When Virtual Network Operator Meets E-Commerce Platform: Advertising via Data Reward

***Abstract*—In China, some e-commerce platform (EP) companies such as Alibaba and JD have been now allowed to partner with network operators (NOs) to act as virtual network operators (VNOs) to provide mobile data services for mobile users (MUs). However, it is a question worth researching on how to generate more profits for all network players after EP companies being VNOs through appropriate integration of the VNO business and the companies' own e-commerce business. To address this issue, in this work we propose a novel incentive mechanism for advertising via mobile data reward, and model it as a three-stage Stackelberg game. In Stage I, the NO decides the price of mobile data for the VNO; in Stage II, the VNO decides its data plan fee for MUs and the ad price for e-commerce merchants (EMs); in Stage III, the MUs make their own decisions on the data plan subscription and the number of ads to be watched, while the EMs decide the number of ad slots they buy from the EP. We obtain the closed-form optimal solution of the Nash equilibrium by backward induction. Simulation results show the impact of the system parameters on the utilities of game players and social welfare, and reveal that the solution can indeed lead to a quadri-win outcome in some cases. At the same time, we summarize some insights that have some economic guidance.**

***Index Terms*—Virtual network operator, e-commerce platform, data reward, Stackelberg game, network economics.**

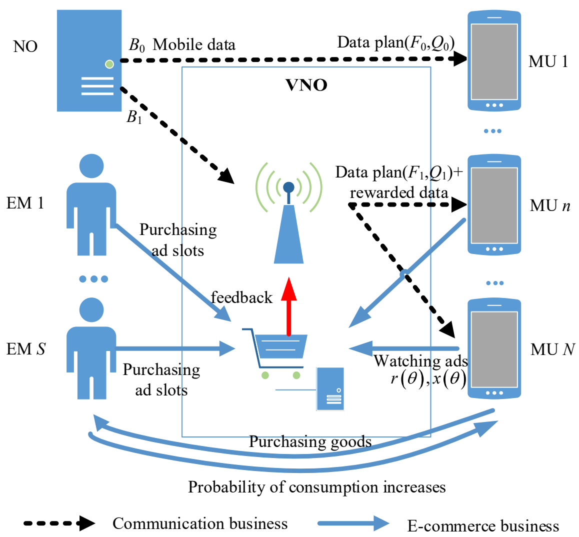

Fig. 1: Research scenario of an EP as a VNO.

I. Introduction

A. Motivation

With the deployment of the 5G networks, in China people's consumption habits have not only been changing from offline to online but also more and more from PC to mobile terminals. During the COVID-19 pandemic in 2020, many people stayed home and relied on e-commerce platforms (EPs) such as Alibaba, JD, and Pinduoduo to purchase fruits, vegetables, and daily necessities. In fact, short video shopping, (interactive) live streaming, VR/AR marketing, and other new applications have increasingly become the daily lives of Chinese people. These applications are occupying more and more network traffic, and it is expected that their traffic in operators' networks will increase unprecedentedly in the future [1]. Correspondingly, e-commerce platforms have become major contributors to network traffic, and the common people are becoming more and more attached to these EPs.

On the other hand, in 2018 China has allowed e-commerce platforms such as Alibaba and JD as virtual network operators (VNOs) to participate in network operations. This gives the e-commerce platforms and traditional network operators (NOs) an opportunity to renew their business model for not only increasing their own revenues and the service satisfaction of mobile users (MUs) but also improving network resource utilization at the same time. Closely related, in recent years the network design that a VNO participates in has been one of the hot issues which the academic community concerns about, for example, spectrum trading among NOs and VNOs [2], cooperative games among VNOs and MUs [3], and competition strategies among multiple VNOs with complete and incomplete information [4]. However, in the research scenarios of the aforementioned works, VNOs do not have the second role of an e-commerce platform at the same time, so the solutions they propose cannot be well applied to the scenario of interest here.

In fact, when an e-commerce platform like Alibaba participates in network design and operation, the number of roles that affect the benefits of all parties in the network will be more than that in the case where only the traditional NOs or NOs and non-EP VNOs participate in together. As shown in Fig. 1, the EP serves both the e-commerce merchants (EMs) and MUs on its platform. When the EP serves as a VNO as well, it can attract MUs to spend on its platform through data rewards [5], thereby increasing the revenue of both the EMs and MUs. However, it also should pay the NO for the data rewards, so the strategies of the NO, VNO, EMs, and MUs are interrelated in this network.

In the new scenario of an EP being a VNO participating in network operations, there are many issues that need to be studied. Some of these problems which in this work we hope to solve include:

- For the NO, how should it price its mobile data or other network resources to maximize its revenue?
- For the VNO, how should it customize its data plan or a data incentive mechanism to attract MUs, and provide services with EMs (e.g., advertising) to attract them to purchase its services for improving their revenue?
- For the EMs, how should they purchase services from the EP to increase their revenue from the e-commerce business?
- For the MUs, how should they select the data plan of the NO and VNO, and act according to the data incentive mechanism of the VNO to maximize their utility?

B. Contribution

In response to the aforementioned issues raised in the new scenario, as shown in Fig. 1 in this work we consider a basic scene consisting of a single NO, a single EP which serves as a VNO, multiple EMs served by the EP, and multiple MUs that can choose mobile service from either the NO or VNO. We illustrate the incentive mechanism for advertising via mobile data reward of this scenario in Fig. 1, where the black dotted arrow and the blue solid arrow represent the communication business and the e-commerce business, respectively.

We model the interactions among the NO, VNO, MUs, and EMs by a three-stage Stackelberg game. In Stage I, the NO decides the price of mobile data (i.e., the payment for purchasing unit mobile data) for the VNO. In Stage II, the VNO decides the data plan fee for the MUs and the ad price (i.e., the payment for purchasing one ad slot) for the EMs. In Stage III, the MUs with different valuations for the mobile service make their data plan subscription and ad watching decisions, in which we consider an α-fair data consumption utility function and uniform distribution of MUs' valuation. Meanwhile, the EMs decide the number of ad slots to purchase, considering the advertising's wear-out effect (i.e., an ad's effectiveness can decrease if it reaches a user who has watched the same ad several times). The main contributions of this paper are summarized as follows:

- For the scenario of a single NO, a single EP as a VNO, multiple EMs, and multiple MUs, we propose a novel incentive mechanism for advertising via mobile data reward, and formulate the interactions of the four types of decision-makers as a three-stage Stackelberg game;
- We obtain the closed-form optimal solution of the Nash equilibrium of the problem by backward induction. Using the optimal solution obtained, we can provide the optimal strategies for the NO, VNO, EMs, and MUs, respectively, in the network design or usage, such as the pricing of the NO's mobile data, the pricing of the VNO's data plan and advertising, the number of advertisements an EM purchasing from the VNO, and the data plan selection and advertisement watching strategies of an MU;
- We conduct extensive simulations to study the impact of some important system parameters, such as the data amount of the VNO's data plan and data reward per ad, on the Nash equilibrium. We also reveal some useful observations which have a meaning of economic guidance.

