Contents

1 Reminder

1.1 Bug List

- 沒開 long long
- 本地編譯請開-Wall -Wextra -Wshadow -fsanitize=address
- 陣列戳出界 / 開不夠大 / 開太大本地 compile 噴怪 error
- 傳之前先確定選對檔案
- 寫好的函式忘記呼叫
- 變數打錯
- 0-base / 1-base
- 忘記初始化
- == 打成 =
- dp[i] 從 dp[i-1] 轉移時忘記特判 i > 0
- std::sort 比較運算子寫成 < 或是讓 = 的情況為 true
- 漏 case / 分 case 要好好想
- 線段樹改值懶標初始值不能設為 0
- 少碰動態開點，能離散化就離散化
- DFS 的時候不小心覆寫到全域變數
- 浮點數誤差
- 記得刪 cerr

1.2 OwO

- 可以構造複雜點的測資幫助思考
- 真的卡太久請跳題
- Enjoy The Contest!

2 Basic

2.1 Vimrc

```
set number relativenumber ai t_Co=256 tabstop=4
set mouse=a shiftwidth=4 encoding=utf8
set bs=2 ruler laststatus=2 cmdheight=2
set clipboard=unnamedplus showcmd autoread
set belloff=all
filetype indent on

inoremap ( ()<Esc>i
inoremap " ""<Esc>i
inoremap [ []<Esc>i
inoremap ' ''<Esc>i
inoremap { {<CR>}<Esc>ko

nnoremap <tab> gt
nnoremap <S-tab> gT
inoremap <C-n> <Esc>:tabnew<CR>
nnoremap <C-n> :tabnew<CR>

inoremap <F9> <Esc>:w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>
nnoremap <F9> :w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>

syntax on
colorscheme desert
set filetype=cpp
set background=dark
hi Normal ctermfg=white ctermbg=black
```

2.2 Runcpp.sh

```
#! /bin/bash
clear
echo "Start compiling $1..."
echo
g++ -O2 -std=c++20 -fsanitize=address -Wall -Wextra -Wshadow $2/$1 -o $2/out
if [ "$?" -ne 0 ]
then
    exit 1
fi
echo
echo "Done compiling"
echo "========================="
echo
echo "Input file:"
echo
cat $2/in.txt
echo
echo "========================="
echo
declare startTime=`date +%s%N`
$2/out < $2/in.txt > $2/out.txt
declare endTime=`date +%s%N`
delta=`expr $endTime - $startTime`
delta=`expr $delta / 1000000`
cat $2/out.txt
echo
echo "time: $delta ms"
```

2.3 Stress

```
g++ gen.cpp -o gen.out
g++ ac.cpp -o ac.out
g++ wa.cpp -o wa.out
for ((i=0;;i++))
do
    echo "$i"
    ./gen.out > in.txt
    ./ac.out < in.txt > ac.txt
    ./wa.out < in.txt > wa.txt
    diff ac.txt wa.txt || break
done
```

2.4 Others

```
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#pragma GCC optimize("trapv")
```

```cpp
mt19937 gen(chrono::steady_clock::now().time_since_epoch().count());
uniform_int_distribution<int> dis(1, 100);
cout << dis(gen) << endl;
shuffle(v.begin(), v.end(), gen);

struct edge {
    int a, b, w;
    friend istream& operator>>(istream& in, edge& x) { in >> x.a >> x.b >> x.w; }
    friend ostream& operator<<(ostream& out, const edge & x) {
        out << "(" << x.a << "," << x.b << "," << x.w << ")";
        return out;
    }
};
struct cmp {
    bool operator()(const edge& x, const edge& y) const { return x.w < y.w; }
};
set<edge, cmp> st;                              // 遞增
map<edge, long long, cmp> mp;                   // 遞增
priority_queue<edge, vector<edge>, cmp> pq;     // 遞減

#include <bits/extc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

// map
tree<int, int, less<>, rb_tree_tag, tree_order_statistics_node_update> tr;
tr.order_of_key(element);
tr.find_by_order(rank);

// set
tree<int, null_type, less<>, rb_tree_tag, tree_order_statistics_node_update> tr;
tr.order_of_key(element);
tr.find_by_order(rank);

// hash table
gp_hash_table<int, int> ht;
ht.find(element);
ht.insert({key, value});
ht.erase(element);

// priority queue
__gnu_pbds::priority_queue<int, less<int>> big_q;           // Big First
__gnu_pbds::priority_queue<int, greater<int>> small_q;      // Small First
q1.join(q2);  // join
```

3 Data Structure

3.1 BIT

```cpp
struct BIT {
    int n;
    long long bit[N];

    void init(int x, vector<long long> &a) {
        n = x;
        for (int i = 1, j; i <= n; i++) {
            bit[i] += a[i - 1], j = i + (i & -i);
            if (j <= n) bit[j] += bit[i];
        }
    }

    void update(int x, long long dif) {
        while (x <= n) bit[x] += dif, x += x & -x;
    }

    long long query(int l, int r) {
        if (l != 1) return query(1, r) - query(1, l - 1);

        long long ret = 0;
        while (l <= r) ret += bit[r], r -= r & -r;
        return ret;
    }
} bm;
```

3.2 Lazy Propagation Segment Tree

```cpp
struct lazy_propagation{
    // 0-based, [l, r], tg[0]->add, tg[1]->set
    ll seg[N * 4], tg[2][N*4];
    void assign (bool op, ll val, int idx){
        if (op == 0){
            if (tg[1][idx]) tg[1][idx] += val;
            else            tg[0][idx] += val;
        }
        else    seg[idx] = 0, tg[0][idx] = 0, tg[1][idx] = val;
    }
    ll sum (int idx, int len){
        if (tg[1][idx]) return tg[1][idx] * len;
        return tg[0][idx] * len + seg[idx];
    }
    void pull (int idx, int len){
        seg[idx] = sum(2*idx, (len+1)/2) + sum(2*idx+1, len/2);
    }
    void push (int idx){
        if (!tg[0][idx] && !tg[1][idx]) return ;
        if (tg[0][idx]){
            assign(0, tg[0][idx], 2*idx);
            assign(0, tg[0][idx], 2*idx+1);
            tg[0][idx] = 0;
        }
        else{
            assign(1, tg[1][idx], 2*idx);
            assign(1, tg[1][idx], 2*idx+1);
            tg[1][idx] = 0;
        }
    }
    void update (bool op, ll val, int gl, int gr, int l, int r, int idx){
        if (r < l || gr < l || r < gl)  return ;
        if (gl <= l && r <= gr){
            assign(op, val, idx);
            return ;
        }

        int mid = (l + r) / 2;
        push(idx);
        update(op, val, gl, gr, l, mid, 2*idx);
        update(op, val, gl, gr, mid+1, r, 2*idx+1);
        pull(idx, r-l+1);
    }
    ll query (int gl, int gr, int l, int r, int idx){
        if (r < l || gr < l || r < gl)  return 0;
        if (gl <= l && r <= gr) return sum(idx, r-l+1);

        push(idx), pull(idx, r-l+1);
        int mid = (l + r) / 2;
        return query(gl, gr, l, mid, 2*idx) + query(gl, gr, mid+1, r, 2*idx+1);
    }
}bm;
```

3.3 Treap

```cpp
mt19937 rng(random_device{}());
struct Treap {
    Treap *l, *r;
    int val, sum, real, tag, num, pri, rev;
    Treap(int k) {
        l = r = NULL;
        val = sum = k;
        num = 1;
        real = -1;
        tag = 0;
        rev = 0;
        pri = rng();
    }
};
int siz(Treap *now) { return now ? now->num : 0ll; }
int sum(Treap *now) {
    if (!now) return 0;
```

```cpp
    if (now->real != -1) return (now->real + now->tag)
        * now->num;
    return now->sum + now->tag * now->num;
}
void pull(Treap *&now) {
    now->num = siz(now->l) + siz(now->r) + 1ll;
    now->sum = sum(now->l) + sum(now->r) + now->val +
        now->tag;
}
void push(Treap *&now) {
    if (now->rev) {
        swap(now->l, now->r);
        now->l->rev ^= 1;
        now->r->rev ^= 1;
        now->rev = 0;
    }
    if (now->real != -1) {
        now->real += now->tag;
        if (now->l) {
            now->l->tag = 0;
            now->l->real = now->real;
            now->l->val = now->real;
        }
        if (now->r) {
            now->r->tag = 0;
            now->r->real = now->real;
            now->r->val = now->real;
        }
        now->val = now->real;
        now->sum = now->real * now->num;
        now->real = -1;
        now->tag = 0;
    } else {
        if (now->l) now->l->tag += now->tag;
        if (now->r) now->r->tag += now->tag;
        now->sum += sum(now);
        now->val += now->tag;
        now->tag = 0;
    }
}
Treap *merge(Treap *a, Treap *b) {
    if (!a || !b) return a ? a : b;
    else if (a->pri > b->pri) {
        push(a);
        a->r = merge(a->r, b);
        pull(a);
        return a;
    } else {
        push(b);
        b->l = merge(a, b->l);
        pull(b);
        return b;
    }
}
void split_size(Treap *rt, Treap *&a, Treap *&b, int
    val) {
    if (!rt) {
        a = b = NULL;
        return;
    }
    push(rt);
    if (siz(rt->l) + 1 > val) {
        b = rt;
        split_size(rt->l, a, b->l, val);
        pull(b);
    } else {
        a = rt;
        split_size(rt->r, a->r, b, val - siz(a->l) - 1)
            ;
        pull(a);
    }
}
void split_val(Treap *rt, Treap *&a, Treap *&b, int val
    ) {
    if (!rt) {
        a = b = NULL;
        return;
    }
    push(rt);
    if (rt->val <= val) {
        a = rt;
        split_val(rt->r, a->r, b, val);
        pull(a);
    } else {
        b = rt;
        split_val(rt->l, a, b->l, val);
        pull(b);
    }
}
```

3.4 Persistent Treap

```cpp
struct node {
    node *l, *r;
    char c;
    int v, sz;
    node(char x = '$') : c(x), v(mt()), sz(1) {
        l = r = nullptr;
    }
    node(node* p) { *this = *p; }
    void pull() {
        sz = 1;
        for (auto i : {l, r})
            if (i) sz += i->sz;
    }
} arr[maxn], *ptr = arr;
inline int size(node* p) { return p ? p->sz : 0; }
node* merge(node* a, node* b) {
    if (!a || !b) return a ?: b;
    if (a->v < b->v) {
        node* ret = new (ptr++) node(a);
        ret->r = merge(ret->r, b), ret->pull();
        return ret;
    } else {
        node* ret = new (ptr++) node(b);
        ret->l = merge(a, ret->l), ret->pull();
        return ret;
    }
}
P<node*> split(node* p, int k) {
    if (!p) return {nullptr, nullptr};
    if (k >= size(p->l) + 1) {
        auto [a, b] = split(p->r, k - size(p->l) - 1);
        node* ret = new (ptr++) node(p);
        ret->r = a, ret->pull();
        return {ret, b};
    } else {
        auto [a, b] = split(p->l, k);
        node* ret = new (ptr++) node(p);
        ret->l = b, ret->pull();
        return {a, ret};
    }
}
```

3.5 Li Chao Tree

```cpp
constexpr int maxn = 5e4 + 5;
struct line {
    ld a, b;
    ld operator()(ld x) { return a * x + b; }
} arr[(maxn + 1) << 2];
bool operator<(line a, line b) { return a.a < b.a; }
#define m ((l + r) >> 1)
void insert(line x, int i = 1, int l = 0, int r = maxn)
    {
    if (r - l == 1) {
        if (x(l) > arr[i](l))
            arr[i] = x;
        return;
    }
    line a = max(arr[i], x), b = min(arr[i], x);
    if (a(m) > b(m))
        arr[i] = a, insert(b, i << 1, l, m);
    else
        arr[i] = b, insert(a, i << 1 | 1, m, r);
}
ld query(int x, int i = 1, int l = 0, int r = maxn) {
    if (x < l || r <= x) return -numeric_limits<ld>::
        max();
    if (r - l == 1) return arr[i](x);
    return max({arr[i](x), query(x, i << 1, l, m),
        query(x, i << 1 | 1, m, r)});
}
#undef m
```

3.6 Sparse Table

```cpp
const int lgmx = 19;

int n, q;
int spt[lgmx][maxn];

void build() {
    FOR(k, 1, lgmx, 1) {
        for (int i = 0; i + (1 << k) - 1 < n; i++) {
            spt[k][i] = min(spt[k - 1][i], spt[k - 1][i + (1 << (k - 1))]);
        }
    }
}

int query(int l, int r) {
    int ln = len(l, r);
    int lg = __lg(ln);
    return min(spt[lg][l], spt[lg][r - (1 << lg) + 1]);
}
```

