2016 American Control Conference (ACC)
Boston Marriott Copley Place
July 6-8, 2016. Boston, MA, USA

Linear Robust Generalized Proportional Integral Control of a Ball and Beam System for Trajectory Tracking Tasks

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***Abstract*—In this article, the ball and beam trajectory tracking problem is tackled using a robust Generalized Proportional Integral (GPI) Controller. The controller is developed on the basis of the tangent linearized system model around the equilibrium point. The approximate linearized system is differentially flat, and hence, controllable. Flatness reveals that the resulting linearized system has an interesting cascade property which allows the simplification of the controller design. Experimental results are included to show the effectiveness of the controller in trajectory tracking and disturbance rejection control.**

I. INTRODUCTION

A classic underactuated system is the Ball and beam system [1], consisting of an actuated beam where a metal ball freely rotates. The associated control problem consists in forcing the ball to track a reference trajectory by means of the movement of the beam. Among the reported solutions, the ball and beam system has been successfully controlled by discontinuous control [2], fuzzy systems [3], [4], neural networks [5], output feedback control [6], and from a game theory perspective [7]. Recently, immersion and invariance [8], have been also used. In [9], a comprehensive comparison study of robust control schemes is reported.

The main reason of the large amount of different proposed controllers is the fact that the ball and beam system is unstable and non feedback linearizable [1]. This increments the complexity of traditional control/observer design [10]. However, as shown in the work of Hauser [1], the approximate linearization allows to avoid singularities but it is necessary to reject the associated nonlinearities as well as the neglected dynamics which may prevent a good performance. To overcome all these effects, robust control schemes, using disturbance observers [11], [12], [13], [14] have been proposed. The scheme improves the closed loop system performance while preserving the advantages of the linearized analysis and design options. The tangent linearization is controllable and, hence, flat, [15]. The flatness of the linearized system allows for a natural cascade disturbance observer decomposition (see [16]), leading to a remarkably noise efficient dynamic feedback controller design scheme.

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This work was partially supported by the SIP-IPN under research project 20150279

The GPI control, or Control based on *Integral Reconstructors*, is a relatively recent development in automatic control (see [17]) whose main departure from traditional state feedback controller design lies in the absence of asymptotic observers, which are substituted by a structural reconstruction of the state. The state of the system is obtained on the basis of inputs, outputs and linear combinations of iterated integrals of such signals. The reconstructed states are off the actual values due to the effect of the neglected initial conditions and of classical perturbations: i.e., constant, ramp, parabolic disturbances. Thanks to the superposition principle, added iterated output, or input, integral error compensation has been shown to complete a remarkably stable feedback design.

As a consequence of this approach, state feedback control laws are implemented with no need for asymptotic observers nor for digital computations based on output samplings. The control laws may be purely analogue.

As carried out in Linear Active Disturbance Rejection Control with Extended State Observers (and also GPI observers) ([18], [16]), the Robust GPI controller can be used in a analogous manner without using observers. Taking external disturbances, un-modeled dynamics, as well as exogenous perturbation terms into a single, lumped, additive disturbance input, their adverse effects may be rejected by a robust version of the GPI controller while only measured outputs are needed.

In this article, an output feedback controller is proposed for a trajectory tracking task in the ball and beam system. The approach uses a robust version of the GPI control scheme for linear systems. The robust GPI controller exploits the flatness of the linearized model of the system while efficiently controlling the nonlinear plant. Thanks to the flatness property, exploited already for the Extended State Observers-ADRC scheme, and resulting in an observer cascade decomposition ([16]), here, the same flatness property dually allows for the parallel decomposition of the overall robust GPI observer.

The remainder of the article is organized as follows: Section II formulates the problem, the flatness of the linearized system is established and the parallel controller structure is derived. The feedback controller design is presented in section III. Section IV provides some illustrative experimental results performed on a laboratory prototype. A rest-to-rest trajectory tracking task is successfully achieved. Finally, some concluding remarks and suggestions for further work are given in the last section.

