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A Novel Robust Control Method for Motion Control of Uncertain Single-Link Flexible-Joint Manipulator

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***Abstract*—Single-link flexible-joint manipulator (FJM) is a kind of time-varying nonlinear system with underactuated characteristics. This paper takes the uncertain single-link FJM (USLFJM) as the object, and presents a novel robust control approach based on equivalent input disturbance (EID) method for motion control (i.e., position control and trajectory tracking control) of the USLFJM. Based on the uncertain dynamic model and desired target of this system, an error system is constructed. The error system is considered as a linear system with a nonlinear virtual disturbance. The motion control objective of the USLFJM can be realized by globally asymptotically stabilizing this linear system and compensating the influence of the nonlinear virtual disturbance. Then, an EID-based control system is designed to realize this control objective. Only position measurements are utilized in this paper, and the stability of the control system is proved. The simulation results are presented to illustrate the validity and the robustness of the proposed control method.**

***Index Terms*—Equivalent input disturbance (EID), robust control, single-link flexible-joint manipulator (FJM), underactuated system, vibration control.**

I. Introduction

FLEXIBLE-JOINT manipulators (FJMs), whose joints are generally made of flexible materials, can avoid physical damage to the object or the manipulators themselves and make the whole operation more safe and flexible [1]. Therefore, there is a great demand for the FJMs in many fields, such as industrial production, aerospace, and so on.

However, the FJMs are a class of strong-coupling and time-varying nonlinear systems [2], [3]. Their degrees of freedom are more than their control input(s), so the FJMs belong to underactuated systems [4]–[6]. Since underactuated variable(s) can only be controlled indirectly depending on the coupling relationship with active variable(s), motion control of underactuated system is a challenging problem [7], [8]. In addition, the joint flexibility of the FJMs makes the system exist vibration. If this mechanical vibration cannot be suppressed effectively in the control process of the FJMs, it will influence the control accuracy of the system or even destroy the system [9]. Thus, unlike the rigid joint manipulators, the vibration suppression should be considered in the motion control (i.e., position control and trajectory tracking control) of the FJMs.

During the past two decades, people have carried out some researches on motion control of the FJMs. For the position control of the FJMs, a simple PD control was devised by requiring full state measurements in [10]. Then it was extended by only using position measurements [11]. Xin and Liu [12] presented an energy-based control. For the trajectory tracking control of the FJMs, Luca and Book [3] presented a static feedback linearization approach. Ge [13] designed an alternative singular perturbation method. In [14], a linear matrix inequality approach for both robust observer design and observer-based control was proposed. In [15], an output feedback controller was designed by observer dynamic surface design technique. Besides, the energy-shaping method [16], the neuro-adaptive observer-based method [17], and the time-scale expansion-based method [18] were also designed. However, in practical applications, the FJMs are usually affected by some uncertainties, caused by parameter perturbations and external disturbances (i.e., unmodeled dynamics, motor vibration, and so on). Therefore, these above methods will not be applied anymore.

To realize the motion control for the uncertain FJMs, some robust control methods have been explored. For example, an extended-state-observer-based control for the trajectory tracking control of the uncertain FJMs was proposed in [19]. A passivity-based controller for compliantly actuated robots was designed in [20]. An adaptive sliding controller with backstepping like design was presented in [21]. In addition, the iterative scheme [22], the neural-learning-based control strategy [23], and some adaptive control methods [24]–[27] were presented for the motion control against parameter perturbations or external disturbances. However, these methods do not have good performance for the FJMs with both parameter perturbations and external disturbances. Moreover, these methods need multiple sensors, which will not only affect the flexibility of joints but also bring extra noise.

In this paper, we choose a single-link FJM with parameter perturbations and external disturbances as the object to study its robust motion control method. For this uncertain single-link FJM (USLFJM), we first construct an error system based on its uncertain dynamic model and desired target. This error

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system can be considered as a linear system with a nonlinear virtual disturbance. To realize both position control and trajectory tracking control of the USLFJM, we present an equivalent input disturbance (EID)-based control method to stabilize the error system. In the EID-based control system, a state feedback controller is designed to globally asymptotically stabilize the linear system. Meanwhile, a full-order observer and an EID estimator are constructed to compensate the influence caused by the nonlinear virtual disturbance of the error system. Clearly, when the error system is stabilized, the motion control objective of the USLFJM is realized. Considering the real-world factors in the industrial production, a converter and a time-derivative tracker are designed in the proposed EID-based control system. Therefore, the controller design process of the system is simplified and only position measurements are used in the control system. The stability of the control system is proved and the simulation results are given to illustrate the validity of the proposed robust control method.

Comparing with other methods for the USLFJM, the main contributions of this paper are characterized as follows.

1) A simple and valid robust control approach-based on the EID method for the USLFJM has been designed.
2) The proposed method shows good performance for motion control and vibration suppression of the SLFJM with both parameter perturbations and external disturbances.
3) Only position measurements have been used here.

This paper is organized as follows. The uncertain dynamic model of the USLFJM and the corresponding error system are presented in Section II. The design of an EID-based control law is introduced in Section III. Finally, simulations are given in Section IV.

