HW5 for GPGN605: Tikhonov regularization

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1 SMALLEST MODEL

For the smallest model, solve the following equation

$$(\mathbf{G}^\top \mathbf{W}^\top{}_d \mathbf{W}_d + \beta \mathbf{I})\mathbf{m} = \mathbf{G}^\top \mathbf{W}^\top{}_d \mathbf{W}_d \mathbf{d}^{obs} \tag{1}$$

Results:

1) Data misfit is shown in Figure 1a;

2) Model objective function is shown in Figure 1b;

3) Tikhonov curve is shown in Figure 1c;

4) Curvature of the Tikhonov curve is shown in Figure 2a;

5) The GCV curve is shown in Figure 2b;

6) Using the discrepancy principle, $\beta_1 = 0.16$;

7) Using the L-curve (maximum curvature of the L-curve), $\beta_2 = 0.208$;

8) Using the GCV curve, $\beta_3 = 0.343$;

9) Constructed model using β_1 (discrepancy principle) is the red curve in Figure 3;

10) Constructed model using β_2 (L-curve) is the green curve in Figure 3;

11) Constructed model using β_3 (GCV) is the orange curve in Figure 3.

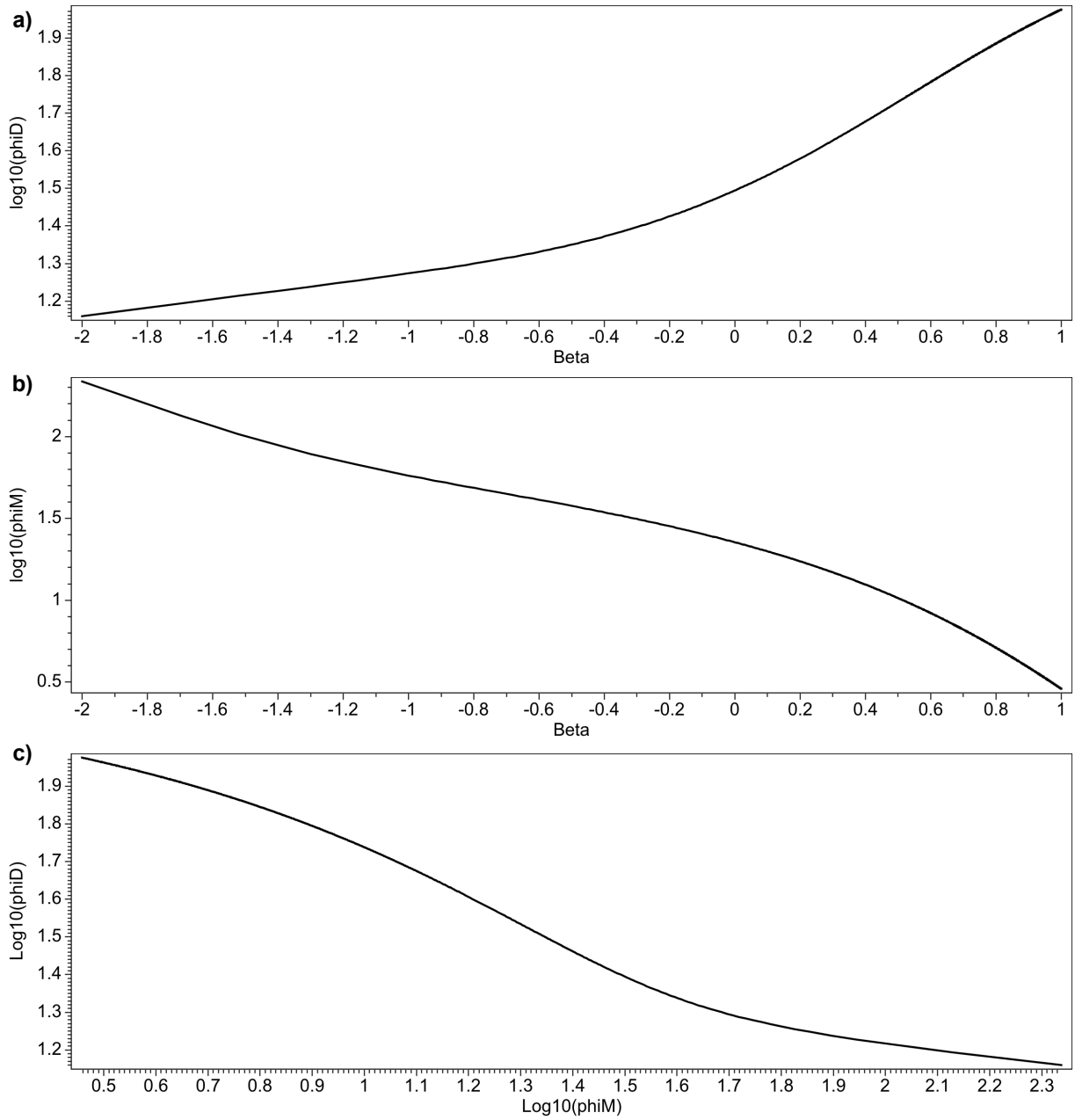

Figure 1. The data misfit (a), model objective function (b) and Tikhonov curve (c) for the smallest model problem.

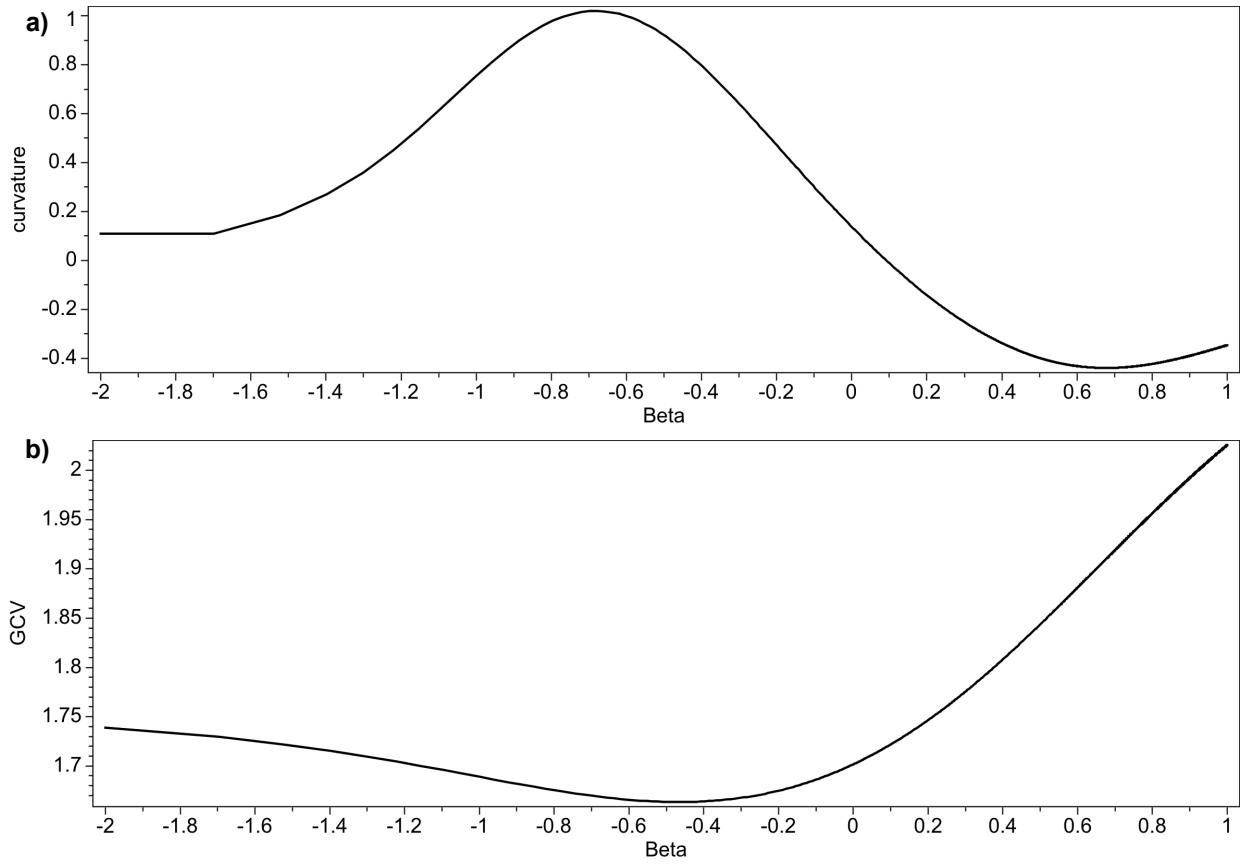

Figure 2. The curvature (a) of the Tikhonov curve (Figure 1c), and GCV curve (b) are used to choose optimal β values for the smallest model problem.