The remainder of this paper is organized as follows. In Section II, we review related works. In Section III, we propose the incentive mechanism for advertising via mobile data reward and introduce the three-stage Stackelberg game. In Section IV, we obtain the closed-form optimal solution of the Nash equilibrium of the problem by backward induction and analyze the impact of the system parameters on social welfare. In Section V, we discuss simulation results to reveal the impact of system parameters on the Nash equilibrium and show some useful observations. Finally, we draw the conclusions in Section VI.

II. Related Works

In the literature, there exist many work studying the access and/or resource allocation problem for wireless MUs in the traditional VNO scenarios, where the resources are usually defined in the frequency domain based on the orthogonal frequency division multiple access technology [2]–[4]. For example, in [2] the authors study a problem in which multiple VNOs under cellular networks can lease spectrum from a NO to provide data offloading services for MUs and model the interaction among VNOs as a Cournot game and a Stackelberg game, respectively. In [3], a two-stage spectrum leasing framework, where a VNO acquires spectrum resources through both advance reservation and on-demand request, is formulated as a tri-level nested optimization problem. The authors in [4] study the competition strategies among VNOs with complete or incomplete information about spectrum inventories and use the Bayesian coalition formation game to formulate the pricing decision problem. However, the aforementioned works only study the three-party game relationship among NOs, VNOs, and MUs from the perspective of communication services, and do not consider the potential second role of a VNO (i.e., an EP), thus ignoring the interaction between the EMs and EP.

From the MU's point of view, although the data plan fee will show a downward trend in the coming period, the changes that occur in the media mode of the e-commerce business make it still a certain tight problem. In this case, sponsored data is considered as a possible solution to achieve multiple-win results [6]. Sponsored data is the provision of a certain amount of data or navigation functionality to a mobile application/website for free to a specific user. There are two main types of sponsored data: zero-rating [7] and data reward [6]. Depending on the role of content providers (CPs) in provisioning of the free services in the data plan, zero-rating implies that the CPs pay the NOs for offering free services individually to MUs [5]. An example of zero-rating applications is the "Free Basics" provided by Facebook which gives free access to a simplified version of some websites

[8], [9]. Alternatively, Aquto and Unlockd are two leading companies providing technical support for data rewarding [10], [11]. They develop mobile APPs that display ads and track the amount of rewarded data. The impact of value-added services is studied in [12] from the perspective of a VNO, but the model there is relatively simple and only the changes to a single VNO's utility is considered. The work in [7] provides a modeling of zero-rating services and their utility impact on NOs and subscribers, without taking VNO into account. The authors in [6] use a two-stage Stackelberg game to model a novel data rewarding ecosystem where a NO offers users data rewards to create new revenue streams, but ignore the potential dual role characteristics of the VNO as we are to study. Of all the sponsored data solutions, data reward is the most user-oriented–with this solution users are the ones who decide when they want to engage with reward programs and how they want to spend the data they won. Therefore, in this work we adopt the data reward scheme in our incentive mechansim design.

In the literature, there exist some works studying advertising through certain incentive mechanisms [13]–[15]. The work in [13] considers venue owners offering advertising to specific users with an access model of both paid access and viewing ads through Wi-Fi deployment in public places. The authors in [14] achieve better incentives through cooperation between online APPs and offline venues to promote revenue for both parties. The work in [15] subsidizes users' data by CPs paying money to Internet service providers (ISPs) for the purpose of increasing their own profits. However, as these works do not consider the fact that the advertising platforms, such as the venue owners, online APPs, and CPs, can also have the potential second role of a VNO, their research scenarios are different from the one which we are interested in.

III. System Model

In this section, we model the strategies of the four types of decision-makers in the incentive mechanism: NO, VNO, MUs, and EMs. Then, we formulate their interactions as a three-stage Stackelberg game.

A. NO

To derive insights into the system design, we focus on a single data plan scenario, which has been widely considered in the literature (e.g., [12], [15]). We consider a monopolistic NO offering a predetermined (monthly) flat-rate data plan (F_0, Q_0) to MUs, where $F_0 > 0$ denotes the subscription fee, and $Q_0 > 0$ denotes the data amount associated with a subscription. The NO decides the price of mobile data c (i.e., the payment for purchasing unit mobile data) for the VNO. Suppose that in the system the network has a total network capacity of B. Let the amount of mobile data used by the NO be B_0 and the amount of mobile data purchased by the VNO be B_1. Thus, we have

$$B_0 + B_1 \leq B. \tag{1}$$

B. VNO

For the communication business, the VNO offers a monthly flat-rate data plan (F_1, Q_1) to MUs, where $F_1 > 0$ denotes the subscription fee, and $Q_1 > 0$ denotes the data amount associated with a subscription. In the real business case, the data amount of the VNO's data plan is usually used for some exclusive applications with large traffic of its own enterprise or for attracting MUs. So this type of data plan usually has the characteristics of more data amount but a lower price for unit mobile data as compared with the NO's, which however results in a higher data plan fee. Without loss of generality, we assume $Q_0 < Q_1$ and $F_0 < F_1$.

To incentivize MUs to choose its data plan while watching ads placed on its EP for EMs, we consider that the VNO will give w mobile data per ad to those subscribers who use its data plan while also watching ads on its EPs.

The VNO needs to decide two variables: (i) data plan fee F_1, while the amount of mobile data Q_1 is supposed to be predetermined; (ii) an ad price p, which is the price that the VNO charges the EMs for buying one ad slot. Here, we consider a price-based mechanism, where the VNO sells the ad slots in advance at a fixed price.

C. MU

We consider a continuum of users, and denote the mass of users by N. Let θ denote a user's type, which parameterizes its valuation for mobile service [6], [13]. We assume that θ is a continuous random variable drawn from $[0, \theta_{\rm m}]$, and its probability density function $g(\theta)$ satisfies $g(\theta) > 0$ for all $\theta \in [0, \theta_{\rm m}]$. For convenience of analysis, similar to [16] we assume that θ is uniformly distributed in $[0, \theta_{\rm m}]$.

Let $r \in \{0, 1\}$ denote a user's data plan subscription decision, and $x \in [0, \infty)$ denote the number of ads that a user chooses to watch (during one month). We allow x and the advertisers' purchasing decisions to be fractional [13], [17]. Then, the amount of data that a user obtains from its subscription and ad watching is

$$z = (Q_1 + wx)r + Q_0(1 - r) \tag{2}$$

where the first term represents the data amount of the MU obtains if he subscribes to the data plan of the VNO and watches x ads, and the second item represents the data amount of the MU obtains if he subscribes to the data plan of the NO.