In this paper, we note that $*^{(i)}$ denotes the ith time derivative of $*$, $\|*\|_\infty$ denotes the infinite norm, $*(s)$ denotes the Laplace transformation of the signal $*(t)$, and $\mathrm{eig}(*)$ denotes the set of the eigenvalues for the matrix $*$.

II. Model Description and Problem Formulation

In this section, the dynamic model of the USLFJM is built. Then an error system about this dynamic model and desired target is constructed to simplify the analysis and design of the control system. The control problem of the USLFJM is also discussed in this section.

A. Dynamic Modeling

The structure of the USLFJM (Fig. 1) contains one flexible joint and one rigid link. The space model of the flexible-joint is shown in Fig. 2. This system has two degrees of freedom but only one control input generated by motor. Therefore, this system is an underactuated system. The effect of gravity for the system is taken into account since this system rotates in a vertical plane. The model parameters and variables of the USLFJM are given as follows.

- θ Angle of motor shaft relative to the vertical axis.
- q Angle of link relative to the vertical axis.
- $\hat{m}$ Mass of link.

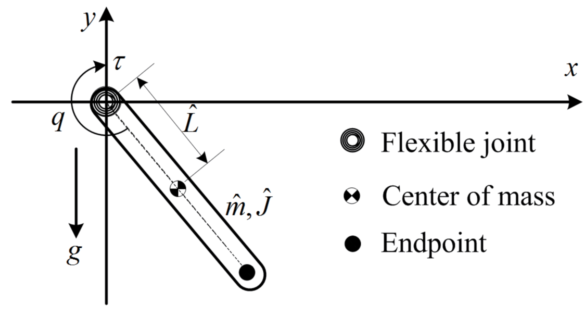

Fig. 1. Physical structure of the USLFJM.

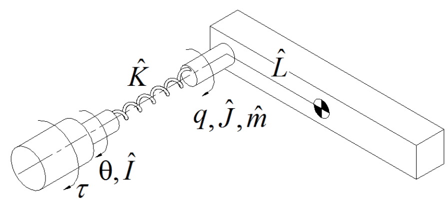

Fig. 2. Flexible-joint space model of the USLFJM.

- $\hat{L}$ Distance from the flexible-joint to the center of mass of the link.
- $\hat{I}$ Moment of inertia relative to motor shaft.
- $\hat{J}$ Moment of inertia relative to link shaft.
- $\hat{K}$ Stiffness of spring.
- τ Torque of motor.

Actually, the uncertainties of the USLFJM usually include parameter perturbations and external disturbances. Therefore, the model parameters of the USLFJM can be expressed as $\hat{m} = m + \Delta m$, $\hat{L} = L + \Delta L$, $\hat{I} = I + \Delta I$, $\hat{J} = J + \Delta J$, and $\hat{K} = K + \Delta K$, where m, L, I, J, and K are certain constants denoting the nominal parts of the model parameters, and Δm, ΔL, ΔI, ΔJ, ΔK are the uncertain constants denoting the perturbation parts of the model parameters. We assume that all the absolute values of the unknown constants have upper bounds.

Then, the kinetic energy, L_k, of the USLFJM is

$$L_k = \frac{1}{2}\hat{I}\dot{\theta}^2 + \frac{1}{2}\hat{J}\dot{q}^2 \tag{1}$$

and the potential energy, L_p, of the USLFJM is

$$L_p = \frac{1}{2}\hat{K}(q-\theta)^2 + \hat{m}g\hat{L}(1-\cos q). \tag{2}$$

We choose $L_a = L_k - L_p$ to be the Lagrangian function of the system. Let $\beta = [q, \theta]^T$. Using the Euler–Lagrange modeling equations

$$\begin{cases} \frac{d}{dt}\left[\frac{\partial L_a}{\partial \dot{q}}\right] - \frac{\partial L_a}{\partial q} = \varepsilon_1 \\ \frac{d}{dt}\left[\frac{\partial L_a}{\partial \dot{\theta}}\right] - \frac{\partial L_a}{\partial \theta} = \tau + \varepsilon_2 \end{cases} \tag{3}$$

we obtain the dynamic model of the USLFJM

$$\hat{M}\ddot{\beta} + \hat{G} = \lambda\tau + \varepsilon \tag{4}$$

where $\lambda = [0, 1]^T$ and $\varepsilon = [\varepsilon_1, \varepsilon_2]^T$ is the bounded external disturbances of the system including the unmodeled dynamics, $\hat{M}$ is the positive-definite inertial matrix, and $\hat{G}$ is the matrix of gravitational force and elastic force. $\hat{M}$ and $\hat{G}$ can be expressed as

$$\hat{M} = \begin{bmatrix} \hat{J} & 0 \\ 0 & \hat{I} \end{bmatrix}, \quad \hat{G} = \begin{bmatrix} \hat{K}(q-\theta) + \hat{m}g\hat{L}\sin q \\ -\hat{K}(q-\theta) \end{bmatrix} \tag{5}$$

