

Generalized Proportional Integral Control (GPI)

Design for a Ball and Beam System

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Abstract

This document describes the analysis and design of a Proportional Integral Generalized (GPI) control strategy for a ball and beam system, such analysis is made under the active disturbance rejection. The principle objective is to show the advantages of the control strategy GPI as to signals tracking and disturbance rejection, compared to conventional control techniques such as the PID control. To validate the process, the modeling of the system was carried out and with this model simulations were carried out under nominal conditions, applying the two control strategies. It was found that the GPI control presented a better performance since it achieves a lower mean squared Error percentage than the PID control, even in the presence of disturbances.

Keywords: PID Control, Generalized Integral Proportional Control, modeling

1. Introduction

Currently many of the control problems that are presented can be controlled easily,

since for a certain input signal, its output is bounded in its majority of constant times; However, due to their nature or design, there are important groups of systems that are unstable, which in order to make them operate safely, is essential to provide feedback to the control method[1].

So it can be said that studying systems that are essentially unstable is of great interest, since it is a control problem that must be analyzed in the laboratory. The difficulty presented in this type of systems is due to the intrinsic nonlinearity thereof; To solve this type of problem a Ball and Beam system is proposed, which is a system consisting of a beam pivoted at one end by an electric motor, which is responsible for varying the angle of inclination of the same in order to maintain in a certain position a ball that moves freely on its surface, this is a device with a simple configuration created for the theoretical-practical study of static and dynamic systems through analog and digital control techniques[2].

The Ball and Beam system presents an interesting control problem since previously said by its nature is an unstable system; that can be extrapolated to the industry. In this type of system it is possible to implement classic control strategies or, as is the objective of this document, modern control strategies such as generalized proportional integral control (GPI).

This document presents a comparative analysis between classic control strategies and GPI control. The objective of implementing this control strategy is to reduce the tracking error and reject internal and external system disturbances.

2. System model

A ball is placed on a beam, see figure 1, where it is allowed to roll with 1 degree of freedom along the length of the beam. A lever arm is attached to the beam at one end and a servo gear at the other. When the servo assisted rotates at a θ angle, the lever changes the angle of the beam by α . When the angle of the horizontal position is changed, gravity causes the ball to roll along the beam [3].

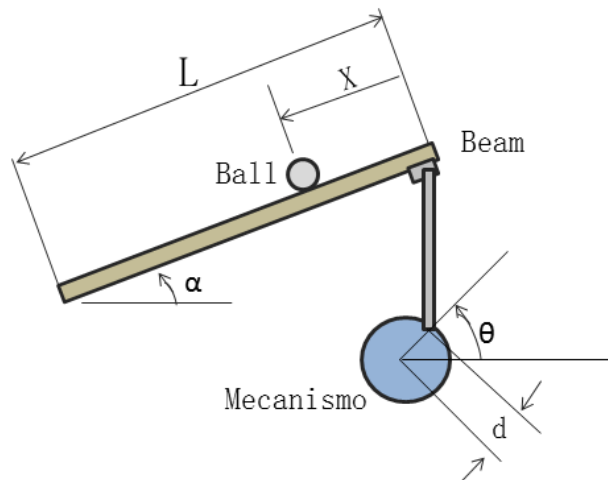


Figure 1. Ball and Beam system diagram

The mathematical model of this system is given by the Lagrangian equation

$$0 = \left(\frac{J}{R^2} + m\right) a + mg \sin(\alpha) - m \dot{x}^2 \quad (1)$$

For the moment when the ball is stable in the system, it will be considered that $\alpha = 0$, so the equation is reduced to:

$$\left(\frac{J}{R^2} + m\right) a = -mg \alpha \quad (2)$$

Then a linear approximation is made with the arc length for the two reference systems: $\alpha \propto \theta$

