Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
C		**2412209**

Enjoy a Cozy and Green Bath

Summary

Keywords: Heat transfer, Thermodynamic system, CFD, Energy conservation

Contents

1 Introduction

1.1 Background

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1.3 Restatement of the Problem

2 Assumptions and Justification

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is properly justified.

3 Notations

Symbols	Description	Unit
h	Convection heat transfer coefficient	W/(m$^2\cdot$ K)
k	Thermal conductivity	W/(m $\cdot$ K)
c_p	Specific heat	J/(kg $\cdot$ K)
ρ	Density	kg/m^2
δ	Thickness	m
t	Temperature	°C, K
τ	Time	s, min, h
q_m	Mass flow	kg/s
Φ	Heat transfer power	W
T	A period of time	s, min, h
V	Volume	m^3, L
M, m	Mass	kg
A	Aera	m^2
a, b, c	The size of a bathtub	m^3

where we define the main parameters while specific value of those parameters will be given later.

4 Model Overview

In our basic model, we aim at three goals: keeping the temperature as even as possible, making it close to the initial temperature and decreasing the water consumption.

We start with the simple sub-model where hot water is added constantly. At first we introduce convection heat transfer control equations in rectangular coordinate system. Then we define the mean temperature of bath water.

Afterwards, we introduce Newton cooling formula to determine heat transfer capacity. After deriving the value of parameters, we get calculating results via formula deduction and simulating results via CFD.

Secondly, we present the complicated sub-model in which hot water is added discontinuously. We define an iteration consisting of two process: heating and standby. As for heating process, we derive control equations and boundary conditions. As for standby process, considering energy conservation law, we deduce the relationship of total heat dissipating capacity and time.

Then we determine the time and amount of added hot water. After deriving the value of parameters, we get calculating results via formula deduction and simulating results via CFD.

At last, we define two criteria to evaluate those two ways of adding hot water. Then we propose optimal strategy for the user in a bathtub. The whole modeling process can be shown as follows.

