Calculus Stewart Ch3 Sec2

26. Suppose that f and g are continuous on $[a, b]$ and differentiable on (a, b). Suppose also that $f(a) = g(a)$ and $f'(x) < g'(x)$ for $a < x < b$. Prove that $f(b) < g(b)$.

Proof:

Let $h(x) = g(x) - f(x)$

By M.V.T., $h(b) - h(a) = h'(c)(b - a)$ for some $c \in (a, b)$

$\because h'(x) = \big(g(x) - f(x)\big)' = g'(x) - f'(x) > 0$

$\therefore h'(c)(b - a) = h(b) - h(a) > 0$

$\therefore h(b) > h(a)$

$\therefore g(b) - f(b) > g(a) - f(a) = 0$

$\therefore g(b) > f(b)$

34. A number a is called a fixed point of a function if $f(a) = a$. Prove that if $f'(x) \neq 1$ for all real numbers x, then f has at most one fixed point.

Proof:

Suppose f has more than one fixed point, say a, b and $a < b$

By M.V.T., $f(b) - f(a) = f'(c)(a - b)$ for some $c \in (a, b)$

$\because$ a and b are distinct fixed points

$\therefore f(b) - f(a) = a - b = f'(c)(a - b) \neq 0$

$\Rightarrow f'(c) = 1$ *contradicts!

$\Rightarrow$ f has at most one fixed point