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1 Reminder

1.1 Bug List

- 沒開 long long
- 陣列戳出界／開不夠大／開太大本地 compile 噴怪 error
- 傳之前先確定選對檔案
- 寫好的函式忘記呼叫
- 變數打錯
- 0-base / 1-base
- 忘記初始化
- == 打成 =
- <= 打成 <+
- dp[i] 從 dp[i-1] 轉移時忘記特判 i > 0
- std::sort 比較運算子寫成 < 或是讓 = 的情況為 true
- 漏 case
- 線段樹改值懶標初始值不能設為 0
- DFS 的時候不小心覆寫到全域變數
- 浮點數誤差
- unsigned int128
- 多筆測資不能沒讀完直接 return
- 記得刪 cerr

1.2 OwO

- 可以構造複雜點的測資幫助思考
- 真的卡太久請跳題
- Enjoy The Contest!

2 Basic

2.1 Vimrc

```

set number relativenumber ai t_Co=256 tabstop=4
set mouse=a shiftwidth=4 encoding=utf8
set bs=2 ruler laststatus=2 cmdheight=2
set clipboard=unnamedplus showcmd autoread
set belloff=all
filetype indent on
"set guifont Hack:h16
":set guifont?

inoremap ( ()<Esc>i
inoremap " "<Esc>i
inoremap [ []<Esc>i
inoremap ' '<Esc>i
inoremap { {<CR><Esc>ko

vmap <C-c> "+y
inoremap <C-v> <Esc>p
nnoremap <C-v> p

nnoremap <tab> gt
nnoremap <S-tab> gT
inoremap <C-n> <Esc>:tabnew<CR>
nnoremap <C-n> :tabnew<CR>

inoremap <F9> <Esc>:w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>
nnoremap <F9> :w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>

syntax on
colorscheme desert

set filetype=cpp
set background=dark
hi Normal ctermfg=white ctermbg=black

```

2.2 Runcpp.sh

```

1 #! /bin/bash
2 clear
3 echo "Start compiling $1..."
4 echo
5 g++ -O2 -std=c++20 -Wall -Wextra -Wshadow $2/$1 -o $2/
6 out
7 if [ "$?" -ne 0 ]

```

```

7 then
8     exit 1
9 fi
10 echo
11 echo "Done compiling"
12 echo "=====
13 echo
14 echo "Input file:"
15 echo
16 cat $2/in.txt
17 echo
18 echo "=====
19 echo
20 declare startTime=`date +%s%N`
21 $2/out < $2/in.txt > $2/out.txt
22 declare endTime=`date +%s%N`
23 delta=`expr $endTime - $startTime`
24 delta=`expr $delta / 1000000`
25 cat $2/out.txt
26 echo
27 echo "time: $delta ms"

```

2.3 Stress

```

1 g++ gen.cpp -o gen.out
2 g++ ac.cpp -o ac.out
3 g++ wa.cpp -o wa.out
4 for ((i=0;;i++))
5 do
6     echo "$i"
7     ./gen.out > in.txt
8     ./ac.out < in.txt > ac.txt
9     ./wa.out < in.txt > wa.txt
10    diff ac.txt wa.txt || break
11 done

```

2.4 PBDS

```

1 #include <bits/extc++.h>
2 using namespace __gnu_pbds;
3
4 // map
5 tree<int, int, less<>, rb_tree_tag,
6     tree_order_statistics_node_update> tr;
7 tr.order_of_key(element);
8 tr.find_by_order(rank);
9
10 // set
11 tree<int, null_type, less<>, rb_tree_tag,
12     tree_order_statistics_node_update> tr;
13 tr.order_of_key(element);
14 tr.find_by_order(rank);
15
16 // priority queue
17 __gnu_pbds::priority_queue<int, less<int> > big_q; //
18     Big First
19 __gnu_pbds::priority_queue<int, greater<int> > small_q;
20 // Small First
21 q1.join(q2); // join

```

2.5 Random

```

1 mt19937 gen(chrono::steady_clock::now().
2     time_since_epoch().count());
3 uniform_int_distribution<int> dis(1, 100);
4 cout << dis(gen) << endl;
5 shuffle(v.begin(), v.end(), gen);

