Calculus Stewart Ch7 Problem Plus

2.

$$\int \frac{dx}{x^7 - x}$$

Hint:

$$\int \frac{\frac{1}{6} dx^6}{x^6(x^6 - 1)}$$

3.

$$\int_0^1 \left(\sqrt[3]{1-x^7} - \sqrt[7]{1-x^3}\right) dx$$

Proof:

Let $u = \sqrt[3]{1-x^7}$, then $x = \sqrt[7]{1-u^3}$

$$\int_0^1 \left(\sqrt[3]{1-x^7} - \sqrt[7]{1-x^3}\right) dx = \int_0^1 u d(\sqrt[7]{1-u^3}) - \int_0^1 \sqrt[7]{1-x^3} dx$$
$$= u\sqrt[7]{1-u^3}\Big|_1^0 - \int_1^0 \sqrt[7]{1-u^3} du - \int_0^1 \sqrt[7]{1-x^3} dx = 0$$

7. A function is defined by

$$f(x) = \int_0^\pi \cos t \cos(x-t)\, dt \quad 0 \le x \le 2\pi$$

Find the minimum value of f.

Proof:

$$f(x) = \int_0^\pi (\cos x + \cos(x - 2t)) dt = \frac{\pi}{2}\cos x + \frac{1}{2}\left(\int_0^\pi \cos x \cos 2t\, dt + \int_0^\pi \sin x \sin 2t\, dt\right)$$
$$= \frac{\pi}{2}\cos x + \frac{1}{2}\sin x$$

The minimum value of f is $\frac{1}{2}\sqrt{\pi^2 + 1}$

14.

$$\int \sqrt{\tan x}\, dx$$

Proof:

Let $u = \sqrt{\tan x}$

$$\int \sqrt{\tan x}\, dx = \int \frac{2u}{\sqrt{u^4+1}} du = \int \frac{du^2}{\sqrt{(u^2)^2+1}} = \sinh^{-1} \tan x + C$$

where C is constant