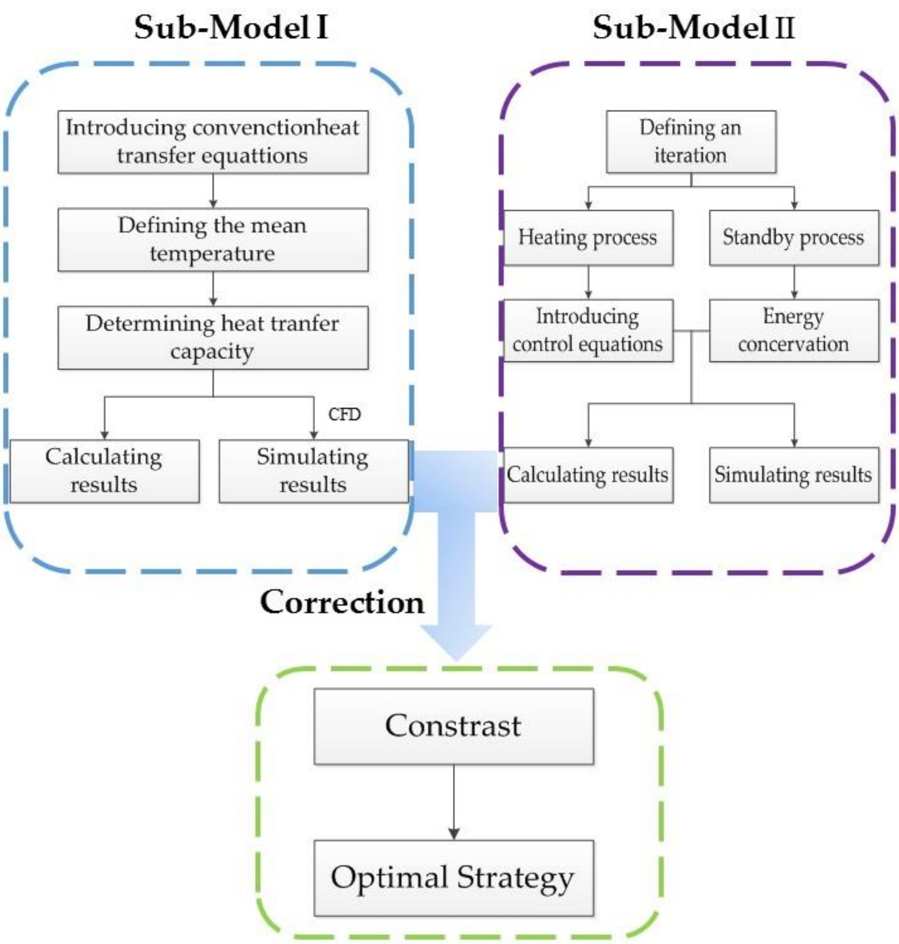

Figure 1: Modeling process

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. . (1) (2) To this end [1] $XY\{x_1, x_2, \cdots, x_m\}\{y_1, y_2, \cdots, y_n\}$ $n_{ij}X = x_iY = y_j$

Table 1:

	y_1	y_2	$\cdots$	y_n
x_1	n_{11}	n_{12}	$\cdots$	n_{1n}
x_2	n_{21}	n_{22}	$\cdots$	n_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_m	n_{m1}	n_{m2}	$\cdots$	n_{mn}

- H_0XY
- H_1XY

$$\chi^2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{(n_{ij} - E_{ij})^2}{E_{ij}}$$

$E_{ij}X = x_iY = y_j, chi^2H_0XY$

XY fig **??** fig **??**.a 1556.61pH_0 fig **??**.b

6

pointppointp0-1pKLKLpp

6.1 Model I: Bradley-Terry

p32323131pBradley-Terryp

6.1.1 based Bradley-Terry

n $\{1, 2, \cdots, n\}$, k $\beta_k(k = 1, 2, \cdots, n), P_{k,j}$ k j , B-T

$$P_{k,j} = \exp(\beta_k - \beta_j) / [1 + \exp(\beta_k - \beta_j)],$$

$P_{k,j}$ β_k β_j . N_{kj} k j , , k j n_{kj}, j k $n_{jk} = N_{kj} - n_{kj}$, $\beta_k(k = 1, 2, \cdots, n)$

$$L(\beta_1, \beta_2, \cdots, \beta_n) = \prod_{1\leqslant k\neq j\leqslant n}\frac{N_{kj}!}{n_{kj}!(N_{kj} - n_{kj})!}\left(\frac{\exp(\beta_k - \beta_j)}{1 + \exp(\beta_k - \beta_j)}\right)^{n_{kj}}\left(\frac{1}{1 + \exp(\beta_k - \beta_j)}\right)^{(N_{kj} - n_{kj})}.$$

, $\sum_{k=1}^{n}\beta_k = 0$, $\beta_k = 0, k \in \{1, 2, \cdots, n\}$.

[1]

6.1.2 Bradley-Terry with

, h , v , i , , , $Y_i = 1,\ Y_i = 0.$, δ B-T (1) ,

$$P\left(Y_i = 1\right) = \exp\left(\delta + \beta_{h_i} - \beta_{v_i}\right) / \left[1 + \exp\left(\delta + \beta_{h_i} - \beta_{v_i}\right)\right].$$

, (2) .