3.7 Time Segment Tree

```cpp
constexpr int maxn = 1e5 + 5;
V<P<int>> arr[(maxn + 1) << 2];
V<int> dsu, sz;
V<tuple<int, int, int>> his;
int cnt, q;
int find(int x) {
    return x == dsu[x] ? x : find(dsu[x]);
};
inline bool merge(int x, int y) {
    int a = find(x), b = find(y);
    if (a == b) return false;
    if (sz[a] > sz[b]) swap(a, b);
    his.emplace_back(a, b, sz[b]), dsu[a] = b, sz[b] += sz[a];
    return true;
};
inline void undo() {
    auto [a, b, s] = his.back();
    his.pop_back();
    dsu[a] = a, sz[b] = s;
}
#define m ((l + r) >> 1)
void insert(int ql, int qr, P<int> x, int i = 1, int l = 0, int r = q) {
    // debug(ql, qr, x); return;
    if (qr <= l || r <= ql) return;
    if (ql <= l && r <= qr) {
        arr[i].push_back(x);
        return;
    }
    if (qr <= m)
        insert(ql, qr, x, i << 1, l, m);
    else if (m <= ql)
        insert(ql, qr, x, i << 1 | 1, m, r);
    else {
        insert(ql, qr, x, i << 1, l, m);
        insert(ql, qr, x, i << 1 | 1, m, r);
    }
}
void traversal(V<int>& ans, int i = 1, int l = 0, int r = q) {
    int opcnt = 0;
    // debug(i, l, r);
    for (auto [a, b] : arr[i])
        if (merge(a, b))
            opcnt++, cnt--;
    if (r - l == 1)
        ans[l] = cnt;
    else {
        traversal(ans, i << 1, l, m);
        traversal(ans, i << 1 | 1, m, r);
    }
    while (opcnt--)
        undo(), cnt++;
    arr[i].clear();
}
#undef m
inline void solve() {
    int n, m;
    cin >> n >> m >> q, q++;
    dsu.resize(cnt = n), sz.assign(n, 1);
    iota(dsu.begin(), dsu.end(), 0);
    // a, b, time, operation
    unordered_map<ll, V<int>> s;
    for (int i = 0; i < m; i++) {
        int a, b;
        cin >> a >> b;
        if (a > b) swap(a, b);
        s[((ll)a << 32) | b].emplace_back(0);
    }
    for (int i = 1; i < q; i++) {
        int op, a, b;
        cin >> op >> a >> b;
        if (a > b) swap(a, b);
        switch (op) {
            case 1:
                s[((ll)a << 32) | b].push_back(i);
                break;
            case 2:
                auto tmp = s[((ll)a << 32) | b].back();
                s[((ll)a << 32) | b].pop_back();
                insert(tmp, i, P<int>{a, b});
        }
    }
    for (auto [p, v] : s) {
        int a = p >> 32, b = p & -1;
        while (v.size()) {
            insert(v.back(), q, P<int>{a, b});
            v.pop_back();
        }
    }
    V<int> ans(q);
    traversal(ans);
    for (auto i : ans)
        cout << i << ' ';
    cout << endl;
}
```

3.8 Dynamic Median

```cpp
struct Dynamic_Median {
    multiset<long long> lo, hi;
    long long slo = 0, shi = 0;
    void rebalance() {
        // keep sz(lo) >= sz(hi) and sz(lo) - sz(hi) <= 1
        while((int)lo.size() > (int)hi.size() + 1) {
            auto it = prev(lo.end());
            long long x = *it;
            lo.erase(it); slo -= x;
            hi.insert(x); shi += x;
        }
        while((int)lo.size() < (int)hi.size()) {
            auto it = hi.begin();
            long long x = *it;
            hi.erase(it); shi -= x;
            lo.insert(x); slo += x;
        }
    }
    void add(long long x) {
        if(lo.empty() || x <= *prev(lo.end())) {
            lo.insert(x); slo += x;
        }
        else {
            hi.insert(x); shi += x;
        }
        rebalance();
    }
    void remove_one(long long x) {
        if(!lo.empty() && x <= *prev(lo.end())) {
            auto it = lo.find(x);
            if(it != lo.end()) {
                lo.erase(it); slo -= x;
            }
            else {
                auto it2 = hi.find(x);
                hi.erase(it2); shi -= x;
            }
        }
        else {
```

```
            auto it = hi.find(x);
            if(it != hi.end()) {
                hi.erase(it); shi -= x;
            }
            else {
                auto it2 = lo.find(x);
                lo.erase(it2); slo -= x;
            }
        }
        rebalance();
    }
};
```

3.9 SOS DP

```
for (int mask = 0; mask < (1 << n); mask++) {
    for (int submask = mask; submask != 0; submask = (
        submask - 1) & mask) {
        int subset = mask ^ submask;
}   }
```

4 Flow / Matching

4.1 Dinic

```
using namespace std;
const int N = 2000 + 5;
int n, m, s, t, level[N], iter[N];
struct edge {int to, cap, rev;};
vector<edge> path[N];
void add(int a, int b, int c) {
    path[a].pb({b, c, sz(path[b])});
    path[b].pb({a, 0, sz(path[a]) - 1});
}
void bfs() {
    memset(level, -1, sizeof(level));
    level[s] = 0;
    queue<int> q;
    q.push(s);
    while (q.size()) {
        int now = q.front();q.pop();
        for (edge e : path[now]) if (e.cap > 0 && level
            [e.to] == -1) {
                level[e.to] = level[now] + 1;
                q.push(e.to);
        }
    }
}
int dfs(int now, int flow) {
    if (now == t) return flow;
    for (int &i = iter[now]; i < sz(path[now]); i++) {
        edge &e = path[now][i];
        if (e.cap > 0 && level[e.to] == level[now] + 1)
             {
            int res = dfs(e.to, min(flow, e.cap));
            if (res > 0) {
                e.cap -= res;
                path[e.to][e.rev].cap += res;
                return res;
            }
        }
    }
    return 0;
}
int dinic() {
    int res = 0;
    while (true) {
        bfs();
        if (level[t] == -1) break;
        memset(iter, 0, sizeof(iter));
        int now = 0;
        while ((now = dfs(s, INF)) > 0) res += now;
    }
    return res;
}
```

4.2 MCMF

```
struct MCMF {
    int n, s, t, par[N + 5], p_i[N + 5], dis[N + 5],
        vis[N + 5];
    struct edge {
        int to, cap, rev, cost;
    };
    vector<edge> path[N];
    void init(int _n, int _s, int _t) {
        n = _n, s = _s, t = _t;
        FOR(i, 0, 2 * n + 5)
        par[i] = p_i[i] = vis[i] = 0;
    }
    void add(int a, int b, int c, int d) {
        path[a].pb({b, c, sz(path[b]), d});
        path[b].pb({a, 0, sz(path[a]) - 1, -d});
    }
    void spfa() {
        FOR(i, 0, n * 2 + 5)
        dis[i] = INF,
        vis[i] = 0;
        dis[s] = 0;
        queue<int> q;
        q.push(s);
        while (!q.empty()) {
            int now = q.front();
            q.pop();
            vis[now] = 0;
            for (int i = 0; i < sz(path[now]); i++) {
                edge e = path[now][i];
                if (e.cap > 0 && dis[e.to] > dis[now] +
                    e.cost) {
                    dis[e.to] = dis[now] + e.cost;
                    par[e.to] = now;
                    p_i[e.to] = i;
                    if (vis[e.to] == 0) {
                        vis[e.to] = 1;
                        q.push(e.to);
                    }
                }
            }
        }
    }
    pii flow() {
        int flow = 0, cost = 0;
        while (true) {
            spfa();
            if (dis[t] == INF)
                break;
            int mn = INF;
            for (int i = t; i != s; i = par[i])
                mn = min(mn, path[par[i]][p_i[i]].cap);
            flow += mn;
            cost += dis[t] * mn;
            for (int i = t; i != s; i = par[i]) {
                edge &now = path[par[i]][p_i[i]];
                now.cap -= mn;
                path[i][now.rev].cap += mn;
            }
        }
        return mp(flow, cost);
    }
};
```

4.3 KM

```
struct KM {
    int n, mx[1005], my[1005], pa[1005];
    int g[1005][1005], lx[1005], ly[1005], sy[1005];
    bool vx[1005], vy[1005];
    void init(int _n) {
        n = _n;
        FOR(i, 1, n + 1)
        fill(g[i], g[i] + 1 + n, 0);
    }
    void add(int a, int b, int c) { g[a][b] = c; }
    void augment(int y) {
        for (int x, z; y; y = z)
            x = pa[y], z = mx[x], my[y] = x, mx[x] = y;
    }
    void bfs(int st) {
        FOR(i, 1, n + 1)
        sy[i] = INF,
        vx[i] = vy[i] = 0;
        queue<int> q;
        q.push(st);
```

```cpp
        for (;;) {
            while (!q.empty()) {
                int x = q.front();
                q.pop();
                vx[x] = 1;
                FOR(y, 1, n + 1)
                if (!vy[y]) {
                    int t = lx[x] + ly[y] - g[x][y];
                    if (t == 0) {
                        pa[y] = x;
                        if (!my[y]) {
                            augment(y);
                            return;
                        }
                        vy[y] = 1, q.push(my[y]);
                    } else if (sy[y] > t)
                        pa[y] = x, sy[y] = t;
                }
            }
            int cut = INF;
            FOR(y, 1, n + 1)
            if (!vy[y] && cut > sy[y]) cut = sy[y];
            FOR(j, 1, n + 1) {
                if (vx[j]) lx[j] -= cut;
                if (vy[j])
                    ly[j] += cut;
                else
                    sy[j] -= cut;
            }
            FOR(y, 1, n + 1) {
                if (!vy[y] && sy[y] == 0) {
                    if (!my[y]) {
                        augment(y);
                        return;
                    }
                    vy[y] = 1;
                    q.push(my[y]);
                }
            }
        }
    }
    int solve() {
        fill(mx, mx + n + 1, 0);
        fill(my, my + n + 1, 0);
        fill(ly, ly + n + 1, 0);
        fill(lx, lx + n + 1, 0);
        FOR(x, 1, n + 1)
        FOR(y, 1, n + 1)
        lx[x] = max(lx[x], g[x][y]);
        FOR(x, 1, n + 1)
        bfs(x);
        int ans = 0;
        FOR(y, 1, n + 1)
        ans += g[my[y]][y];
        return ans;
    }
};
```

4.4 Hopcroft-Karp

```cpp
struct HopcroftKarp {
    // id: X = [1, nx], Y = [nx+1, nx+ny]
    int n, nx, ny, m, MXCNT;
    vector<vector<int> > g;
    vector<int> mx, my, dis, vis;
    void init(int nnx, int nny, int mm) {
        nx = nnx, ny = nny, m = mm;
        n = nx + ny + 1;
        g.clear();
        g.resize(n);
    }
    void add(int x, int y) {
        g[x].emplace_back(y);
        g[y].emplace_back(x);
    }
    bool dfs(int x) {
        vis[x] = true;
        Each(y, g[x]) {
            int px = my[y];
            if (px == -1 ||
                (dis[px] == dis[x] + 1 &&
                 !vis[px] && dfs(px))) {
                mx[x] = y;
                my[y] = x;
                return true;
            }
        }
        return false;
    }
    void get() {
        mx.clear();
        mx.resize(n, -1);
        my.clear();
        my.resize(n, -1);

        while (true) {
            queue<int> q;
            dis.clear();
            dis.resize(n, -1);
            for (int x = 1; x <= nx; x++) {
                if (mx[x] == -1) {
                    dis[x] = 0;
                    q.push(x);
                }
            }
            while (!q.empty()) {
                int x = q.front();
                q.pop();
                Each(y, g[x]) {
                    if (my[y] != -1 && dis[my[y]] ==
                        -1) {
                        dis[my[y]] = dis[x] + 1;
                        q.push(my[y]);
                    }
                }
            }

            bool brk = true;
            vis.clear();
            vis.resize(n, 0);
            for (int x = 1; x <= nx; x++)
                if (mx[x] == -1 && dfs(x))
                    brk = false;

            if (brk) break;
        }
        MXCNT = 0;
        for (int x = 1; x <= nx; x++)
            if (mx[x] != -1) MXCNT++;
    }
} hk;
```

4.5 Blossom

```cpp
const int N=5e2+10;
struct Graph{
    int to[N],bro[N],head[N],e;
    int lnk[N],vis[N],stp,n;
    void init(int _n){
        stp=0;e=1;n=_n;
        FOR(i,0,n+1)head[i]=lnk[i]=vis[i]=0;
    }
    void add(int u,int v){
        to[e]=v,bro[e]=head[u],head[u]=e++;
        to[e]=u,bro[e]=head[v],head[v]=e++;
    }
    bool dfs(int x){
        vis[x]=stp;
        for(int i=head[x];i;i=bro[i])
        {
            int v=to[i];
            if(!lnk[v])
            {
                lnk[x]=v;lnk[v]=x;
                return true;
            }
            else if(vis[lnk[v]]<stp)
            {
                int w=lnk[v];
                lnk[x]=v,lnk[v]=x,lnk[w]=0;
                if(dfs(w))return true;
                lnk[w]=v,lnk[v]=w,lnk[x]=0;
            }
        }
```

```cpp
                return false;
    }
    int solve(){
        int ans=0;
        FOR(i,1,n+1){
            if(!lnk[i]){
                stp++;
                ans+=dfs(i);
            }
        }
        return ans;
    }
    void print_matching(){
        FOR(i,1,n+1)
            if(i<graph.lnk[i])
                cout<<i<<" "<<graph.lnk[i]<<endl;
    }
};
```

4.6 Cover / Independent Set

```
V(E) Cover: choose some V(E) to cover all E(V)
V(E) Independ: set of V(E) not adj to each other

M = Max Matching
Cv = Min V Cover
Ce = Min E Cover
Iv = Max V Ind
Ie = Max E Ind (equiv to M)

M = Cv (Konig Theorem)
Iv = V \ Cv
Ce = V - M

Construct Cv:
1. Run Dinic
2. Find s-t min cut
3. Cv = {X in T} + {Y in S}
```

