Probability

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Three Axioms of Probability

1. $0 \leq P(A) \leq 1$ for any event A.
2. $P(\Omega) = 1$ (Ω: sample space).
3. $A_1, A_2, \ldots$ are mutually exclusive events $\Rightarrow P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$.
   `> mutually exclusive:` $A_i \cap A_j = \emptyset$ `for` $i \neq j$.

Derived from three axioms

- $P(\emptyset) = 0$ Empty set
- $P(A) + P(\bar{A} = 1)$: Complement
- $P(A) = P(A - B) + P(A \cap B)$: DeMorgan's Law
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$: Union and Intersection
- If $A \subseteq B \Rightarrow P(A) \leq P(B)$: Inclusion-Exclusion Principle
- Bool's Inequality: $P(A \cup B) \leq P(A) + P(B)$
- Bonferroni's Inequality: $P(A \cap B) \geq P(A) + P(B) - 1$ (not sure)

Events Relations and Probability Rules

- Dependent: A event is affected by another event.
- Independent: A event is not affected by another event.
- Mutually Exclusive: Two events cannot happen at the same time.

Counting Principles

Rule of Sum, Rule of Product, Permutations, Combinations

Rule of Product

Permutations

$$P(n,r) = n \cdot (n-1) \cdot \cdots \cdot (n-r+1) = \frac{n!}{(n-r)!}$$

Theorem: Number of permutations of n distinct objects arranged in a **circle** is $(n-1)!$.

Combinations

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!}$$