

Calculus Stewart Ch15 Problem Plus

8. Show that

$$\int_0^{\infty} \frac{\arctan \pi x - \arctan x}{x} dx = \frac{\pi}{2} \ln \pi$$

by first expressing the integral as an iterated integral.

Solution:

$$\begin{aligned} \int_0^{\infty} \frac{\arctan \pi x - \arctan x}{x} dx &= \int_0^{\infty} \int_1^{\pi} \frac{1}{1+x^2 y^2} dy dx = \int_1^{\pi} \int_0^{\infty} \frac{1}{1+x^2 y^2} dx dy = \int_1^{\pi} \frac{\pi}{2y} dy \\ &= \frac{\pi}{2} \ln \pi \end{aligned}$$