4.7 Hungarian Algorithm

```cpp
const int N = 2e3;
int match[N];
bool vis[N];
int n;
vector<int> ed[N];
int match_cnt;
bool dfs(int u) {
    vis[u] = 1;
    for(int i : ed[u]) {
        if(match[i] == 0 || !vis[match[i]] && dfs(match[i])) {
            match[i] = u;
            return true;
        }
    }
    return false;
}
void hungary() {
    memset(match, 0, sizeof(match));
    match_cnt = 0;
    for(int i = 1; i <= n; i++) {
        memset(vis, 0, sizeof(vis));
        if(dfs(i)) match_cnt++;
    }
}
```

5 Graph

5.1 Heavy-Light Decomposition

```cpp
const int N = 2e5 + 5;
int n, dfn[N], son[N], top[N], num[N], dep[N], p[N];
vector<int> path[N];
struct node {
    int mx, sum;
} seg[N << 2];
void update(int x, int l, int r, int qx, int val) {
    if (l == r) {
        seg[x].mx = seg[x].sum = val;
        return;
    }
    int mid = (l + r) >> 1;
    if (qx <= mid)update(x << 1, l, mid, qx, val);
    else update(x << 1 | 1, mid + 1, r, qx, val);
    seg[x].mx = max(seg[x << 1].mx, seg[x << 1 | 1].mx);
    seg[x].sum = seg[x << 1].sum + seg[x << 1 | 1].sum;
}
int big(int x, int l, int r, int ql, int qr) {
    if (ql <= l && r <= qr) return seg[x].mx;
    int mid = (l + r) >> 1;
    int res = -INF;
    if (ql <= mid) res = max(res, big(x << 1, l, mid, ql, qr));
    if (mid < qr) res = max(res, big(x << 1 | 1, mid + 1, r, ql, qr));
    return res;
}
int ask(int x, int l, int r, int ql, int qr) {
    if (ql <= l && r <= qr) return seg[x].sum;
    int mid = (l + r) >> 1;
    int res = 0;
    if (ql <= mid) res += ask(x << 1, l, mid, ql, qr);
    if (mid < qr) res += ask(x << 1 | 1, mid + 1, r, ql, qr);
    return res;
}
void dfs1(int now) {
    son[now] = -1;
    num[now] = 1;
    for (auto i : path[now]) {
        if (!dep[i]) {
            dep[i] = dep[now] + 1;
            p[i] = now;
            dfs1(i);
            num[now] += num[i];
            if (son[now] == -1 || num[i] > num[son[now]]) son[now] = i;
        }
    }
}
int cnt;
void dfs2(int now, int t) {
    top[now] = t;
    cnt++;
    dfn[now] = cnt;
    if (son[now] == -1) return;
    dfs2(son[now], t);
    for (auto i : path[now])
        if (i != p[now] && i != son[now])dfs2(i, i);
}
int path_big(int x, int y) {
    int res = -INF;
    while (top[x] != top[y]) {
        if (dep[top[x]] < dep[top[y]]) swap(x, y);
        res = max(res, big(1, 1, n, dfn[top[x]], dfn[x]));
        x = p[top[x]];
    }
    if (dfn[x] > dfn[y]) swap(x, y);
    res = max(res, big(1, 1, n, dfn[x], dfn[y]));
    return res;
}
int path_sum(int x, int y) {
    int res = 0;
    while (top[x] != top[y]) {
        if (dep[top[x]] < dep[top[y]]) swap(x, y);
        res += ask(1, 1, n, dfn[top[x]], dfn[x]);
        x = p[top[x]];
    }
    if (dfn[x] > dfn[y]) swap(x, y);
    res += ask(1, 1, n, dfn[x], dfn[y]);
    return res;
}
void buildTree() {
    FOR(i, 0, n - 1) {
        int a, b;
        cin >> a >> b;
        path[a].pb(b);
        path[b].pb(a);
    }
}
```

```cpp
void buildHLD(int root) {
    dep[root] = 1;
    dfs1(root);
    dfs2(root, root);
    FOR(i, 1, n + 1) {
        int now;
        cin >> now;
        update(1, 1, n, dfn[i], now);
    }
}
```

5.2 Centroid Decomposition

```cpp
#include <bits/stdc++.h>
using namespace std;
const int N = 1e5 + 5;
vector<int> a[N];
int sz[N], lv[N];
bool used[N];
int f_sz(int x, int p) {
    sz[x] = 1;
    for (int i : a[x])
        if (i != p && !used[i])
            sz[x] += f_sz(i, x);
    return sz[x];
}
int f_cen(int x, int p, int total) {
    for (int i : a[x]) {
        if (i != p && !used[i] && 2 * sz[i] > total)
            return f_cen(i, x, total);
    }
    return x;
}
void cd(int x, int p) {
    int total = f_sz(x, p);
    int cen = f_cen(x, p, total);
    lv[cen] = lv[p] + 1;
    used[cen] = 1;
    // cout << "cd: " << x << " " << p << " " << cen <<
        "\n";
    for (int i : a[cen]) {
        if (!used[i])
            cd(i, cen);
    }
}
int main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n;
    cin >> n;
    for (int i = 0, x, y; i < n - 1; i++) {
        cin >> x >> y;
        a[x].push_back(y);
        a[y].push_back(x);
    }
    cd(1, 0);
    for (int i = 1; i <= n; i++)
        cout << (char)('A' + lv[i] - 1) << " ";
    cout << "\n";
}
```

5.3 Bellman-Ford + SPFA

```cpp
int n, m;

// Graph
vector<vector<pair<int, ll> > > g;
vector<ll> dis;
vector<bool> negCycle;

// SPFA
vector<int> rlx;
queue<int> q;
vector<bool> inq;
vector<int> pa;
void SPFA(vector<int>& src) {
    dis.assign(n + 1, LINF);
    negCycle.assign(n + 1, false);
    rlx.assign(n + 1, 0);
    while (!q.empty()) q.pop();
    inq.assign(n + 1, false);
    pa.assign(n + 1, -1);

    for (auto& s : src) {
        dis[s] = 0;
        q.push(s);
        inq[s] = true;
    }

    while (!q.empty()) {
        int u = q.front();
        q.pop();
        inq[u] = false;
        if (rlx[u] >= n) {
            negCycle[u] = true;
        } else
            for (auto& e : g[u]) {
                int v = e.first;
                ll w = e.second;
                if (dis[v] > dis[u] + w) {
                    dis[v] = dis[u] + w;
                    rlx[v] = rlx[u] + 1;
                    pa[v] = u;
                    if (!inq[v]) {
                        q.push(v);
                        inq[v] = true;
                    }
                }
            }
    }
}

// Bellman-Ford
queue<int> q;
vector<int> pa;
void BellmanFord(vector<int>& src) {
    dis.assign(n + 1, LINF);
    negCycle.assign(n + 1, false);
    pa.assign(n + 1, -1);

    for (auto& s : src) dis[s] = 0;

    for (int rlx = 1; rlx <= n; rlx++) {
        for (int u = 1; u <= n; u++) {
            if (dis[u] == LINF) continue;  // Important
                !!
            for (auto& e : g[u]) {
                int v = e.first;
                ll w = e.second;
                if (dis[v] > dis[u] + w) {
                    dis[v] = dis[u] + w;
                    pa[v] = u;
                    if (rlx == n) negCycle[v] = true;
                }
            }
        }
    }
}

// Negative Cycle Detection
void NegCycleDetect() {
    /* No Neg Cycle: NO
    Exist Any Neg Cycle:
    YES
    v0 v1 v2 ... vk v0 */

    vector<int> src;
    for (int i = 1; i <= n; i++)
        src.emplace_back(i);

    SPFA(src);
    // BellmanFord(src);

    int ptr = -1;
    for (int i = 1; i <= n; i++)
        if (negCycle[i]) {
            ptr = i;
            break;
        }

    if (ptr == -1) {
        return cout << "NO" << endl, void();
    }
```

```cpp
    cout << "YES\n";
    vector<int> ans;
    vector<bool> vis(n + 1, false);

    while (true) {
        ans.emplace_back(ptr);
        if (vis[ptr]) break;
        vis[ptr] = true;
        ptr = pa[ptr];
    }
    reverse(ans.begin(), ans.end());

    vis.assign(n + 1, false);
    for (auto& x : ans) {
        cout << x << ' ';
        if (vis[x]) break;
        vis[x] = true;
    }
    cout << endl;
}

// Distance Calculation
void calcDis(int s) {
    vector<int> src;
    src.emplace_back(s);
    SPFA(src);
    // BellmanFord(src);

    while (!q.empty()) q.pop();
    for (int i = 1; i <= n; i++)
        if (negCycle[i]) q.push(i);

    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (auto& e : g[u]) {
            int v = e.first;
            if (!negCycle[v]) {
                q.push(v);
                negCycle[v] = true;
            }
        }
    }
}
```

5.4 BCC - AP

```cpp
int n, m;
int low[maxn], dfn[maxn], instp;
vector<int> E, g[maxn];
bitset<maxn> isap;
bitset<maxm> vis;
stack<int> stk;
int bccnt;
vector<int> bcc[maxn];
inline void popout(int u) {
    bccnt++;
    bcc[bccnt].emplace_back(u);
    while (!stk.empty()) {
        int v = stk.top();
        if (u == v) break;
        stk.pop();
        bcc[bccnt].emplace_back(v);
    }
}
void dfs(int u, bool rt = 0) {
    stk.push(u);
    low[u] = dfn[u] = ++instp;
    int kid = 0;
    Each(e, g[u]) {
        if (vis[e]) continue;
        vis[e] = true;
        int v = E[e] ^ u;
        if (!dfn[v]) {
            // tree edge
            kid++;
            dfs(v);
            low[u] = min(low[u], low[v]);
            if (!rt && low[v] >= dfn[u]) {
                // bcc found: u is ap
                isap[u] = true;
                popout(u);
            }
        } else {
            // back edge
            low[u] = min(low[u], dfn[v]);
        }
    }
    // special case: root
    if (rt) {
        if (kid > 1) isap[u] = true;
        popout(u);
    }
}
void init() {
    cin >> n >> m;
    fill(low, low + maxn, INF);
    REP(i, m) {
        int u, v;
        cin >> u >> v;
        g[u].emplace_back(i);
        g[v].emplace_back(i);
        E.emplace_back(u ^ v);
    }
}
void solve() {
    FOR(i, 1, n + 1, 1) {
        if (!dfn[i]) dfs(i, true);
    }
    vector<int> ans;
    int cnt = 0;
    FOR(i, 1, n + 1, 1) {
        if (isap[i]) cnt++, ans.emplace_back(i);
    }
    cout << cnt << endl;
    Each(i, ans) cout << i << ' ';
    cout << endl;
}
```

5.5 BCC - Bridge

```cpp
int n, m;
vector<int> g[maxn], E;
int low[maxn], dfn[maxn], instp;
int bccnt, bccid[maxn];
stack<int> stk;
bitset<maxm> vis, isbrg;
void init() {
    cin >> n >> m;
    REP(i, m) {
        int u, v;
        cin >> u >> v;
        E.emplace_back(u ^ v);
        g[u].emplace_back(i);
        g[v].emplace_back(i);
    }
    fill(low, low + maxn, INF);
}
void popout(int u) {
    bccnt++;
    while (!stk.empty()) {
        int v = stk.top();
        if (v == u) break;
        stk.pop();
        bccid[v] = bccnt;
    }
}
void dfs(int u) {
    stk.push(u);
    low[u] = dfn[u] = ++instp;

    Each(e, g[u]) {
        if (vis[e]) continue;
        vis[e] = true;

        int v = E[e] ^ u;
        if (dfn[v]) {
            // back edge
            low[u] = min(low[u], dfn[v]);
        } else {
            // tree edge
            dfs(v);
            low[u] = min(low[u], low[v]);
            if (low[v] == dfn[v]) {
```

```cpp
                isbrg[e] = true;
                popout(u);
            }
        }
    }
}
void solve() {
    FOR(i, 1, n + 1, 1) {
        if (!dfn[i]) dfs(i);
    }
    vector<pii> ans;
    vis.reset();
    FOR(u, 1, n + 1, 1) {
        Each(e, g[u]) {
            if (!isbrg[e] || vis[e]) continue;
            vis[e] = true;
            int v = E[e] ^ u;
            ans.emplace_back(mp(u, v));
        }
    }
    cout << (int)ans.size() << endl;
    Each(e, ans) cout << e.F << ' ' << e.S << endl;
}
```