6.2 Model II: B-T

6.2.1 B-T

6.2.2 EWMA B-T

, , , $\mathrm{B-T}$ EWMA , . $\beta_{h_i}\left(t_i\right)$ t_i i h_i , EWMA

$$\beta_{h_i}\left(t_i\right) = \alpha_1\lambda_1 r_{h_i}\left(t_i^{(-1)}\right) + \left(1-\lambda_1\right)\beta_{h_i}\left(t_i^{(-1)}\right).$$

, α_1 , $\lambda_1 \in [0,1]$, $r_{h_1}\left(t_i^{(-1)}\right)$ h_i $t_i^{(-1)}$. 4) , $\alpha_1\bar{r}_h$, , $\bar{r}_h$. h_i t_i K , (4)

$$\beta_{h_i}\left(t_i\right) = \alpha_1\left\{\lambda_1\sum_{k=0}^{K-1}\left(1-\lambda_1\right)^k r_{h_i}\left(t_i^{(-k-1)}\right) + \left(1-\lambda_1\right)^K\bar{r}_h\right\} = \alpha_1 x_{h_i}\left(t_i;\lambda_1\right).$$

, $\beta_{h_i}\left(t_i\right)$ $r_{h_i}\left(t_i^{(-1)}\right), r_{h_i}\left(t_i^{(-2)}\right), \cdots, r_{h_i}\left(t_i^{(-K)}\right)$. , $\beta_{v_i}\left(t_i\right)$

$$\beta_{v_i}\left(t_i\right) = \alpha_2\left\{\lambda_1\sum_{k=0}^{K-1}\left(1-\lambda_2\right)^k r_{v_i}\left(t_i^{(-k-1)}\right) + \left(1-\lambda_2\right)^K\bar{r}_v\right\} = \alpha_2 x_{v_i}\left(t_i;\lambda_2\right).$$

(5) (6) (3),

$$P\left(Y_i = y_i \mid Y_{i-1} = y_{i-1}, \cdots, Y_1 = y_1\right) = \frac{\exp\left\{\delta_{y_i} + \beta_{h_i}\left(t_i\right) - \beta_{v_i}\left(t_i\right)\right)}{1 + \exp\left\{\delta_{y_i} + \beta_{h_i}\left(t_i\right) - \beta_{v_i}\left(t_i\right)\right\}}.$$

$\mathrm{B-T}$. $\gamma = \left(\alpha_1, \alpha_2, \delta\right)^{\mathrm{T}},\ \lambda = \left(\lambda_1, \lambda_2\right)^{\mathrm{T}},\ \theta = \left(\gamma^{\mathrm{T}}, \lambda^{\mathrm{T}}\right)^{\mathrm{T}}$, , (7)

$$P\left(Y_i = y_i \mid Y_{i-1} = y_{i-1}, \cdots, Y_1 = y_1; \theta\right) = \frac{\exp\left\{\delta_{y_i} + \alpha_1 x_{h_i}\left(t_i;\lambda_1\right) - \alpha_2 x_{v_i}\left(t_i;\lambda_2\right)\right\}}{1 + \exp\left\{\delta_{y_i} + \alpha_1 x_{h_i}\left(t_i;\lambda_1\right) - \alpha_2 x_{v_i}\left(t_i;\lambda_2\right)\right\}}.$$

$$L(\theta; y) = P\left(Y_1 = y_1; \theta\right)\prod_{i=2}^{n} P\left(Y_i = y_i \mid Y_{i-1} = y_{i-1}, \cdots, Y_1 = y_1; \theta\right).$$

7 Sub-model II:

In order to establish the unsteady sub-model, we recall on the working principle of air conditioners. The heating performance of air conditions consist of two processes: heating and standby. After the user set a temperature, the air conditioner will begin to heat until the expected temperature is reached. Then it will go standby. When the temperature get below the expected temperature, the air conditioner begin to work again. As it works in this circle, the temperature remains the expected one.

Inspired by this, we divide the bathtub working into two processes: adding hot water until the expected temperature is reached, then keeping this condition for a while unless the temperature is lower than a specific value. Iterating this circle ceaselessly will ensure the temperature kept relatively stable.

7.1 Heating Model

7.1.1 Control Equations and Boundary Conditions

7.1.2 Determination of Inflow Time and Amount

7.2 Standby Model

7.3 Results

We first give the value of parameters based on others studies. Then we get the calculation results and simulating results via those data.