Where $s_1 = d\theta$ y $s_2 = L \alpha$ then it is said that $s_1 = s_2$ resulting in equation 3:

$$\alpha = \frac{d}{L} * \theta \quad (3)$$

Replacing equation 3 in equation 2 results in:

$$\left(\frac{J}{R^2} + m\right) \ddot{x} = -mg \frac{d}{L} \theta \quad (4)$$

Where the known data are presented in table 1

Table 1: Known data

<i>m</i>	0.02327 Kg	Ball mass
<i>R</i>	33.95 mm	Ball radius
<i>d</i>	50.9 mm	Crank length
<i>g</i>	9.81 m/s ²	Gravitational acceleration
<i>L</i>	0.275 m	Beam length
<i>J</i>	10.7284 Kg*mm ²	Moment of inertia of the ball

3. GPI controller design

This type of control strategies, such as GPI techniques, are capable of rejecting different types of disturbances that adhere to the output of the system, being more robust in the face of unknown perturbations [4]. Considering the above it can be said that for a non-linear SISO system of order n , disturbed and soft, of the form:

$$y_{(t)}^{(n)} = \psi\left(t, y_{(t)}, \dot{y}_{(t)}, \dots, y_{(t)}^{(n-1)}\right) + \varphi\left(t, y_{(t)}\right)u_{(t)} + \delta_{(t)} \quad (5)$$

It is said that the system is not disturbed for $\delta_{(t)} \equiv 0$ and that it is differentially flat since it is possible to express all the variables of the system, including $u_{(t)}$, in terms of differential functions of the flat output $y_{(t)}$ i.e., functions of $y_{(t)}$ and a finite number

of its temporal derivatives [5][6]. It is assumed that the exogenous perturbation $\delta_{(t)}$ is uniformly bounded.

Considering the above, it can be said that the system described in equation 4 is differentially flat since it can be expressed as:

$$y_{(t)}^{(n)} = \kappa u_{(t)} + \delta_{(t)} \quad (6)$$

Where $\delta_{(t)}$ collects the external and internal perturbations and also the uncertainties of the system proper to the unmodeled dynamics thereof, is m -differentiable and uniformly bounded i.e. $\sup |\xi^{(m)}(t)| \leq \kappa$. Where $\kappa = \frac{-mgd}{(\frac{J}{R^2}+m)L}$.

$$\ddot{x} = \frac{-mgd}{(\frac{J}{R^2}+m)L} u_{(t)} + \delta_{(t)} \quad (7)$$

The GPI control is designed within the framework of "active disturbance rejection" [7], and it includes a polynomial model in time of the state-dependent disturbances and those that are of an exogenous nature without any special structure [8]. In equation 8, the structure of the GPI control is shown, where $\kappa = \frac{-mgd}{(\frac{J}{R^2}+m)L}$ and the output x of the system is the position of the ball; **m is the order of the polynomial with which the disturbance approximates**; n is the order of the system; K_{n+m}, \dots, K_1, K_0 correspond the gains of such polynomial; and r .

$$u_{(t)} = \frac{1}{\kappa} \left[r^{(n)} - \left(\frac{K_{n+m}s^{n+m} + \dots + K_1s + K_0}{s^{m+1}(s^{n-1} + K_{2n+m-1}s^{n-2} + \dots + K_{n+m+1})} \right) (x - r) \right] \quad (8)$$

For the case study, it is assumed that $\frac{d^{m+1}\delta_{(t)}}{dt^{m+1}} = 0$, for $m=4$, the structure of the GPI control can be written as follows:

$$u_{(t)} = \frac{1}{\kappa} \left[r^{(2)} - \left(\frac{K_6s^6 + K_5s^5 + K_4s^4 + K_3s^3 + K_2s^2 + K_1s + K_0}{s^5(s + K_7)} \right) (x - r) \right] \quad (9)$$

Replacing equation (9) in equation (7) it is obtained:

$$x^{(2)} = r^{(2)} - \left(\frac{K_6s^6 + K_5s^5 + K_4s^4 + K_3s^3 + K_2s^2 + K_1s + K_0}{s^5(s + K_7)} \right) (x - r) + \delta_{(t)} \quad (10)$$

Then

$$\delta_{(t)} = (x^{(2)} - r^{(2)}) - \left(\frac{K_6s^6 + K_5s^5 + K_4s^4 + K_3s^3 + K_2s^2 + K_1s + K_0}{s^5(s + K_7)} \right) (x - r) \quad (11)$$