5.6 SCC - Tarjan

```cpp
// 2-SAT
vector<int> E, g[maxn];  // 1~n, n+1~2n
int low[maxn], in[maxn], instp;
int sccnt, sccid[maxn];
stack<int> stk;
bitset<maxn> ins, vis;
int n, m;
void init() {
    cin >> m >> n;
    E.clear();
    fill(g, g + maxn, vector<int>());
    fill(low, low + maxn, INF);
    memset(in, 0, sizeof(in));
    instp = 1;
    sccnt = 0;
    memset(sccid, 0, sizeof(sccid));
    ins.reset();
    vis.reset();
}
inline int no(int u) {
    return (u > n ? u - n : u + n);
}
int ecnt = 0;
inline void clause(int u, int v) {
    E.eb(no(u) ^ v);
    g[no(u)].eb(ecnt++);
    E.eb(no(v) ^ u);
    g[no(v)].eb(ecnt++);
}
void dfs(int u) {
    in[u] = instp++;
    low[u] = in[u];
    stk.push(u);
    ins[u] = true;

    Each(e, g[u]) {
        if (vis[e]) continue;
        vis[e] = true;

        int v = E[e] ^ u;
        if (ins[v])
            low[u] = min(low[u], in[v]);
        else if (!in[v]) {
            dfs(v);
            low[u] = min(low[u], low[v]);
        }
    }
    if (low[u] == in[u]) {
        sccnt++;
        while (!stk.empty()) {
            int v = stk.top();
            stk.pop();
            ins[v] = false;
            sccid[v] = sccnt;
            if (u == v) break;
        }
    }
}
int main() {
    init();
    REP(i, m) {
        char su, sv;
        int u, v;
        cin >> su >> u >> sv >> v;
        if (su == '-') u = no(u);
        if (sv == '-') v = no(v);
        clause(u, v);
    }
    FOR(i, 1, 2 * n + 1, 1) {
        if (!in[i]) dfs(i);
    }
    FOR(u, 1, n + 1, 1) {
        int du = no(u);
        if (sccid[u] == sccid[du]) {
            return cout << "IMPOSSIBLE\n", 0;
        }
    }
    FOR(u, 1, n + 1, 1) {
        int du = no(u);
        cout << (sccid[u] < sccid[du] ? '+' : '-') << ' ';
    }
    cout << endl;
}
```

5.7 SCC - Kosaraju

```cpp
const int N = 1e5 + 10;
vector<int> ed[N], ed_b[N];  // 反邊
vector<int> SCC(N);          // 最後SCC的分組
bitset<N> vis;
int SCC_cnt;
int n, m;
vector<int> pre;  // 後序遍歷

void dfs(int x) {
    vis[x] = 1;
    for (int i : ed[x]) {
        if (vis[i]) continue;
        dfs(i);
    }
    pre.push_back(x);
}

void dfs2(int x) {
    vis[x] = 1;
    SCC[x] = SCC_cnt;
    for (int i : ed_b[x]) {
        if (vis[i]) continue;
        dfs2(i);
    }
}

void kosaraju() {
    for (int i = 1; i <= n; i++) {
        if (!vis[i]) {
            dfs(i);
        }
    }
    SCC_cnt = 0;
    vis = 0;
    for (int i = n - 1; i >= 0; i--) {
        if (!vis[pre[i]]) {
            SCC_cnt++;
            dfs2(pre[i]);
        }
    }
}
```

5.8 Eulerian Path - Undir

```cpp
// from 1 to n
#define gg return cout << "IMPOSSIBLE\n", void();

int n, m;
vector<int> g[maxn];
bitset<maxn> inodd;
```

```cpp

void init() {
    cin >> n >> m;
    inodd.reset();
    for (int i = 0; i < m; i++) {
        int u, v;
        cin >> u >> v;
        inodd[u] = inodd[u] ^ true;
        inodd[v] = inodd[v] ^ true;
        g[u].emplace_back(v);
        g[v].emplace_back(u);
    }
}
stack<int> stk;
void dfs(int u) {
    while (!g[u].empty()) {
        int v = g[u].back();
        g[u].pop_back();
        dfs(v);
    }
    stk.push(u);
}
```

5.9 Eulerian Path - Dir

```cpp
// from node 1 to node n
#define gg return cout << "IMPOSSIBLE\n", 0

int n, m;
vector<int> g[maxn];
stack<int> stk;
int in[maxn], out[maxn];

void init() {
    cin >> n >> m;
    for (int i = 0; i < m; i++) {
        int u, v;
        cin >> u >> v;
        g[u].emplace_back(v);
        out[u]++, in[v]++;
    }
    for (int i = 1; i <= n; i++) {
        if (i == 1 && out[i] - in[i] != 1) gg;
        if (i == n && in[i] - out[i] != 1) gg;
        if (i != 1 && i != n && in[i] != out[i]) gg;
    }
}
void dfs(int u) {
    while (!g[u].empty()) {
        int v = g[u].back();
        g[u].pop_back();
        dfs(v);
    }
    stk.push(u);
}
void solve() {
    dfs(1) for (int i = 1; i <= n; i++) if ((int)g[i].
        size()) gg;
    while (!stk.empty()) {
        int u = stk.top();
        stk.pop();
        cout << u << ' ';
    }
}
```

5.10 Hamilton Path

```cpp
// top down DP
// Be Aware Of Multiple Edges
int n, m;
ll dp[maxn][1<<maxn];
int adj[maxn][maxn];

void init() {
    cin >> n >> m;
    fill(dp[0], dp[maxn-1]+(1<<maxn), -1);
}

void DP(int i, int msk) {
    if (dp[i][msk] != -1) return;
    dp[i][msk] = 0;
    REP(j, n) if (j != i && (msk & (1<<j)) && adj[j][i
        ]) {
        int sub = msk ^ (1<<i);
        if (dp[j][sub] == -1) DP(j, sub);
        dp[i][msk] += dp[j][sub] * adj[j][i];
        if (dp[i][msk] >= MOD) dp[i][msk] %= MOD;
    }
}


int main() {
    WiwiHorz
    init();

    REP(i, m) {
        int u, v;
        cin >> u >> v;
        if (u == v) continue;
        adj[--u][--v]++;
    }

    dp[0][1] = 1;
    FOR(i, 1, n, 1) {
        dp[i][1] = 0;
        dp[i][1|(1<<i)] = adj[0][i];
    }
    FOR(msk, 1, (1<<n), 1) {
        if (msk == 1) continue;
        dp[0][msk] = 0;
    }


    DP(n-1, (1<<n)-1);
    cout << dp[n-1][(1<<n)-1] << endl;

    return 0;
}
```

5.11 Kth Shortest Path

```cpp
// time: O(|E| \lg |E|+|V| \lg |V|+K)
// memory: O(|E| \lg |E|+|V|)
struct KSP {  // 1-base
    struct nd {
        int u, v;
        ll d;
        nd(int ui = 0, int vi = 0, ll di = INF) {
            u = ui;
            v = vi;
            d = di;
        }
    };
    struct heap {
        nd* edge;
        int dep;
        heap* chd[4];
    };
    static int cmp(heap* a, heap* b) { return a->edge->
        d > b->edge->d; }
    struct node {
        int v;
        ll d;
        heap* H;
        nd* E;
        node() {}
        node(ll _d, int _v, nd* _E) {
            d = _d;
            v = _v;
            E = _E;
        }
        node(heap* _H, ll _d) {
            H = _H;
            d = _d;
        }
        friend bool operator<(node a, node b) { return
            a.d > b.d; }
    };
    int n, k, s, t, dst[N];
    nd* nxt[N];
    vector<nd*> g[N], rg[N];
    heap *nullNd, *head[N];
    void init(int _n, int _k, int _s, int _t) {
```

```cpp
        n = _n;
        k = _k;
        s = _s;
        t = _t;
        for (int i = 1; i <= n; i++) {
            g[i].clear();
            rg[i].clear();
            nxt[i] = NULL;
            head[i] = NULL;
            dst[i] = -1;
        }
    }
    void addEdge(int ui, int vi, ll di) {
        nd* e = new nd(ui, vi, di);
        g[ui].push_back(e);
        rg[vi].push_back(e);
    }
    queue<int> dfsQ;
    void dijkstra() {
        while (dfsQ.size()) dfsQ.pop();
        priority_queue<node> Q;
        Q.push(node(0, t, NULL));
        while (!Q.empty()) {
            node p = Q.top();
            Q.pop();
            if (dst[p.v] != -1) continue;
            dst[p.v] = p.d;
            nxt[p.v] = p.E;
            dfsQ.push(p.v);
            for (auto e : rg[p.v]) Q.push(node(p.d + e->d, e->u, e));
        }
    }
    heap* merge(heap* curNd, heap* newNd) {
        if (curNd == nullNd) return newNd;
        heap* root = new heap;
        memcpy(root, curNd, sizeof(heap));
        if (newNd->edge->d < curNd->edge->d) {
            root->edge = newNd->edge;
            root->chd[2] = newNd->chd[2];
            root->chd[3] = newNd->chd[3];
            newNd->edge = curNd->edge;
            newNd->chd[2] = curNd->chd[2];
            newNd->chd[3] = curNd->chd[3];
        }
        if (root->chd[0]->dep < root->chd[1]->dep)
            root->chd[0] = merge(root->chd[0], newNd);
        else
            root->chd[1] = merge(root->chd[1], newNd);
        root->dep = max(root->chd[0]->dep,
                        root->chd[1]->dep) +
                    1;
        return root;
    }
    vector<heap*> V;
    void build() {
        nullNd = new heap;
        nullNd->dep = 0;
        nullNd->edge = new nd;
        fill(nullNd->chd, nullNd->chd + 4, nullNd);
        while (not dfsQ.empty()) {
            int u = dfsQ.front();
            dfsQ.pop();
            if (!nxt[u])
                head[u] = nullNd;
            else
                head[u] = head[nxt[u]->v];
            V.clear();
            for (auto&& e : g[u]) {
                int v = e->v;
                if (dst[v] == -1) continue;
                e->d += dst[v] - dst[u];
                if (nxt[u] != e) {
                    heap* p = new heap;
                    fill(p->chd, p->chd + 4, nullNd);
                    p->dep = 1;
                    p->edge = e;
                    V.push_back(p);
                }
            }
            if (V.empty()) continue;
            make_heap(V.begin(), V.end(), cmp);
#define L(X) ((X << 1) + 1)
#define R(X) ((X << 1) + 2)
            for (size_t i = 0; i < V.size(); i++) {
                if (L(i) < V.size())
                    V[i]->chd[2] = V[L(i)];
                else
                    V[i]->chd[2] = nullNd;
                if (R(i) < V.size())
                    V[i]->chd[3] = V[R(i)];
                else
                    V[i]->chd[3] = nullNd;
            }
            head[u] = merge(head[u], V.front());
        }
    }
    vector<ll> ans;
    void first_K() {
        ans.clear();
        priority_queue<node> Q;
        if (dst[s] == -1) return;
        ans.push_back(dst[s]);
        if (head[s] != nullNd)
            Q.push(node(head[s], dst[s] + head[s]->edge->d));
        for (int _ = 1; _ < k and not Q.empty(); _++) {
            node p = Q.top(), q;
            Q.pop();
            ans.push_back(p.d);
            if (head[p.H->edge->v] != nullNd) {
                q.H = head[p.H->edge->v];
                q.d = p.d + q.H->edge->d;
                Q.push(q);
            }
            for (int i = 0; i < 4; i++)
                if (p.H->chd[i] != nullNd) {
                    q.H = p.H->chd[i];
                    q.d = p.d - p.H->edge->d + p.H->chd[i]->edge->d;
                    Q.push(q);
                }
        }
    }
    void solve() {  // ans[i] stores the i-th shortest path
        dijkstra();
        build();
        first_K();  // ans.size() might less than k
    }
} solver;
```

5.12 System of Difference Constraints

```cpp
vector<vector<pair<int, ll>>> G;
void add(int u, int v, ll w) {
    G[u].emplace_back(make_pair(v, w));
}
```

- $x_u - x_v \le c \Rightarrow$ add(v, u, c)
- $x_u - x_v \ge c \Rightarrow$ add(u, v, -c)
- $x_u - x_v = c \Rightarrow$ add(v, u, c), add(u, v -c)
- $x_u \ge c \Rightarrow$ add super vertex $x_0 = 0$, then $x_u - x_0 \ge c \Rightarrow$ add(u, 0, -c)
- Don't for get non-negative constraints for every variable if specified implicitly.
- Interval sum $\Rightarrow$ Use prefix sum to transform into differential constraints. Don't for get $S_{i+1} - S_i \ge 0$ if x_i needs to be non-negative.
- $\frac{x_u}{x_v} \le c \Rightarrow \log x_u - \log x_v \le \log c$

6 String

6.1 Aho Corasick

```
struct ACautomata {
    struct Node {
        int cnt; // 停在此節點的數量
        Node *go[26], *fail, *dic;
        // 子節點 fail指標 最近的模式結尾
        Node() {
            cnt = 0;
            fail = 0;
            dic = 0;
            memset(go, 0, sizeof(go));
        }
    } pool[1048576], *root;
    int nMem;
    Node *new_Node() {
        pool[nMem] = Node();
        return &pool[nMem++];
    }
    void init() {
        nMem = 0;
        root = new_Node();
    }
    void add(const string &str) { insert(root, str, 0); }
    void insert(Node *cur, const string &str, int pos) {
        for (int i = pos; i < str.size(); i++) {
            if (!cur->go[str[i] - 'a'])
                cur->go[str[i] - 'a'] = new_Node();
            cur = cur->go[str[i] - 'a'];
        }
        cur->cnt++;
    }
    void make_fail() { // 全部 add 完做
        queue<Node *> que;
        que.push(root);
        while (!que.empty()) {
            Node *fr = que.front();
            que.pop();
            for (int i = 0; i < 26; i++) {
                if (fr->go[i]) {
                    Node *ptr = fr->fail;
                    while (ptr && !ptr->go[i]) ptr = ptr->fail;
                    fr->go[i]->fail = ptr = (ptr ? ptr->go[i] : root);
                    fr->go[i]->dic = (ptr->cnt ? ptr : ptr->dic);
                    que.push(fr->go[i]);
                }
            }
        }
    }
    // 出現過不同string的總數
    int query_unique(const string& text) {
        Node* p = root;
        int ans = 0;
        for(char ch : text) {
            int i = ch - 'a';
            while(p && !p->go[i]) p = p ->fail;
            p = p ? p->go[i] : root;
            if(p->cnt) {ans += p->cnt, p->cnt = 0;}
            for(Node* t = p->dic; t; t = t->dic) if(t->cnt) {
                ans += t->cnt; t->cnt = 0;
            }
        }
        return ans;
    }
} AC;
```