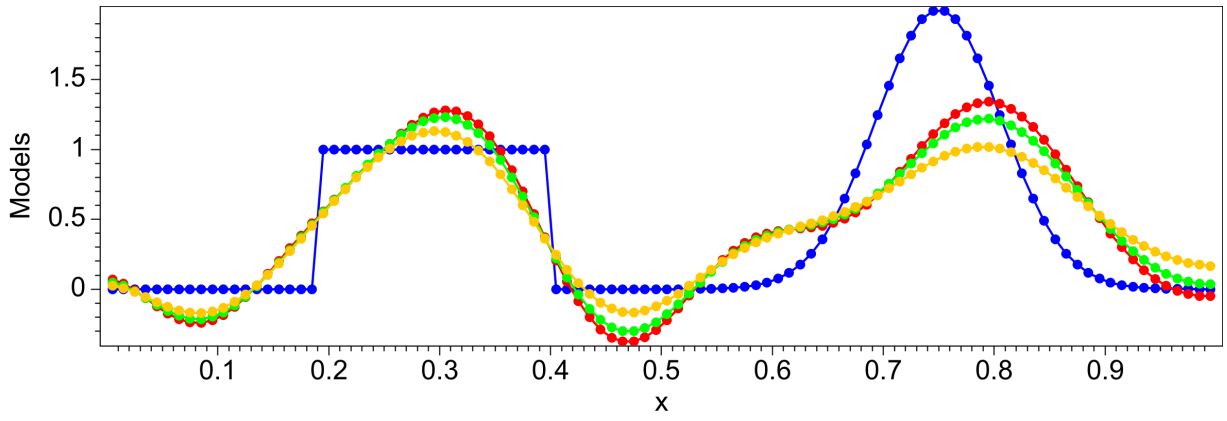

Figure 3. In the smallest model problem, the true model (blue curve) is compared to the constructed models using the β values computed (red curve) using the discrepancy principle, (green curve) using the L-curve, and (orange curve) using the GCV curve, respectively.