II. Mathematical Model of the Ball and Beam System

Consider the Ball and Beam system, illustrated in Figure 1. The system consists on a ball freely rolling over a beam which exhibits an angular displacement possibility. The beam is directly connected to a pulley, which is actuated by a DC motor. The dynamics of the system is described by:

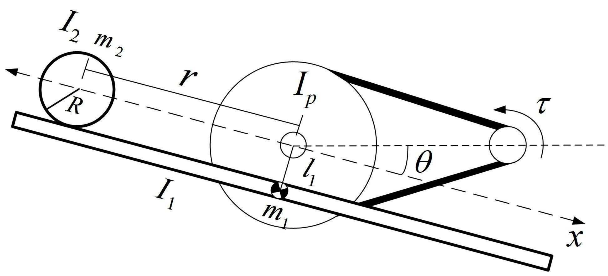

Fig. 1. Schematics of the ball and beam system

$$(m_2 + \frac{I_2}{R^2})\ddot{r} - m_2 r\dot{\theta}^2 + m_2 g \sin\theta = 0 \quad (1)$$

$$(m_2 r^2 + m_1 l_1^2 + I_1 + I_p)\ddot{\theta} + 2 m_2 r\dot{r}\dot{\theta} + \\ + m_2 g r \cos\theta + m_1 g l_1 \sin\theta = N\tau \quad (2)$$

where r represents the position of the ball, measured from the center of the beam, θ is the angular position of the beam, m_2 and R denote, respectively, the mass of the ball and its radius. $I_2 = \frac{2}{5} m_2 R^2$ is the ball inertia, I_1 is the beam inertia; m_1 denotes the mass of the beam and I_p represents the pulley inertia. The control input is the motor torque τ, in terms of its voltage input given by $\tau(t) = \frac{k_\tau}{R_a} V(t)$, where k_τ and R_a represent the torque constant and the armature resistance of the motor. The equilibrium point of the system is just:

$$\bar{r} = 0, \ \bar{\dot{r}} = 0, \ \bar{\theta} = 0, \ \bar{\dot{\theta}} = 0, \ \bar{V} = 0 \quad (3)$$

The approximate linearization of the system (1)-(2) around the equilibrium point (3) is obtained as:

$$(m_2 + \frac{I_2}{R^2})\ddot{r}_\delta + m_2 g \theta_\delta = 0 \quad (4)$$

$$(m_1 l_1^2 + I_1 + I_p)\ddot{\theta}_\delta + m_2 g r_\delta + m_1 g l_1 \theta_\delta = \frac{k_\tau N}{R_a} V_\delta \quad (5)$$

where $r_\delta = r - \bar{r} = r$, $\dot{r}_\delta = \dot{r} - \bar{\dot{\theta}} = \dot{r}$, $\theta_\delta = \theta - \bar{\theta} = \theta$, $\dot{\theta}_\delta = \dot{\theta} - \bar{\dot{\theta}} = \dot{\theta}$ y $V_\delta = V - \bar{V} = V$, represent the incremental variables of the linear system. The resulting linearized model (4)-(5) is flat [15], with incremental flat output $r_\delta = r - \bar{r} = r$. From equation (4), the following relations are obtained:

$$\theta_\delta = -\frac{(m_2 + \frac{I_2}{R^2})}{m_2 g} \ddot{r}_\delta \quad (6)$$

$$\dot{\theta}_\delta = -\frac{(m_2 + \frac{I_2}{R^2})}{m_2 g} r_\delta^{(3)} \quad (7)$$

$$\frac{k_\tau N}{R_a} V_\delta(t) = \frac{(m_1 l_1^2 + I_1 + I_p)}{\alpha} r_\delta^{(4)} + \\ + m_2 g r_\delta + \frac{m_1 g l_1}{\alpha} \ddot{r}_\delta \quad (8) \\ \alpha = -m_2 g/(m_2 + I/_2/R^2)$$