7.3.1 Determination of Parameters

After establishing the model, we have to determine the value of some important parameters.

As scholar Beum Kim points out, the optimal temperature for bath is between 41 and 45°C [1]. Meanwhile, according to Shimodozono's study, 41°C warm water bath is the perfect choice for individual health [2]. So it is reasonable for us to focus on $41°C \sim 45°C$. Because adding hot water continuously is a steady process, so the mean temperature of bath water is supposed to be constant. We value the temperature of inflow and outflow water with the maximum and minimum temperature respectively.

The values of all parameters needed are shown as follows:

.....

7.3.2 Calculating Results

Putting the above value of parameters into the equations we derived before, we can get the some data as follows:

Table 2: The calculating results

Variables	Values	Unit
A_1	1.05	m^2
A_2	2.24	m^2
Φ_1	189.00	W
Φ_2	43.47	W
Φ	232.47	W
q_m	0.014	g/s

From Table **??**, ...

......

8 Correction and Contrast of Sub-Models

After establishing two basic sub-models, we have to correct them in consideration of evaporation heat transfer. Then we define two evaluation criteria to compare the two sub-models in order to determine the optimal bath strategy.

8.1 Correction with Evaporation Heat Transfer

Someone may confuse about the above results: why the mass flow in the first sub-model is so small? Why the standby time is so long? Actually, the above two sub-models are based on ideal conditions without consideration of the change of boundary conditions, the motions made by the person in bathtub and the evaporation of bath water, etc. The influence of personal motions will be discussed later. Here we introducing the evaporation of bath water to correct sub-models.

8.2 Contrast of Two Sub-Models

Firstly we define two evaluation criteria. Then we contrast the two submodels via these two criteria. Thus we can derive the best strategy for the person in the bathtub to adopt.

9 Model Analysis and Sensitivity Analysis

9.1 The Influence of Different Bathtubs

Definitely, the difference in shape and volume of the tub affects the convection heat transfer. Examining the relationship between them can help people choose optimal bathtubs.

9.1.1 Different Volumes of Bathtubs

In reality, a cup of water will be cooled down rapidly. However, it takes quite long time for a bucket of water to become cool. That is because their volume is different and the specific heat of water is very large. So that the decrease of temperature is not obvious if the volume of water is huge. That also explains why it takes 45 min for 320 L water to be cooled by 1°C.

In order to examine the influence of volume, we analyze our sub-models by conducting sensitivity Analysis to them.

We assume the initial volume to be 280 L and change it by $\pm 5\%$, $\pm 8\%$, $\pm 12\%$ and $\pm 15\%$. With the aid of sub-models we established before, the variation of some parameters turns out to be as follows

10 Strength and Weakness

10.1 Strength

- We analyze the problem based on thermodynamic formulas and laws, so that the model we established is of great validity.
- Our model is fairly robust due to our careful corrections in consideration of real-life situations and detailed sensitivity analysis.
- Via Fluent software, we simulate the time field of different areas throughout the bathtub. The outcome is vivid for us to understand the changing process.

Table 3: Variation of some parameters

V	A_1	A_2	T_2	q_{m1}	q_{m2}	Φ_q
-15.00%	-5.06%	-9.31%	-12.67%	-2.67%	-14.14%	-5.80%
-12.00%	-4.04%	-7.43%	-10.09%	-2.13%	-11.31%	-4.63%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%
-8.00%	-2.68%	-4.94%	-6.68%	-1.41%	-7.54%	-3.07%

- We come up with various criteria to compare different situations, like water consumption and the time of adding hot water. Hence an overall comparison can be made according to these criteria.
- Besides common factors, we still consider other factors, such as evaporation and radiation heat transfer. The evaporation turns out to be the main reason of heat loss, which corresponds with other scientists experimental outcome.

10.2 Weakness

- Having knowing the range of some parameters from others essays, we choose a value from them to apply in our model. Those values may not be reasonable in reality.
- Although we investigate a lot in the influence of personal motions, they are so complicated that need to be studied further.
- Limited to time, we do not conduct sensitivity analysis for the influence of personal surface area.