We use $\theta u(z)$ to capture a type-θ user's utility of using the mobile service. Here, $u(z), z \geq 0$, is the same for all users, and can be any strictly increasing, strictly concave, and twice differentiable functions that satisfy $u(0) = 0$ and $\lim_{z\to\infty} u'(z) = 0$. The concavity of $u(z)$ captures the diminishing marginal return with respect to the data amount. Different utility functions have been considered in recent several works, such as α-fair function [15], logarithmic function [16], and exponential function [18]. To simplify analysis, we consider a specific α-fair utility function: $u(z) = 2\sqrt{z}$. Although extension to other similar utility functions is straightforward, in this work to obtain insights analytically we focus on the aforementioned utility function.

Hence, a type-θ user's utility can be expressed as

$$\Gamma^{\rm MU} = \theta u((Q_1 + wx)r + Q_0(1-r)) - (F_1 + hx)r - F_0(1-r) \tag{3}$$

where h denotes the user's average disutility (e.g., inconvenience) of watching one ad. We assume that the total disutility of watching ads linearly increases with the number of watched ads [19], [20]. In Section III-A we will analyze the user's optimal decisions r^* and x^*. In the Nash equilibrium, we denote the total number of ads decided by all MUs G as

$$G = N \int_0^{\theta_{\mathrm{m}}} x^*(\theta) g(\theta) d\theta. \tag{4}$$

D. EM

We consider S homogeneous EMs, which implies that the utilities of EMs are the same. The EMs have an incentive to purchase ad slots in the EP to promote their own business by displaying ads for MUs. Hence, the EMs need to decide the required number of ad slots m_s where $s \in \{1, 2, ..., S\}$. Consisdering that the EP adopts a random strategy for all ads to N MUs, the probability that an MU sees the ad of EM s each time he watches an ad is $\frac{m_s}{G}$ [6], [13]. Notice that the EP can affect the total purchase volume of EMs by adjusting the price per unit of ad p to make sure $\sum_s m_s \leq G$. If $\sum_s m_s = G$, a user will always see an EM's ad at random; if $\sum_s m_s < G$, there is a certain probability that a user will not see any EMs' ad, in which case we consider replacing it with the EP's ad, or not showing any ad to the user.

In the related works of advertising business model, the utility of EMs increases and then decreases with the increment of $\frac{m_s}{G}$, which reflects the advertising's wear-out effect. This is because too much repetition may make the user have a bad impression of the product. Some studies, such as [19] and [21], explicitly consider a quadratic relation between the ad repetition and the advertising's effectiveness. Hence, we adopt a quadratic function as well to model the utility of the EMs:

$$\Gamma^{\mathrm{EM}} = A\frac{m_s}{G} - \left(\frac{m_s}{G}\right)^2 - pm_s \tag{5}$$

where $A > 0$ is a system parameter. Note that a smalller A in (5) reflects a stronger degree of wear-out effect.

E. Three-Stage Stackelberg Game

We model the interactions among the NO, VNO, MUs, and EMs by a three-stage Stackelberg game as illustrated in Fig. 2. In Stage I, the NO decides the price of mobile data (i.e., the payment for purchasing unit mobile data) for the VNO. In Stage II, the VNO decides the data plan fee for the MUs and the ad price (i.e., the payment for purchasing one ad slot) for the EMs. In Stage III, the MUs with different valuations for the mobile service make their data plan subscription and ad watching decisions. Meanwhile, the EMs decide the number of ad slots to be purchased.

IV. Three-Stage Game Analysis

In this section, we analyze the three-stage Stackelberg game by backward induction [22].

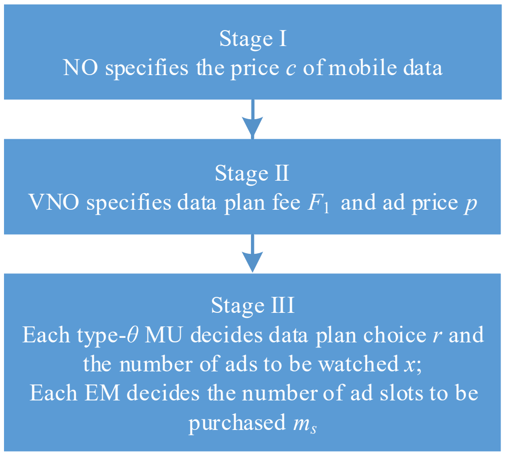

Fig. 2: Three-stage Stackelberg game.

A. Stage III: MUs' Access and EMs' Advertising

In this subsection, we analyze each MU's optimal data plan choice and the number of ads chosen to watch in the EP, and the EMs' optimal advertising strategy in Stage III. The MUs and EMs make their decisions by responding to the NO's price of mobile data c in Stage I, and to the VNO's decisions of F_1 and p in Stage II.

1) MUs' Optimal Access: To maximize the utility of mobile service, a type-θ user's decision strategy can be formulated as the following optimization problem (OP):

$$\text{OP 1}: \max_{r \in \{0,1\}, x \in [0,\infty)} \Gamma^{\mathrm{MU}} \tag{6}$$

where Γ^{MU} is given in (3). Although OP 1 is a mixed-integer OP, we can characterize the MU's decision in the following proposition by detailed case-by-case analysis.

Proposition 1: The optimal decisions of a type-θ user ($\theta \in [0, \theta_{\mathrm{m}}]$) are as follows:

Case 1: If $F_0 < F_1 < 2\frac{h\sqrt{Q_1}}{w}(\sqrt{Q_1} - \sqrt{Q_0}) + F_0$, $x^* = \begin{cases} 0 & \theta \in [0, \theta_2] \\ 0 & \theta \in [\theta_2, \theta_1] \\ \frac{\theta^2 w}{h^2} - \frac{Q_1}{w} & \theta \in [\theta_1, \theta_{\mathrm{m}}] \end{cases}$, $r^* = \begin{cases} 0 & \theta \in [0, \theta_2] \\ 1 & \theta \in [\theta_2, \theta_1] \\ 1 & \theta \in [\theta_1, \theta_{\mathrm{m}}] \end{cases}$;

Case 2: If $2\frac{h\sqrt{Q_1}}{w}(\sqrt{Q_1} - \sqrt{Q_0}) + F_0 \leq F_1 \leq \frac{{\theta_{\mathrm{m}}}^2 w}{h} - 2\theta_{\mathrm{m}}\sqrt{Q_0} + \frac{hQ_1}{w} + F_0$, $x^* = \begin{cases} 0 & \theta \in [0, \theta_3] \\ \frac{\theta^2 w}{h^2} - \frac{Q_1}{w} & \theta \in [\theta_3, \theta_{\mathrm{m}}] \end{cases}$, $r^* = \begin{cases} 0 & \theta \in [0, \theta_3] \\ 1 & \theta \in [\theta_3, \theta_{\mathrm{m}}] \end{cases}$;