where $\hat{M} = M + \Delta M$ and $\hat{G} = G + \Delta G$, where M and G are the certain matrices and ΔM and ΔG are the uncertain matrices. The matrices M, G, ΔM, and ΔG can be expressed as follows:

$$\begin{aligned} M &= \begin{bmatrix} J & 0 \\ 0 & I \end{bmatrix}, \quad \Delta M = \begin{bmatrix} \Delta J & 0 \\ 0 & \Delta I \end{bmatrix} \\ G &= \begin{bmatrix} K(q-\theta) + mgL\sin q \\ -K(q-\theta) \end{bmatrix} \\ \Delta G &= \begin{bmatrix} \Delta K(q-\theta) + \Delta\mu\sin q \\ -\Delta K(q-\theta) \end{bmatrix} \end{aligned} \tag{6}$$

where $\Delta\mu = (\Delta mL + m\Delta L + \Delta m\Delta L)g$.

Let $x = [x_1, x_2, x_3, x_4]^T = [q, \theta, \dot{q}, \dot{\theta}]^T$. The state space equation of the uncertain dynamic model (4) is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ -\frac{K}{J}(x_1 - x_2) - \frac{mgL}{J}\sin x_1 + \psi_1 \\ \frac{K}{I}(x_1 - x_2) + \frac{\tau}{I} + \psi_2 \end{bmatrix} \tag{7}$$

where

$$\begin{aligned} \psi_1 &= \frac{\Delta J}{J}\ddot{x}_1 + \frac{\Delta K}{J}(x_1 - x_2) + \frac{\Delta\mu}{J}\sin x_1 - \frac{\varepsilon_1}{J} \\ \psi_2 &= \frac{\Delta I}{I}\ddot{x}_2 - \frac{\Delta K}{I}(x_1 - x_2) - \frac{\varepsilon_2}{I}. \end{aligned} \tag{8}$$

B. Problem Formulation

The most common control objective for the USLFJM is motion control, which consists of position control and trajectory tracking control.

Let q_d be the desired objective of the link angle, and let $e_1 = q - q_d$. Based on the uncertain dynamic model (4), the time derivative of the error e_1 satisfies

$$\begin{bmatrix} \dot{e}_1 \\ \ddot{e}_1 \\ e_1^{(3)} \\ e_1^{(4)} \end{bmatrix} = \begin{bmatrix} \dot{q} - \dot{q}_d \\ -\frac{K}{J}(q-\theta) - \frac{mgL}{J}\sin q - \ddot{q}_d + \eta_1 \\ -\frac{K}{J}(\dot{q}-\dot{\theta}) - \frac{mgL}{J}\dot{q}\cos q - q_d^{(3)} + \eta_2 \\ -\frac{K}{J}\ddot{q} + \frac{K}{IJ}\tau - q_d^{(4)} + \xi + \frac{K^2}{IJ}(q-\theta) + \alpha \end{bmatrix} \tag{9}$$

where

$$\begin{aligned} \xi &= \frac{mgL}{J}(\dot{q}^2\sin q - \ddot{q}\cos q), \alpha = \frac{K}{J}\psi_2 + \eta_3 \\ \eta_1 &= \frac{\Delta J}{J}\ddot{q} + \frac{\Delta K}{J}(q-\theta) + \frac{\Delta\mu}{J}\sin q - \frac{\varepsilon_1}{J}. \end{aligned} \tag{10}$$

η_2 and η_3 are uncertain terms involving parameter perturbations and external disturbances. Since the external disturbances may not be continuous and differentiable, η_2 and η_3 cannot be expressed concretely here.

Let $e = [e_1, e_2, e_3, e_4]^T = [e_1, \dot{e}_1, \ddot{e}_1, e_1^{(3)}]^T$. In order to simplify the controller design process, based on (9), we design

$$\tau = \frac{IJ}{K}(u_e + q_d^{(4)} - \xi) - K(q-\theta) + I\ddot{q}_d. \tag{11}$$

Next, an error system is constructed from (9) and (11)

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} e_2 \\ e_3 \\ e_4 \\ -\frac{K}{J}e_3 + u_e + \alpha \end{bmatrix} \tag{12}$$

where u_e is the input of the error system.

Remark 1: For the position control, q_d is the desired angle of the link, which is a constant, hence $\dot{q}_d, \ddot{q}_d$, and $q_d^{(3)}$ are equal to zero. As for the trajectory tracking control, q_d is the desired trajectory of the link angle. And we assume that q_d is a given smooth function containing time variable t, which means $\dot{q}_d$, $\ddot{q}_d$, and $q_d^{(3)}$ are computable.

In the process of designing the actual law (11), q, θ, and the time derivative of link angle, $\dot{q}$, $\ddot{q}$, and $q^{(3)}$ (i.e., $q^{(i)}, i = 1, 2, 3$), should be measured for the USLFJM system. Noted that $q^{(3)}$ is related to the design of u_e, which is illustrated in Section III. In general, the angles of the FJMs can be accurately measured by encoders. However, the angular velocities cannot be accurately obtained by tachometers, because the tachometers usually bring noise, which affects the control performance of the system [28]. Therefore, we only measure the position measurements, q, θ, by encoders, and we estimate $q^{(i)}$ by the designing of high-order time-derivative tracker.