11 Further Discussion

In this part, we will focus on different distribution of inflow faucets. Then we discuss about the real-life application of our model.

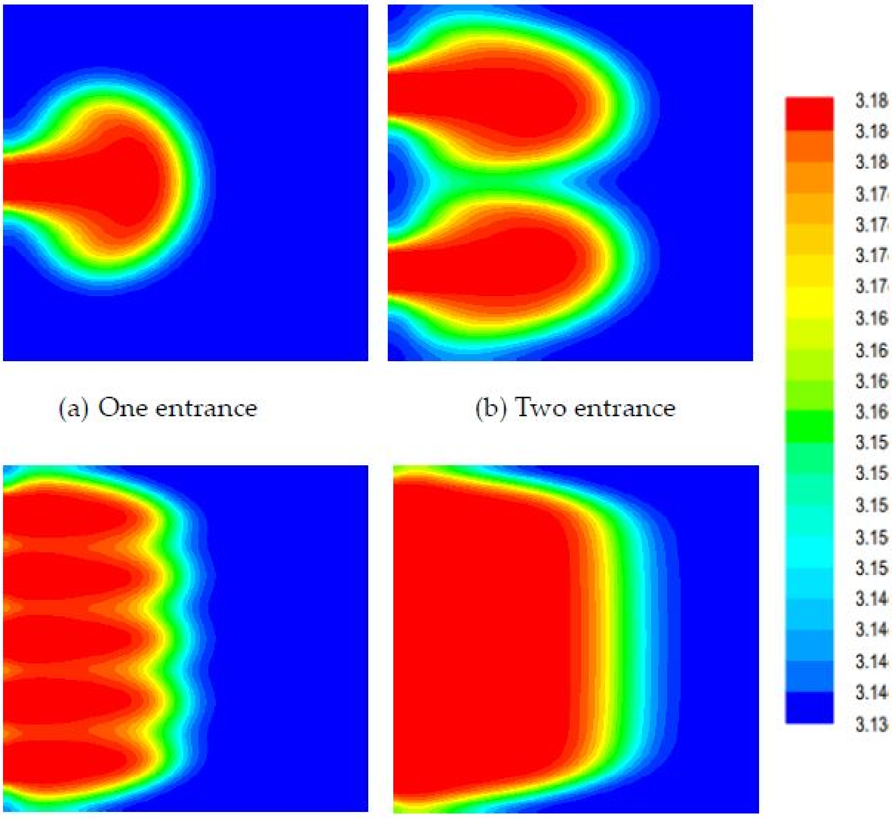
- Different Distribution of Inflow Faucets

  In our before discussion, we assume there being just one entrance of inflow.

  From the simulating outcome, we find the temperature of bath water is hardly even. So we come up with the idea of adding more entrances.

  The simulation turns out to be as follows

  From the above figure, the more the entrances are, the evener the temperature will be. Recalling on the before simulation outcome, when there is only one entrance for inflow, the temperature of corners is quietly lower than the middle area.



(a) One entrance

(b) Two entrance

(c) Five entrance

(d) The whole side as entrance

Figure 2: The simulation results of different ways of arranging entrances

In conclusion, if we design more entrances, it will be easier to realize the goal to keep temperature even throughout the bathtub.

- Model Application

  Our before discussion is based on ideal assumptions. In reality, we have to make some corrections and improvement.

1) Adding hot water continually with the mass flow of 0.16 kg/s. This way can ensure even mean temperature throughout the bathtub and waste less water.
2) The manufacturers can design an intelligent control system to monitor the temperature so that users can get more enjoyable bath experience.
3) We recommend users to add bubble additives to slow down the water being cooler and help cleanse. The additives with lower thermal conductivity are optimal.
4) The study method of our establishing model can be applied in other area relative to convection heat transfer, such as air conditioners.