Case 3: If $F_1 > \frac{{\theta_{\mathrm{m}}}^2 w}{h} - 2\theta_{\mathrm{m}}\sqrt{Q_0} + \frac{hQ_1}{w} + F_0$, $x^* = 0$, $r^* = 0, \theta \in [0, \theta_{\mathrm{m}}]$, where

$$\theta_0 = \frac{h\sqrt{Q_0}}{w} \tag{7a}$$

$$\theta_1 = \frac{h\sqrt{Q_1}}{w} \tag{7b}$$

$$\theta_2 = \frac{F_1 - F_0}{2(\sqrt{Q_1} - \sqrt{Q_0})} \tag{7c}$$

$$\theta_3 = \frac{h}{w}\left(\sqrt{Q_0} + \sqrt{Q_0 - Q_1 - \frac{w}{h}(F_0 - F_1)}\right). \quad (7d)$$

In Case 3, all MUs choose to subscribe to the data plan of the NO, which is contrary to the motivation of the incentive mechanism we design. Therefore, in the following part of the analysis, we focus on Case 1 and Case 2.

2) EMs' Optimal Advertising: To maximize the utility of advertising, each EM should solve the following OP:

$$\text{OP 2} : \max_{m_s} \ \Gamma^{\text{EM}} \quad (8a)$$

$$\text{s.t. } m_s \geq 0 \quad (8b)$$

where Γ^{EM} is given in (5). Because OP 2 is a convex OP, we can find its optimal solution easily as follows.

Proposition 2: In both Case 1 and Case 2, the optimal decision for each EM can be expressed as

$$m_s^* = G\frac{A - pG}{2}. \quad (9)$$

B. Stage II: VNO's Prices of Data Plan and Advertising

The VNO obtains revenue from both the mobile data market and the ad market. Next, we analyze the VNO's strategy by considering the two cases that MUs be categorized, as found in Proposition 1.

1) Case 1: In the mobile data market, each MU with $\theta \in [\theta_2, \theta_{\rm m}]$ who subscribes to the data plan of the VNO should pay F_1 to it, and the VNO should pay for mobile data to the NO. The VNO's corresponding revenue is

$$\Gamma^{\text{data}} = N\left(1 - \frac{\theta_2}{\theta_{\rm m}}\right)F_1 - cB_1. \quad (10)$$

In the ad market, each EM pays p for each purchased ad slot. The VNO's corresponding revenue is

$$\Gamma^{\text{tax}} = Spm_s^* = SpG\frac{A - pG}{2}. \quad (11)$$

Recall that B_1 denotes the total data demand of the VNO's subscribers, i.e., the total amount of mobile data that MUs request (by subscription and watching ads) given reward w. Based on Proposition 1, we can compute B_1 as

$$B_1 = N\frac{\theta_1 - \theta_2}{\theta_{\rm m}}Q_1 + N\int_{\theta_1}^{\theta_{\rm m}} \frac{\theta^2 w^2}{h^2} g(\theta) d\theta \quad (12)$$

where the first item means that the MUs with $\theta \in [\theta_2, \theta_1]$ will get Q_1 amount of data from the VNO and the second item means that the MUs with $\theta \in [\theta_1, \theta_{\rm m}]$ will get $\frac{\theta^2 w^2}{h^2}$ amount of data. Because only the MUs with $\theta \in [\theta_1, \theta_{\rm m}]$ will choose to watch ads, we can compute G as

$$G = N\int_{\theta_1}^{\theta_{\rm m}} \left(\frac{\theta^2 w}{h^2} - \frac{Q_1}{w}\right)\frac{1}{\theta_{\rm m}} d\theta. \quad (13)$$

To maximize the total utility of both data and ad markets, the VNO's strategy design in Case 1 can be formulated as the following OP:

$$\text{OP 3} : \quad \max_{F_1, p} \quad \Gamma^{\text{VNO}} = \Gamma^{\text{data}} + \Gamma^{\text{tax}} \quad (14a)$$

$$\text{s.t.} \quad S\sum_s m_s^* \leq G. \quad (14b)$$

Here, constraint (14b) implies that the total number of ads purchased by the EMs should not exceed the total number of ads decided by the MUs. As OP 3 is a convex OP, with some manipulation, we have the following proposition.

Proposition 3: In Case 1, the optimal strategy for VNO can be expressed as

$$F_1^* = \theta_{\rm m}\left(\sqrt{Q_1} - \sqrt{Q_0}\right) + \frac{F_0}{2} + \frac{cQ_1}{2} \triangleq F_{\rm A} \quad (15a)$$

$$p^* = \frac{A - \frac{2}{S}}{G}. \quad (15b)$$

2) Case 2: Similarly, we study the VNO's strategy design in Case 2 below. In the mobile data market, each MU with $\theta \in [\theta_3, \theta_{\rm m}]$ who subscribes to the data plan of the VNO should pay F_1 to it, and the VNO should pay for mobile data to the NO. The VNO's mobile data revenue is

$$\Gamma^{\text{data}} = N\left(1 - \frac{\theta_3}{\theta_{\rm m}}\right)F_1 - cB_1. \quad (16)$$

In the ad market, the VNO's revenue for selling advertising is the same with (11). In Case 2, each MU with $\theta \in [\theta_3, \theta_{\rm m}]$ who subscribes to the data plan of the VNO will get $\frac{\theta^2 w^2}{h^2}$ amount of data. Then, we can compute B_1 as

$$B_1 = N\int_{\theta_3}^{\theta_{\rm m}} \frac{\theta^2 w^2}{h^2} g(\theta) d\theta. \quad (17)$$

Here, because only the MUs with $\theta \in [\theta_3, \theta_{\rm m}]$ will choose to watch ads, we can compute G as

$$G = N\int_{\theta_3}^{\theta_{\rm m}} \left(\frac{\theta^2 w}{h^2} - \frac{Q_1}{w}\right)\frac{1}{\theta_{\rm m}} d\theta. \quad (18)$$