Let $q_s^{(i)}$ be the estimated value of $q^{(i)}$ and $z = [z_1, z_2, z_3, z_4]^T = [q_s, \dot{q}_s, \ddot{q}_s, q_s^{(3)}]^T$. Here, we introduce a high-order time-derivative tracker [29] to estimate $q^{(i)}$. The time-derivative tracker can estimate its input signal without any knowledge of its dynamics. The state space equation of the high-order time-derivative tracker is

$$\begin{bmatrix} \dot{z}_i \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_{i+1} \\ -\gamma^4(z_1 - v) - C_4^3\gamma^3 z_2 - C_4^2\gamma^2 z_3 - C_4^1\gamma z_4 \end{bmatrix} \tag{13}$$

where $v = q$ is the input signal, C_4^k $(k = 1, 2, 3, 4)$ is the binomial coefficient, and $\gamma > 0$ is constant. According to [29], we know that

$$\lim_{\gamma\to\infty} q_s^{(i)} = q^{(i)}. \tag{14}$$

Let $e_{js} = q_s^{(i)} - q_d^{(i)} (j = 2, 3, 4)$ be the estimated value of e_j in (12). Clearly, e_{js} approach e_j as γ approaches infinity. Therefore, the control input for the system (7) is rewritten as

$$\tau = \frac{IJ}{K}(u_e + q_d^{(4)} - \xi_s) - K(q-\theta) + I\ddot{q}_d \tag{15}$$

where $\xi_s = (mgL/J)(\dot{q}_s^2\sin q - \ddot{q}_s\cos q)$. After that, we can get the control input by designing the control law u_e from (15). In other words, (15) can be considered as a converter from u_e to τ.

Then we consider the system (12) as a linear system with a nonlinear virtual disturbance. System (12) can be rewritten as

$$\begin{cases} \dot{e} = Ae + Bu_e + d \\ y = Ce \end{cases} \tag{16}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{K}{J} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad d = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \alpha + \sigma_4 \end{bmatrix}. \tag{17}$$

$C \in \mathbb{R}^{4\times 4}$ is an identity matrix, the nonlinear virtual disturbance d contains the nonlinear term of the error system (12) and the disturbances, $\sigma_k(k = 1, 2, 3, 4)$, caused by the time-derivative tracker.

Therefore, our control objective has been transformed to design the control law, u_e, to asymptotically stabilize the linear system (A, B, C) at the origin and compensate the influence of the nonlinear virtual disturbance in (16).

III. EID-Based Control System Design

In this section, we design an EID-based control system to realize our control objective.

It is not difficult to prove that the system (A, B, C) in (16) has the following two characteristics: 1) the system (A, B, C) is controllable and observable and 2) the system (A, B, C) has no zeros, because the Smith–Macmillan standard form of the system (A, B, C) is $[1/(s^4 + s^2K/J), 0, 0, 0]$. When the system (A, B, C) in (16) satisfies the above two specific characteristics, according to the EID existence conclusion given in [30], we know that there always exists an EID, δ, in the control input channel. Besides, the influence that the EID produced is the same as the influence that the nonlinear virtual disturbance d [in (16)] produced. That is to say, the system (16) can be expressed as

$$\begin{cases} \dot{e} = Ae + B(u_e + \delta) \\ y = Ce. \end{cases} \tag{18}$$

That means the nonlinear virtual disturbance, d, in (16) is introduced into the input channel of the system (18) in the form of EID, δ. In this way, if the EID δ can be calculated or estimated to actively compensate the influence of nonlinear virtual disturbance, we can design a feedback controller to globally asymptotically stabilize the linear system (A, B, C) at the origin.

Therefore, we present an EID-based control system, which contains full-order observer, EID estimator, and state feedback controller. Every part is illustrated as follows.

A. Design of Full-Order Observer

In order to build the EID estimator, a full-order observer is constructed first as an auxiliary. The full-order state observer is presented as

$$\begin{cases} \dot{\hat{e}} = A\hat{e} + B\hat{u}_e + K_l(y - \hat{y}) \\ \hat{y} = C\hat{e} \end{cases} \tag{19}$$

where $\hat{e}$ is the estimated value of e, $K_l \in \mathbb{R}^{4\times 4}$ is the observer gain, and $\hat{u}_e$ is the input of the observer.

Since this observer (19) needs provide estimation state to both EID estimator and feedback controller as accurate as possible, the convergence rate of the observer must be faster than the system (18). Here, we use an effective approach to turn the full-order observer into a high gain observer [30]. The design process of the high gain observer is briefly described to make this paper self-sufficient.

Since the system (A, B) is controllable and (C, A) is observable, the observability of the system (19) can be transformed into the controllability of its dual system. Thus, the optimal estimation problem of the system (18) can be transformed to the optimal control problem of its dual system [31]. The dual system of the system (A, B, C) can be easily obtained

$$\begin{cases} \dot{e}_l = A^T e_l + C^T \hat{u}_{el} \\ y_l = B^T e_l \end{cases} \tag{20}$$

and the quadratic cost function is

$$J_1 = \int_0^{\infty} \{(\rho {e_l}^T Q_1 e_l + \hat{u}_{el}^T R_1 \hat{u}_{el})\} dt \tag{21}$$

where $Q_1 \in \mathbb{R}^{4\times 4}$ is a positive-definite symmetric matrix, and R_1, $\rho > 0$ are scalar weighted parameters.

