

Calculus Stewart Ch5 Sec1

61.

Proof:

$$\begin{aligned}\pi \int_{-r}^r \left((R + \sqrt{r^2 - x^2})^2 - (R - \sqrt{r^2 - x^2})^2 \right) dx &= \pi \int_{-r}^r 4R\sqrt{r^2 - x^2} dx = 4\pi R \int_{-r}^r \sqrt{r^2 - x^2} dx \\ &= 4\pi R \frac{\pi r^2}{2} = 2\pi^2 r^2 R\end{aligned}$$

64.

Proof:

$$8 \int_0^r \sqrt{r^2 - x^2}^2 dx = 8 \left(r^3 - \frac{1}{3} r^3 \right) = \frac{16}{3} r^3$$