6.2 KMP

```
vector<int> f;
// 沒匹配到可以退回哪裡
void buildFailFunction(string &s) {
    f.resize(s.size(), -1);
    for (int i = 1; i < s.size(); i++) {
        int now = f[i - 1];
        while (now != -1 and s[now + 1] != s[i]) now = f[now];
        if (s[now + 1] == s[i]) f[i] = now + 1;
    }
}

void KMPmatching(string &a, string &b) {
    for (int i = 0, now = -1; i < a.size(); i++) {
        while (a[i] != b[now + 1] and now != -1) now = f[now];
        if (a[i] == b[now + 1]) now++;
        if (now + 1 == b.size()) {
            cout << "found a match start at position "
                << i - now << endl;
            now = f[now];
        }
    }
}
```

6.3 Z Value

```
string is, it, s;
// is: 被搜尋 it: 要找的
int n;
vector<int> z;
// 計算每個位置 i 開始的字串，和 s 的共農前綴長度
void init() {
    cin >> is >> it;
    s = it + '0' + is;
    n = (int)s.size();
    z.resize(n, 0);
}
void solve() {
    int ans = 0;
    z[0] = n;
    for (int i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r) z[i] = min(z[i - l], r - i + 1);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i]++;
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
        if (z[i] == (int)it.size()) ans++;
    }
    cout << ans << endl;
}
```

6.4 Manacher

```
// 找最長回文
int n;
string S, s;
vector<int> m;
void manacher() {
    s.clear();
    s.resize(2 * n + 1, '.');
    for (int i = 0, j = 1; i < n; i++, j += 2) s[j] = S[i];
    m.clear();
    m.resize(2 * n + 1, 0);
    // m[i] := max k such that s[i-k, i+k] is palindrome
    int mx = 0, mxk = 0;
    for (int i = 1; i < 2 * n + 1; i++) {
        if (mx - (i - mx) >= 0) m[i] = min(m[mx - (i - mx)], mx + mxk - i);
        while (0 <= i - m[i] - 1 && i + m[i] + 1 < 2 * n + 1 &&
               s[i - m[i] - 1] == s[i + m[i] + 1]) m[i]++;
        if (i + m[i] > mx + mxk) mx = i, mxk = m[i];
    }
}
void init() {
    cin >> S;
    n = (int)S.size();
}
void solve() {
    manacher();
    int mx = 0, ptr = 0;
    for (int i = 0; i < 2 * n + 1; i++)
        if (mx < m[i]) {
            mx = m[i];
            ptr = i;
        }
    for (int i = ptr - mx; i <= ptr + mx; i++)
```

```
        if (s[i] != '.') cout << s[i];
    cout << endl;
}
```

6.5 Suffix Array

```
#define F first
#define S second
struct SuffixArray {  // don't forget s += "$";
    int n;
    string s;
    vector<int> suf, lcp, rk;
    // 後綴陣列：suf[i] = 第 i 小的後綴起點
    // LCP 陣列：lcp[i] = suf[i] 與 suf[i-1] 的最長共同
        前綴長度
    // rank 陣列：rk[i] = 起點在 i 的後綴的名次
    vector<int> cnt, pos;
    vector<pair<pair<int, int>, int> > buc[2];
    void init(string _s) {
        s = _s;
        n = (int)s.size();
        // resize(n): suf, rk, cnt, pos, lcp, buc[0~1]
        suf.assign(n, 0);
        rk.assign(n, 0);
        lcp.assign(n, 0);
        cnt.assign(n, 0);
        pos.assign(n, 0);
        buc[0].assign(n, {{0,0},0});
        buc[1].assign(n, {{0,0},0});
    }
    void radix_sort() {
        for (int t : {0, 1}) {
            fill(cnt.begin(), cnt.end(), 0);
            for (auto& i : buc[t]) cnt[(t ? i.F.F : i.F
                .S)]++;
            for (int i = 0; i < n; i++)
                pos[i] = (!i ? 0 : pos[i - 1] + cnt[i -
                    1]);
            for (auto& i : buc[t])
                buc[t ^ 1][pos[(t ? i.F.F : i.F.S)]++]
                    = i;
        }
    }
    bool fill_suf() {
        bool end = true;
        for (int i = 0; i < n; i++) suf[i] = buc[0][i].
            S;
        rk[suf[0]] = 0;
        for (int i = 1; i < n; i++) {
            int dif = (buc[0][i].F != buc[0][i - 1].F);
            end &= dif;
            rk[suf[i]] = rk[suf[i - 1]] + dif;
        }
        return end;
    }
    void sa() {
        for (int i = 0; i < n; i++)
            buc[0][i] = make_pair(make_pair(s[i], s[i])
                , i);
        sort(buc[0].begin(), buc[0].end());
        if (fill_suf()) return;
        for (int k = 0; (1 << k) < n; k++) {
            for (int i = 0; i < n; i++)
                buc[0][i] = make_pair(make_pair(rk[i],
                    rk[(i + (1 << k)) % n]), i);
            radix_sort();
            if (fill_suf()) return;
        }
    }
    void LCP() {
        int k = 0;
        for (int i = 0; i < n - 1; i++) {
            if (rk[i] == 0) continue;
            int pi = rk[i];
            int j = suf[pi - 1];
            while (i + k < n && j + k < n && s[i + k]
                == s[j + k]) k++;
            lcp[pi] = k;
            k = max(k - 1, 0);
        }
    }
};
SuffixArray suffixarray;
```

6.6 Suffix Automaton

```
struct SAM {
    struct State {
        int next[26];
        int link, len;
        // suffix link，指向最長真後綴所對應的狀態
        // 該狀態代表的字串集合中的最長字串長度
        State() : link(-1), len(0) { memset(next, -1,
            sizeof next); }
    };
    vector<State> st;
    int last;
    vector<long long> occ; // 每個狀態的出現次數 (
        endpos 個數)
    vector<int> first_bkpos; // 出現在哪裡
    SAM(int maxlen = 0) {
        st.reserve(2 * maxlen + 5); st.push_back(State
            ()); last = 0;
        occ.reserve(2 * maxlen + 5); occ.push_back(0);
        first_bkpos.push_back(-1);
    }
    void extend(int c) {
        int cur = (int)st.size();
        st.push_back(State());
        occ.push_back(0);
        first_bkpos.push_back(0);
        st[cur].len = st[last].len + 1;
        first_bkpos[cur] = st[cur].len - 1;
        int p = last;
        while (p != -1 && st[p].next[c] == -1) {
            st[p].next[c] = cur;
            p = st[p].link;
        }
        if (p == -1) {
            st[cur].link = 0;
        } else {
            int q = st[p].next[c];
            if (st[p].len + 1 == st[q].len) {
                st[cur].link = q;
            } else {
                int clone = (int)st.size();
                st.push_back(st[q]);
                first_bkpos.push_back(first_bkpos[q]);
                occ.push_back(0);
                st[clone].len = st[p].len + 1;
                while (p != -1 && st[p].next[c] == q) {
                    st[p].next[c] = clone;
                    p = st[p].link;
                }
                st[q].link = st[cur].link = clone;
            }
        }
        last = cur;
        occ[cur] += 1;
    }
    void finalize_occ() {
        int m = (int)st.size();
        vector<int> order(m);
        iota(order.begin(), order.end(), 0);
        sort(order.begin(), order.end(), [&](int a, int
             b){ return st[a].len > st[b].len; });
        for (int v : order) {
            int p = st[v].link;
            if (p != -1) occ[p] += occ[v];
        }
    }
};
```

6.7 Minimum Rotation

```
// rotate(begin(s), begin(s)+minRotation(s), end(s))
// 找出字串的最小字典序旋轉
int minRotation(string s) {
    int a = 0, n = s.size();
    s += s;
    for (int b = 0; b < n; b++)
        for (int k = 0; k < n; k++) {
```

```cpp
            if (a + k == b || s[a + k] < s[b + k]) {
                b += max(0, k - 1);
                break;
            }
            if (s[a + k] > s[b + k]) {
                a = b;
                break;
            }
        }
    return a;
}
```

6.8 Lyndon Factorization

```cpp
// Duval: 將字串唯一分解為字典序非遞增的 Lyndon 子字串
vector<string> duval(string const& s) {
    int n = s.size();
    int i = 0;
    vector<string> factorization;
    while (i < n) {
        int j = i + 1, k = i;
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i;
            else
                k++;
            j++;
        }
        while (i <= k) {
            factorization.push_back(s.substr(i, j - k));
            i += j - k;
        }
    }
    return factorization;  // O(n)
}
```

6.9 Rolling Hash

```cpp
const ll C = 27;
inline int id(char c) { return c - 'a' + 1; }
struct RollingHash {
    string s;
    int n;
    ll mod;
    vector<ll> Cexp, hs;
    RollingHash(string& _s, ll _mod) : s(_s), n((int)_s.size()), mod(_mod) {
        Cexp.assign(n, 0);
        hs.assign(n, 0);
        Cexp[0] = 1;
        for (int i = 1; i < n; i++) {
            Cexp[i] = Cexp[i - 1] * C;
            if (Cexp[i] >= mod) Cexp[i] %= mod;
        }
        hs[0] = id(s[0]);
        for (int i = 1; i < n; i++) {
            hs[i] = hs[i - 1] * C + id(s[i]);
            if (hs[i] >= mod) hs[i] %= mod;
        }
    }
    inline ll query(int l, int r) {
        ll res = hs[r] - (l ? hs[l - 1] * Cexp[r - l + 1] : 0);
        res = (res % mod + mod) % mod;
        return res;
    }
};
```

6.10 Trie

```cpp
pii a[N][26];

void build(string &s) {
    static int idx = 0;
    int n = s.size();
    for (int i = 0, v = 0; i < n; i++) {
        pii &now = a[v][s[i] - 'a'];
        if (now.first != -1)
            v = now.first;
        else
            v = now.first = ++idx;
        if (i == n - 1)
            now.second++;
    }
}
```

7 Geometry

7.1 Basic Operations

```cpp
// typedef long long T;
typedef long double T;
const long double eps = 1e-12;

short sgn(T x) {
    if (abs(x) < eps) return 0;
    return x < 0 ? -1 : 1;
}

struct Pt {
    T x, y;
    Pt(T _x = 0, T _y = 0) : x(_x), y(_y) {}
    Pt operator+(Pt a) { return Pt(x + a.x, y + a.y); }
    Pt operator-(Pt a) { return Pt(x - a.x, y - a.y); }
    Pt operator*(T a) { return Pt(x * a, y * a); }
    Pt operator/(T a) { return Pt(x / a, y / a); }
    T operator*(Pt a) { return x * a.x + y * a.y; }
    T operator^(Pt a) { return x * a.y - y * a.x; }
    bool operator<(Pt a) { return x < a.x || (x == a.x && y < a.y); }
    // return sgn(x-a.x) < 0 || (sgn(x-a.x) == 0 && sgn(y-a.y) < 0); }
    bool operator==(Pt a) { return sgn(x - a.x) == 0 && sgn(y - a.y) == 0; }
};

Pt mv(Pt a, Pt b) { return b - a; }
T len2(Pt a) { return a * a; }
T dis2(Pt a, Pt b) { return len2(b - a); }
Pt rotate(Pt u) { return {-u.y, u.x}; }
Pt unit(Pt x) { return x / sqrtl(x * x); }
short ori(Pt a, Pt b) { return ((a ^ b) > 0) - ((a ^ b) < 0); }
bool onseg(Pt p, Pt l1, Pt l2) {
    Pt a = mv(p, l1), b = mv(p, l2);
    return ((a ^ b) == 0) && ((a * b) <= 0);
}
inline T cross(const Pt &a, const Pt &b, const Pt &c) {
    return (b.x - a.x) * (c.y - a.y)
         - (b.y - a.y) * (c.x - a.x);
}

long double polar_angle(Pt ori, Pt pt){
    return atan2(pt.y - ori.y, pt.x - ori.x);
}
// slope to degree atan(Slope) * 180.0 / acos(-1.0);
bool argcmp(Pt u, Pt v) {
    auto half = [](const Pt& p) {
        return p.y > 0 || (p.y == 0 && p.x >= 0);
    };
    if (half(u) != half(v)) return half(u) < half(v);
    return sgn(u ^ v) > 0;
}
int ori(Pt& o, Pt& a, Pt& b) {
    return sgn((a - o) ^ (b - o));
}
struct Line {
    Pt a, b;
    Pt dir() { return b - a; }
};
int PtSide(Pt p, Line L) {
    return sgn(ori(L.a, L.b, p)); // for int
    return sgn(ori(L.a, L.b, p) / sqrt(len2(L.a - L.b)));
}
bool PtOnSeg(Pt p, Line L) {
    return PtSide(p, L) == 0 and sgn((p - L.a) * (p - L.b)) <= 0;
}
Pt proj(Pt& p, Line& l) {
    Pt d = l.b - l.a;
```

```
    T d2 = len2(d);
    if (sgn(d2) == 0) return l.a;
    T t = ((p - l.a) * d) / d2;
    return l.a + d * t;
}
struct Cir {
    Pt o;
    T r;
};
bool disjunct(Cir a, Cir b) {
    return sgn(sqrtl(len2(a.o - b.o)) - a.r - b.r) >=
        0;
}
bool contain(Cir a, Cir b) {
    return sgn(a.r - b.r - sqrtl(len2(a.o - b.o))) >=
        0;
}
```