Where $(x - r)$ represents the tracking error e and $(x^{(2)} - r^{(2)})$ is the second derivative thereof.

$$\delta_{(t)} = \ddot{e} - \left(\frac{K_6 s^6 + K_5 s^5 + K_4 s^4 + K_3 s^3 + K_2 s^2 + K_1 s + K_0}{s^5(s+K_7)} \right) e \quad (12)$$

Solving equation 12 and applying the Laplace transform, it is obtained:

$$(s^5 \delta_{(t)})(s + K_7) = (s^8 + K_7 s^7 + K_6 s^6 + K_5 s^5 + K_4 s^4 + K_3 s^3 + K_2 s^2 + K_1 s + K_0) e \quad (13)$$

Where $(s^5 \delta_{(t)}) = 0$, then:

$$e(s^8 + K_7 s^7 + K_6 s^6 + K_5 s^5 + K_4 s^4 + K_3 s^3 + K_2 s^2 + K_1 s + K_0) = 0 \quad (14)$$

Equation 14 shows the dynamics of the error e , if $\delta_{(t)}^{(5)}$ is bounded and if the coefficients K_i , for $(i = 1, 2, \dots, 7)$, are chosen in such a way that the roots of the polynomial of equation 14 are on the left side of the complex plane. The tracking error will be bounded and the dimension can be made as small as wanted by moving the roots of this one away from the imaginary axis.

4. Results

To test the control strategy applied to the Ball and Beam system, a sinusoidal reference signal with a period of 20s and a step type signal were proposed. The tracking error was measured with the root-mean-square error (RMSE) and the results were compared with a PID controller.

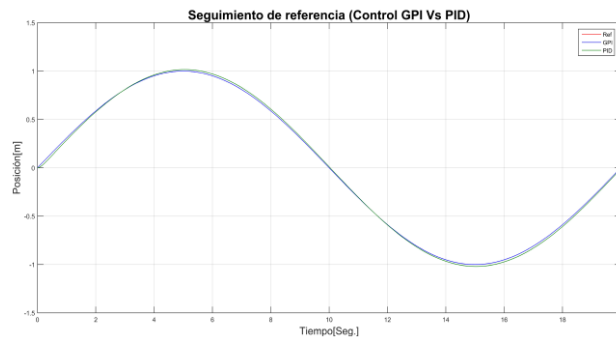


Figure 2. Reference tracking of the PID controller and the GPI controller

Eight poles were selected for the GPI controller with values of $[-2.0 - 2.5 - 3.0 - 3.5 - 4.0 - 4.5 - 5.0 - 5.5] \times 10^2$, resulting in Figure 2 and Figure 3; these results are shown in table 2.

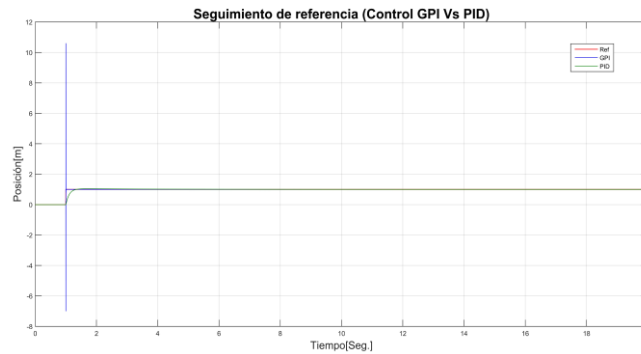


Figure 3. Reference tracking of the PID controller and the GPI controller.

Tabla 2: Base de reglas Base of rules

Est. de ctrl	PECM(Sin)	PECM(Step)
GPI	$6.1756 \times 10^{-13}\%$	0.2222%
PID	0.0515%	0.2802%

For the disturbance rejection, a step type signal and a 2Hz sinusoidal signal were added to the input of the plant at 10s, and the mean square error percentage was calculated.