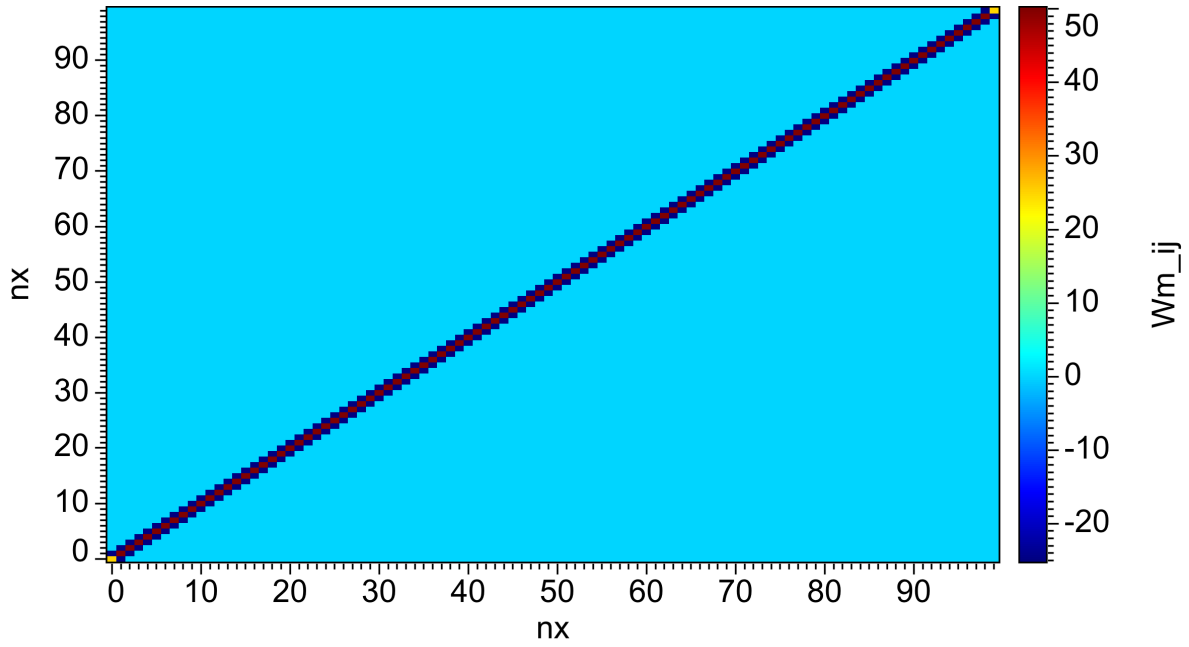

Figure 4. The model weighting matrix $\mathbf{W}^{\top}{}_m\mathbf{W}_m$.

2 FLATTEST MODEL

For the flattest model, solve the following equation

$$(\mathbf{G}^{\top}\mathbf{W}^{\top}{}_d\mathbf{W}_d + \beta\mathbf{W}^{\top}{}_m\mathbf{W}_m)\mathbf{m} = \mathbf{G}^{\top}\mathbf{W}^{\top}{}_d\mathbf{W}_d\mathbf{d}^{obs} \tag{2}$$

Results:

1) Matrix $\mathbf{W}^{\top}{}_m\mathbf{W}_m$ is shown in Figure 4, this matrix is tridiagonal and SPD;

2) Data misfit is shown in Figure 5a;

3) Model objective function is shown in Figure 5b;

4) Tikhonov curve is shown in Figure 5c;

5) Curvature of the Tikhonov curve is shown in Figure 6a;

6) The GCV curve is shown in Figure 6b;

7) Using the discrepancy principle, $\beta_1 = 0.28$;

8) Using the L-curve (maximum curvature of the L-curve), $\beta_2 = 0.724$;

9) Using the GCV curve, $\beta_3 = 0.639$;

10) Constructed model using β_1 (discrepancy principle) is the red curve in Figure 7;

11) Constructed model using β_2 (L-curve) is the green curve in Figure 7;

12) Constructed model using β_3 (GCV) is the orange curve in Figure 7.

Comparison:

1) Smallest model and flattest model:

In this example, comparing the Figures 3 and 7, we observe that the constructed models using the flattest model object function (equation 2) is better than the ones computed using the smallest model objective function (equation 1). Because all the constructed models using different optimal β values for the flattest objective function are more stable, while those for the smallest objective function are more fluctuable.

2) Different regularization parameters in flattest model:

In the flattest model problem, the optimal $\beta_2 = 0.724$ and $\beta_3 = 0.639$ computed using the L-curve and GCV curve, respectively, are close to each other, and hence the constructed models (green and orange curves in Figure 7) using β_2 and β_3 are close. However, the optimal $\beta_1 = 0.28$ computed using the discrepancy principle is more different from β_2 and β_3, and hence the constructed model (red curve in Figure 7) is more different from others.