The optimal control law of the dual system (20) is

$$\hat{u}_{el} = K_l^T e_l. \tag{22}$$

By using the Ricatti matrix differential equation to minimize (21), the controller gain ${K_l}^T$ satisfies

$$\begin{cases} AS_1 + S_1A^T - S_1C^TR_1^{-1}CS_1 + \rho Q_1 = 0 \\ {K_l}^T = \frac{1}{2}{R_1}^{-1}CS_1 \end{cases} \tag{23}$$

which yields

$$(A - K_lC)^T S_1 + S_1(A - K_lC) + \rho Q_1 = 0 \tag{24}$$

where $S_1 = S_1^T > 0$ and $S_1 \in \mathbb{R}^{4\times 4}$ are the solution of the Ricatti matrix differential equation. That means quicker convergence rate of the observer will be obtained by the increase of ρ [32].

In addition, we define the observation errors of the state and the output as

$$\begin{cases} \Delta e = \hat{e} - e \\ \Delta y = \hat{y} - y. \end{cases} \tag{25}$$

Lemma 1 [33]: γ is a stability matrix (all real parts of γ are less than zero), if and only if for any $Q > 0$, there exists a unique solution $P > 0$ for the Lyapunov equation

$$P\gamma + \gamma^T P + Q = 0. \tag{26}$$

Obviously, $A - K_lC$ in (24) is similar to γ in (26). Thus, based on Lemma 1, we know that the K_l obtained from (23) makes $A - K_lC$ stable.

Combining (18), (19), and (25) yields

$$\Delta\dot{e} = (A - K_lC)\Delta e + \hat{u}_e - u_e - \delta. \tag{27}$$

So, if we can obtain the value of δ, we can design $u_e = \hat{u}_e - \delta$ to compensate disturbances. Then we have $\Delta\dot{e} = (A - K_lC)\Delta e$. Clearly, when $A - K_lC$ is stable, $\Delta e \to 0$ ($\hat{e} \to e$) as $t \to \infty$.

Therefore, in the next section, we obtain the value of δ by designing an EID estimator.

B. Design of EID Estimator

In fact, the EID, δ, can be obtained directly by stable-inversion approach [34]. However, this approach is pretty complex. Here, we utilized a simpler estimator, EID estimator, to indirectly estimate and compensate the disturbance. Different from the traditional disturbance-observer method as [35]–[37], this EID estimator requires no inverse dynamics.

Combining (18), (19), and (25) also yields

$$\dot{\hat{e}} = A\hat{e} + Bu_e + [B\delta + (\Delta\dot{e} - A\Delta e)]. \tag{28}$$

Assume that there is a $\Delta\delta$ that satisfies

$$\Delta\dot{e} - A\Delta e = B\Delta\delta. \tag{29}$$

Substituting (28) into (29) yields

$$\dot{\hat{e}} = A\hat{e} + B(u_e + \hat{\delta}) \tag{30}$$

where

$$\hat{\delta} = \delta + \Delta\delta \tag{31}$$

is the estimated value of the EID. Combining (19), (25), and (30), we have

$$\hat{\delta} = -B^+K_lC\Delta e + \hat{u}_e - u_e \tag{32}$$

where $B^+ = (B^TB)^{-1}B^T$.

In addition, for the EID estimator, there should be a term in u_e to offset the disturbance. Here, we use a filter $F(s)$. After filtering, $\hat{\delta}$ is recorded as $\tilde{\delta}$, and it satisfies

$$\tilde{D}(s) = F(s)\hat{D}(s) \tag{33}$$

where $\tilde{D}(s)$ and $D(s)$ are the Laplace transforms of $\tilde{\delta}$ and $\hat{\delta}$, respectively. The filter $F(s)$ is designed as

$$|F(jw)| \approx 1 \ \forall w \in [0, w_r] \tag{34}$$

where w_r denotes the highest angular frequency of the disturbance. Besides, the cut-off angular frequency of $F(s)$ should be five to ten times larger than w_r. Thus, the control law is set as

$$u_e = \hat{u}_e - \tilde{\delta}. \tag{35}$$

Based on (18), (19), and (25), we also obtain

$$\begin{cases} \Delta\dot{e} = A\Delta e - K_l\Delta y + B(\hat{u}_e - \delta - u_e) \\ \Delta y = C\Delta e. \end{cases} \tag{36}$$

In the s-domain, substituting (35) into (36) yields

$$\Delta e(s) = [sI_4 - (A - K_lC)]^{-1}B\big[\tilde{D}(s) - D(s)\big]. \tag{37}$$

From (32) and (35), there is the following relationship between $\tilde{\delta}$ and $\hat{\delta}$:

$$\hat{\delta} = -B^+K_lC\Delta e + \tilde{\delta}. \tag{38}$$

From (33), (37), and (38), in the s-domain, we obtain that

$$[I - F(s)]\tilde{D}(s) = -F(s)B^+K_lW(s)[D(s) - \tilde{D}(s)] \tag{39}$$

where $W(s) = [sI_4 - (A - K_lC)]^{-1}B$. Therefore, based on (34) and (39), $\tilde{D}(jw)$ approaches $D(jw)$ when $F(jw)B^+K_lW(jw)$ has a full column rank for almost all $\forall w \in [0, w_r]$ [38]. Because a first-order filter is helpful enough [30], we employ this filter as

$$F(s) = \frac{1}{Ts + 1} \tag{40}$$

where T is a time constant.

