & (\phi^\wedge)^{n-1} \rho \\ 0^T & 0 \end{bmatrix}\right)$$

需要展开计算

$$\exp\left(\begin{bmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{bmatrix}\right) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\begin{bmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{bmatrix}\right)^n$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{n!} \left(\begin{bmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{bmatrix}\right)^n = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} + \sum_{n=1}^{\infty} \frac{1}{n!} \left(\begin{bmatrix} (\phi^\wedge)^n & (\phi^\wedge)^{n-1} \rho \\ 0^T & 0 \end{bmatrix}\right)$$

思路提示

- Q5：证明好难

首先证明：$\boldsymbol{R a}^{\wedge} \boldsymbol{R}^{\mathrm{T}}=(\boldsymbol{R a})^{\wedge}$

推荐下面这个方法（**from 黑熊**）：

假设有三个向量 $a,b,c \in \mathbb{R}^3$，且$a = b^\wedge c$

步骤1：三个向量同时旋转R到另一个坐标系下

$$(Ra)(Rb)=(Rc)$$

$$\Downarrow$$

$$a = R^{-1}(Rb)^\wedge (Rc)$$

步骤2：与原始关系对比

$$\begin{cases} a = b^\wedge c \\ a = R^{-1}(Rb)^\wedge (Rc) \end{cases}$$

$$\Downarrow$$

$$b^\wedge = R^{-1}(Rb)^\wedge R$$

步骤3：接着变换（R的正交性）

$$R^{-1} = R^T$$

$$Rb^\wedge R^T = (Rb)^\wedge$$

$$\Downarrow$$

$$Ra^\wedge R^T = (Ra)^\wedge$$

思路提示

- Q5：证明好难

接着证明：$\boldsymbol{R}\exp\left(\boldsymbol{p}^{\wedge}\right)\boldsymbol{R}^{\mathrm{T}}=\exp\left((\boldsymbol{R}\boldsymbol{p})^{\wedge}\right)$

然后就很简单了（**from 黑熊**）：

步骤1：利用罗格里斯公式展开 $\exp\left(p^{\wedge}\right)$

$$R\exp\left(p^{\wedge}\right)R^{T}=R\left[\cos\theta\cdot I+(1-\cos\theta)\cdot aa^{T}+\sin\theta\cdot a^{\wedge}\right]R^{T}$$

$$=\cos\theta\cdot RR^{T}+(1-\cos\theta)\cdot Raa^{T}R^{T}+\sin\theta\cdot Ra^{\wedge}R^{T}$$

$$\boxed{=\cos\theta\cdot I+(1-\cos\theta)\cdot Ra(Ra)^{T}+\sin\theta\cdot(Ra)^{\wedge}}$$

还是罗格里斯的展开形式

步骤2：Ra看作一个向量

$$R\exp\left(p^{\wedge}\right)R^{T}=\exp\left((R\theta a)^{\wedge}\right)$$

$$=\exp\left((Rp)^{\wedge}\right)$$

$$p=\theta a$$

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- Q6：证明好难

BCH近似很重要： →

$$\frac{\partial \ln\left(R_1R_2\right)^{\vee}}{\partial R_1}$$

1. exp近似（右扰动）：

$$\exp\left(\Delta\varphi^{\wedge}\right) = \mathrm{I} + \Delta\varphi^{\wedge}$$

$$\exp\left(\varphi^{\wedge} + \Delta\varphi^{\wedge}\right) = \exp\left(\varphi^{\wedge}\right)\exp\left(J_r\left(\varphi^{\wedge}\right)\cdot\Delta\varphi^{\wedge}\right)$$

$$\exp\left(\varphi^{\wedge}\right)\exp\left(\Delta\varphi^{\wedge}\right) = \exp\left(\varphi^{\wedge}\cdot J_r^{-1}\left(\varphi^{\wedge}\right)\cdot\Delta\varphi^{\wedge}\right)$$

2. log近似（右扰动）：

$$\begin{aligned}\ln\left(R\cdot\Delta R\right)^{\vee} &= \ln\left(\exp\left(\varphi^{\wedge}\right)\cdot\exp\left(\Delta\varphi^{\wedge}\right)\right)\\ &= \ln\left(\exp\left(\varphi^{\wedge} + J_r^{-1}\left(\varphi^{\wedge}\right)\cdot\Delta\varphi^{\wedge}\right)\right)\\ &= \varphi^{\wedge} + J_r^{-1}\left(\varphi^{\wedge}\right)\cdot\Delta\varphi^{\wedge}\end{aligned}$$

