Introduction to the Error-state Kalman filter

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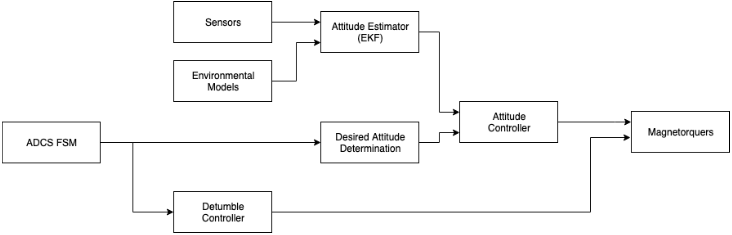
Overview

Motivation



* Note that all submodules of the system will communicate with the FSM. Only the most explicit connections are drawn here.

The controller needs to know the attitude in order to control it, but there is no way to measure it directly $\rightarrow$ we have to estimate it!

State space models

Want to represent an arbitrary system of differential equations in vector form. In general: $\dot{x} = f(x, u)$

Continous LTI state space model

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

Discrete LTI state space model

$$\begin{aligned} x[k+1] &= Ax[k] + Bu[k] \\ y[k] &= Cx[k] + Du[k] \end{aligned} \tag{2}$$

Mass-spring-damper example

How you are used to seeing it

$$m\ddot{x} + d\dot{x} + ky = u \tag{3}$$

State space representation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u \tag{4}$$

Luenberger observer

Let's say we have a state space model of our system. How would we try to estimate the states of the system?

The logical first try (open-loop observer)

$$\dot{\hat{x}} = A\hat{x} + Bu \tag{5}$$

But because of modeling uncertainty our estimate will quickly diverge from the real value $\rightarrow$ include a correction term based on measurements (closing the loop) $\rightarrow$ Luenberger observer

The Luenberger observer

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}), \quad \hat{y} = C\hat{x} \tag{6}$$

But how do we decide the gain L?

The Kalman filter

Let us first assume that our process model and measurement model includes **normally distributed** noise:

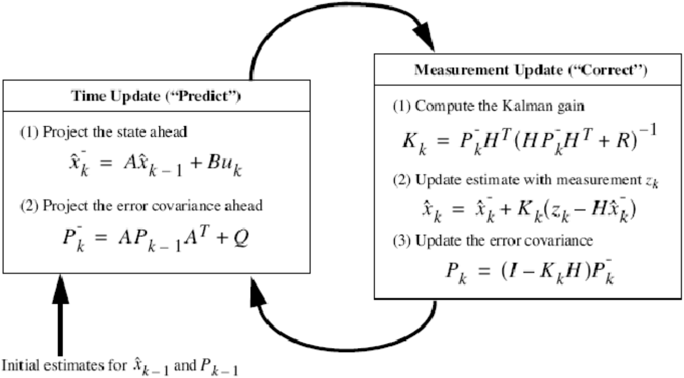
Stochastic LTI system

$$\begin{aligned} \dot{x} &= Ax + Bu + w \\ y &= Cx + Du + v \end{aligned} \tag{7}$$

The Kalman Filter is the optimal Luenberger observer for this system, in the sense that it minimizes **mean squared error**, i.e. $E\{(x - \hat{x})^2\}$.

The Kalman filter equations

For the discrete case (which is what you would implement on a microcontroller) the Kalman filter equations are:



The details are not important here, but note the predict + correct steps.

The satellite kinematics and kinetics

Let's try to apply the Kalman filter to our satellite:

Satellite kinematics

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{q} \otimes \boldsymbol{\omega} \tag{8}$$

Satellite kinetics

$$\dot{\omega} = J^{-1}\left[\mathbf{L} - \omega \times (J\omega)\right] \tag{9}$$

Problem: the system is highly nonlinear, so the Kalman filter cannot be directly applied (since it assumes a linear model).

Extended Kalman filter

The easiest solution to this problem would be to linearize the nonlinear dynamics at each timestep $\rightarrow$ Extended Kalman filter (EKF).

Problem: modeling uncertainty - the kinetics require that we know the inertia matrix of the satellite and since the dynamics are highly nonlinear the EKF might diverge:(

$\rightarrow$ Drop the kinetics, only use the kinematics: $\dot{\mathbf{q}} = \frac{1}{2}\mathbf{q} \otimes \boldsymbol{\omega}$.
We let ω be the "control input", which we measure with the IMU.

Error-state Kalman filter

Now we are getting closer to a somewhat usage algorithm, but the dynamics are still highly nonlinear, which means the EKF will behave poorly.

Paragraphs of Text

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Bullet Points

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- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
- Nam cursus est eget velit posuere pellentesque
- Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block 1

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Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

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Table

Treatments	**Response 1**	**Response 2**
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

Thank you for coming to my TED talk