C. Design of State Feedback Controller

Finally, we design a state feedback controller to ensure the asymptotically stability of the closed-loop system (18). Here, we use the linear quadratic regulation optimal control method. There ia a matrix $K_f \in \mathbb{R}^{1\times 4}$ that makes the feedback control law be

$$\hat{u}_e = K_f\hat{e}. \tag{41}$$

Minimizing the quadratic cost function

$$J_2 = \int_0^\infty (\hat{e}^TQ_2\hat{e} + \hat{u}^TR_2\hat{u})dt \tag{42}$$

gives us

$$S_2A + A^TP - PBR_2^{-1}B^TS_2 + Q_2 + \dot{S}_2 = 0 \tag{43}$$

and

$$K_f = -R_2^{-1}B^TS_2 \tag{44}$$

where $Q_2 \in \mathbb{R}^{4\times 4}$ is a positive-definite symmetric matrix, R_2 is a scalar positive parameter, and $S_2 \in \mathbb{R}^{4\times 4}$ is the solution of the Ricatti matrix differential equation. The optimal $\hat{u}_e$ satisfies (43) and (44).

For the system (18), since (A, B) is controllable, combining (43) and (44), we can get that the system (A, B, C) is asymptotically stable when $A + BK_f$ is stable.

Noted that the full-order observer and feedback controller should be designed that do not destroy the stability of the error system. Regarding stability, if we let $d = 0$, the transfer function of $\hat{\delta}$ to $\tilde{\delta}$ in the s-domain can be obtained from (37) and (38)

$$G(s) = B^+(sI_4 - A)N \tag{45}$$

where

$$N = [sI_4 - (A - K_lC)]^{-1}B. \tag{46}$$

According to the separation theorem [32] and the small gain theorem [31], the stability of the system (18) can be guaranteed under the following conditions.

1) $A - K_lC$ and $A + BK_f$ are stable.
2) $\|GF\|_\infty = \sup_{\omega\geq 0} \varepsilon_{\max}[G(j\omega)F(j\omega)] < 1$, where $\varepsilon_{\max}[\cdot]$ is the maximum singular-value function.

Condition 1) has been proved in the previous design. In (23), according to the concept of perfect regulation [39], we can design K_l^T to ensure

$$\lim_{\rho\to\infty} \{[sI_4 - (A - K_lC)]^{-1}B\}^T = 0 \tag{47}$$

where $I_4 \in \mathbb{R}^{4\times 4}$ is the identity matrix. From (45)–(47), we find the left part of (47) is a part of (46). Thus, for a given filter, $F(s)$, condition 2) is guaranteed by designing K_l under a large enough ρ.

The structure diagram of the EID-based control system is shown in Fig. 3.

IV. Simulation

In this section, we apply the EID-based control method to realize motion control of the USLFJM. By using MATLAB, the simulation results of the position control and trajectory tracking control for the USLFJM are presented as follows.

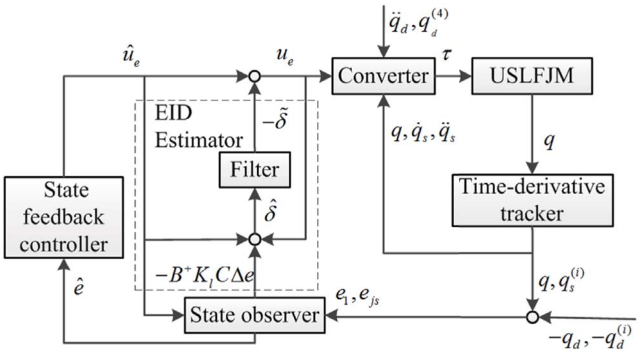

Fig. 3. Structure diagram of EID-based control system.

A. Position Control

For the position control, the USLFJM can be stabilized at any desired position from any initial position by using the EID-based control method. Here, we rotate this manipulator from the initial position, $q(0) = 0$, to the desired position, $q_d = (\pi/2)$, and suppress its vibration ($q_d \to (\pi/2)$, $\dot{q}_d \to 0$).