步骤1：添加右扰动（注意谁是变量）

$$\frac{\partial \ln\left(R_1R_2\right)^{\vee}}{\partial R_1} = \lim_{\Delta\varphi\to 0}\frac{\ln\left(R_1\exp\left(\Delta\varphi^{\wedge}\right)R_2\right)^{\vee} - \ln\left(R_1R_2\right)^{\vee}}{\Delta\varphi}$$

步骤2：Vector近似，把扰动从exp内提取出来

$$= \lim_{\Delta\varphi\to 0}\frac{\ln\left(R_1\left(\mathrm{I}+\Delta\varphi^{\wedge}\right)R_2\right)^{\vee} - \ln\left(R_1R_2\right)^{\vee}}{\Delta\varphi}$$

$$= \lim_{\Delta\varphi\to 0}\frac{\ln\left(R_1R_2 + R_1\Delta\varphi^{\wedge}R_2\right)^{\vee} - \ln\left(R_1R_2\right)^{\vee}}{\Delta\varphi}$$

思路提示

- Q6：证明好难

BCH近似很重要： → $\dfrac{\partial \ln\left(R_1 R_2\right)^\vee}{\partial R_1}$

1. exp近似（右扰动）：

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$$\begin{aligned}\ln\left(R\cdot\Delta R\right)^\vee &= \ln\left(\exp\left(\varphi^\wedge\right)\cdot\exp\left(\Delta\varphi^\wedge\right)\right)\\ &= \ln\left(\exp\left(\varphi^\wedge + J_r^{-1}\left(\varphi^\wedge\right)\cdot\Delta\varphi^\wedge\right)\right)\\ &= \varphi^\wedge + J_r^{-1}\left(\varphi^\wedge\right)\cdot\Delta\varphi^\wedge\end{aligned}$$

步骤3：把R_1R_2放在同一边才能继续展开，所以

上一题的结论：$\boxed{\boldsymbol{R}\boldsymbol{a}^\wedge\boldsymbol{R}^{\mathrm{T}} = (\boldsymbol{R}\boldsymbol{a})^\wedge}$

$$= \lim_{\Delta\varphi\to 0}\frac{\ln\left(R_1R_2 + R_1\Delta\varphi^\wedge R_2\right)^\vee - \ln\left(R_1R_2\right)^\vee}{\Delta\varphi}$$

$$= \lim_{\Delta\varphi\to 0}\frac{\ln\left(R_1R_2 + \boxed{R_1\Delta\varphi^\wedge R_1^T R_1 R_2}\right)^\vee - \ln\left(R_1R_2\right)^\vee}{\Delta\varphi}$$

$$= \lim_{\Delta\varphi\to 0}\frac{\ln\left(\left(I + \left(R_1\Delta\varphi\right)^\wedge\right)R_1R_2\right)^\vee - \ln\left(R_1R_2\right)^\vee}{\Delta\varphi}$$

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- Q6：证明好难

BCH近似很重要：→ $\dfrac{\partial \ln\left(R_1R_2\right)^{\vee}}{\partial R_1}$

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$$\exp\left(\varphi^{\wedge}\right)\exp\left(\Delta\varphi^{\wedge}\right) = \exp\left(\varphi^{\wedge}\cdot J_r^{-1}\left(\varphi^{\wedge}\right)\cdot\Delta\varphi^{\wedge}\right)$$

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$$\ln\left(R\cdot\Delta R\right)^{\vee} = \ln\left(\exp\left(\varphi^{\wedge}\right)\cdot\exp\left(\Delta\varphi^{\wedge}\right)\right)$$

$$= \ln\left(\exp\left(\varphi^{\wedge} + J_r^{-1}\left(\varphi^{\wedge}\right)\cdot\Delta\varphi^{\wedge}\right)\right)$$

$$= \varphi^{\wedge} + J_r^{-1}\left(\varphi^{\wedge}\right)\cdot\Delta\varphi^{\wedge}$$