Similar to OP 3, to maximize the total utility of both data and ad markets, the VNO's OP in Case 2 can be formulated as:

$$\text{OP 4} : \quad \max_{F_1, p} \quad \Gamma^{\text{VNO}} = \Gamma^{\text{data}} + \Gamma^{\text{tax}} \quad (19a)$$

$$\text{s.t.} \quad S\sum_s m_s^* \leq G. \quad (19b)$$

OP 4 is a non-convex OP. However, recalling that $\theta_3 = \frac{h}{w}\left(\sqrt{Q_0} + \sqrt{Q_0 - Q_1 - \frac{w}{h}(F_0 - F_1)}\right)$, we can denote F_1 with a function of θ_3 as

$$F_1 = \frac{h}{w}\left[\left(\frac{\theta_3 w}{h} - \sqrt{Q_0}\right)^2 + Q_1 - Q_0\right] + F_0 \triangleq \phi(\theta_3). \quad (20)$$

Then, we can transform OP 4 into a non-convex OP about variables θ_3 and p as

$$\text{OP 5} : \quad \max_{\theta_3, p} \quad N\left(1 - \frac{\theta_3}{\theta_{\rm m}}\right)\phi(\theta_3) - cB_1 + Spm_s^* \quad (21a)$$

$$\text{s.t.} \quad S\sum_s m_s^* \leq G \quad (21b)$$

$$\hat{\Gamma}^{\text{VNO}}(\theta_1) > \hat{\Gamma}^{\text{VNO}}(\theta_{\rm m}) \quad (21c)$$

$$\theta_3 \in [\theta_1, \theta_{\rm m}], \quad (21d)$$

where $\hat{\Gamma}^{\rm VNO}(\theta_3) = \Gamma^{\rm VNO}(F_1 = \phi(\theta_3))$. Constraint (21c) means that we do not expect $\theta_3 = \theta_{\rm m}$, which implies that no MUs will subscribe to the data plan of the VNO. The objective function of OP 5 is a cubic function and constraints (21b)-(21d) are linear constraints. Then, its optimal solution must be obtained at the endpoints of interval $[\theta_1, \theta_{\rm m}]$ or at the stationary points. Thus, we have the following proposition.

Proposition 4: In Case 2, the optimal strategy for the VNO can be expressed as

$$\theta_3{}^* = \arg\max\{\hat{\Gamma}^{\rm VNO}(\theta_1), \hat{\Gamma}^{\rm VNO}(\theta_4)\} \tag{22a}$$

$$p^* = \frac{A - \frac{2}{S}}{G} \tag{22b}$$

where θ_4 is the root of equation

$$\frac{d\hat{\Gamma}^{\rm VNO}(\theta_3)}{d\theta_3} = 0. \tag{23}$$

For the sake of convenience to express, we define $\phi(\theta_1) \triangleq F_{\rm B}$ and $\phi(\theta_4) \triangleq F_{\rm C}$. Given an arbitrary price c of the NO, the VNO should compare its optimal utilities in Case 1 and Case 2 to decide the data plan fee F_1, so the optimal decision of the VNO is affected by price c of the NO and we have the following proposition.

Proposition 5: From the perspective of the NO, the optimal strategy of the VNO in Stage II can be summed up as

$$F_1^* = \begin{cases} F_{\rm A} & c \in \Omega_1 \quad \text{Case A} \\ F_{\rm B} & c \in \Omega_2 \quad \text{Case B} \\ F_{\rm C} & c \in \Omega_3 \quad \text{Case C} \end{cases} \tag{24a}$$

$$p^* = \frac{A - \frac{2}{S}}{G} \tag{24b}$$

where intervals $\Omega_i, i \in \{1, 2, 3\}$ are divided by comparing the VNO's utilities $\Gamma^{\rm VNO}(F_1^* = F_{\rm A})$ in Case 1, $\Gamma^{\rm VNO}(F_1^* = F_{\rm B})$ and $\Gamma^{\rm VNO}(F_1^* = F_{\rm C})$ in Case 2. For example, when $c \in \Omega_1$, we have $\Gamma^{\rm VNO}(F_1^* = F_{\rm A}) > \max\{\Gamma^{\rm VNO}(F_1^* = F_{\rm B}), \Gamma^{\rm VNO}(F_1^* = F_{\rm C})\}$. Intervals Ω_i's can be found numerically with Newton's method or by the *fzeros()* function in MATLAB.

Substituting the VNO's optimal strategy in formula (24b) into the EMs' optimal strategy in formula (9) and utility function of formula (5), we can obtain the following proposition:

Proposition 6: Given the optimal strategy of the VNO, the optimal strategy and utility of the EMs can be derived as

$$m_s^* = \frac{G}{S} \tag{25a}$$

$$\Gamma^{\rm EM}(F_1^*, p^*) = \frac{1}{S^2}. \tag{25b}$$

Equation (25a) implies that as long as both VNO and MUs make their optimal decisions in the proposed business model, the optimal strategies of all homogeneous EMs are the same and only affected by the number of competing EMs and the total number of ads decided by all MUs. Equation (25b) implies that the optimal utilities of all homogeneous EMs are the same and only affected by the number of EMs, regardless of any other system parameters. It is also noteworthy that although the optimal utility of each EM is relatively small, the EMs can always achieve their goals of positive revenues in the Nash equilibrium.

C. Stage I: NO's Price of Mobile Data

The NO obtains revenue from both the MUs who subscribe to it and the VNO in the mobile data market. Next, we analyze the NO's strategy by considering the three cases that the VNO's optimal strategy be categorized, as found in Proposition 5.

1) Case A: In this case, to maximize the revenue of the NO, we study the following OP:

$$\text{OP 6}: \quad \max_{c} \quad \Gamma^{\rm NO} = N\frac{\theta_2}{\theta_{\rm m}}F_0 + cB_1 \tag{26a}$$

$$\text{s.t.} \quad N\frac{\theta_2}{\theta_{\rm m}}Q_0 + B_1 \le B \tag{26b}$$

$$c \in \Omega_1 \tag{26c}$$

where θ_2 given in Proposition 1 is a function of $F_1^* = F_{\rm A} = \theta_{\rm m}\left(\sqrt{Q_1} - \sqrt{Q_0}\right) + \frac{F_0}{2} + \frac{cQ_1}{2}$ that is derived for the VNO in Case 1 of Stage II, and B_1 given in (12) is a function of θ_2. Constraint (26b) implies that the total demand of mobile data cannot exceed the network capacity B of the NO. OP 6 is convex, thus can be solved by CVX-toolbox in MATLAB. Let c_1^* denote its optimal solution.

2) Case B: In this case, to find the NO's optimal strategy we solve the following OP similar to OP 6:

$$\text{OP 7}: \quad \max_{c} \quad \Gamma^{\rm NO} = N\frac{\theta_1}{\theta_{\rm m}}F_0 + cB_1 \tag{27a}$$

$$\text{s.t.} \quad N\frac{\theta_1}{\theta_{\rm m}}Q_0 + B_1 \le B \tag{27b}$$

$$c \in \Omega_2. \tag{27c}$$

Notice that, in OP 7 θ_1 is given in Proposition 1, and B_1 is given in (17) which is a function of θ_3 thus a function of $F_1^* = F_{\rm B} = \phi(\theta_1)$ given in (24a) in Case 2 of Stage II. As OP 7 is a linear OP, we can solve it by CVX-toolbox in MATLAB easily as well. Let c_2^* denote the optimal solution to the OP.