The nominal parameters are $m = 2$ kg, $L = 1$ m, $K = 10$ N $\cdot$ m/rad, $I = 0.5$ kg $\cdot$ m^2, and $J = 2$ kg $\cdot$ m^2. In order to illustrate the robustness of the EID-based control method, we made a change of 10% in the model nominal parameters. The actual model parameters and the external disturbances are set as

$$\begin{cases} \hat{m} = 1.1m, \ \hat{L} = 1.1L, \hat{I} = 1.1I, \ \hat{K} = 1.1K, \ \hat{J} = \hat{m}\hat{L}^2 \\ \varepsilon_1 = 0.2\sin t, \ \varepsilon_2 = 0.1\sin 2t. \end{cases} \quad (48)$$

The initial states are set as

$$[q(0), \theta(0), \dot{q}(0), \dot{\theta}(0)]^T = [0, 0, 0, 0]^T. \quad (49)$$

The parameter of the time-derivative tracker is chosen as $\gamma = 300$. The design parameters of the controller are set as

$$\begin{cases} w_r = 5 \text{ rad/s}, T = 0.03 \\ Q_1 = 3000I_4, R_1 = I_4, \rho = 10^6 \\ Q_2 = 150I_4, R_2 = 1. \end{cases} \quad (50)$$

We can obtain that

$$\begin{cases} \text{eig}(A - K_l C) = -9.1324 \pm 0.4278i, -9.1849 \pm 2.8473i \\ \text{eig}(A + BK_f) = -11.7790, -1.0688, -0.7171 \pm 0.6772i \\ \|GF\|_\infty = 0.7708 < 1. \end{cases} \quad (51)$$

That means the system (18) meets the requirements of stability.

The simulation results are shown in Fig. 4. Under the influence of parameter perturbations and external disturbances, the angle of the link reaches the desired position in approximately 6 s, and then the vibration of the USLFJM is well suppressed by using the proposed EID-based control method. Therefore, the EID-based control method we proposed has a good robustness.

B. Trajectory Tracking Control

To show the superiority of the presented control method, we compare our trajectory tracking performances with [17]. The model parameters, desired trajectory, and initial states are

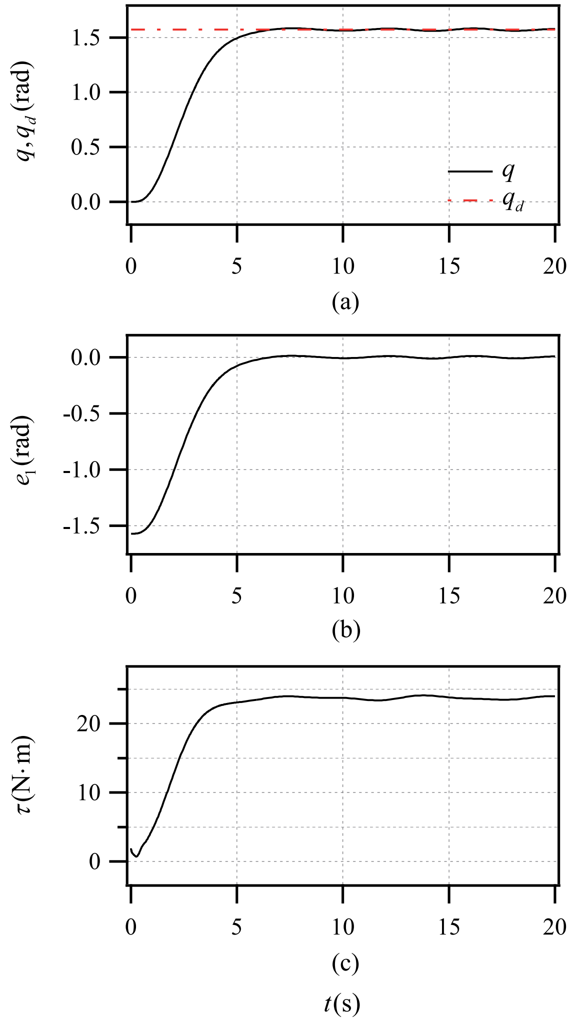

Fig. 4. Position control of the USLFJM. (a) Angles. (b) Angular error. (c) Control torque.

the same as [17], i.e., $m = 2$ kg, $L = 1$ m, $K = 10$ N $\cdot$ m/rad, $I = 0.5$ kg $\cdot$ m^2, and $J = 2$ kg $\cdot$ m^2. The desired trajectory is

$$\ddot{q}_d = 2\sin 1.5t - 0.5q_d - 1.5\dot{q}_d. \quad (52)$$

The initial states are

$$\begin{cases} [q(0), \theta(0), \dot{q}(0), \dot{\theta}(0)]^T = [0.2, 0.2, 0, 0]^T \\ [q_d(0), \dot{q}_d(0)]^T = [0.5, 0.2]^T. \end{cases} \quad (53)$$

The control strategy in [17] did not consider the effect on the perturbed parameters and external disturbances. To further illustrate the validity of our method, in this simulation, we added the unmodeled dynamics (e.g., viscous damping of both the link and actuator) in the external disturbances on the basis of (48). Generally speaking, the viscous damping in FJM system is usually expressed as the product of angular velocity and viscous damping coefficients. Therefore, the actual model parameters and the external disturbances are given as

$$\begin{cases} \hat{m} = 1.1m, \ \hat{L} = 1.1L, \ \hat{I} = 1.1I, \ \hat{K} = 1.1K, \ \hat{J} = \hat{m}\hat{L}^2 \\ \varepsilon_1 = 0.4\dot{q} + 0.2\sin t, \ \varepsilon_2 = 0.6\dot{\theta} + 0.1\sin 2t. \end{cases} \quad (54)$$

