

Introduction to Vectors

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April 29, 2024

$$A\underline{x} = \underline{b}$$

- A is a matrix
- \underline{x} is a vector
- \underline{b} is an another vector

Remark. *If the three column vectors do not lie in the same plain, we can solve $A\underline{x} = \underline{b}$ for every \underline{b}*

Guassain Elimination

$$2x + 4y - 2z = 2 \quad 2 \text{ is pivot} \quad (1)$$

$$4x + 9y - 3z = 8 \quad -2 \times (1) \quad (2)$$

$$-2x - 3y + 7z = 10 \quad +(1) \quad (3)$$

\Rightarrow

$$2x + 4y - 2z = 2 \quad (4)$$

$$y + z = 4 \quad (5)$$

$$y + 5z = 12 \quad (6)$$

\Rightarrow

$$2x + 4y - 2z = 2 \quad (7)$$

$$y + z = 4 \quad (8)$$

$$4z = 8 \quad (9)$$

pivots: 2, 1, 4 (They are meaningful)

Do backward substitution to get: $z = 2, y = 2, x = -1$

$$\begin{bmatrix} \boxed{2} & 4 & -2 & \vdots & 2 \\ 4 & 9 & -3 & \vdots & 8 \\ -2 & -3 & 7 & \vdots & 10 \end{bmatrix}$$

\Rightarrow

$$\begin{bmatrix} 2 & 4 & -2 & \vdots & 2 \\ 0 & \boxed{1} & 1 & \vdots & 4 \\ 0 & 1 & 5 & \vdots & 12 \end{bmatrix}$$

\Rightarrow

$$\begin{bmatrix} 2 & 4 & -2 & \vdots & 2 \\ 0 & 1 & 1 & \vdots & 4 \\ 0 & 0 & \boxed{4} & \vdots & 8 \end{bmatrix}$$

In general,

$$[A:b] \Rightarrow [\boxed{}:\underline{c}]$$

Remark. $\boxed{}$ means upper-triangular matrix

Remark. In order to solve the unknowns, pivots cannot be 0