3) Case C: In Case C, to derive the optimal strategy of the NO, we study the following OP:

$$\text{OP 8}: \quad \max_{c} \quad \Gamma^{\rm NO} = N\frac{\theta_4}{\theta_{\rm m}}F_0 + cB_1 \tag{28a}$$

$$\text{s.t.} \quad N\frac{\theta_4}{\theta_{\rm m}}Q_0 + B_1 \le B \tag{28b}$$

$$c \in \Omega_3 \tag{28c}$$

where θ_4 is given in (23), and B_1 given in (17) is a function of θ_3 thus a function of $F_1^* = F_{\rm C} = \phi(\theta_4)$ given in (24a) in Case 2 of Stage II. Unfortunately, different from OPs 6 and 7, OP 8 is a non-convex OP. However, similar to OP 4, we can transform it into a non-convex OP with a cubic objective function of θ_4. Then, the optimal solution to OP 8 must be obtained at the endpoints of interval Ω_3 or at the stationary points. Let c_3^* denote its optimal solution.

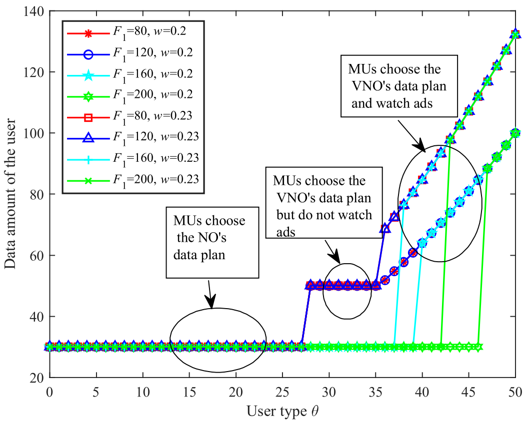

Fig. 3: Data amount of an MU with type θ.

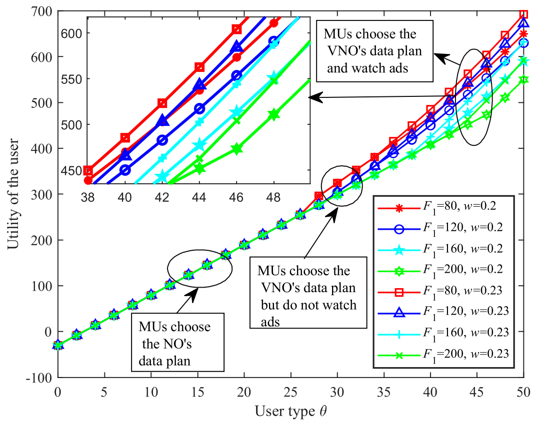

Fig. 4: Utility of an MU with type θ.

Finally, the NO can compare its local optimal utilities in Cases A, B, and C to find its global optimal strategy, as summarized in the following proposition.

Proposition 7: The global optimal solution of the NO in Stage I can be derived as

$$c^* = \arg\max_{c} \left\{ \Gamma^{\mathrm{NO}}\left(c_1^*\right), \Gamma^{\mathrm{NO}}\left(c_2^*\right), \Gamma^{\mathrm{NO}}\left(c_3^*\right) \right\}. \quad (29)$$

D. Social Welfare

In this subsection, we study the social welfare (SW) of the whole system at the equilibrium, which consists of the NO's revenue, the VNO's revenue, the MUs' total utility, and the EMs' total payoff. The social welfare analysis is important for understanding how much the entire system benefits from the proposed incentive mechanism for advertising via mobile data reward, and how it is affected by different system parameters. Specifically, we can compute the SW as:

$$\begin{aligned} SW =& \Gamma^{\mathrm{NO}}\left(c^*\right) \\ &+ \Gamma^{\mathrm{VNO}}\left(c^*, F_1^*, p^*\right) + S\Gamma^{\mathrm{EM}}\left(c^*, F_1^*, p^*\right) \\ &+ N\int_0^{\theta_{\mathrm{m}}} \Gamma^{\mathrm{MU}}\left(\theta, c^*, F_1^*, p^*\right) g\left(\theta\right) d\theta. \end{aligned} \quad (30)$$

Since it is difficult to analytically judge the monotonicity of the SW regarding system parameters, such as the VNO's data plan fee, data reward per ad, and data amount of the VNO's data plan, we study the impact of system parameters on the SW based on simulation results in Section V.

V. Numerical Results

In this section, we provide numerical results to study the NO's revenue, the VNO's revenue, the MUs' utilities, the EMs' utilities, and the social welfare with different values of the VNO's data plan fee, data reward per ad, and data amount of the VNO's data plan. The default values of important variables used in the simulation are summarized in Table I.

TABLE I: Simulation default parameters

Variable	Default value
Data plan fee of the NO F_0	30
Data amount of the NO's data plan Q_0	30 GB
Total network capacity of the NO B	10^{11} GB
User type parameter θ_{m}	50
Number of MUs N	10^8
Users' average disutility of watching one ad h	1
Number of EMs S	10
Parameter A of EMs' utility function	10^{10}
Data amount of the VNO's data plan Q_1	50 GB
Amount of data reward for watching per ad w	0.2 GB

A. MUs' Optimal Decision

Fig. 3 shows the data amount different MUs obtain under the VNO's different data plan fees F_1's and data rewards per ad w's. In general, there are three types of MUs in the figure–the first type of MUs choosing the NO's data plan, the second type of MUs choosing the VNO's data plan but without watching ads, and the last ones choosing the VNO's data plan and watching ads. If the VNO's data plan fee is relatively low (e.g., $F_1 = 80$ or 120), the MUs' decisions belong to Case 1 in Proposition 1, i.e., all three types of MUs appear in the network. If the VNO's data plan fee F_1 increases for example to 160 or 200, the MUs' decisions fall into Case 2 in Proposition 1, i.e., only the first and third types of MUs appear in the network. Further, the larger the VNO's data plan fee, the more the MUs subscribe to the NO but the more the ads an MU who subscribe to the VNO will watch. This is rational because the increase of the VNO's data plan fee makes MUs who subscribe to it tend to earn data rewards by watching ads to compensate the high data plan fee. It is also noteworthy that, when MUs decide to watch ads, the number of ads they will watch mainly depends on the data reward w per ad and their user type (see Proposition 1), rather than network data plan fee.