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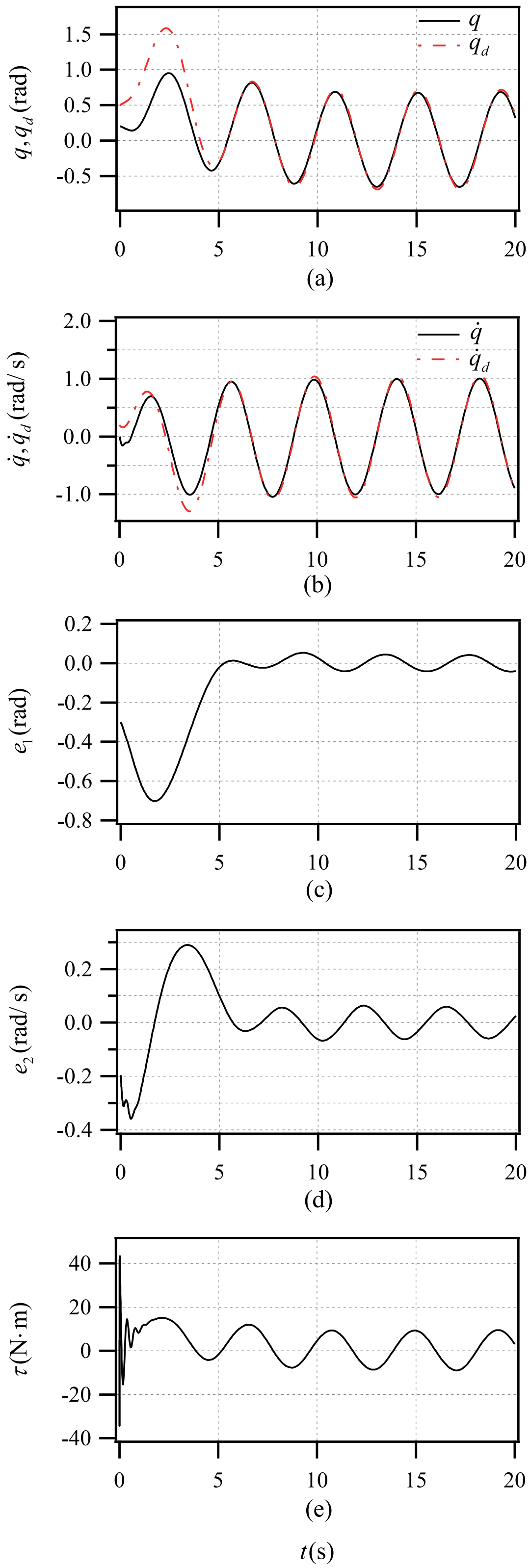

Fig. 5. Trajectory tracking control of the USLFJM. (a) Angles. (b) Angular velocities. (c) Angular error. (d) Angular velocity error. (e) Control torque.

In addition, the design parameters of the time-derivative tracker and the controller are chosen as

$$\begin{cases} \gamma = 1000, \ w_r = 5 \text{ rad/s}, T = 0.02 \\ Q_1 = 3000 I_4, \ R_1 = I_4, \rho = 5 \times 10^5 \\ Q_2 = 300 I_4, R_2 = 1. \end{cases} \tag{55}$$

We can get that

$$\begin{cases} \text{eig}(A - K_l C) = -6.4602 \pm 0.4277i, -6.5322 \pm 2.8335i \\ \text{eig}(A + BK_f) = -16.9964, -1.0321, -0.7126 \pm 0.6926i \\ \|GF\|_\infty = 0.8729 < 1. \end{cases} \tag{56}$$

Therefore, the system (18) meets the requirements of stability.

The trajectory tracking results in Fig. 5 show that the link angle, q, can follow the trajectory in approximately 5 s, and trajectory errors are converging to a small neighborhood of zero. In contrast, [17] achieved approximate tracking effect only for nominal single-link FJM system. Our results show the superiority of the EID-based control method in robustness.

V. Conclusion

In this paper, a novel robust control method has been presented for motion control (i.e., position control and trajectory tracking control) of a single link FJM with parameter perturbations and external disturbances. Meantime, only position measurements have been used in this paper. Here, we constructed an error system based on the uncertain dynamic model of the USLFJM. The stability of this EID-based control system was proved. The simulation results illustrated the validity of the proposed control method. In addition, the control method proposed in this paper is aimed at USLFJM. Extending the proposed method to the multilink FJMs and other underactuated systems with similar characteristics is an important direction of our future efforts.

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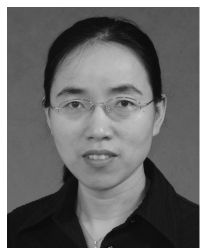

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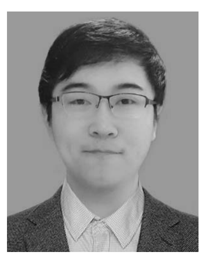

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