Conditional Probability

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Def. $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ is the probability of X given Y.

1. X: event of interest
2. Y: event that has been observed (condition)

Remark. *if outcome X and Y are mutually exclusive, then $P(X|Y) = 0$*

$$P(A \cap B) = P(A \mid B) \cdot P(B) = P(B \mid A) \cdot P(A)$$

Total Probability Theorem

If C_1, C_2, ..., C_n are a set of **mutually exclusive and exhaustive events**, i.e. $C_1 \cup C_2 \cup C_3 \cup ... \cup C_n = S$, then for any event A:

$$P(A) = \sum_{i=1}^{n} P(A \mid C_i)P(C_i) = P(A \mid C)P(C) + P(A \mid \bar{C})P(\bar{C})$$

Benefit: It can separate an event into a set of mutually exclusive and exhaustive sub-events.

Bayes' Theorem

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

When to use? Help us infer unknown conditional probabilities based on the given conditions.

$\Rightarrow$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \quad \text{(insert Total Probability Theorem)}$$

$$= \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid \bar{A})P(\bar{A})}$$