Fig. 4 shows the relationship between MUs' utility and their user type θ under the VNO's different data plan fees F_1's and data rewards per ad w's. It can be seen from the figure that,

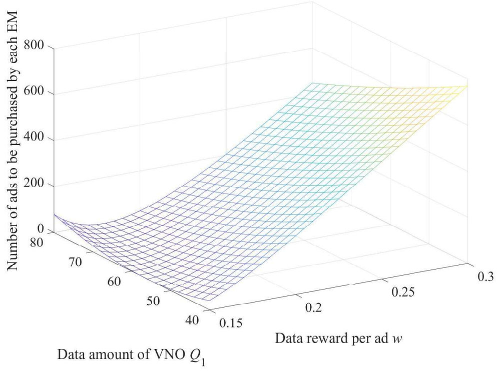

Fig. 5: The number of ad slots to be purchased by each EM.

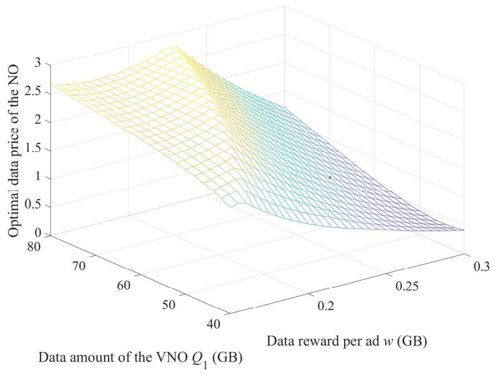

Fig. 6: The optimal data price of the NO under the VNO's different data amounts and data rewards per ad.

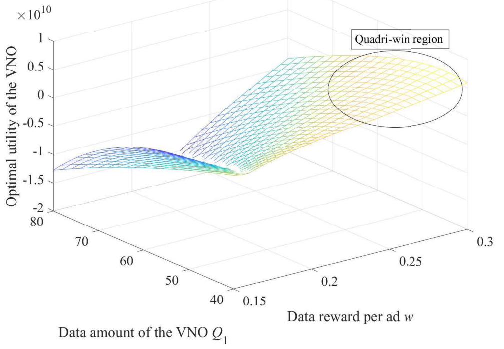

Fig. 7: The optimal utility of the VNO with its different data amounts and data rewards per ad.

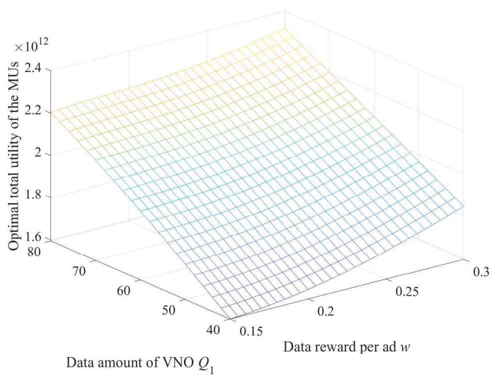

Fig. 8: Optimal total utility of the MUs.

given a fixed data reward, if user type $\theta \leq \theta_2$ ($\theta > \theta_2$), the utility of an MU remains unchanged (reduces) as the VNO's data plan fee increases, for example, $\theta_2 = 27$ when $F_1 = 80$ and $w = 0.2$ in Case 1. This is rational because the MUs with $\theta \leq \theta_2$ will subscribe to the NO's data plan and the MUs with $\theta > \theta_2$ will choose the VNO's data plan according to Proposition 1. Furthermore, one can notice that, given a fixed VNO's data plan fee, before user type θ increases to a certain threshold (e.g., $\theta = 35$ for $F_1 = 80$), the utility of an MU remains unchanged, if the data reward increases from 0.2 to 0.23. This is because the MUs with $\theta_2 < \theta \leq \theta_1$ will choose the VNO's data plan but do not watch ads, so the data amount of these MUs is not affected by the data reward per ad. However, after the user type beyond the threshold, the utility of an MU increases as the data reward per ad increases, since for these MUs, they will choose the VNO's data plan and watch ads, thus can obtain more mobile data according to Proposition 1.

B. Impact of System Parameters on the Nash Equilibrium

Because we are mainly concerned about the impact of an EP as a VNO in the proposed system, in this subsection we study the trends of the number of ad slots to be purchased by each EM, the optimal data price of the NO, the optimal utility of the VNO, the optimal total utility of the MUs, the optimal utility of the NO, and the social welfare under the VNOs different data amounts of the data plan Q_1's and data rewards per ad w's.

Fig. 5 shows the number of ad slots to be purchased by each EM in the Nash equilibrium. For example, when $Q_1 = 50$ GB and $w = 0.2$ GB, the number of ad slots purchased by each EM is about 3.45×10^8. From Fig. 3, it can be found that about 3×10^7 MUs choose to watch ads at this setting. Therefore, every month each MU who chooses to watch ads will watch 11.5 ads on average. It can be observed that the number of ad slots to be purchased by each EM has different changing trends with the VNO's data reward per ad if the VNO chooses different data amounts of its data plan. Specifically, for a smaller data amount of the VNO's data plan (e.g., 40 GB), the number of ad slots to be purchased by each EM increases when the VNO's data reward per ad increases. This is because the data amount of the VNO's data plan is small. So more MUs are willing to choose the VNO's data plan and

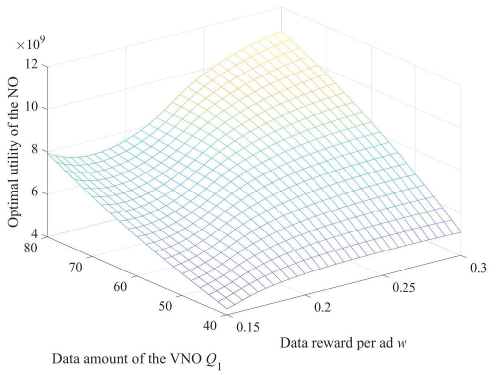

Fig. 9: The optimal utility of the NO under the VNO's different data amounts and data rewards per ad.

