

Calculus Stewart Ch2 Problem Plus

18. If f and g are differentiable function with $f(0) = g(0) = 0$ and $g'(0) \neq 0$, show that

$$\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{f'(0)}{g'(0)}$$

Proof:

$$\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 0} \frac{\frac{f(x)}{x}}{\frac{g(x)}{x}} = \lim_{x \rightarrow 0} \frac{\frac{f(x) - f(0)}{x - 0}}{\frac{g(x) - g(0)}{x - 0}} = \frac{\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0}}{\lim_{x \rightarrow 0} \frac{g(x) - g(0)}{x - 0}} = \frac{f'(0)}{g'(0)}$$