步骤4：近似反推

$$= \lim_{\Delta\varphi\to 0}\frac{\ln\left(\left(I + \left(R_1\Delta\varphi\right)^{\wedge}\right)R_1R_2\right)^{\vee} - \ln\left(R_1R_2\right)^{\vee}}{\Delta\varphi}$$

$$= \lim_{\Delta\varphi\to 0}\frac{\ln\left(\exp\left(R_1\Delta\varphi\right)^{\wedge} R_1R_2\right)^{\vee} - \ln\left(R_1R_2\right)^{\vee}}{\Delta\varphi}$$

思路提示

- Q6：证明好难

BCH近似很重要：

$$\frac{\partial \ln(R_1R_2)^\vee}{\partial R_1}$$

1. exp近似（右扰动）：

$$\exp(\Delta\varphi^\wedge) = \mathrm{I} + \Delta\varphi^\wedge$$

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2. log近似（右扰动）：

$$\ln(R\cdot\Delta R)^\vee = \ln\left(\exp(\varphi^\wedge)\cdot\exp(\Delta\varphi^\wedge)\right)$$

$$= \ln\left(\exp\left(\varphi^\wedge + J_r^{-1}(\varphi^\wedge)\cdot\Delta\varphi^\wedge\right)\right)$$

$$= \varphi^\wedge + J_r^{-1}(\varphi^\wedge)\cdot\Delta\varphi^\wedge$$

步骤5：把指数部分看作一个左扰动

$$= \lim_{\Delta\varphi\to 0}\frac{\ln\left(\exp(R_1\Delta\varphi)^\wedge R_1R_2\right)^\vee - \ln(R_1R_2)^\vee}{\Delta\varphi}$$

$$= \lim_{\Delta\varphi\to 0}\frac{\ln(R_1R_2) + J_l^{-1}(R_1R_2)(R_1\Delta\varphi) - \ln(R_1R_2)^\vee}{\Delta\varphi}$$

$$= J_l^{-1}\left(\boxed{R_1R_2}\right)R_1$$

注意这里是说左雅可比里面的旋转

思路提示

- Q6：轨迹绘制

头文件：

```
// TODO inlcude thirdparty
#include <Eigen/Core>
#include <sophus/se3.hpp>          // Change se3.h->se3.hpp if install with template
#include <pangolin/pangolin.h>
```

类型更改：

```
// function for plotting trajectory, don't edit this code
// start point is red and end point is blue
void DrawTrajectory(vector<Sophus::SE3d, Eigen::aligned_allocator<Sophus::SE3d>>);
```

思路提示

- Q7：轨迹绘制

建议：要多用glog、gflags呀！

gflags设定输入路径：

```
DEFINE_string(traj_path,
              "/home/jamesgzl/桌面/SLAM_tutorial/lesson3/draw_traj/trajectory.txt",
              "Path to load trajectory.txt");
```

glog打印error：

```
(base) [redacted]:~/桌面/SLAM_tutorial/lesson3/draw_traj/build/Output$ ./draw_traj --traj_path=test.txt
E20220919 18:59:25.565698 19582 draw_trajectory.cpp:40] Please check out the path of the trajectory.txt!
(base) [redacted]:~/桌面/SLAM_tutorial/lesson3/draw_traj/build/Output$
```

glog打印info：

```
I20220919 18:49:53.051337 18347 draw_trajectory.cpp:55] Find trajectory.txt at ../../estimated.txt
I20220919 18:49:53.054145 18347 draw_trajectory.cpp:55] Find trajectory.txt at ../../groundtruth.txt
I20220919 18:49:53.057397 18347 draw_trajectory.cpp:40] Total estimate trajectory pose length: 613
I20220919 18:49:53.057426 18347 draw_trajectory.cpp:41] Total groundtruth trajectory pose length: 613
```

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● Q7：轨迹绘制

建议：可以简化代码

类型声明typedef：

```
typedef vector<Sophus::SE3d, Eigen::aligned_allocator<Sophus::SE3d>> SE3_vector;
```

Main函数代码多放在function里写

● Q8：误差计算

```
Tangent log() const
```

Logarithmic map.

Computes the logarithm, the inverse of the group exponential which maps element of the group (rigid body transformations) to elements of the tangent space (twist).

To be specific, this function computes `vee(logmat(.))` with `logmat(.)` being the matrix logarithm and `vee(.)` the vee-operator of SE(3).

$$e_i = \| \log(\boldsymbol{T}_{gi}^{-1}\boldsymbol{T}_{ei})^{\vee} \|_2.$$

在log函数中默认已经使用了vee