

Probability

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Three Axioms of Probability

1. $0 \leq P(A) \leq 1$ for any event A .
2. $P(\Omega) = 1$ (Ω : sample space).
3. A_1, A_2, \dots are mutually exclusive events $\Rightarrow P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$
> mutually exclusive: $A_i \cap A_j = \emptyset$ for $i \neq j$.

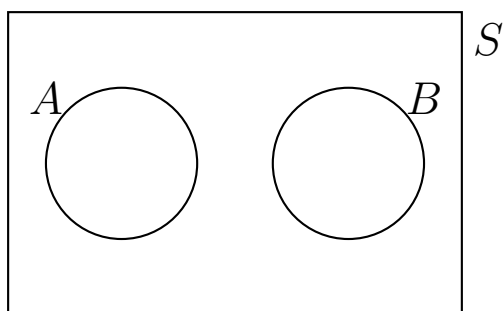
Derived from three axioms

- $P(\emptyset) = 0$ Empty set
- $P(A) + P(\bar{A}) = 1$: Complement
- $P(A) = P(A - B) + P(A \cap B)$: DeMorgan's Law
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$: Union and Intersection
- If $A \subseteq B \Rightarrow P(A) \leq P(B)$: Inclusion-Exclusion Principle
- Bool's Inequality: $P(A \cup B) \leq P(A) + P(B)$
- Bonferroni's Inequality: $P(A \cap B) \geq P(A) + P(B) - 1$ (not sure)

Events Relations and Probability Rules

- Dependent: A event is affected by another event.
- Independent: A event is not affected by another event.
- Mutually Exclusive: Two events cannot happen at the same time.

Mutually Exclusive in Venn diagram:



$$A \cap B = \emptyset$$

Independent Events

We usually use $P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$ to check if two events are independent.

$$P(A \cap B) = P(A) \cdot P(B)$$

Counting Principles

Rule of Sum, Rule of Product, Permutations, Combinations

Rule of Product

Permutations

$$P(n, r) = n \cdot (n - 1) \cdot \dots \cdot (n - r + 1) = \frac{n!}{(n - r)!}$$

Theorem: Number of permutations of n distinct objects arranged in a **circle** is $(n - 1)!$.

Combinations

$$C(n, r) = \frac{P(n, r)}{r!} = \frac{n!}{r!(n - r)!}$$

Finding Probability

Generally, we need to list and find sample space, then count the number of favorable outcomes and find the probability.

If the experiment meets some attributes, we can use the following methods:

Binomial Distribution

If the experiment meets the following conditions:

- The experiment is repeated n times.
- Each trial has two outcomes: success or failure.
- The probability of success is p and the probability of failure is $q = 1 - p$.
- The trials are independent.