Calculus Stewart Ch9 Problem Plus

1. Find all functions f such that f' is continuous and

$$f^2(x) = 100 + \int_0^x \{f^2(t) + f'(t)^2\}dt \ \ \forall x \in \mathbb{R}$$

Proof:

$2ff' = f^2 + f'^2 \Rightarrow (f - f')^2 = 0 \Rightarrow f' = f$

$\Rightarrow f = Ae^x = \pm 10e^x$

4. Find all function f that satisfy the equation

$$\left(\int f(x)dx\right)\left(\int \frac{1}{f(x)}dx\right) = -1$$

Proof:

Let $S = \int f(x)dx$

$f(x)\left(\int \frac{1}{f(x)}dx\right) + \left(\int f(x)dx\right)\frac{1}{f(x)} = 0$

$\Rightarrow f \cdot \left(-\frac{1}{S}\right) + \frac{S}{f} = 0 \Rightarrow f^2 = S^2$

$\Rightarrow \int f(x)dx = \pm f(x) \Rightarrow f(x) = \pm f'(x)$

$\Rightarrow f(x) = Ae^{\pm x}$

9.

$$x\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Proof:

Let $z = y'$

$xz' = \sqrt{1 + z^2}$

$\Rightarrow \sinh^{-1} z = \ln|x| + C$

$\Rightarrow z = \sinh(\ln|x| + C)$

$\Rightarrow y = \int \sinh(\ln|x| + C)\, dx$