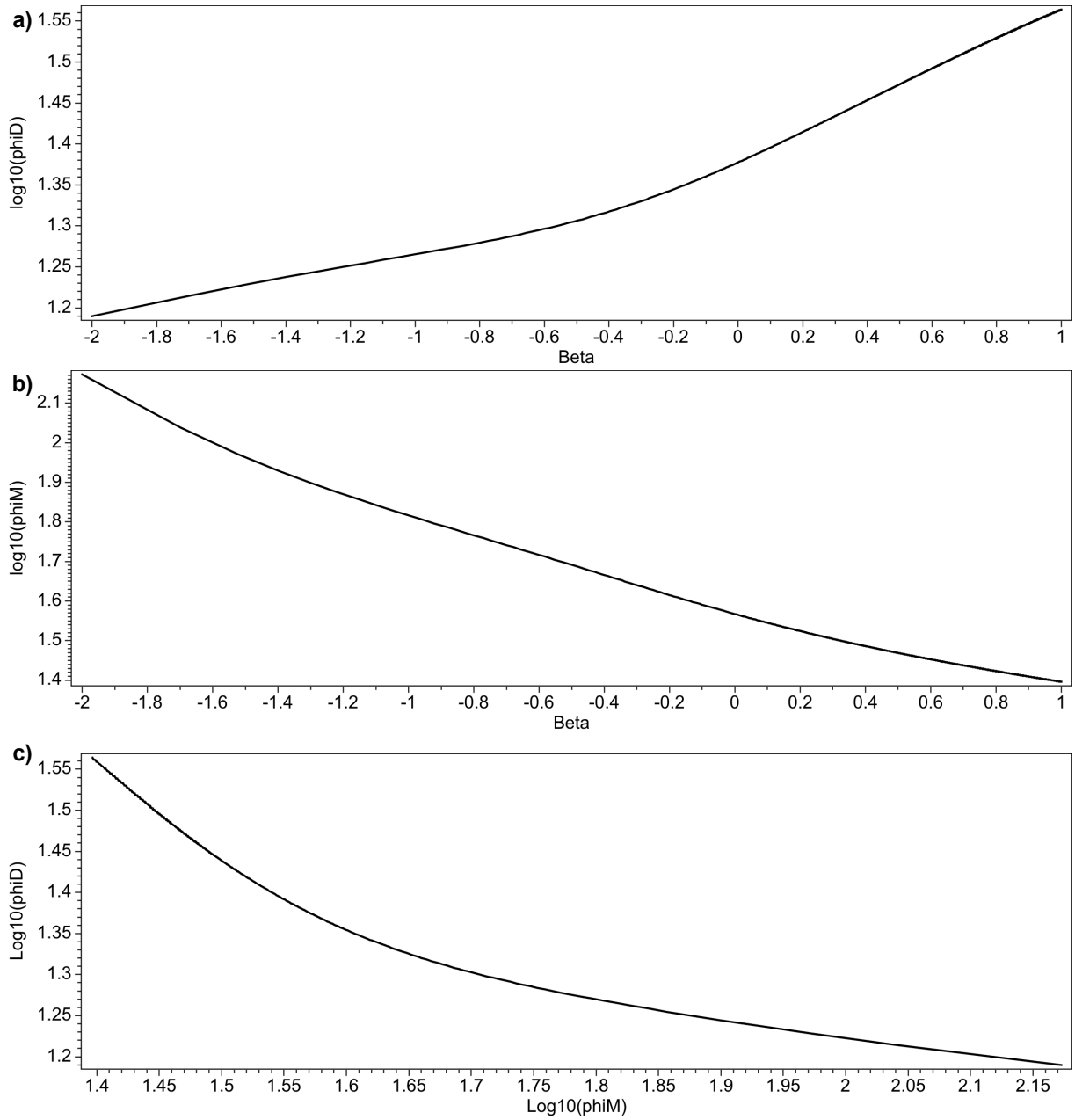

Figure 5. The data misfit (a), model objective function (b) and Tikhonov curve (c) for the flattest model problem.