```

3 Python

3.1 I/O

```

1 import sys
2 input = sys.stdin.readline
3
4 # Input
5 def readInt():

```

```

6     return int(input())
7 def readList():
8     return list(map(int,input().split()))
9 def readStr():
10    s = input()
11    return list(s[:len(s) - 1])
12 def readVars():
13    return map(int,input().split())
14
15 # Output
16 sys.stdout.write(string)
17
18 # faster
19 def main():
20     pass
21 main()

```

3.2 Decimal

```

1 from decimal import *
2 getcontext().prec = 2500000
3 getcontext().Emax = 2500000
4 a,b = Decimal(input()),Decimal(input())
5 a*=b
6 print(a)

```

4 Data Structure

4.1 Heavy Light Decomposition

```

1 const int N=2e5+5;
2 int n,dfn[N],son[N],top[N],num[N],dep[N],p[N];
3 vector<int>path[N];
4 struct node
5 {
6     int mx,sum;
7 }seg[N<<2];
8 void update(int x,int l,int r,int qx,int val)
9 {
10     if(l==r)
11     {
12         seg[x].mx=seg[x].sum=val;
13         return;
14     }
15     int mid=(l+r)>>1;
16     if(qx<=mid)update(x<<1,l,mid,qx,val);
17     else update(x<<1|1,mid+1,r,qx,val);
18     seg[x].mx=max(seg[x<<1].mx,seg[x<<1|1].mx);
19     seg[x].sum=seg[x<<1].sum+seg[x<<1|1].sum;
20 }
21 int big(int x,int l,int r,int ql,int qr)
22 {
23     if(ql<=l&&r<=qr)return seg[x].mx;
24     int mid=(l+r)>>1;
25     int res=-INF;
26     if(ql<=mid)res=max(res,big(x<<1,l,mid,ql,qr));
27     if(mid<qr)res=max(res,big(x<<1|1,mid+1,r,ql,qr));
28     return res;
29 }
30 int ask(int x,int l,int r,int ql,int qr)
31 {
32     if(ql<=l&&r<=qr)return seg[x].sum;
33     int mid=(l+r)>>1;
34     int res=0;
35     if(ql<=mid)res+=ask(x<<1,l,mid,ql,qr);
36     if(mid<qr)res+=ask(x<<1|1,mid+1,r,ql,qr);
37     return res;
38 }
39 void dfs1(int now)
40 {
41     son[now]=-1;
42     num[now]=1;
43     for(auto i:path[now])
44     {
45         if(!dep[i])
46         {
47             dep[i]=dep[now]+1;
48             p[i]=now;
49             dfs1(i);

```

```

50         num[now]+=num[i];
51         if(son[now]==-1||num[i]>num[son[now]])son[
            now]=i;
52     }
53 }
54 }
55 int cnt;
56 void dfs2(int now,int t)
57 {
58     top[now]=t;
59     cnt++;
60     dfn[now]=cnt;
61     if(son[now]==-1)return;
62     dfs2(son[now],t);
63     for(auto i:path[now])
64         if(i!=p[now]&&i!=son[now])
65             dfs2(i,i);
66 }
67 int path_big(int x,int y)
68 {
69     int res=-INF;
70     while(top[x]!=top[y])
71     {
72         if(dep[top[x]]<dep[top[y]])swap(x,y);
73         res=max(res,big(1,1,n,dfn[top[x]],dfn[x]));
74         x=p[top[x]];
75     }
76     if(dfn[x]>dfn[y])swap(x,y);
77     res=max(res,big(1,1,n,dfn[x],dfn[y]));
78     return res;
79 }
80 int path_sum(int x,int y)
81 {
82     int res=0;
83     while(top[x]!=top[y])
84     {
85         if(dep[top[x]]<dep[top[y]])swap(x,y);
86         res+=ask(1,1,n,dfn[top[x]],dfn[x]);
87         x=p[top[x]];
88     }
89     if(dfn[x]>dfn[y])swap(x,y);
90     res+=ask(1,1,n,dfn[x],dfn[y]);
91     return res;
92 }
93 void buildTree()
94 {
95     FOR(i,0,n-1)
96     {
97         int a,b;cin>>a>>b;
98         path[a].pb(b);
99         path[b].pb(a);
100     }
101 }
102 void buildHLD(int root)
103 {
104     dep[root]=1;
105     dfs1(root);
106     dfs2(root,root);
107     FOR(i,1,n+1)
108     {
109         int now;cin>>now;
110         update(1,1,n,dfn[i],now);
111     }
112 }