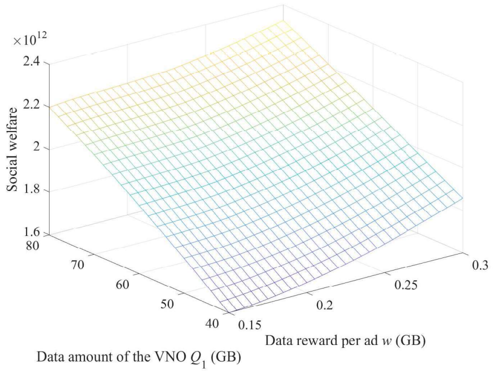

Fig. 10: The social welfare with the VNO's different data amounts and data rewards per ad.

these MUs will watch more ads to earn more mobile data. However, for a larger data amount of the VNO's data plan (e.g., 80 GB), the number of ad slots to be purchased by each EM decreases and then increases when the data reward per ad increases. This is because at the beginning, as the data reward per ad increases, the MUs are reluctant to watch too many ads because the rewarded data is enough to use. When the data reward per ad further increases, the MUs are attached to watch more ads for more revenue. It is also noteworthy that the number of ad slots to be purchased by each EM has different changing trends with the data amount of the VNO's data plan given the VNO's different data rewards per ad. Specifically, for a larger data reward per ad (e.g., 0.3 GB), the number of ad slots to be purchased by each EM decreases with the increase of the data amount of the VNO's data plan. It is rational because according to Proposition 1, fewer MUs are willing to watch ads and these MUs will watch fewer ads. On the other hand, for a smaller data reward per ad (e.g., 0.15 GB), the number of ad slots to be purchased by each EM decreases and then increases when the data amount of the VNO's data plan increases. This is because at first, fewer MUs are willing to watch ads and these MUs will watch fewer ads as the rewarded data is unable to make up for the disutility of watching ads. However, when the data amount of the VNO's data plan further increases, the subscribers of the VNO have to watch more ads to compensate the high data plan fee.

Fig. 6 shows the NO's optimal data price c^* in the Nash equilibrium. It can be observed that given a fixed data reward per ad the NO's optimal data price increases and then decreases with the increase of the data amount of the VNO's data plan. This is because at the beginning, as the data amount of the VNO's data plan increases, the utility that subscribers can get from the VNO's data plan increases, thus more MUs are willing to choose the VNO's data plan, making the VNO willing to buy more data from the NO at a relatively high price. However, when the data amount of the VNO's data plan increases to a certain threshold (e.g., $Q_1 = 62$ for $w = 0.2$), a further increase in the NO's data price makes the VNO keep increasing its data plan fee, which, on the other hand, makes fewer MUs willing to subscribe to the VNO's data plan and in turn makes the total utility of the VNO decrease. So, the NO's data price decreases correspondingly. Similarly, given a fixed data amount of the VNO's data plan, the optimal data price of the NO also increases and then decreases with the increase of the VNO's data reward per ad.

Fig. 7 shows the VNO's optimal utility in the Nash equilibrium. It can be found that given a fixed data reward per ad the VNO's optimal utility decreases with the increase of its data amount of the data plan. This is because the VNO needs to pay more data costs to the NO as its data amount of the data plan increases. On the other hand, given a fixed value of the VNO's data amount of data plan, the utility of the VNO decreases and then increases with the increase of the VNO's data reward per ad. This is because, as seen in Fig. 6, the optimal data price of the NO increases and then decreases when the VNO's data reward per ad increases. So the data cost paid by the VNO to the NO also increases and then decreases. It is also noteworthy that the optimal utility of the VNO may be negative when the data reward per ad is small and the data amount of the data plan is large. Therefore, to avoid the situation that the VNO will not join this business model, the system parameters need to be reasonably tuned to achieve a quadri-win result as shown in Fig. 7. In contrast to the VNO's optimal utility, Fig. 8 shows that the optimal total utility of the MUs in the Nash equilibrium always increases with the data amount of the VNO's data plan or its data reward per ad, simply because such a generous behavior of the VNO makes the MUs enjoy more mobile services, as illustrated in (3).

In Fig. 9, we study the NO's optimal utility in the Nash equilibrium. It can be seen that the NO's optimal utility increases with the increment of the VNO's data amount of data plan given the latter's any fixed data reward per ad. This is because as the VNO's data amount of the data plan increases, the utility that an MU can get from the VNO's data plan increases, and therefore more MUs are willing to choose the VNO's data plan, making the VNO willing to buy more data from the NO at a high price. It is also noteworthy that the optimal utility of the NO has different changing trends with the VNO's data reward per ad if the VNO chooses different data amounts of its data plan. Specifically, for a smaller data amount of the VNO's data plan (e.g., 40 GB), the optimal utility of the NO increases and then decreases when the VNO's data reward per ad increases. This is because at the beginning, as the data reward per ad increases, the utility of MUs increases and they are more willing to choose the VNO's data plan, thus increasing the NO's revenue in the data market. However, a larger data reward per ad makes the VNO's data cost increase and profit decrease, which in turn increases its data plan fee and makes the number of its subscribers decrease and eventually the NO's profit in the data market decrease. On the other hand, for a large data amount of the VNO's data plan (e.g., 80 GB), the NO's optimal utility decreases and then increases as the VNO's data reward per ad increases. This is because at first, as the data reward per ad increases, the data price of the NO also increases, making the VNO's utility decrease and thus reduce the number of its subscribers, which eventually reduces the NO's profit. However, as the VNO's data reward per ad further increases, the data price of the NO starts to decrease. Therefore, the VNO's utility starts to increase and gradually starts to adjust the data plan fee to increase the number of subscribers, which makes the utility of the NO's data market increase.

C. Impact of System Parameters on Social Welfare

Fig. 10 shows how the social welfare changes in the Nash equilibrium. It can be observed that the social welfare increases with the increase of the VNO's data amount of the data plan for the same data reward per ad, and it aslo increases with the increase of the VNO's data reward per ad for the same data amount of data plan. As can be observed from Figs. 7-9 and Proposition 6, this is because the social welfare is mainly composed of the total utility of MUs, instead of the utilities of other game players. When the VNO's data amount of data plan and data reward per ad are larger, the total utility of MUs becomes larger. However, as can be observed from Fig. 7, for the VNO, making its data amount of data plan and data reward per ad as large as possible is not the optimal strategy for itself. In our system parameter setting, the VNO should increase the data reward per ad and decrease the data amount of its data plan to increase its optimal utility in Nash equilibrium. At the same time, as shown in Fig. 7, the four types of decision-makers in the incentive mechanism can have a quadri-win result only in this region.

VI. Conclusion

In this paper, for the scenario with a single NO and a single VNO which also is an EP serving multiple e-commerce merchants, we have proposed a novel incentive mechanism for advertising via mobile data reward and modeled it as a three-stage Stackelberg game. We have obtained the closed-form optimal solution of the Nash equilibrium by backward induction. The simulation results present the impact of the system parameters on the utilities of game players and social welfare. They also reveal that the solution can indeed lead to a quadri-win outcome with a small value of data amount of the VNO's data plan and a big value of data reward per ad in our system parameter setting. At the same time, we summarize some insights that have some economic guidance. For the future work, we will consider a more complex business model in the VNO combined with EP scenario, for example, multiple VNOs and multiple NOs. Moreover, we will consider to apply multi-agent reinforcement learning to solve the game problem in this topic.

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