Calculus (II) Final Examination June 7, 2016

CALCULUS (II)

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Show your work, otherwise no credit will be granted.

1. (15 points) Evaluate the integral (i) $\int_0^\infty e^{-x^2}dx$, (ii) $\int_0^\infty x^{5/2}e^{-x}dx$.

2. (10 points) Evaluate the integral $\int_0^1\int_0^1 e^{\max\{x^2,y^2\}}dxdy$.

3. (10 points) Find the volume of the solid that lies outside the cone $z=\sqrt{x^2+y^2}$ and inside the hemisphere $z=\sqrt{1-x^2-y^2}$.

4. (10 points) Evaluate the integral $\iint_\Omega (x+y)^2 dxdy$, where Ω is the parallelogram bounded by the lines: $x+y=0$, $x+y=1$, $2x-y=0$ and $2x-y=3$.

5. (10 points) Find the area of the pentagon with vertices $(0,0),(-1,2),(0,6),(2,4)$ and $(3,1)$.

6. (10 points) Evaluate the surface integral $\iint_S \vec{F}\cdot\vec{n}dS$, where $\vec{F}(x,y,z)=(0,y,-z)$ and the surface S is given by the paraboloid $y=x^2+z^2$, $0\le y\le 1$. Here, use the normal $\vec{n}$ that has negative y component.

7. (10 points) Find the positively oriented, piecewise smooth, simple closed curve for which the value of the line integral $\int_C (y^3-y)dx-2x^3dy$ is a maximum.

8. (10 points) Evaluate the surface integral $\iint_S (x^4+y^4+z^4)dS$, where $S=\{(x,y,z)\mid x^2+y^2+z^2=1\}$ is the unit sphere.

9. (10 points) Evaluate the line integral $\int_C (ye^x)dx+(2y\cos y+e^x)dy$ along the curve C: $\vec{r}(t)=(\frac{\pi}{2}\cos t,\frac{\pi}{2}+\frac{\pi}{2}\sin t)$, where t ranges from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

10. (5 points) Write down your opinions or comments about this course.

Solutions:

2.

$\int_0^1\int_0^1 e^{\max\{x^2,y^2\}}dx\,dy = 2\iint_D e^{x^2}dA$ where $D=\{(x,y)\in[0,1]\times[0,1] \mid x\ge y\}$

$$= 2\int_0^1\int_0^x e^{x^2}dy\,dx = 2\int_0^1 xe^{x^2}dx = e-1$$

3.

$$\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^1 \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = \frac{\sqrt{2}}{3}\pi$$

4.

Let $u = x + y, v = 2x - y$

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{\left|\frac{\partial(u,v)}{\partial(x,y)}\right|} = \frac{1}{\left|\det\begin{pmatrix}1 & 1\\ 2 & -1\end{pmatrix}\right|} = \frac{1}{3}$$

$$\iint\limits_{\Omega} (x+y)^2 dxdy = \int_0^3 \int_0^1 u^2 \cdot \frac{1}{3} du\, dv = \frac{1}{3}$$

8.

$$\iint\limits_{S} (x^4 + y^4 + z^4) dS = \iint\limits_{S} (x^3, y^3, z^3) \cdot (x, y, z) dS$$

By Gaussian divergence theorem,

$$= \iiint\limits_{E} div(x^3, y^3, z^3) dV = \iiint\limits_{E} 3(x^2 + y^2 + z^2) dV = 3 \int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho^2 \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta = \frac{12}{5}\pi$$