The ball acceleration can be expressed in terms of the measurable angular position of the beam as follows:

$$\ddot{r}_\delta = \alpha \theta_\delta$$

This property is exploited in the achievement of a parallel decomposition of the robust GPI controller for the overall system [16]. The input - flat output relation (8) is denoted as:

$$r_\delta^{(4)} = \frac{\alpha k_\tau N}{(m_1 l_1^2 + I_1 + I_p) R_a} V_\delta(t) - \frac{\alpha m_2 g}{(m_1 l_1^2 + I_1 + I_p)} r_\delta \\ - \frac{m_1 g l_1}{(m_1 l_1^2 + I_1 + I_p)} \ddot{r}_\delta \quad (9)$$

Notice that the linearized system is decomposed into a cascade connection of two independent blocks: The first one, controlled by the voltage input $V_\delta(t)$ whose corresponding output is given as the incremental acceleration $\ddot{r}_\delta$. This output coincides with the incremental angular position of the beam θ_δ, scaled by a factor α. It is obtained the following relation, $\ddot{r}_\delta = \alpha\theta_\delta$. The signal $\alpha\theta_\delta$, acts as a measurable auxiliary input to the second block, which consists of a chain of two integrators, associated,respectively, with the phase variables: $\dot{r}_\delta$ and r_δ (see Figure 2). The last variable, r_δ, is the output of the second block and the variable which represents the output of the complete system.

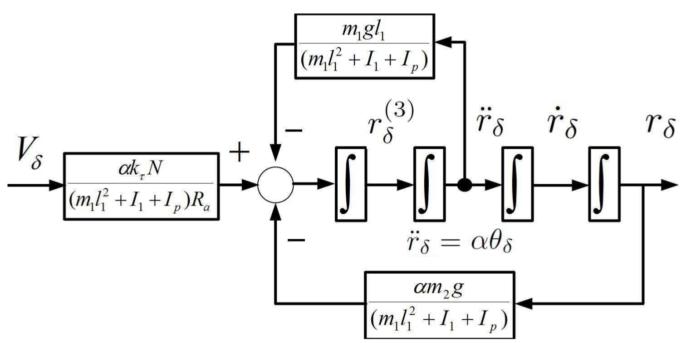

Fig. 2. Decoupled cascade structure of the system.

A. Problem Formulation

Given the Ball and Beam system, modeled by (1)-(2), and approximately linearized using the incremental coordinates $r_\delta^*(t) = r^*(t) - \bar{r} = r^*(t)$, devise a robust control law such that it forces the ball's position to track a smooth rest to rest trajectory from an initial equilibrium value, $r_\delta(0)$, towards a final desired equilibrium within a prescribed time interval $[0, t_{final}]$. The task must be carried out in spite of external disturbance inputs, neglected dynamics and other un-modeled disturbances.

B. Construction of a simplified system

In order to apply the Active Disturbance Rejection philosophy ([14], [19]), it is necessary to express the system as a disturbed chain of integrators, where the disturbance input lumps the set of nonlinearities, neglected dynamics, nonmodelled dynamics, external disturbances.

For the case of the Ball and Beam system, the incremental (linearized) system can be expressed as:

$$r_\delta^{(4)} = \frac{\alpha k_\tau N}{(m_1 l_1^2 + I_1 + I_p) R_a} V_\delta(t) - \frac{\alpha m_2 g}{(m_1 l_1^2 + I_1 + I_p)} r_\delta - \frac{m_1 g l_1}{(m_1 l_1^2 + I_1 + I_p)} \ddot{r}_\delta + \textbf{H.O.T.} \tag{10}$$

where the expression **H.O.T.** represents the "High Order Terms" neglected in the linearization process. The trajectory tracking error of the flat output is defined as follows:

$$e_{r_\delta} := r_\delta - r^*(t) = r - \bar{r} - (r^*(t) - \bar{r}) \tag{11}$$