```

4.2 Skew Heap

```

1 struct node{
2     node *l,*r;
3     int v;
4     node(int x):v(x){
5         l=r=nullptr;
6     }
7 };
8 node* merge(node* a,node* b){
9     if(!a||!b) return a?:b;
10    // min heap
11    if(a->v>b->v) swap(a,b);
12    a->r=merge(a->r,b);
13    swap(a->l,a->r);
14    return a;

```

4.3 Leftist Heap

```

1 struct node{
2     node *l,*r;
3     int d, v;
4     node(int x):d(1),v(x){
5         l=r=nullptr;
6     }
7 };
8 static inline int d(node* x){return x?x->d:0;}
9 node* merge(node* a,node* b){
10     if(!a||!b) return a?:b;
11    // min heap
12    if(a->v>b->v) swap(a,b);
13    a->r=merge(a->r,b);
14    if(d(a->l)<d(a->r))
15        swap(a->l,a->r);
16    a->d=d(a->r)+1;
17    return a;
18 }

```

4.4 Treap

```

1 mt19937 rng(random_device{}());
2 struct Treap
3 {
4     Treap *l,*r;
5     int val,num,pri;
6     Treap(int k)
7     {
8         l=r=NULL;
9         val=k;
10        num=1;
11        pri=rng();
12    }
13 };
14 int siz(Treap *now){return now?now->num:0;}
15 void pull(Treap *&now)
16 {
17     now->num=siz(now->l)+siz(now->r)+1;
18 }
19 Treap* merge(Treap *a,Treap *b)
20 {
21     if(!a||!b)return a?:b;
22     else if(a->pri>b->pri)
23     {
24         a->r=merge(a->r,b);
25         pull(a);
26         return a;
27     }
28     else
29     {
30         b->l=merge(a,b->l);
31         pull(b);
32         return b;
33     }
34 }
35 void split_size(Treap *rt,Treap *&a,Treap *&b,int val)
36 {
37     if(!rt)
38     {
39         a=b=NULL;
40         return;
41     }
42     if(siz(rt->l)+1>val)
43     {
44         b=rt;
45         split_size(rt->l,a,b->l,val);
46         pull(b);
47     }
48     else
49     {
50         a=rt;
51         split_size(rt->r,a->r,b,val-siz(a->l)-1);
52         pull(a);
53     }
54 }
55 void split_val(Treap *rt,Treap *&a,Treap *&b,int val)

```

```

56 {
57     if(!rt)
58     {
59         a=b=NULL;
60         return;
61     }
62     if(rt->val<=val)
63     {
64         a=rt;
65         split_val(rt->r,a->r,b,val);
66         pull(a);
67     }
68     else
69     {
70         b=rt;
71         split_val(rt->l,a,b->l,val);
72         pull(b);
73     }
74 }
75 void treap_dfs(Treap *now)
76 {
77     if(!now) return;
78     treap_dfs(now->l);
79     cout<<now->val<<" ";
80     treap_dfs(now->r);
81 }

```

4.5 Persistent Treap

```

1 struct node {
2     node *l, *r;
3     char c; int v, sz;
4     node(char x = '$'): c(x), v(mt()), sz(1) {
5         l = r = nullptr;
6     }
7     node(node* p) { *this = *p; }
8     void pull() {
9         sz = 1;
10        for (auto i : {l, r})
11            if (i) sz += i->sz;
12    }
13 } arr[maxn], *ptr = arr;
14 inline int size(node* p) { return p ? p->sz : 0; }
15 node* merge(node* a, node* b) {
16     if (!a || !b) return a ? b;
17     if (a->v < b->v) {
18         node* ret = new(ptr++) node(a);
19         ret->r = merge(ret->r, b); ret->pull();
20         return ret;
21     }
22     else {
23         node* ret = new(ptr++) node(b);
24         ret->l = merge(a, ret->l); ret->pull();
25         return ret;
26     }
27 }
28 P<node*> split(node* p, int k) {
29     if (!p) return {nullptr, nullptr};
30     if (k >= size(p->l) + 1) {
31         auto [a, b] = split(p->r, k - size(p->l) - 1);
32         node* ret = new(ptr++) node(p);
33         ret->r = a; ret->pull();
34         return {ret, b};
35     }
36     else {
37         auto [a, b] = split(p->l, k);
38         node* ret = new(ptr++) node(p);
39         ret->l = b; ret->pull();
40         return {a, ret};
41     }
42 }

