M6803 Computational Methods in Engineering

AY 2024/2025 Assignment 1 Due Date: **Saturday 14 Sep 2024**

For some questions, you will need to do extra reading beyond what is covered in the class. Please submit your solutions in soft copy through the course site, using the same link where you obtained this question paper. Please submit the source code of your programs in separate files.

1. Explain the representation of negative integers in terms of (i) a sign bit, (ii) ones complement and (iii) twos complement. Write the binary number of the decimal integer -147 using the three different ways.

2. Write a program that converts a given floating point binary number with a 24-bit normalized mantissa and an 8-bit exponent to its decimal (i.e. base 10) equivalent. For the mantissa, use the representation that has a hidden bit, and for the exponent use a bias of 127 instead of a sign bit. Of course, you need to take care of negative numbers in the mantissa also. Use your program to answer the following questions:

   (a) Mantissa: 11110010 11000101 01101010, exponent: 01011000. What is the base-10 number?
   (b) What is the largest number (in base 10) the system can represent?
   (c) What is the smallest non-zero positive base-10 number the system can represent?
   (d) What is the smallest difference between two such numbers? Give your answer in base 10.
   (e) How many significant base-10 digits can we trust using such a representation?

   Your program should do the computations from first principle, and not use functions already available in Python, or whatever language you use.

3. The sine function can be computed as the sum of an infinite power series

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \qquad |x| < \infty,\ x \text{ is in radians.}$$

   Compute the value of $\sin(x)$ for these three x values: 0.5, 5, 50. Note that this is an infinite series and you can only terminate the computation at a term that is very small (i.e. below a certain tolerance which you can set).
   i. Discuss your observations in doing these computations. Do you see any problem with the direct evaluation of this series on a computer? Why?
   ii. Suggest any remedy you may have in overcoming the problem(s).

   You may evaluate the series by writing a Python program.

4. Using the Taylor series, write (i) the first order approximation and (ii) the second order approximation to the function $f(x) = x^2 + \sin(2x)$. Given the value of $f(1)$, which you can evaluate accurately, find the first order and the second order Taylor series approximation value of $f(1.2)$. (Again, x is in radians. Manual calculation suffices. Not need for a program here.)

5. Write a program to evaluate an arithmetic expression given the equivalent postfix string of the expression. Provide at least three test results with different postfix strings of different lengths in your submission. You may store the postfix string in a Python list.

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