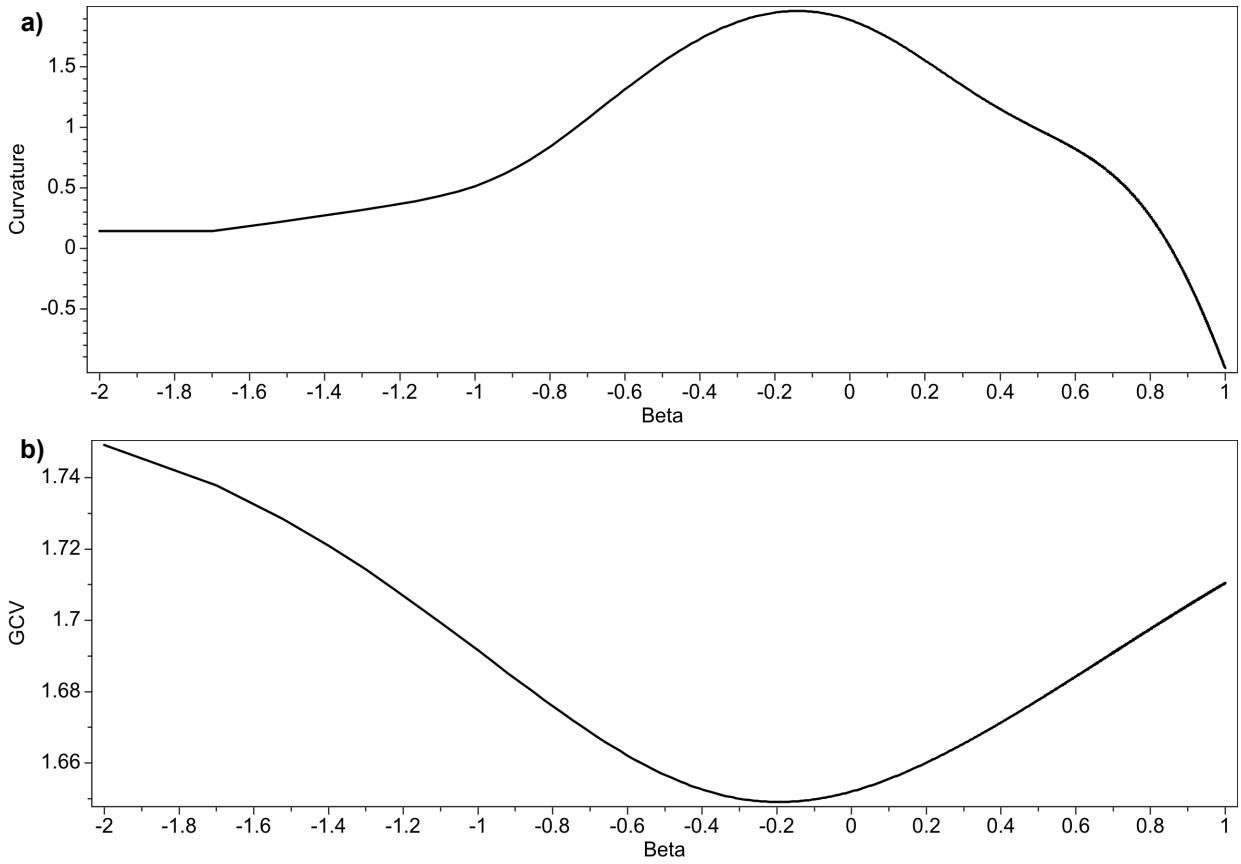

Figure 6. The curvature (a) of the Tikhonov curve (Figure 5c), and GCV curve (b) are used to choose optimal β values for the flattest model problem.

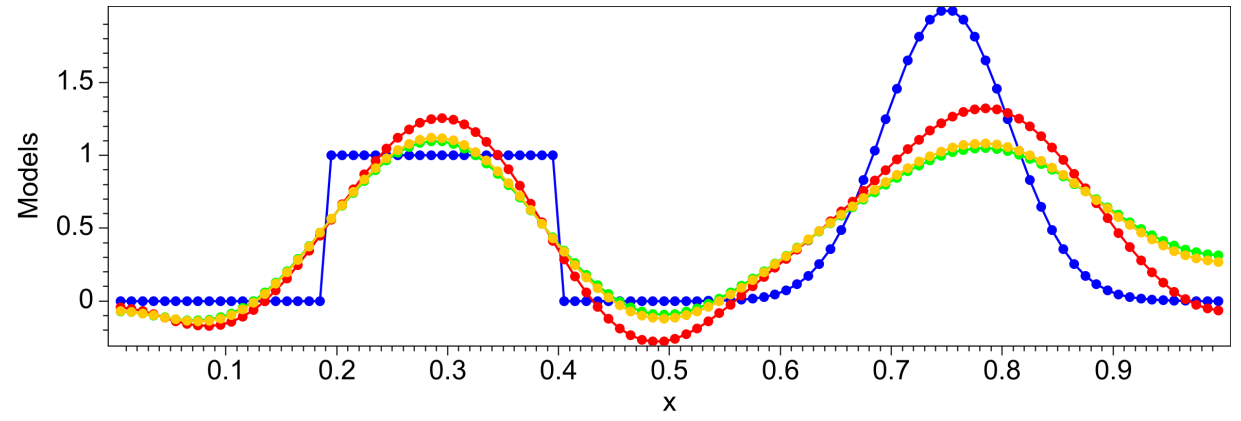

Figure 7. In the flattest model problem, the true model (blue curve) is compared to the constructed models using the β values computed (red curve) using the discrepancy principle, (green curve) using the L-curve, and (orange curve) using the GCV curve, respectively.