Clearly, due to the nature of the equilibrium point and the control input, this term coincides, precisely, with the trajectory tracking error $r - r^*(t)$. On the other hand, the feedforward input is considered as $V_\delta^*(t)$ (in this case, simply denoted by V^*, to be used further ahead). We have the following nominal relation:

$$[r^*]_\delta^{(4)} = \frac{\alpha k_\tau N}{(m_1 l_1^2 + I_1 + I_p) R_a} V_\delta^*(t) - \frac{\alpha m_2 g}{(m_1 l_1^2 + I_1 + I_p)} r_\delta^* - \frac{m_1 g l_1}{(m_1 l_1^2 + I_1 + I_p)} \ddot{r}_\delta^* \tag{12}$$

For the incremental input voltage, the following relation is obtained:

$$\begin{aligned} e_V &= V_\delta(t) - V_\delta^*(t) \\ &= V(t) - \bar{V} - (V^*(t) - \bar{V}) = V - V^*(t) \end{aligned} \tag{13}$$

Using the last equations, the dynamics of the trajectory tracking error of the flat output is obtained as follows:

$$e_{r\delta}^{(4)} = \frac{\alpha k_\tau N}{(m_1 l_1^2 + I_1 + I_p) R_a} e_V(t) - \frac{\alpha m_2 g}{(m_1 l_1^2 + I_1 + I_p)} e_{r\delta} + - \frac{m_1 g l_1}{(m_1 l_1^2 + I_1 + I_p)} \ddot{e}_{r\delta} + \textbf{H.O.T.}$$

According to the ADRC procedure ([16], [18]), the dynamics of the trajectory tracking error is re-written as the following simplified perturbed model:

$$e_{r_\delta}^{(4)} = \kappa e_V(t) + \xi(t) \tag{14}$$

where $\kappa = (\alpha k_\tau N)/((m_1 l_1^2 + I_1 + I_p) R_a)$, and $\xi(t)$ is the lumped disturbance signal. In other words:

$$\begin{aligned} \xi(t) = &- \frac{\alpha m_2 g}{(m_1 l_1^2 + I_1 + I_p)} e_{r\delta}(t) \\ &- \frac{m_1 g l_1}{(m_1 l_1^2 + I_1 + I_p)} \ddot{e}_{r\delta}(t) + \textbf{H.O.T.} \end{aligned}$$

III. Generalized Proportional Integral Controller Design

For the GPI controller design, it is assumed that the disturbance input $\xi(t)$ is smooth (in practice, this condition can be relaxed). It is also assumed to be bounded and with a locally time polynomial approximation of a certain degree p, to be selected by engineering judgement (in the present case study, we adopt $p = 4$).

A state feedback controller for the linearized Ball and Beam flat output (20) is to regulate a fourth order system ($n = 4$). This can be easily achieved as follows [1]:

$$e_V(t) = -\frac{1}{\kappa}\left[k_7 e_{r\delta}^{(3)} + k_6 \ddot{e}_{r\delta} + k_5 \dot{e}_{r\delta} + k_4 e_{r\delta}\right] \tag{15}$$

However, to avoid the presence of up to third order time derivatives in the controller, one uses integral reconstructors based on neglecting the disturbance term $\xi(t)$ in the simplified model 14. Thus,

$$\begin{aligned} &\hat{e}_{r\delta}^{(3)} = \kappa\left(\int e_V\right), \quad \hat{e}_{r\delta}^{(2)} = \kappa\left(\int^{(2)} e_V\right), \\ &\hat{e}_{r\delta}^{(1)} = \kappa\left(\int^{(3)} e_V\right) \end{aligned} \tag{16}$$

Recall that these integral reconstructor estimates are off with respect to the true value of the underlying time derivative of the tracking error. We proceed to compensate with a finite number of iterated integrals of the tracking error. The controller may be written as:

$$\begin{aligned} e_V(t) = -\frac{1}{\kappa}\Bigg[&k_7 \hat{e}_{r\delta}^{(3)} + k_6 \hat{e}_{r\delta}^{(2)} + k_5 \hat{e}_{r\delta}^{(1)} + k_4 e_{r\delta} \\ &+ k_3\left(\int^{(1)} e_{r\delta}\right) + k_2\left(\int^{(2)} e_{r\delta}\right) + k_1\left(\int^{(3)} e_{r\delta}\right) \\ &+ k_0\left(\int^{(4)} e_{r\delta}\right)\Bigg] \end{aligned} \tag{17}$$