```

4.6 Li Chao Tree

```

1 constexpr int maxn = 5e4 + 5;
2 struct line {
3     ld a, b;
4     ld operator()(ld x) { return a * x + b; }
5 } arr[(maxn + 1) << 2];
6 bool operator<(line a, line b) { return a.a < b.a; }

```

```

7 #define m ((l+r)>>1)
8 void insert(line x, int i = 1, int l = 0, int r = maxn)
9 {
10     if (r - l == 1) {
11         if (x(l) > arr[i](l))
12             arr[i] = x;
13         return;
14     }
15     line a = max(arr[i], x), b = min(arr[i], x);
16     if (a(m) > b(m))
17         arr[i] = a, insert(b, i << 1, l, m);
18     else
19         arr[i] = b, insert(a, i << 1 | 1, m, r);
20 }
21 ld query(int x, int i = 1, int l = 0, int r = maxn) {
22     if (x < l || r <= x) return -numeric_limits<ld>::
23         max();
24     if (r - l == 1) return arr[i](x);
25     return max({arr[i](x), query(x, i << 1, l, m),
26         query(x, i << 1 | 1, m, r)});
27 }
28 #undef m

```

4.7 Time Segment Tree

```

1 constexpr int maxn = 1e5 + 5;
2 V<P<int>> arr[(maxn + 1) << 2];
3 V<int> dsu, sz;
4 V<tuple<int, int, int>> his;
5 int cnt, q;
6 int find(int x) {
7     return x == dsu[x] ? x : find(dsu[x]);
8 }
9 inline bool merge(int x, int y) {
10     int a = find(x), b = find(y);
11     if (a == b) return false;
12     if (sz[a] > sz[b]) swap(a, b);
13     his.emplace_back(a, b, sz[b]); dsu[a] = b, sz[b] +=
14         sz[a];
15     return true;
16 }
17 inline void undo() {
18     auto [a, b, s] = his.back(); his.pop_back();
19     dsu[a] = a, sz[b] = s;
20 }
21 #define m ((l + r) >> 1)
22 void insert(int ql, int qr, P<int> x, int i = 1, int l
23     = 0, int r = q) {
24     // debug(ql, qr, x); return;
25     if (qr <= l || r <= ql) return;
26     if (ql <= l && r <= qr) { arr[i].push_back(x);
27         return; }
28     if (qr <= m)
29         insert(ql, qr, x, i << 1, l, m);
30     else if (m <= ql)
31         insert(ql, qr, x, i << 1 | 1, m, r);
32     else {
33         insert(ql, qr, x, i << 1, l, m);
34         insert(ql, qr, x, i << 1 | 1, m, r);
35     }
36 }
37 void traversal(V<int>& ans, int i = 1, int l = 0, int r
38     = q) {
39     int opcnt = 0;
40     // debug(i, l, r);
41     for (auto [a, b] : arr[i])
42         if (merge(a, b))
43             opcnt++, cnt--;
44     if (r - l == 1) ans[l] = cnt;
45     else {
46         traversal(ans, i << 1, l, m);
47         traversal(ans, i << 1 | 1, m, r);
48     }
49     while (opcnt--)
50         undo(), cnt++;
51     arr[i].clear();
52 }
53 #undef m
54 inline void solve() {
55     int n, m; cin>>n>>m>>q,q++;
56     dsu.resize(cnt = n), sz.assign(n, 1);