References

[1] Jing Zhang, Jie Tang, Bangyong Liang, Zi Yang, Sijie Wang, Jingjing Zuo, and Juanzi Li. Recommendation over a heterogeneous social network. In *Web-Age Information Management, 2008. WAIM'08. The Ninth International Conference on*, pages 309–316. IEEE, 2008.

Enjoy Your Bath Time!

From simulation results of real-life situations, we find it takes a period of time for the inflow hot water to spread throughout the bathtub. During this process, the bath water continues transferring heat into air, bathtub and the person in bathtub. The difference between heat transfer capacity makes the temperature of various areas to be different. So that it is difficult to get an evenly maintained temperature throughout the bath water.

In order to enjoy a comfortable bath with even temperature of bath water and without wasting too much water, we propose the following suggestions.

- Adding hot water consistently
- Using smaller bathtub if possible
- Decreasing motions during bath
- Using bubble bath additives
- Arranging more faucets of inflow

Sincerely yours,

Your friends

Appendices

Appendix A First appendix

In addition, your report must include a letter to the Chief Financial Officer (CFO) of the Goodgrant Foundation, Mr. Alpha Chiang, that describes the optimal investment strategy, your modeling approach and major results, and a brief discussion of your proposed concept of a return-on-investment (ROI). This letter should be no more than two pages in length.

Here are simulation programmes we used in our model as follow.

Input matlab source:

```
function [t,seat,aisle]=OI6Sim(n,target,seated)
pab=rand(1,n);
for i=1:n
    if pab(i)<0.4
       aisleTime(i)=0;
    else
        aisleTime(i)=trirnd(3.2,7.1,38.7);
    end
end
```

Appendix B Second appendix

some more text **Input C++ source:**

```
//===========================================================================
// Name        : Sudoku.cpp
// Author      : wzlf11
// Version     : a.0
// Copyright   : Your copyright notice
// Description : Sudoku in C++.
//===========================================================================

#include <iostream>
#include <cstdlib>
#include <ctime>

using namespace std;

int table[9][9];

int main() {

    for(int i = 0; i < 9; i++){
        table[0][i] = i + 1;
    }

    srand((unsigned int)time(NULL));

    shuffle((int *)&table[0], 9);

    while(!put_line(1))
```

```
    {
        shuffle((int *)&table[0], 9);
    }

    for(int x = 0; x < 9; x++){
        for(int y = 0; y < 9; y++){
            cout << table[x][y] << " ";
        }

        cout << endl;
    }

    return 0;
}
```

Report on Use of AI

This report aims to provide a detailed explanation of the AI techniques we utilized during the MCM/ICM mathematical modeling competition and elucidate their advantages and disadvantages, as well as their application in our modeling and analysis process.

Utilized AI Techniques

The following AI techniques were employed during our modeling and analysis:

- Neural Networks
- Decision Trees
- Cluster Analysis

Neural networks were used to predict trends and changes related to the problem under investigation, decision trees were utilized to determine correlations between factors, and cluster analysis was applied to identify patterns within the dataset.

Application of Techniques

For each technique, we elaborate on their pros and cons and justify our decision for selecting these particular methods. Additionally, we outline the approaches and strategies we implemented for data preprocessing, feature engineering, model training, and tuning.

Results and Conclusions

Using AI techniques, we derived a series of results and conclusions regarding the problem at hand. We evaluated the reliability and accuracy of these outcomes. Through our analysis, we arrived at the following conclusions:

Conclusion 1 Conclusion 2 ...

Discussion

We discuss the future prospects and trends of these techniques, highlighting potential limitations and challenges. Furthermore, we present suggestions and future research directions.

Summary

This report comprehensively outlines the AI techniques employed during the MCM/ICM competition, detailing their application, obtained results and conclusions, and future research directions. We hope this report serves as a valuable reference for understanding the utilization of AI techniques in modeling and analysis.