Project: Discrete logistic growth

Suppose a rabbit colony has a population $x(n)$ at month n, where x is measured in thousands. If the population were growing in an unbounded environment, the population obeys

$$x(n+1) = x(n) + r \cdot x(n) \tag{1}$$

where r is the per-capita growth rate. Suppose if instead the population is in a bounded environment (like an island), growth is limited, and the population obeys

$$x(n+1) = x(n) + r\left(1 - \frac{x(n)}{K}\right)x(n) \tag{2}$$

where K is a parameter we refer to as the carrying capacity.

Suppose $r = 0.1$ and $K = 0.6$.

a. According to your intuition, what population sizes are *steady states*, meaning that if the population had that value at time $n = 0$, then it would remain at that value?

b. Sketch your intuition for the population $x(t)$ from a starting population $x(1) = 0.2$.

Write code to solve the dynamical system, and answer the following questions:

c. Suppose $r = 0.1$ and $K = 0.6$. Generate time series of the populations for a few starting populations $x(1)$. Does it match your intuition?

d. Suppose $r = 2.1$ and $K = 0.6$. Generate time series of the populations for a few starting populations $x(1)$.

In a discrete-time dynamical system, if the population cycles between two values, the solution is called a two-cycle. Cycling between N values is called an N-cycle.

e. Check that at $r = 2.5$ and $K = 0.6$ there is a 4-cycle.

f. (Optional) Can you find a value of r, K and $x(1)$ that gives a 3-cycle?

g. In this part, we will do a parameter sweep for $0 < r < 3.0$, with fixed $K = 0.6$. The goal is to generate a diagram where the horizontal axis is the parameter value r. On the vertical axis, if there is a stable steady state, plot the steady-state population. If there is an N-cycle, plot the N values of x that it cycles through. [1]

- Hint: One way to plot the steady state or the N-cycle is to simulate the system until $n_{\max}$, and plot the last half values of $x(n)$. You need to choose $n_{\max}$ large enough so that the dynamics have settled into their steady state (or steady cycle) by $n_{\max}/2$.
- Hint: How many r values should you explore?

[1] This type of behavior in a dynamical system is called *chaos*! This particular type of chaos is called period-doubling chaos.