Calculus (II) Second Midterm Examination May 3, 2016

CALCULUS (II)

provided by chengscott 鄭余玄 at http://chengscott.github.io/

SECOND MIDTERM EXAMINATION; MAY 3, 2016

Show your work, otherwise no credit will be granted.

1. (10 points) Find the point of maximal curvature on the curve $y = \ln x$.

2. (10 points) Find the tangential and normal components of the acceleration vector of a particle with the position function $\vec{r}(t) = (t, 2t, t^2)$ when $t = 2$.

3. (10 points) Find the curvature of a polar curve $r = f(\theta)$.

4. (10 points) Find equations of the tangent plane and the normal line to the surface $xyz^2 = 6$ at $(3, 2, 1)$.

5. (10 points) Define $g(x, y) = \frac{x^6y^2}{(x^4+y^2)^2}$ if $(x, y) \neq (0, 0)$ and $g(0, 0) = 0$. Is g continuous at $(0, 0)$? Prove or disprove it.

$\nabla_u f = \langle \nabla f, u \rangle$.

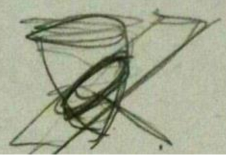
6. (10 points) Let $f : \mathbb{R}^2 \to \mathbb{R}$. Suppose that f has a directional derivative at $(0, 0)$ in the direction of any unit vector $\vec{u} = (a, b)$. Is f continuous at $(0, 0)$? Prove or disprove it.

7. (10 points) Let f be a continuous function on $\mathbb{R}^2$ such that $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial x \partial y}$ are all continuous. Prove that $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$.

8. (15 points) Find the global maximum and minimum values of $f(x, y) = 4xy^2 - x^2y^2 - xy^3$ on the closed triangular region in the xy-plane with vertices $(0, 0)$, $(0, 6)$ and $(6, 0)$.

9. (15 points) The plane $x + y - z = 1$ intersects the upper half of the cone $z^2 = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are closest to and farthest from the origin.

$z = x+y-1$



$z = \sqrt{x^2+y^2}$

$f(\ldots) + \ldots - f(x_0, y_0)$.

Solutions:

3.

$$r(\theta) = (f(\theta)\cos\theta, f(\theta)\sin\theta)$$

$$\kappa = \frac{\|r' \times r''\|}{\|r'\|^3} = \frac{|2r'^2 - rr'' + r^2|}{(r'^2 + r^2)^{\frac{3}{2}}}$$

9. (Give away)