Note that the following relations hold:

[1]In this work, the following notation for the nested integrals is adopted: $\left(\int^i f\right)$, is equivalent to $\int_0^t \int_0^{t_1} \cdots \int_0^{t_{i-1}} f(\tau_i) d\tau_i d\tau_{i-1} \cdots d\tau_1$

$$
\begin{aligned}
e_{r\delta} &= r - r^*(t) \\
\hat{e}_{r\delta}^{(1)} &= \dot{r} - \dot{r}^*(t) = \alpha\left(\int^1 e_{\theta\delta}\right) := \alpha\left(\int^1 e_{\theta\delta}\right) \\
\hat{e}_{r\delta}^{(2)} &= \alpha\theta_\delta - \ddot{r}^*(t) = \alpha(\theta - \theta^*(t)) = \alpha e_{\theta\delta} \\
\hat{e}_{r\delta}^{(3)} &= \kappa\left(\int^1 e_V\right)
\end{aligned}
$$

The proposed controller can be represented in the frequency domain as follows:

$$
\begin{aligned}
e_V(s) = &-\frac{1}{\kappa}\left[k_7\kappa\frac{e_V(s)}{s} + \left(k_6 + \frac{k_5}{s}\right)\alpha e_{\theta\delta}\right] \\
&-\frac{1}{\kappa}\left[\frac{k_4}{s} + \frac{k_3}{s} + \frac{k_2}{s^2} + \frac{k_1}{s^3} + \frac{k_0}{s^4}\right]e_{r\delta}(s)
\end{aligned}
\quad (18)
$$

Algebraic manipulations lead to the following parallel controller structure

$$
\begin{aligned}
e_V(s) \quad = \quad &-\frac{\alpha}{\kappa}\left(\frac{k_6 s + k_5}{s + k_7}\right)e_{\theta\delta} \\
&-\frac{1}{k}\left(\frac{k_4 s^4 + k_3 s^3 + k_2 s^2 + k_1 s + k_0}{s^3(s + k_7)}\right)e_{r\delta}
\end{aligned}
\quad (19)
$$

The closed loop tracking error system, expressed in the frequency domain, leads, after using $\alpha e_{\theta\delta} = \ddot{e}_{r\delta}$, to the following closed loop system for the simplified perturbed system:

$$
\left[s^8 + k_7 s^7 + k_6 s^6 \ldots + k_2 s^2 + k_1 s + k_0\right]e_{r\delta}(s) = s^4\xi(s)
$$

A necessary and sufficient condition for the convergence of the trajectory tracking error, $e_{r\delta}$, as well as the associated phase variables, $\dot{e}_{r\delta}, \ddot{e}_{r\delta}, \ldots, e_{r\delta}^{(p+n)}$, to a neighborhood of the origin of the phase space, consists in choosing the controller gains, say, k_i, $\{i = 0, ..., p-1+n\}$ so that the linear dominant dynamics of the closed loop system is Hurwitz (see [20]).

In this case, due to the high order of the system dynamics, the gain selection procedure hast to take into account a fast desirable response, while avoiding peaking effects, as well as reducing the effects of measurement noises which may affect the controller response. Then, the efficient pole placement procedure developed by *Kim et al* [21], was adopted.

The pole placement approach is described as follows: Consider a characteristic polynomial of $n+p$ degree. Let T be a parameter of desired setting time *or generalized time constant*. Denote $\alpha_1 \in \mathbb{R}$ as the *rate of desirable damping*, to be taken as an adjustable constant parameter such that $\alpha_1 > 2$.

