

Calculus Stewart Ch6 Problem Plus

11. For what value of a is the following equation true?

$$\lim_{x \rightarrow \infty} \left(\frac{x+a}{x-a} \right)^x = e$$

Proof:

Let $u = x - a$

$$\lim_{u \rightarrow \infty} \left(1 + \frac{2a}{u} \right)^{u+a} = \lim_{u \rightarrow \infty} \left(1 + \frac{2a}{u} \right)^u \left(1 + \frac{2a}{u} \right)^a = \lim_{u \rightarrow \infty} \left(1 + \frac{2a}{u} \right)^u = e^{2a} = e$$

$$\therefore a = \frac{1}{2}$$

18. For what values of c does the curve $y = cx^3 + e^x$ have inflection points?

Proof:

$$y' = 3cx^2 + e^x$$

$$y'' = 6cx + e^x = 0$$

$$e^x = -6cx$$

(1)

Consider $c > 0$ has one inflection point

(2)

Consider $c < 0$,

Suppose tangent at (a, e^a)

$$T(x) = e^a(x - a) + e^a$$

$$-ae^a + e^a = 0$$

$$\therefore a = 1$$

$$\therefore c = -\frac{e}{6} \text{ has one inflection point and } c < -\frac{e}{6} \text{ has two inflection points}$$