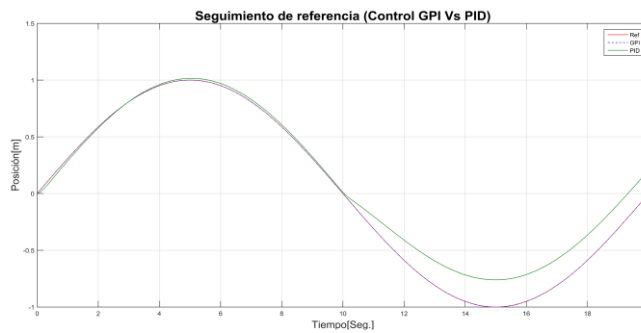


Figure 4. Disturbance Rejection of the PID controller and the GPI controller

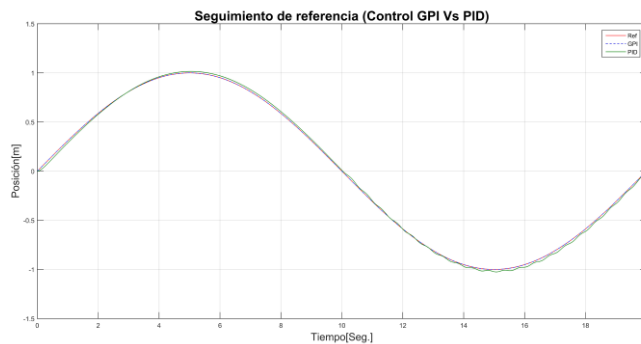


Figure 5. Disturbance Rejection of the PID controller and the GPI controller

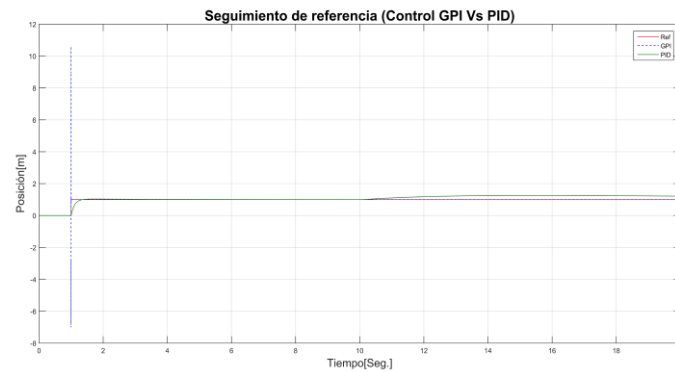


Figure 6. Disturbance Rejection of the PID controller and the GPI controller

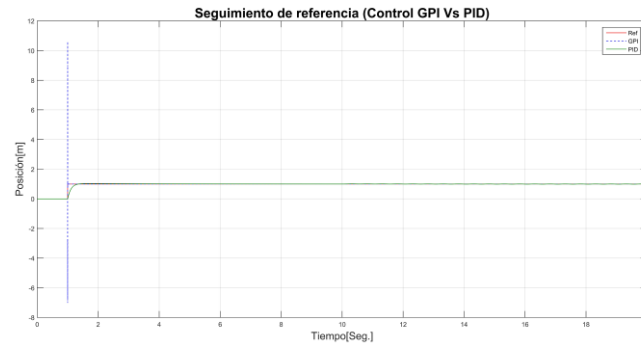


Figure 7. Disturbance Rejection of the PID controller and the GPI controller

Table 3: Step type disturbance

Est. de ctrl	PECM(Sin)	PECM(Step)
GPI	$6.1756 \times 10^{-13}\%$	0.2222%
PID	4.3065%	2.8472%

Table 4: Sinusoidal type disturbance

Est. de ctrl	PECM(Sin)	PECM(Step)
GPI	$6.1756 \times 10^{-13}\%$	0.2222%
PID	0.0562%	0.2821%

5. Conclusions

The GPI control techniques present a very good response to different external disturbances.

It is possible to affirm that the error tends asymptotically to zero provided that the gains of the error polynomial are sufficiently large.

Compared to PID control, the proposed control presents better performance in terms of monitoring and disturbance rejection. The results of the PECM calculation show a clear advantage of the GPI control over the PID control.

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