

## HW2 for GPGN658: Displacement from potentials

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### 1 POTENTIALS FROM A DISPLACEMENT FIELD

A displacement field  $\mathbf{u}(\mathbf{x}, t)$  can be expressed as a sum of curl-free and divergence-free forms by using the Helmholtz decomposition

$$\mathbf{u} = \nabla\theta + \nabla \times \boldsymbol{\psi}. \quad (1)$$

Given a displacement field  $\mathbf{u}$ , we can easily compute its corresponding scalar potential

$$\Theta = \nabla \cdot \mathbf{u} = \nabla \cdot (\nabla\theta), \quad (2)$$

and vector potential

$$\boldsymbol{\Psi} = \nabla \times \mathbf{u} = \nabla \times (\nabla \times \boldsymbol{\psi}). \quad (3)$$

### 2 DISPLACEMENT FROM POTENTIALS

Given the scalar and vector potentials  $\Theta$  and  $\boldsymbol{\Psi}$ , we can also reconstruct its corresponding displacement field  $\mathbf{u}$ .

We know that the vector Laplacian of the displacement field  $\mathbf{u}$  can be expressed as

$$\begin{aligned} \nabla^2 \mathbf{u} &= \nabla(\nabla \cdot \mathbf{u}) - \nabla \times (\nabla \times \mathbf{u}) \\ &= \nabla\Theta - \nabla \times \boldsymbol{\Psi}. \end{aligned} \quad (4)$$

Both the left-hand and right-hand sides of the above equations are vectors. Given the scalar and vector potentials  $\Theta$  and  $\Psi$ , we can first compute the vector on the right-hand side, then we can compute all the three components of the displacement field  $\mathbf{u}$  by solving three Laplacian equations with a specific boundary condition.

Generally, such a Laplacian equation can be solved by using the Conjugate Gradient method because a negative Laplacian operator is symmetric positive definite.