Calculus Stewart Ch7 Sec4

71. Suppose F, G, and Q are polynomials and

$$\frac{F(x)}{Q(x)} = \frac{G(x)}{Q(x)}$$

for all x except when $Q(x) = 0$. Prove that $F(x) = G(x)$ for all x.

Proof:

(1)

$\forall x \in \mathbb{R}$ s.t. $Q(x) \neq 0$

$\frac{F(x)}{Q(x)} = \frac{G(x)}{Q(x)} \Rightarrow F(x) = G(x)$

(2)

Since Q is a polynomial, there are finite $x_0 \in \mathbb{R}$ s.t. $Q(x_0) = 0$

$\because f$ is continuous

give $\varepsilon > 0\ \exists \delta > 0$ s.t.

$|F(x) - F(x_0)| < \varepsilon$ for $|x - x_0| < \delta$

$\Rightarrow$ for $|x - x_0| < \delta$

$|F(x) - F(x_0)| = |G(x) - F(x_0)| < \varepsilon$

$\Rightarrow \lim_{x \to x_0} G(x) = F(x_0)$

$\because g$ is continuous

$\lim_{x \to x_0} G(x) = G(x_0) = F(x_0)$

By (1)(2) $\Rightarrow F = G\ \forall x \in \mathbb{R}$

Calculus Stewart Ch7 Sec8

71.

(a) $\mathcal{L}\{1\} = \frac{1}{s}, s > 0$

(b) $\mathcal{L}\{e^t\} = \frac{1}{s-1}, s > 1$

(c) $\mathcal{L}\{t\} = \frac{1}{s^2}, s > 0$

79. Suppose f is continuous on $[0, \infty)$ and $\lim_{x \to \infty} f(x) = 1$. Is it possible that $\int_0^\infty f(x)dx$ is convergent?

Proof: NO

Give $\varepsilon > 0$ $\exists N > 0$ s.t.

$|f(x) - 1| < \varepsilon$ for $n > N$

$$\int_0^\infty f(x)dx = \int_0^N f(x)dx + \int_N^\infty f(x)dx \geq \int_0^N f(x)dx + \int_N^\infty (1-\varepsilon)dx$$

$\int_N^\infty (1-\varepsilon)dx$ is divergent

Calculus Stewart Ch7 Review

71. Use the Comparison Theorem to determine whether the integral is convergent or divergent.

(a) $\int_1^\infty \frac{2+\sin x}{\sqrt{x}}dx$

(b) $\int_1^\infty \frac{1}{\sqrt{1+x^4}}dx$

Proof:

(a)

$$\int_1^\infty \frac{2+\sin x}{\sqrt{x}}dx > \int_1^\infty \frac{1}{\sqrt{x}}dx = 2\sqrt{x}\Big|_1^\infty$$

is divergent

(b)

$$\int_1^\infty \frac{1}{\sqrt{1+x^4}}dx < \int_1^\infty \frac{1}{\sqrt{x^4}}dx = \frac{1}{x}\Big|_1^\infty$$

is convergent

79.

$$\int_0^\infty \frac{\ln x}{1+x^2}dx$$

Proof:

Let $u = \frac{1}{x}$

$$\lim_{M\to\infty}\int_0^M \frac{\ln x}{1+x^2}dx = \lim_{M\to\infty}\int_M^0 \frac{-\ln u}{1+\frac{1}{u^2}}\left(-\frac{1}{u^2}\right)du = \lim_{M\to\infty} -\int_0^M \frac{\ln u}{1+u^2}du$$

$$\Rightarrow \lim_{M\to\infty}\int_0^M \frac{\ln x}{1+x^2}dx = 0$$