HW2 Written Part

Zhicheng Guo
CSCI 4270

September 27, 2019

Problem 1
(a)
$x(t) = x_1 t + x_0$
$y(t) = y_1 t + y_0$
$z(t) = z_1 t + z_0$
we have $< x, y, z >=< x_1, y_1, z_1 > t+ < x_0, y_0, z_0 >$
(b)
looking along z axis and project onto the image plane we have:
$u_0 = \frac{f(x_1 t_0 + x_0)}{z}, v_0 = \frac{f(y_1 t_0 + y_0)}{z}$
$u_1 = \frac{f(x_1 t_1 + x_0)}{z}, v_1 = \frac{f(y_1 t_1 + y_0)}{z}$
(c)
in the following steps, $(z_1 t_1 + z_0)(z_1 t_0 + z_0)$ will be replace with Z
$a = \frac{f(y_1 t_1 + y_0)}{(z_1 t_1 + z_0)} - \frac{f(y_1 t_0 + y_0)}{z_1 t_0 + z_0} \to a = \frac{f}{Z}[y_1 z_0 (t_1 - t_0) + y_0 z_1 (t_0 - t_1)] \to a = \frac{f}{Z}(y_0 z_1 - y_1 z_0)(t_0 - t_1)$
$b = \frac{f(x_1 t_0 + x_0)}{z_1 t_0 + z_0} - \frac{f(x_1 t_1 + x_0)}{(z_1 t_1 + z_0)} \to b = \frac{f}{Z}[x_1 z_0 (t_0 - t_1) + x_0 z_1 (t_1 - t_0)] \to b = \frac{f}{Z}(x_1 z_0 - x_0 z_1)(t_0 - t_1)$
$c = \frac{f(y_1 t_0 + y_0)}{z_1 t_0 + z_0} * \frac{f(x_1 t_1 + x_0)}{(z_1 t_1 + z_0)} - \frac{f(x_1 t_0 + x_0)}{z_1 t_0 + z_0} * \frac{f(y_1 t_1 + y_0)}{(z_1 t_1 + z_0)} \to \frac{f^2}{Z}(x_0 y_1 (t_0 - t_1) + x_1 y_0 (t_1 - t_0)) \to c = \frac{f^2}{Z}(x_0 y_1 - x_1 y_0)(t_0 - t_1)$
so we have line in implicit form:
$au + bv + c = \frac{f}{Z}(y_0 z_1 - y_1 z_0)(t_0 - t_1)u + \frac{f}{Z}(x_1 z_0 - x_0 z_1)(t_0 - t_1)v + \frac{f^2}{Z}(x_0 y_1 - x_1 y_0)(t_0 - t_1)$
assume the projection is a line, we have $ax + by + c = 0$, in this case it is $au + bv + c = 0$
based on the assumption, we have $\frac{f}{Z}(y_0 z_1 - y_1 z_0)(t_0 - t_1)u + \frac{f}{Z}(x_1 z_0 - x_0 z_1)(t_0 - t_1)v + \frac{f^2}{Z}(x_0 y_1 - x_1 y_0)(t_0 - t_1) = 0$, by eliminating $(t_0 - t_1)$ and $\frac{f}{Z}$ from both side of the equation we have $(y_0 z_1 - y_1 z_0)u + (x_1 z_0 - x_0 z_1)v + f(x_0 y_1 - x_1 y_0) = 0$, so there are such parameter (a, b, c) that enables $au + bv + c = 0$
as we can see there is no t in the function, the parameters left are all constant, the assumption is true and it is valid that the projection is a line
(d)
when $\frac{x_0}{z_0} = \frac{x_1}{z_1} \&\& \frac{y_0}{z_0} = \frac{y_1}{z_1}$, it will be a point.

Problem 2
$A = USV^T$
$A^T A = (USV^T)^T (USV^T)$
$A^T A = VS^T U^T USV^T$

$A^T A = V S^T S V^T$
$A^T A = V S^2 V^T$
V is eigenvectors of $A^T A$ and for all non-zero elements in S, they are square root of the non-zero elements in $A^T A$