Introduction to Vectors

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$A\underline{x} = \underline{b}$

- A is a matrix
- $\underline{x}$ is a vector
- $\underline{b}$ is an another vector

Remark. *If the three column vectors do not lie in the same plain, we can solve $A\underline{x} = \underline{b}$ for every $\underline{b}$*

Guassian Elimination

$$\begin{aligned} 2x + 4y - 2z &= 2 \quad 2 \text{ is pivot} && (1)\\ 4x + 9y - 3z &= 8 \quad -2 \times (1) && (2)\\ -2x - 3y + 7z &= 10 \quad +(1) && (3) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} 2x + 4y - 2z &= 2 && (4)\\ y + z &= 4 && (5)\\ y + 5z &= 12 && (6) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} 2x + 4y - 2z &= 2 && (7)\\ y + z &= 4 && (8)\\ 4z &= 8 && (9) \end{aligned}$$

pivots: 2, 1, 4 (They are meaningful)
Do backward substitution to get: $z = 2, y = 2, x = -1$

$$\left[\begin{array}{ccc:c} \boxed{2} & 4 & -2 & 2 \\ 4 & 9 & -3 & 8 \\ -2 & -3 & 7 & 10 \end{array}\right]$$

$\Rightarrow$

$$\left[\begin{array}{cccc:c} 2 & 4 & -2 & & 2 \\ 0 & \boxed{1} & 1 & & 4 \\ 0 & 1 & 5 & & 12 \end{array}\right]$$

$\Rightarrow$

$$\left[\begin{array}{cccc:c} 2 & 4 & -2 & & 2 \\ 0 & 1 & 1 & & 4 \\ 0 & 0 & \boxed{4} & & 8 \end{array}\right]$$

In general,

$[A \vdots b] \Rightarrow [\bigsqcup \vdots \underline{c}]$

Remark. $\bigsqcup$ *means upper-triangualr matrix*

Remark. *In order to solve the unknowns, pivots cannot be 0*