Let us define:

$$
\alpha_i = \left[\frac{\sin\left(\frac{i\pi}{p+n}\right) + \sin\left(\frac{\pi}{p+n}\right)}{2\sin\left(\frac{i\pi}{p+n}\right)}\right]\alpha_1, \qquad i = 2, 3, ..., p-1+n \quad (20)
$$

For a_0, being an arbitrary strictly positive constant, it is chosen

$$
k_0 = \frac{Ta_0}{a_{p+n}} \quad (21)
$$

with

$$
a_{p+n} = \frac{T^{p+n}a_0}{\alpha_{p-1+n}\alpha_{p-2+n}^2\alpha_{p-3+n}^3\cdots\alpha_1^{p-1+n}} \quad (22)
$$

The rest of parameters, k_i, is computed through the following expression:

$$
k_i = \frac{T^i a_0}{\alpha_{i-1}\alpha_{i-2}^2\alpha_{i-3}^3\cdots\alpha_1^{i-1}k_{p+n}}, \quad i = 1, 2, 3, ..., p-1+n \quad (23)
$$

Thus, a fast convergence of the estimation error, while avoiding as possible the peaking effects can be produced (see [22]).

IV. Experimental Results

In order to show the effectiveness of the proposed controller, some experimental tests were carried out on a laboratory prototype shown in figure 3. The experimental device consists of a DC motor from NISCA Motors (model NC5475B) which drives an aluminum beam via a synchronous belt and a pulley with a ratio $N = 6:1$. The angular position of the beam is measured using an incremental optical encoder of 2500 pulses per revolution. A linear sensor, fixed on the beam, consists on a track etched wire made of a nickel-chromium wire on which a stainless steel ball can roll with negligible friction. The ball acts as a wiper, similar to that of a potentiometer. The position along the beam can be obtained by measuring the output voltage of the linear sensor with a resolution of 25 [mm/V].

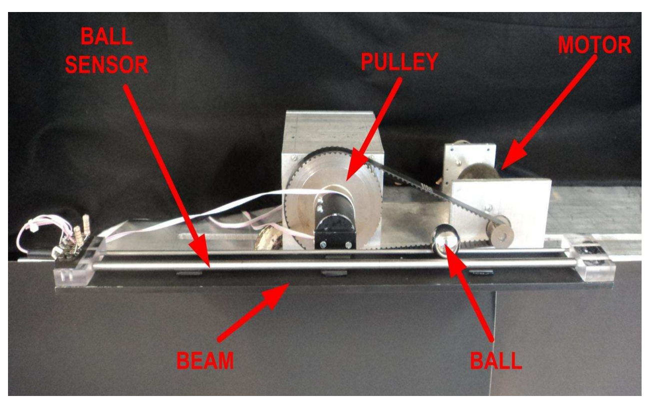

Fig. 3. Prototype of the ball and beam system

A power amplifier from STMicroelectronics model TDA7293 drives the motor. The data acquisition is carried out through a data card, model QPIDe, from Quanser. This card reads signals from the optical incremental encoder, the linear sensor voltage and, in turn, supplies control voltage to the power amplifiers. The control strategy was implemented

Parameter	Value
Beam inertia I_1	0.0045 [Kg-m^2]
Beam mass m_1	0.065 [Kg]
Distance from the beam's center of mass to the rotation center l_1	0.015[m]
Pulley inertia I_p	0.001 [Kg-m^2]
Ball mass m_2	0.065 [Kg]
Ball inertia I_2	0.0045 [Kg-m^2]
Ball radius R	0.0127 [m]
Armature resistance R_a	12.1 [Ω]
Torque constant k_τ	0.0724 [Nm/A]

TABLE I
EXPERIMENTAL PROTOTYPE PARAMETERS

in the Matlab-Simulink Quarc platform. Finally, the sampling time was set to be 0.001[s]. The parameters of the ball and beam system, as well as the actuator parameters are detailed in table I. Figure 4 depicts a block diagram of the experimental arrangement.