7.2 Sort by Angle

```
int ud(Pt a) {  // up or down half plane
    if (a.y > 0) return 0;
    if (a.y < 0) return 1;
    return (a.x >= 0 ? 0 : 1);
}
sort(pts.begin(), pts.end(), [&](const Pt& a, const Pt&
     b) {
    if (ud(a) != ud(b)) return ud(a) < ud(b);
    return (a ^ b) > 0;
});
```

7.3 Intersection

```
bool line_intersect_check(Pt p1, Pt p2, Pt q1, Pt q2) {
    if (onseg(p1, q1, q2) || onseg(p2, q1, q2) || onseg
        (q1, p1, p2) || onseg(q2, p1, p2)) return true;
    Pt p = mv(p1, p2), q = mv(q1, q2);
    return (ori(p, mv(p1, q1)) * ori(p, mv(p1, q2)) <
        0) && (ori(q, mv(q1, p1)) * ori(q, mv(q1, p2))
        < 0);
}
// long double
Pt line_intersect(Pt a1, Pt a2, Pt b1, Pt b2) {
    Pt da = mv(a1, a2), db = mv(b1, b2);
    T det = da ^ db;
    if (sgn(det) == 0) { // parallel
        // return Pt(NAN, NAN);
    }
    T t = ((b1 - a1) ^ db) / det;
    return a1 + da * t;
}
vector<Pt> CircleInter(Cir a, Cir b) {
    double d2 = len2(a.o - b.o), d = sqrt(d2);
    if (d < max(a.r, b.r) - min(a.r, b.r) || d > a.r +
        b.r) return {};
    Pt u = (a.o + b.o) / 2 + (a.o - b.o) * ((b.r * b.r
        - a.r * a.r) / (2 * d2));
    double A = sqrt((a.r + b.r + d) * (a.r - b.r + d) *
         (a.r + b.r - d) * (-a.r + b.r + d));
    Pt v = rotate(b.o - a.o) * A / (2 * d2);
    if (sgn(v.x) == 0 and sgn(v.y) == 0) return {u};
    return {u - v, u + v}; // counter clockwise of a
}
vector<Pt> CircleLineInter(Cir c, Line l) {
    Pt H = proj(c.o, l);
    Pt dir = unit(l.b - l.a);
    T h = sqrtl(len2(H - c.o));
    if (sgn(h - c.r) > 0) return {};
    T d = sqrtl(max((T)0, c.r * c.r - h * h));
    if (sgn(d) == 0) return {H};
    return {H - dir * d, H + dir * d};
}
```

7.4 Polygon Area

```
// 2 * area
T dbPoly_area(vector<Pt>& e) {
    T res = 0;
    int sz = e.size();
    for (int i = 0; i < sz; i++) {
        res += e[i] ^ e[(i + 1) % sz];
    }
    return abs(res);
}
```

7.5 Convex Hull

```
vector<Pt> convexHull(vector<Pt> pts) {
    vector<Pt> hull;
    sort(pts.begin(), pts.end());
    for (int i = 0; i < 2; i++) {
        int b = hull.size();
        for (auto ei : pts) {
            while (hull.size() - b >= 2 && ori(mv(hull[
                hull.size() - 2], hull.back()), mv(hull
                [hull.size() - 2], ei)) == -1) {
                hull.pop_back();
            }
            hull.emplace_back(ei);
        }
        hull.pop_back();
        reverse(pts.begin(), pts.end());
    }
    return hull;
}
```

7.6 Point In Convex

```
bool point_in_convex(const vector<Pt> &C, Pt p, bool
     strict = true) {
    // only works when no three point are collinear
    int n = C.size();
    int a = 1, b = n - 1, r = !strict;
    if (n == 0) return false;
    if (n < 3) return r && onseg(p, C[0], C.back());
    if (ori(mv(C[0], C[a]), mv(C[0], C[b])) > 0) swap(a
        , b);
    if (ori(mv(C[0], C[a]), mv(C[0], p)) >= r || ori(mv
        (C[0], C[b]), mv(C[0], p)) <= -r) return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        if (ori(mv(C[0], C[c]), mv(C[0], p)) > 0) b = c
            ;
        else a = c;
    }
    return ori(mv(C[a], C[b]), mv(C[a], p)) < r;
}
```

7.7 Point Segment Distance

```
double point_segment_dist(Pt q0, Pt q1, Pt p) {
    if (q0 == q1) {
        double dx = double(p.x - q0.x);
        double dy = double(p.y - q0.y);
        return sqrt(dx * dx + dy * dy);
    }
    T d1 = (q1 - q0) * (p - q0);
    T d2 = (q0 - q1) * (p - q1);
    if (d1 >= 0 && d2 >= 0) {
        double area = fabs(double((q1 - q0) ^ (p - q0))
            );
        double base = sqrt(double(dis2(q0, q1)));
        return area / base;
    }
    double dx0 = double(p.x - q0.x), dy0 = double(p.y -
         q0.y);
    double dx1 = double(p.x - q1.x), dy1 = double(p.y -
         q1.y);
    return min(sqrt(dx0 * dx0 + dy0 * dy0), sqrt(dx1 *
        dx1 + dy1 * dy1));
}
```

7.8 Point in Polygon

```
short inPoly(vector<Pt>& pts, Pt p) {
    // 0=Bound 1=In -1=Out
    int n = pts.size();
    for (int i = 0; i < pts.size(); i++) if (onseg(p,
        pts[i], pts[(i + 1) % n])) return 0;
    int cnt = 0;
    for (int i = 0; i < pts.size(); i++) if (
        line_intersect_check(p, Pt(p.x + 1, p.y + 2e9),
         pts[i], pts[(i + 1) % n])) cnt ^= 1;
```

```cpp
    return (cnt ? 1 : -1);
}
```

7.9 Minimum Euclidean Distance

```cpp
long long Min_Euclidean_Dist(vector<Pt> &pts) {
    sort(pts.begin(), pts.end());
    set<pair<long long, long long>> s;
    s.insert({pts[0].y, pts[0].x});
    long long l = 0, best = LLONG_MAX;
    for (int i = 1; i < (int)pts.size(); i++) {
        Pt now = pts[i];
        long long lim = (long long)ceil(sqrtl((long double)best));
        while (now.x - pts[l].x > lim) {
            s.erase({pts[l].y, pts[l].x}); l++;
}
        auto low = s.lower_bound({now.y - lim, LLONG_MIN});
        auto high = s.upper_bound({now.y + lim, LLONG_MAX});
        for (auto it = low; it != high; it++) {
            long long dy = it->first - now.y;
            long long dx = it->second - now.x;
            best = min(best, dx * dx + dy * dy);
        }
        s.insert({now.y, now.x});
    }
    return best;
}
```

7.10 Minkowski Sum

```cpp
void reorder(vector <Pt> &P) {
  rotate(P.begin(), min_element(P.begin(), P.end(), [&](Pt a, Pt b) { return make_pair(a.y, a.x) < make_pair(b.y, b.x); }), P.end());
}
vector <Pt> Minkowski(vector <Pt> P, vector <Pt> Q) {
  // P, Q: convex polygon
  reorder(P), reorder(Q);
  int n = P.size(), m = Q.size();
  P.push_back(P[0]), P.push_back(P[1]), Q.push_back(Q[0]), Q.push_back(Q[1]);
  vector <Pt> ans;
  for (int i = 0, j = 0; i < n || j < m; ) {
    ans.push_back(P[i] + Q[j]);
    auto val = (P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]);
    if (val >= 0) i++;
    if (val <= 0) j++;
  }
  return ans;
}
```

7.11 Lower Concave Hull

```cpp
struct Line {
  mutable ll m, b, p;
  bool operator<(const Line& o) const { return m < o.m; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = inf; return false; }
    if (x->m == y->m) x->p = x->b > y->b ? inf : -inf;
    else x->p = div(y->b - x->b, x->m - y->m);
    return x->p >= y->p;
  }
  void add(ll m, ll b) {
    auto z = insert({m, b, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.m * x + l.b;
  }
};
```

7.12 Pick's Theorem

Consider a polygon which vertices are all lattice points.

Let i = number of points inside the polygon.

Let b = number of points on the boundary of the polygon.

Then we have the following formula:

$$Area = i + \frac{b}{2} - 1$$

7.13 Rotating SweepLine

```cpp
double cross(const Pt &a, const Pt &b) {
    return a.x*b.y - a.y*b.x;
}
int rotatingCalipers(const vector<Pt>& hull) {
    int m = hull.size();
    if (m < 2) return 0;
    int j = 1;
    T maxd = 0;
    for (int i = 0; i < m; ++i) {
        int ni = (i + 1) % m;
        while (abs(cross({hull[ni].x - hull[i].x, hull[ni].y - hull[i].y}, {hull[(j+1)%m].x - hull[i].x, hull[(j+1)%m].y - hull[i].y})) > abs(cross({hull[ni].x - hull[i].x, hull[ni].y - hull[i].y}, {hull[j].x - hull[i].x, hull[j].y - hull[i].y}))) {
            j = (j + 1) % m;
        }
        maxd = max(maxd, dis2(hull[i], hull[j]));
        maxd = max(maxd, dis2(hull[ni], hull[j]));
    }
    return maxd; // TODO
}
```

7.14 Half Plane Intersection

```cpp
bool cover(Line& L, Line& P, Line& Q) {
    long double u = (Q.a - P.a) ^ Q.dir();
    long double v = P.dir() ^ Q.dir();
    long double x = P.dir().x * u + (P.a - L.a).x * v;
    long double y = P.dir().y * u + (P.a - L.a).y * v;
    return sgn(x * L.dir().y - y * L.dir().x) * sgn(v) >= 0;
}
vector<Line> HPI(vector<Line> P) {
    sort(P.begin(), P.end(), [&](Line& l, Line& m) {
        if (argcmp(l.dir(), m.dir())) return true;
        if (argcmp(m.dir(), l.dir())) return false;
        return ori(m.a, m.b, l.a) > 0;
    });

    int l = 0, r = -1;
    for (size_t i = 0; i < P.size(); ++i) {
        if (i && !argcmp(P[i - 1].dir(), P[i].dir())) continue;
        while (l < r && cover(P[i], P[r - 1], P[r])) --r;
        while (l < r && cover(P[i], P[l], P[l + 1])) ++l;
        P[++r] = P[i];
    }
    while (l < r && cover(P[l], P[r - 1], P[r])) --r;
    while (l < r && cover(P[r], P[l], P[l + 1])) ++l;

    if (r - l <= 1 || !argcmp(P[l].dir(), P[r].dir()))
        return {};
    if (cover(P[l + 1], P[l], P[r])) return {};

    return vector<Line>(P.begin() + l, P.begin() + r + 1);
}
```

```cpp
}
```

7.15 Minimum Enclosing Circle

```cpp
const int INF = 1e9;
Pt circumcenter(Pt A, Pt B, Pt C) {
    // a1(x-A.x) + b1(y-A.y) = c1
    // a2(x-A.x) + b2(y-A.y) = c2
    // solve using Cramer's rule
    T a1 = B.x - A.x, b1 = B.y - A.y, c1 = dis2(A, B) / 2.0;
    T a2 = C.x - A.x, b2 = C.y - A.y, c2 = dis2(A, C) / 2.0;
    T D = Pt(a1, b1) ^ Pt(a2, b2);
    T Dx = Pt(c1, b1) ^ Pt(c2, b2);
    T Dy = Pt(a1, c1) ^ Pt(a2, c2);
    if (D == 0) return Pt(-INF, -INF);
    return A + Pt(Dx / D, Dy / D);
}
Pt center;
T r2;
void minEncloseCircle(vector<Pt> pts) {
    mt19937 gen(chrono::steady_clock::now().time_since_epoch().count());
    shuffle(pts.begin(), pts.end(), gen);
    center = pts[0], r2 = 0;

    for (int i = 0; i < pts.size(); i++) {
        if (dis2(center, pts[i]) <= r2) continue;
        center = pts[i], r2 = 0;
        for (int j = 0; j < i; j++) {
            if (dis2(center, pts[j]) <= r2) continue;
            center = (pts[i] + pts[j]) / 2.0;
            r2 = dis2(center, pts[i]);
            for (int k = 0; k < j; k++) {
                if (dis2(center, pts[k]) <= r2) continue;
                center = circumcenter(pts[i], pts[j], pts[k]);
                r2 = dis2(center, pts[i]);
            }
        }
    }
}
```