```



```

64 } } } } }
65
66
67 // Negative Cycle Detection
68 void NegCycleDetect() {
69     /* No Neg Cycle: NO
70     Exist Any Neg Cycle:
71     YES
72     v0 v1 v2 ... vk v0 */
73
74     vector<int> src;
75     for (int i = 1; i <= n; i++)
76         src.emplace_back(i);
77
78     SPFA(src);
79     // BellmanFord(src);
80
81     int ptr = -1;
82     for (int i = 1; i <= n; i++) if (negCycle[i])
83         { ptr = i; break; }
84
85     if (ptr == -1) { return cout << "NO" << endl, void
86         (); }
87
88     cout << "YES\n";
89     vector<int> ans;
90     vector<bool> vis(n+1, false);
91
92     while (true) {
93         ans.emplace_back(ptr);
94         if (vis[ptr]) break;
95         vis[ptr] = true;
96         ptr = pa[ptr];
97     }
98     reverse(ans.begin(), ans.end());
99
100     vis.assign(n+1, false);
101     for (auto& x : ans) {
102         cout << x << ' ';
103         if (vis[x]) break;
104         vis[x] = true;
105     }
106     cout << endl;
107 }
108
109 // Distance Calculation
110 void calcDis(int s) {
111     vector<int> src;
112     src.emplace_back(s);
113     SPFA(src);
114     // BellmanFord(src);
115
116     while (!q.empty()) q.pop();
117     for (int i = 1; i <= n; i++)
118         if (negCycle[i]) q.push(i);
119
120     while (!q.empty()) {
121         int u = q.front(); q.pop();
122         for (auto& e : g[u]) {
123             int v = e.first;
124             if (!negCycle[v]) {
125                 q.push(v);
126                 negCycle[v] = true;
127             }
128         }
129     }
130 }

```

6.2 BCC - AP

```

1 int n, m;
2 int low[maxn], dfn[maxn], instp;
3 vector<int> E, g[maxn];
4 bitset<maxn> isap;
5 bitset<maxn> vis;
6 stack<int> stk;
7 int bccnt;
8 vector<int> bcc[maxn];
9 inline void popout(int u) {
10     bccnt++;
11     bcc[bccnt].emplace_back(u);
12     while (!stk.empty()) {
13         int v = stk.top();
14         if (u == v) break;

```

```

15         stk.pop();
16         bcc[bccnt].emplace_back(v);
17     }
18 }
19 void dfs(int u, bool rt = 0) {
20     stk.push(u);
21     low[u] = dfn[u] = ++instp;
22     int kid = 0;
23     Each(e, g[u]) {
24         if (vis[e]) continue;
25         vis[e] = true;
26         int v = E[e]^u;
27         if (!dfn[v]) {
28             // tree edge
29             kid++; dfs(v);
30             low[u] = min(low[u], low[v]);
31             if (!rt && low[v] >= dfn[u]) {
32                 // bcc found: u is ap
33                 isap[u] = true;
34                 popout(u);
35             }
36         } else {
37             // back edge
38             low[u] = min(low[u], dfn[v]);
39         }
40     }
41     // special case: root
42     if (rt) {
43         if (kid > 1) isap[u] = true;
44         popout(u);
45     }
46 }
47 void init() {
48     cin >> n >> m;
49     fill(low, low+maxn, INF);
50     REP(i, m) {
51         int u, v;
52         cin >> u >> v;
53         g[u].emplace_back(i);
54         g[v].emplace_back(i);
55         E.emplace_back(u^v);
56     }
57 }
58 void solve() {
59     FOR(i, 1, n+1, 1) {
60         if (!dfn[i]) dfs(i, true);
61     }
62     vector<int> ans;
63     int cnt = 0;
64     FOR(i, 1, n+1, 1) {
65         if (isap[i]) cnt++, ans.emplace_back(i);
66     }
67     cout << cnt << endl;
68     Each(i, ans) cout << i << ' ';
69     cout << endl;
70 }

```

6.3 BCC - Bridge

```

1 int n, m;
2 vector<int> g[maxn], E;
3 int low[maxn], dfn[maxn], instp;
4 int bccnt, bccid[maxn];
5 stack<int> stk;
6 bitset<maxn> vis, isbrg;
7 void init() {
8     cin >> n >> m;
9     REP(i, m) {
10         int u, v;
11         cin >> u >> v;
12         E.emplace_back(u^v);
13         g[u].emplace_back(i);
14         g[v].emplace_back(i);
15     }
16     fill(low, low+maxn, INF);
17 }
18 void popout(int u) {
19     bccnt++;
20     while (!stk.empty()) {
21         int v = stk.top();
22         if (v == u) break;

