Calculus Stewart Ch8 Sec2

29. (a) The ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ a>b$$

is rotated about the x-axis to form a surface called an ellipsoid or prolate spheroid. Find the surface area of this ellipsoid.

(b) If the ellipse in part (a) is rotated about its minor axis (the y-axis), the resulting ellipsoid is called an oblate spheroid. Find the surface area of this ellipsoid.

Proof:

(a)

$$2\pi\ \cdot 2\int_0^a y\sqrt{1+y'^2}dx=\frac{2\pi a^2 b}{\sqrt{a^2-b^2}}\sin^{-1}\frac{\sqrt{a^2-b^2}}{a}+2\pi b^2$$

(b)

$$2\pi\ \cdot 2\int_0^b x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\ dx=\frac{2\pi ab^2}{\sqrt{a^2-b^2}}\sinh^{-1}\frac{\sqrt{a^2-b^2}}{b}+2\pi a^2$$

33. Find the area of the surface obtained by rotating the circle $x^2+y^2=r^2$ about the line $y=r$.

Proof:

$$2\pi\left(2\cdot\int_0^r\left(r+\sqrt{r^2-x^2}\right)\sqrt{1+\frac{x^2}{y^2}}dx+2\cdot\int_0^r\left(r-\sqrt{r^2-x^2}\right)\sqrt{1+\frac{x^2}{y^2}}dx\right)=4\pi^2r^2$$

Discovery Project

5. Find a formula for the area of the surface obtained by rotating C about the line $y=mx+b$.

$$2\pi\int_p^q|f(x)-mx-b|\frac{\sqrt{1+f'(x)^2}}{\sqrt{1+m^2}}dx=2\pi\int_p^q\frac{|f(x)-mx-b|}{\sqrt{1+m^2}}dS$$