7.16 Union of Circles

```cpp
// Area[i] : area covered by at least i circle
vector<T> CircleUnion(const vector<Cir> &C) {
    const int n = C.size();
    vector<T> Area(n + 1);
    auto check = [&](int i, int j) {
        if (!contain(C[i], C[j]))
            return false;
        return sgn(C[i].r - C[j].r) > 0 or (sgn(C[i].r - C[j].r) == 0 and i < j);
    };
    struct Teve {
        double ang; int add; Pt p;
        bool operator<(const Teve &b) { return ang < b.ang; }
    };
    auto ang = [&](Pt p) { return atan2(p.y, p.x); };
    for (int i = 0; i < n; i++) {
        int cov = 1;
        vector<Teve> event;
        for (int j = 0; j < n; j++) if (i != j) {
            if (check(j, i)) cov++;
            else if (!check(i, j) and !disjunct(C[i], C[j])) {
                auto I = CircleInter(C[i], C[j]);
                assert(I.size() == 2);
                double a1 = ang(I[0] - C[i].o), a2 = ang(I[1] - C[i].o);
                event.push_back({a1, 1, I[0]});
                event.push_back({a2, -1, I[1]});
                if (a1 > a2) cov++;
            }
        }
        if (event.empty()) {
            Area[cov] += acos(-1) * C[i].r * C[i].r;
            continue;
        }
        sort(event.begin(), event.end());
        event.push_back(event[0]);
        for (int j = 0; j + 1 < event.size(); j++) {
            cov += event[j].add;
            Area[cov] += (event[j].p ^ event[j + 1].p) / 2.;
            double theta = event[j + 1].ang - event[j].ang;
            if (theta < 0) theta += 2 * acos(-1);
            Area[cov] += (theta - sin(theta)) * C[i].r * C[i].r / 2.;
        }
    }
    return Area;
}
```

7.17 Area Of Circle Polygon

```cpp
double AreaOfCirclePoly(Cir C, vector<Pt> &P) {
    auto arg = [&](Pt p, Pt q) { return atan2l(p ^ q, p * q); };
    double r2 = (double)(C.r * C.r / 2);
    auto tri = [&](Pt p, Pt q) {
        Pt d = q - p;
        T a = (d * p) / (d * d);
        T b = ((p * p) - C.r * C.r) / (d * d);
        T det = a * a - b;
        if (det <= 0) return (double)(arg(p, q) * r2);
        T s = max((T)0.0L, -a - sqrtl(det));
        T t = min((T)1.0L, -a + sqrtl(det));
        if (t < 0 || 1 <= s) return (double)(arg(p, q) * r2);
        Pt u = p + d * s, v = p + d * t;
        return (double)(arg(p, u) * r2 + (u ^ v) / 2 + arg(v, q) * r2);
    };
    long double sum = 0.0L;
    for (int i = 0; i < (int)P.size(); i++)
        sum += tri(P[i] - C.o, P[(i + 1) % P.size()] - C.o);
    return (double)fabsl(sum);
}
```

7.18 3D Point

```cpp
struct Pt {
  double x, y, z;
  Pt(double _x = 0, double _y = 0, double _z = 0): x(_x), y(_y), z(_z){}
  Pt operator + (const Pt &o) const
  { return Pt(x + o.x, y + o.y, z + o.z); }
  Pt operator - (const Pt &o) const
  { return Pt(x - o.x, y - o.y, z - o.z); }
  Pt operator * (const double &k) const
  { return Pt(x * k, y * k, z * k); }
  Pt operator / (const double &k) const
  { return Pt(x / k, y / k, z / k); }
  double operator * (const Pt &o) const
  { return x * o.x + y * o.y + z * o.z; }
  Pt operator ^ (const Pt &o) const
  { return {Pt(y * o.z - z * o.y, z * o.x - x * o.z, x * o.y - y * o.x)}; }
};
double abs2(Pt o) { return o * o; }
double abs(Pt o) { return sqrt(abs2(o)); }
Pt cross3(Pt a, Pt b, Pt c)
{ return (b - a) ^ (c - a); }
double area(Pt a, Pt b, Pt c)
{ return abs(cross3(a, b, c)); }
double volume(Pt a, Pt b, Pt c, Pt d)
{ return cross3(a, b, c) * (d - a); }
bool coplaner(Pt a, Pt b, Pt c, Pt d)
{ return sign(volume(a, b, c, d)) == 0; }
Pt proj(Pt o, Pt a, Pt b, Pt c) // o proj to plane abc
{ Pt n = cross3(a, b, c);
  return o - n * ((o - a) * (n / abs2(n)));}
Pt line_plane_intersect(Pt u, Pt v, Pt a, Pt b, Pt c) {
  // intersection of line uv and plane abc
  Pt n = cross3(a, b, c);
  double s = n * (u - v);
  if (sign(s) == 0) return {-1, -1, -1}; // not found
```

```cpp
  return v + (u - v) * ((n * (a - v)) / s); }
Pt rotateAroundAxis(Pt v, Pt axis, double theta) {
    axis = axis / abs(axis); // axis must be unit vector
    double cosT = cos(theta);
    double sinT = sin(theta);
    Pt term1 = v * cosT;
    Pt term2 = (axis ^ v) * sinT;
    Pt term3 = axis * ((axis * v) * (1 - cosT));
    return term1 + term2 + term3;
}
```

8 Number Theory

8.1 FFT

```cpp
typedef complex<double> cp;

const double pi = acos(-1);
const int NN = 131072;

struct FastFourierTransform {
    /*
            Iterative Fast Fourier Transform
            How this works? Look at this
            0th recursion 0(000)   1(001)   2(010)   3(011)   4(100)   5(101)   6(110)   7(111)
            1th recursion 0(000)   2(010)   4(100)   6(110) | 1(011)   3(011)   5(101)   7(111)
            2th recursion 0(000)   4(100) | 2(010)   6(110) | 1(011)   5(101) | 3(011)   7(111)
            3th recursion 0(000) | 4(100) | 2(010) | 6(110) | 1(011) | 5(101) | 3(011) | 7(111)
            All the bits are reversed => We can save the reverse of the numbers in an array!
    */
    int n, rev[NN];
    cp omega[NN], iomega[NN];
    void init(int n_) {
        n = n_;
        for (int i = 0; i < n_; i++) {
            // Calculate the nth roots of unity
            omega[i] = cp(cos(2 * pi * i / n_), sin(2 * pi * i / n_));
            iomega[i] = conj(omega[i]);
        }
        int k = __lg(n_);
        for (int i = 0; i < n_; i++) {
            int t = 0;
            for (int j = 0; j < k; j++) {
                if (i & (1 << j)) t |= (1 << (k - j - 1));
            }
            rev[i] = t;
        }
    }

    void transform(vector<cp> &a, cp *xomega) {
        for (int i = 0; i < n; i++)
            if (i < rev[i]) swap(a[i], a[rev[i]]);
        for (int len = 2; len <= n; len <<= 1) {
            int mid = len >> 1;
            int r = n / len;
            for (int j = 0; j < n; j += len)
                for (int i = 0; i < mid; i++) {
                    cp tmp = xomega[r * i] * a[j + mid + i];
                    a[j + mid + i] = a[j + i] - tmp;
                    a[j + i] = a[j + i] + tmp;
                }
        }
    }

    void fft(vector<cp> &a) { transform(a, omega); }
    void ifft(vector<cp> &a) {
        transform(a, iomega);
        for (int i = 0; i < n; i++) a[i] /= n;
    }
} FFT;

const int MAXN = 262144;
// (must be 2^k)
// 262144, 524288, 1048576, 2097152, 4194304
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx;  // real() ,imag()
const ld PI = acosl(-1);
const cplx I(0, 1);
cplx omega[MAXN + 1];
void pre_fft() {
    for (int i = 0; i <= MAXN; i++) {
        omega[i] = exp(i * 2 * PI / MAXN * I);
    }
}
// n must be 2^k
void fft(int n, cplx a[], bool inv = false) {
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN - (i * theta % MAXN) : i * theta % MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^= k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if (inv) {
        for (i = 0; i < n; i++) a[i] /= n;
    }
}
cplx arr[MAXN + 1];
inline void mul(int _n, long long a[], int _m, long long b[], long long ans[]) {
    int n = 1, sum = _n + _m - 1;
    while (n < sum) n <<= 1;
    for (int i = 0; i < n; i++) {
        double x = (i < _n ? a[i] : 0), y = (i < _m ? b[i] : 0);
        arr[i] = complex<double>(x + y, x - y);
    }
    fft(n, arr);
    for (int i = 0; i < n; i++) arr[i] = arr[i] * arr[i];
    fft(n, arr, true);
    for (int i = 0; i < sum; i++) ans[i] = (long long int)(arr[i].real() / 4 + 0.5);
}

long long a[MAXN];
long long b[MAXN];
long long ans[MAXN];
int a_length;
int b_length;
```

8.2 Pollard's rho

```cpp
ll add(ll x, ll y, ll p) {
    return (x + y) % p;
}
ll qMul(ll x, ll y, ll mod) {
    ll ret = x * y - (ll)((long double)x / mod * y) * mod;
    return ret < 0 ? ret + mod : ret;
}
ll f(ll x, ll mod) { return add(qMul(x, x, mod), 1, mod); }
ll pollard_rho(ll n) {
    if (!(n & 1)) return 2;
```

```cpp
    while (true) {
        ll y = 2, x = rand() % (n - 1) + 1, res = 1;
        for (int sz = 2; res == 1; sz *= 2) {
            for (int i = 0; i < sz && res <= 1; i++) {
                x = f(x, n);
                res = __gcd(llabs(x - y), n);
            }
            y = x;
        }
        if (res != 0 && res != n) return res;
    }
}
vector<ll> ret;
void fact(ll x) {
    if (miller_rabin(x)) {
        ret.push_back(x);
        return;
    }
    ll f = pollard_rho(x);
    fact(f);
    fact(x / f);
}
```

8.3 Miller Rabin

```cpp
// n < 4,759,123,141        3 :  2, 7, 61
// n < 1,122,004,669,633    4 :  2, 13, 23, 1662803
// n < 3,474,749,660,383          6 :  pirmes <= 13
// n < 2^64                       7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool witness(ll a, ll n, ll u, int t) {
    if (!(a %= n)) return 0;
    ll x = mypow(a, u, n);
    for (int i = 0; i < t; i++) {
        ll nx = mul(x, x, n);
        if (nx == 1 && x != 1 && x != n - 1) return 1;
        x = nx;
    }
    return x != 1;
}
bool miller_rabin(ll n, int s = 100) {
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if (n < 2) return 0;
    if (!(n & 1)) return n == 2;
    ll u = n - 1;
    int t = 0;
    while (!(u & 1)) u >>= 1, t++;
    while (s--) {
        ll a = randll() % (n - 1) + 1;
        if (witness(a, n, u, t)) return 0;
    }
    return 1;
}
```

8.4 Fast Power

Note: $a^n \equiv a^{(n \bmod (p-1))} (\bmod p)$

8.5 Extend GCD

```cpp
ll GCD;
pll extgcd(ll a, ll b) {
    if (b == 0) {
        GCD = a;
        return pll{1, 0};
    }
    pll ans = extgcd(b, a % b);
    return pll{ans.S, ans.F - a / b * ans.S};
}
pll bezout(ll a, ll b, ll c) {
    bool negx = (a < 0), negy = (b < 0);
    pll ans = extgcd(abs(a), abs(b));
    if (c % GCD != 0) return pll{-LLINF, -LLINF};
    return pll{ans.F * c / GCD * (negx ? -1 : 1),
               ans.S * c / GCD * (negy ? -1 : 1)};
}
ll inv(ll a, ll p) {
    if (p == 1) return -1;
    pll ans = bezout(a % p, -p, 1);
    if (ans == pll{-LLINF, -LLINF}) return -1;
    return (ans.F % p + p) % p;
}
```

8.6 Mu + Phi

```cpp
const int maxn = 1e6 + 5;
ll f[maxn];
vector<int> lpf, prime;
void build() {
    lpf.clear();
    lpf.resize(maxn, 1);
    prime.clear();
    f[1] = ...; /* mu[1] = 1, phi[1] = 1 */
    for (int i = 2; i < maxn; i++) {
        if (lpf[i] == 1) {
            lpf[i] = i;
            prime.emplace_back(i);
            f[i] = ...; /* mu[i] = 1, phi[i] = i-1 */
        }
        for (auto& j : prime) {
            if (i * j >= maxn) break;
            lpf[i * j] = j;
            if (i % j == 0)
                f[i * j] = ...; /* 0, phi[i]*j */
            else
                f[i * j] = ...; /* -mu[i], phi[i]*phi[j
                    ] */
            if (j >= lpf[i]) break;
        }
    }
}
```

8.7 Discrete Log

```cpp
long long mod_pow(long long a, long long e, long long p
    ){
    long long r = 1 % p;
    while(e){
        if(e & 1) r = (__int128)r * a % p;
        a = (__int128)a * a % p;
        e >>= 1;
    }
    return r;
}
long long mod_inv(long long a, long long p){
    return mod_pow((a%p+p)%p, p-2, p);
}
// BSGS: solve a^x = y (mod p), gcd(a,p)=1, p prime,
    return minimal x>=0, or -1 if no solution
long long bsgs(long long a, long long y, long long p){
    a%=p; y%=p;
    if(y==1%p) return 0;          // x=0
    long long m = (long long)ceil(sqrt((long double)p))
        ;
    // baby steps: a^j
    unordered_map<long long,long long> table;
    table.reserve(m*2);
    long long cur = 1%p;
    for(long long j=0;j<m;++j){
        if(!table.count(cur)) table[cur]=j;
        cur = (__int128)cur * a % p;
    }
    long long am = mod_pow(a, m, p);
    long long am_inv = mod_inv(am, p);
    long long gamma = y % p;
    for(long long i=0;i<=m;++i){
        auto it = table.find(gamma);
        if(it != table.end()){
            long long x = i*m + it->second;
            return x;
        }
        gamma = (__int128)gamma * am_inv % p;
    }
    return -1;
}
```

