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ECE 681

Homework III: Curse of Dimensionality

Part I

A. Consider the special case Q = P = D = 1, where we flip one of each coin exactly once. Then the question "are these data separable?" is equivalent to the question "did the two coins produce different results?" because clearly we cannot even try to tell which coin is which if the only features they show are identical. If they produce different results, however, we could learn, for example, that quarters usually land heads, while pennies usually land tails.

   Answer: .5 (50%)

B. Now consider the case where Q=2 and P =D=1. Note that the events 1) in which the first quarter and the penny produce the same result, and 2) in which the second quarter and the penny produce the same result are independent. What is the probability that neither quarter produces the same result as the penny?

   Answer: .25 (25%)

C. If P = D = 1, what is the probability that no quarter produces the same result as the penny, as a function of Q?

   Answer: .25 (25%)

D. Now consider P = 2 and D = 1. Given that the first penny produced a different result from all of the quarters, what is the probability that the second penny did as well (as a function of Q)? Then, what is the probability that neither of the pennies produced the same result as any of the quarters?

   Answer: 1 / (2^Q)

E. For D = 1, what is the probability that with Q quarters and P pennies, no pair of unlike coins produced the same result, as a function of Q and P? This is the probability that these two classes are separable in 1 dimension.

Answer: .5 (50%)

F. Now consider Q = P = 1 and D = 2. That is, we flip one quarter and one penny two times each. Each coin now has four possible results: [0, 0], [0, 1], [1, 0], and [1, 1]. What is the probability that the two coins produce different results?

Answer: .75 (75%)

G. For P = 1 and Q = D = 2, what is the probability that neither quarter produces the same result as the penny?

Answer: 9/16 (56%)

H. Finally, for P = 1 and D = 2, what is the probability that no quarter produces the same result as the penny, as a function of Q?

Answer: (2^ (Q+1) - 2) /(2 ^ (Q + 1))^2

Part II

A. What trend do you see in the probability of class separability as the number of observations increases?

Answer: Probability of separation decreases with more observations

B. What trend do you see in the probability of class separability as the number of features increases?

Answer: Probability of separation increases with more features

C. How do changes in the number of observations and features impact the potential for overfitting?

Answer: Increasing the number of features leads to a better chance of a better fit, but too many features could cause overfitting. While increasing the number of observations helps reduce the chance of overfitting as you avoid fitting to a potential coincidental pattern in the data.

Part III

A. Build a simulation of this coin-flipping problem. Verify your results from Part 1, and estimate the probability of class separability for Q = P = D = 5.
Answer: See coin_simulator and calc_sep functions attached. When running 1000 trials we get separability to range anywhere from 42% to 48% or approximately on average 45%.

Extra Credit

Derive an expression for the probability that no quarter and penny share the same result (i.e. the classes are separable; not necessarily linearly separable, just separable in the absolute sense) for arbitrary Q, P , and D. Warning: this is very difficult.

Answer:
1 - ((2^(Q+P) - 2)/(2^(Q+P)))^D

Total probability - Probability of **No** Separation = Probability of Separation