Calculus Stewart Ch1 Problem Plus

12. A fixed point of a function f is a number c in its domain such that $f(c) = c$. Use the Intermediate Value Theorem to prove that any continuous function with domain $[0,1]$ and range in $[0,1]$ must have a fixed point.

Proof:

If $f(0) = 0$ or $f(1) = 1$, proved.

Otherwise,

Let $g(x) = f(x) - x$, then

$g(0) = f(0) - 0 > 0$

$g(1) = f(1) - 1 < 0$

$\therefore g(0)g(1) < 0$

$\therefore$ By I.V.T., $\exists c \in (0,1)$ s.t.

$g(c) = 0$

i.e. $f(c) - c = 0 \Rightarrow f(c) = c$