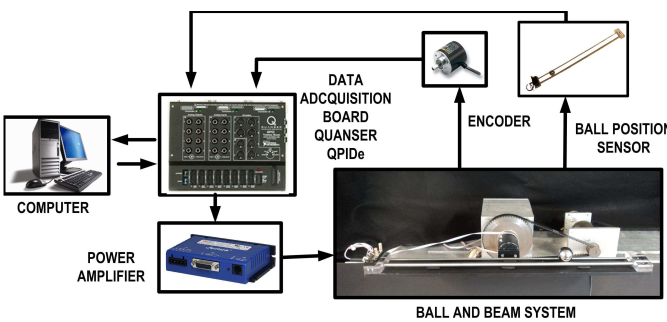

Fig. 4. Schematics of the implementation of the GPI controller

The initial conditions of the ball position, as well as the beam angular position were given by $[r = 0, \theta = 0]$. The GPI control gain parameters (according to the tuning algorithm) for the incremental position error control were set to be $n = 8$, $T = 6$, $a_0 = 4$, $\alpha_1 = 3$. Figure 5 shows the tracking results on a rest to rest trajectory, obtained with the help of a Bézier polynomial of lower degree 9. The ball starts on an initial position $r_\delta(0) = 0$, and then it is moved to a first rest position $r_\delta(4.5) = -0.14$[m] during the interval $[2, 4.5][s]$. Once the ball is stabilized, it is taken, at time $t = 9$[s] towards a final position $r_\delta(13.5) = 0.14$[m]. At the time $t = 16.5$, the system is perturbed by an impulsive external force directly applied to the ball. The reaction of the GPI controller allows the ball to practically remain at the stable position $r^*(t)$, while avoiding the external disturbance input effects. As shown in the results (see Figure 6), the trajectory tracking error for the ball position is restricted to the interval of $[-0.03, 0.02]$ [m], demonstrating the remarkable robustness of the proposal.

Figure 7 shows the trajectory tracking results with respect to the ball acceleration which, thanks to flatness, is proportional to the angular position of the beam. Thus, the measurement of the ball acceleration is indirectly carried out by means of the angular beam position. The evolution of the control input voltage, V_δ is depicted in Figure 8. The experimental results illustrate the fact that the applied voltage on the DC motor is bounded. On the other hand, the corrective reaction of the robust GPI control against the impulsive disturbance input is performed within a reasonably short amount of time.

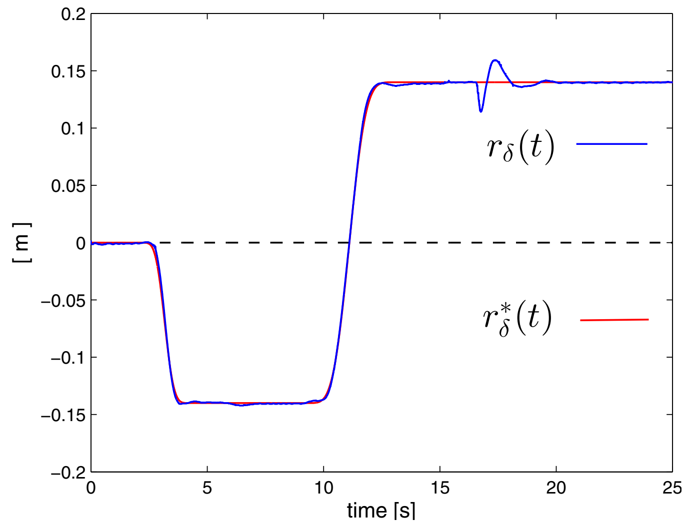

Fig. 5. Behavior of the ball's position trajectory tracking.