8.8 sqrt mod

```cpp
// the Jacobi symbol is a generalization of the
    Legendre symbol,
// such that the bottom doesn't need to be prime.
// (n|p) -> same as legendre
// (n|ab) = (n|a)(n|b)
// work with long long
int Jacobi(int a, int m) {
```

```
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}
// solve x^2 = a (mod p)
// 0: a == 0
// -1: a isn't a quad res of p
// else: return X with X^2 % p == a
// doesn't work with long long
int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    if (int jc = Jacobi(a, p); jc <= 0) return jc;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1
                * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1
             % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}
```

8.9 Primitive Root

```
unsigned long long primitiveRoot(ull p) {
    auto fac = factor(p - 1);
    sort(all(fac));
    fac.erase(unique(all(fac)), fac.end());
    auto test = [p, fac](ull x) {
        for(ull d : fac)
        if (modpow(x, (p - 1) / d, p) == 1)
            return false;
        return true;
    };
    uniform_int_distribution<unsigned long long> unif
        (1, p - 1);
    unsigned long long root;
    while(!test(root = unif(rng)));
    return root;
}
```

8.10 Other Formulas

- Inversion:
  $aa^{-1} \equiv 1 \pmod m$. a^{-1} exists iff $\gcd(a, m) = 1$.
- Linear inversion:
  $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod m$
- Fermat's little theorem:
  $a^p \equiv a \pmod p$ if p is prime.
- Euler function:
  $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$
- Euler theorem:
  $a^{\phi(n)} \equiv 1 \pmod n$ if $\gcd(a, n) = 1$.
- Extended Euclidean algorithm:
  $ax + by = \gcd(a, b) = \gcd(b, a \bmod b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$
- Divisor function:
  $\sigma_x(n) = \sum_{d|n} d^x$. $n = \prod_{i=1}^r p_i^{a_i}$.
  $\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$ if $x \neq 0$. $\sigma_0(n) = \prod_{i=1}^r (a_i + 1)$.
- Chinese remainder theorem (Coprime Moduli):
  $x \equiv a_i \pmod{m_i}$.
  $M = \prod m_i$. $M_i = M/m_i$. $t_i = M_i^{-1}$.
  $x = kM + \sum a_i t_i M_i$, $k \in \mathbb{Z}$.
- Chinese remainder theorem:
  $x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2} \Rightarrow x = m_1 p + a_1 = m_2 q + a_2 \Rightarrow m_1 p - m_2 q = a_2 - a_1$
  Solve for (p, q) using ExtGCD.
  $x \equiv m_1 p + a_1 \equiv m_2 q + a_2 \pmod{lcm(m_1, m_2)}$
- Avoiding Overflow: $ca \bmod cb = c(a \bmod b)$
- Dirichlet Convolution: $(f * g)(n) = \sum_{d|n} f(n) g(n/d)$
- Important Multiplicative Functions + Proterties:
  1. $\epsilon(n) = [n = 1]$
  2. $1(n) = 1$
  3. $id(n) = n$
  4. $\mu(n) = 0$ if n has squared prime factor
  5. $\mu(n) = (-1)^k$ if $n = p_1 p_2 \cdots p_k$
  6. $\epsilon = \mu * 1$
  7. $\phi = \mu * id$
  8. $[n = 1] = \sum_{d|n} \mu(d)$
  9. $[gcd = 1] = \sum_{d|gcd} \mu(d)$
- Möbius inversion: $f = g * 1 \Leftrightarrow g = f * \mu$

8.11 Polynomial

```
const int maxk = 20;
const int maxn = 1<<maxk;
const ll LINF = 1e18;

/* P = r*2^k + 1
P                   r    k    g
998244353           119  23   3
1004535809          479  21   3

P                   r    k    g
3                   1    1    2
5                   1    2    2
17                  1    4    3
97                  3    5    5
193                 3    6    5
257                 1    8    3
7681                15   9    17
12289               3    12   11
40961               5    13   3
65537               1    16   3
786433              3    18   10
5767169             11   19   3
7340033             7    20   3
23068673            11   21   3
104857601           25   22   3
167772161           5    25   3
469762049           7    26   3
1004535809          479  21   3
2013265921          15   27   31
2281701377          17   27   3
3221225473          3    30   5
75161927681         35   31   3
77309411329         9    33   7
206158430209        3    36   22
2061584302081       15   37   7
2748779069441       5    39   3
6597069766657       3    41   5
39582418599937      9    42   5
79164837199873      9    43   5
263882790666241     15   44   7
```

```cpp
1231453023109121    35  45  3
1337006139375617    19  46  3
3799912185593857    27  47  5
4222124650659841    15  48  19
7881299347898369    7   50  6
31525197391593473   7   52  3
180143985094819841  5   55  6
1945555039024054273 27  56  5
4179340454199820289 29  57  3
9097271247288401921 505 54  6 */

const int g = 3;
const ll MOD = 998244353;

ll pw(ll a, ll n) { /* fast pow */ }

#define siz(x) (int)x.size()

template<typename T>
vector<T>& operator+=(vector<T>& a, const vector<T>& b) {
    if (siz(a) < siz(b)) a.resize(siz(b));
    for (int i = 0; i < min(siz(a), siz(b)); i++) {
        a[i] += b[i];
        a[i] -= a[i] >= MOD ? MOD : 0;
    }
    return a;
}

template<typename T>
vector<T>& operator-=(vector<T>& a, const vector<T>& b) {
    if (siz(a) < siz(b)) a.resize(siz(b));
    for (int i = 0; i < min(siz(a), siz(b)); i++) {
        a[i] -= b[i];
        a[i] += a[i] < 0 ? MOD : 0;
    }
    return a;
}

template<typename T>
vector<T> operator-(const vector<T>& a) {
    vector<T> ret(siz(a));
    for (int i = 0; i < siz(a); i++) {
        ret[i] = -a[i] < 0 ? -a[i] + MOD : -a[i];
    }
    return ret;
}

vector<ll> X, iX;
vector<int> rev;

void init_ntt() {
    X.clear(); X.resize(maxn, 1);  // x1 = g^((p-1)/n)
    iX.clear(); iX.resize(maxn, 1);

    ll u = pw(g, (MOD-1)/maxn);
    ll iu = pw(u, MOD-2);

    for (int i = 1; i < maxn; i++) {
        X[i] = X[i-1] * u;
        iX[i] = iX[i-1] * iu;
        if (X[i] >= MOD) X[i] %= MOD;
        if (iX[i] >= MOD) iX[i] %= MOD;
    }

    rev.clear(); rev.resize(maxn, 0);
    for (int i = 1, hb = -1; i < maxn; i++) {
        if (!(i & (i-1))) hb++;
        rev[i] = rev[i ^ (1<<hb)] | (1<<(maxk-hb-1));
} }

template<typename T>
void NTT(vector<T>& a, bool inv=false) {

    int _n = (int)a.size();
    int k = __lg(_n) + ((1<<__lg(_n)) != _n);
    int n = 1<<k;
    a.resize(n, 0);

    short shift = maxk-k;
    for (int i = 0; i < n; i++)
        if (i > (rev[i]>>shift))
            swap(a[i], a[rev[i]>>shift]);

    for (int len = 2, half = 1, div = maxn>>1; len <= n; len<<=1, half<<=1, div>>=1) {
        for (int i = 0; i < n; i += len) {
            for (int j = 0; j < half; j++) {
                T u = a[i+j];
                T v = a[i+j+half] * (inv ? iX[j*div] : X[j*div]) % MOD;
                a[i+j] = (u+v >= MOD ? u+v-MOD : u+v);
                a[i+j+half] = (u-v < 0 ? u-v+MOD : u-v);
    } } }

    if (inv) {
        T dn = pw(n, MOD-2);
        for (auto& x : a) {
            x *= dn;
            if (x >= MOD) x %= MOD;
} } }

template<typename T>
inline void resize(vector<T>& a) {
    int cnt = (int)a.size();
    for (; cnt > 0; cnt--) if (a[cnt-1]) break;
    a.resize(max(cnt, 1));
}

template<typename T>
vector<T>& operator*=(vector<T>& a, vector<T> b) {
    int na = (int)a.size();
    int nb = (int)b.size();
    a.resize(na + nb - 1, 0);
    b.resize(na + nb - 1, 0);

    NTT(a); NTT(b);
    for (int i = 0; i < (int)a.size(); i++) {
        a[i] *= b[i];
        if (a[i] >= MOD) a[i] %= MOD;
    }
    NTT(a, true);

    resize(a);
    return a;
}

template<typename T>
void inv(vector<T>& ia, int N) {
    vector<T> _a(move(ia));
    ia.resize(1, pw(_a[0], MOD-2));
    vector<T> a(1, -_a[0] + (-_a[0] < 0 ? MOD : 0));

    for (int n = 1; n < N; n<<=1) {
        // n -> 2*n
        // ia' = ia(2-a*ia);

        for (int i = n; i < min(siz(_a), (n<<1)); i++)
            a.emplace_back(-_a[i] + (-_a[i] < 0 ? MOD : 0));

        vector<T> tmp = ia;
        ia *= a;
        ia.resize(n<<1);
        ia[0] = ia[0] + 2 >= MOD ? ia[0] + 2 - MOD : ia[0] + 2;
        ia *= tmp;
        ia.resize(n<<1);
    }
    ia.resize(N);
}

template<typename T>
void mod(vector<T>& a, vector<T>& b) {
    int n = (int)a.size()-1, m = (int)b.size()-1;
    if (n < m) return;

    vector<T> ra = a, rb = b;
    reverse(ra.begin(), ra.end()); ra.resize(min(n+1, n-m+1));
    reverse(rb.begin(), rb.end()); rb.resize(min(m+1, n-m+1));
```

```

    inv(rb, n-m+1);

    vector<T> q = move(ra);
    q *= rb;
    q.resize(n-m+1);
    reverse(q.begin(), q.end());

    q *= b;
    a -= q;
    resize(a);
}

/* Kitamasa Method (Fast Linear Recurrence):
Find a[K] (Given a[j] = c[0]a[j-N] + ... + c[N-1]a[j
    -1])
Let B(x) = x^N - c[N-1]x^(N-1) - ... - c[1]x^1 - c[0]
Let R(x) = x^K mod B(x)   (get x^K using fast pow and
    use poly mod to get R(x))
Let r[i] = the coefficient of x^i in R(x)
=> a[K] = a[0]r[0] + a[1]r[1] + ... + a[N-1]r[N-1] */
```

9 Linear Algebra

9.1 Gaussian-Jordan Elimination

```
int n;
vector<vector<ll>> v;
void gauss(vector<vector<ll>>& v) {
    int r = 0;
    for (int i = 0; i < n; i++) {
        bool ok = false;
        for (int j = r; j < n; j++) {
            if (v[j][i] == 0) continue;
            swap(v[j], v[r]);
            ok = true;
            break;
        }
        if (!ok) continue;
        ll div = inv(v[r][i]);
        for (int j = 0; j < n + 1; j++) {
            v[r][j] *= div;
            if (v[r][j] >= MOD) v[r][j] %= MOD;
        }
        for (int j = 0; j < n; j++) {
            if (j == r) continue;
            ll t = v[j][i];
            for (int k = 0; k < n + 1; k++) {
                v[j][k] -= v[r][k] * t % MOD;
                if (v[j][k] < 0) v[j][k] += MOD;
            }
        }
        r++;
    }
}
```

9.2 Determinant

1. Use GJ Elimination, if there's any row consists of only 0, then det = 0, otherwise det = product of diagonal elements.

2. Properties of det:

   - Transpose: Unchanged
   - Row Operation 1 - Swap 2 rows: $-det$
   - Row Operation 2 - $k\overrightarrow{r_i}$: $k \times det$
   - Row Operation 3 - $k\overrightarrow{r_i}$ add to $\overrightarrow{r_j}$: Unchaged

10 Combinatorics

10.1 Catalan Number

$$C_0 = 1, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}, C_n = C_n^{2n} - C_{n-1}^{2n}$$

0	1	1	2	5
4	14	42	132	429
8	1430	4862	16796	58786
12	208012	742900	2674440	9694845

10.2 Burnside's Lemma

Let X be the original set.
Let G be the group of operations acting on X.
Let X^g be the set of x not affected by g.
Let X/G be the set of orbits.
Then the following equation holds:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

11 Special Numbers

11.1 Fibonacci Series

1	1	1	2	3
5	5	8	13	21
9	34	55	89	144
13	233	377	610	987
17	1597	2584	4181	6765
21	10946	17711	28657	46368
25	75025	121393	196418	317811
29	514229	832040	1346269	2178309
33	3524578	5702887	9227465	14930352

$f(45) \approx 10^9, f(88) \approx 10^{18}$

11.2 Prime Numbers

- First 50 prime numbers:

1	2	3	5	7	11
6	13	17	19	23	29
11	31	37	41	43	47
16	53	59	61	67	71
21	73	79	83	89	97
26	101	103	107	109	113
31	127	131	137	139	149
36	151	157	163	167	173
41	179	181	191	193	197
46	199	211	223	227	229

- Very large prime numbers:
  1000001333 1000500889 2500001909
  2000000659 900004151 850001359

- $\pi(n) \equiv$ Number of primes $\leq n \approx n/((\ln n) - 1)$
  $\pi(100) = 25, \pi(200) = 46$
  $\pi(500) = 95, \pi(1000) = 168$
  $\pi(2000) = 303, \pi(4000) = 550$
  $\pi(10^4) = 1229, \pi(10^5) = 9592$
  $\pi(10^6) = 78498, \pi(10^7) = 664579$