Calculus Stewart Ch4 Sec2

68. (a) If f is continuous on $[a, b]$, show that

$$\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$$

(b) Use the result of part (a) to show that

$$\left|\int_0^{2\pi} f(x)\sin 2x\,dx\right| \leq \int_0^{2\pi} |f(x)|dx$$

Proof:

(a)

$-|f(x)| \leq f(x) \leq |f(x)|$

$$\int_a^b -|f(x)|dx < \int_a^b f(x)dx < \int_a^b |f(x)|dx$$

$$\Rightarrow \left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$$

(b)

$-|f(x)| \leq f(x)\sin 2x \leq |f(x)|$

$$\int_0^{2\pi} -|f(x)|dx < \int_0^{2\pi} f(x)\sin 2x\,dx < \int_0^{2\pi} |f(x)|dx$$

$$\Rightarrow \left|\int_0^{2\pi} f(x)\sin 2x\,dx\right| \leq \int_0^{2\pi} |f(x)|dx$$

69. Let $f(x) = 0$ if x is any rational number and $f(x) = 1$ if x is any irrational number. Show that f is not integrable on $[0,1]$.

Proof:

Let $\mathcal{P}$ be a partition on $[0,1]$

$0 \leq f(x) \leq 1$

$$U(\mathcal{P}, f) = \sum_{k=1}^{n} 1 \cdot \Delta x_k = 1 - 0 = 1$$

$$L(\mathcal{P}, f) = \sum_{k=1}^{n} 0 \cdot \Delta x_k = 0$$

$\inf_{\mathcal{P}} U(\mathcal{P}, f) \neq \sup_{\mathcal{P}} L(\mathcal{P}, f)$

$\therefore$ f is not integrable oon $[0,1]$

70. Let $f(0) = 0$ and $f(x) = \frac{1}{x}$ if $0 < x \leq 1$. Show that f is not integrable on $[0,1]$.

Proof:

Divide $[0,1]$ into n subintervals of even length

The first term of Riemann sum is $f(x_1)\frac{1}{n} = \frac{1}{x_1 n}$ for $x_1 \in (0,1)$ can be arbitrary large when $x_1 \to 0$.

So the sum does not exist

$\therefore$ f is not integrable on $[0,1]$

Calculus Stewart Ch4 Sec3

64. If f is continuous and g and h are differentiable functions, find a formula for

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt$$

Proof:

Let F be an antiderivative of f

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt = \frac{d}{dx}\left(\int_{0}^{h(x)} f(t)dt - \int_{0}^{g(x)} f(t)dt\right) = \frac{d}{dx}(F(h(x) - F\big(g(x)\big)$$
$$= f\big(h(x)\big)h'(x) - f\big(g(x)\big)g'(x)$$