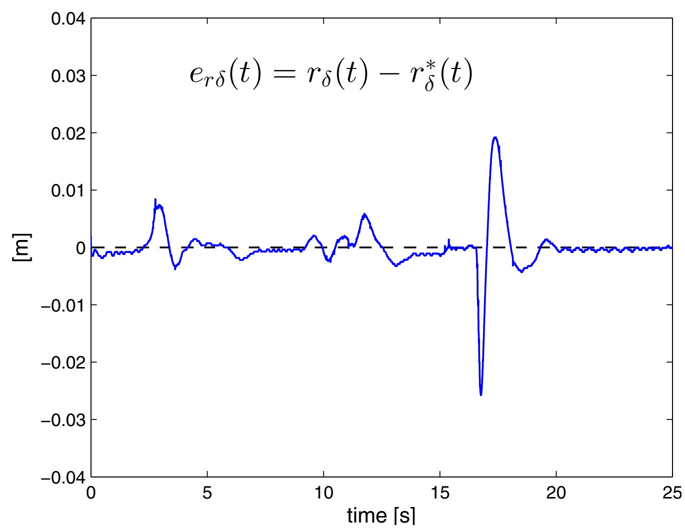

Fig. 6. Tracking error for the ball's position trajectory tracking.

V. CONCLUDING REMARKS

In this article, the problem of trajectory tracking control for the ball and beam system was successfully solved by means of robust GPI control. A cascade property, evident from the flatness of the tangent linearized system, allows the use of a parallel control scheme based only on position measurements. Robust Generalized Proportional Integral controller includes extra integration terms in order to reject an ultra local internal model of the lumped additive disturbance input of finite low order. This is in close correspondence with the Active Disturbance rejection Control philosophy, except for the use

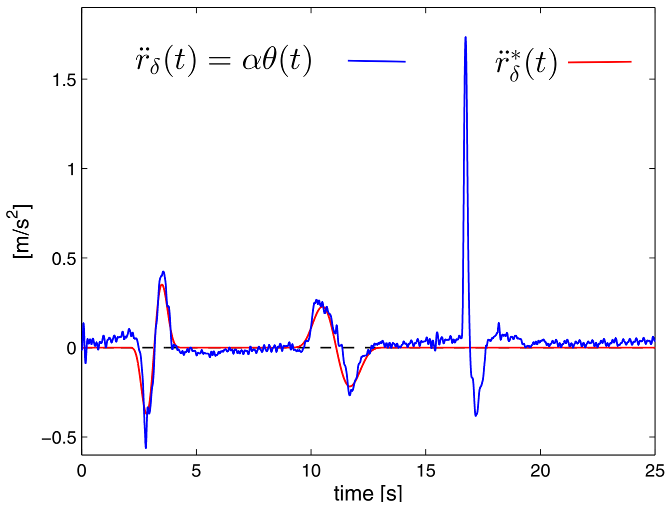

Fig. 7. Trajectory tracking results for the ball's acceleration.

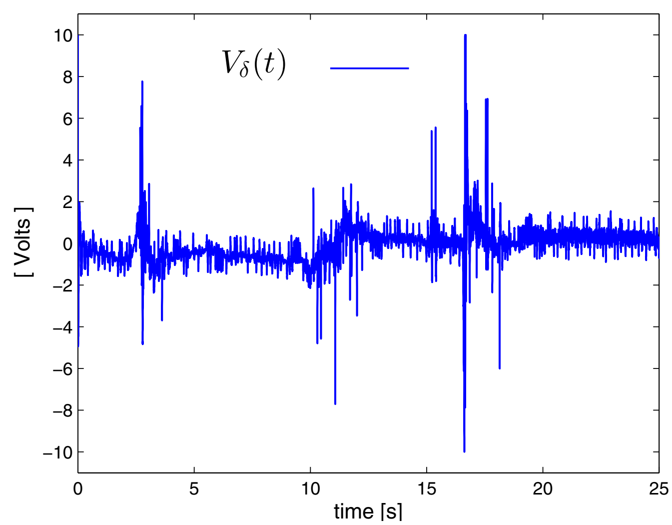

Fig. 8. Evolution of the voltage control input.

of Extended State Observers. The cascade property of the flat parametrization allows reducing the controller complexity and reduces the measurement noise sensitivity in the closed loop response. Experimental results demonstrated a rather small trajectory tracking error even in the presence of impulsive external disturbance inputs. Robustness is evident in spite of the neglected nonlinearities in the linearization process, the presence of un-modeled frictions in the driving belt, the un-modeled motor dynamics and the measurement noises.

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