```

```

23     stk.pop();
24     bccid[v] = bccnt;
25 }
26 }
27 void dfs(int u) {
28     stk.push(u);
29     low[u] = dfn[u] = ++instp;
30
31     Each(e, g[u]) {
32         if (vis[e]) continue;
33         vis[e] = true;
34
35         int v = E[e]^u;
36         if (dfn[v]) {
37             // back edge
38             low[u] = min(low[u], dfn[v]);
39         } else {
40             // tree edge
41             dfs(v);
42             low[u] = min(low[u], low[v]);
43             if (low[v] == dfn[v]) {
44                 isbrg[e] = true;
45                 popout(u);
46             }
47         }
48     }
49 }
50 void solve() {
51     FOR(i, 1, n+1, 1) {
52         if (!dfn[i]) dfs(i);
53     }
54     vector<pii> ans;
55     vis.reset();
56     FOR(u, 1, n+1, 1) {
57         Each(e, g[u]) {
58             if (!isbrg[e] || vis[e]) continue;
59             vis[e] = true;
60             int v = E[e]^u;
61             ans.emplace_back(mp(u, v));
62         }
63     }
64     cout << (int)ans.size() << endl;
65     Each(e, ans) cout << e.F << ' ' << e.S << endl;
66 }

```

6.4 SCC - Tarjan

```

1 // 2-SAT
2 vector<int> E, g[maxn]; // 1~n, n+1~2n
3 int low[maxn], in[maxn], instp;
4 int sccnt, sccid[maxn];
5
6 stack<int> stk;
7 bitset<maxn> ins, vis;
8
9 int n, m;
10
11 void init() {
12     cin >> m >> n;
13     E.clear();
14     fill(g, g+maxn, vector<int>());
15     fill(low, low+maxn, INF);
16     memset(in, 0, sizeof(in));
17     instp = 1;
18     sccnt = 0;
19     memset(sccid, 0, sizeof(sccid));
20     ins.reset();
21     vis.reset();
22 }
23
24 inline int no(int u) {
25     return (u > n ? u-n : u+n);
26 }
27
28 int ecnt = 0;
29 inline void clause(int u, int v) {
30     E.pb(no(u)^v);
31     g[no(u)].pb(ecnt++);
32     E.pb(no(v)^u);
33     g[no(v)].pb(ecnt++);
34 }

```

```

35
36 void dfs(int u) {
37     in[u] = instp++;
38     low[u] = in[u];
39     stk.push(u);
40     ins[u] = true;
41
42     Each(e, g[u]) {
43         if (vis[e]) continue;
44         vis[e] = true;
45
46         int v = E[e]^u;
47         if (ins[v]) low[u] = min(low[u], in[v]);
48         else if (!in[v]) {
49             dfs(v);
50             low[u] = min(low[u], low[v]);
51         }
52     }
53
54     if (low[u] == in[u]) {
55         sccnt++;
56         while (!stk.empty()) {
57             int v = stk.top();
58             stk.pop();
59             ins[v] = false;
60             sccid[v] = sccnt;
61             if (u == v) break;
62         }
63     }
64 }
65
66 int main() {
67     WiWiHorz
68     init();
69
70     REP(i, m) {
71         char su, sv;
72         int u, v;
73         cin >> su >> u >> sv >> v;
74         if (su == '-') u = no(u);
75         if (sv == '-') v = no(v);
76         clause(u, v);
77     }
78
79     FOR(i, 1, 2*n+1, 1) {
80         if (!in[i]) dfs(i);
81     }
82
83     FOR(u, 1, n+1, 1) {
84         int du = no(u);
85         if (sccid[u] == sccid[du]) {
86             return cout << "IMPOSSIBLE\n", 0;
87         }
88     }
89
90     FOR(u, 1, n+1, 1) {
91         int du = no(u);
92         cout << (sccid[u] < sccid[du] ? '+' : '-') << '
93         ' << endl;
94     }
95     cout << endl;
96     return 0;
97 }
98

```

6.5 Eulerian Path - Undir

```

1 // from 1 to n
2 #define gg return cout << "IMPOSSIBLE\n", void();
3
4 int n, m;
5 vector<int> g[maxn];
6 bitset<maxn> inodd;
7
8 void init() {
9     cin >> n >> m;
10    inodd.reset();
11    for (int i = 0; i < m; i++) {
12        int u, v; cin >> u >> v;
13        inodd[u] = inodd[u] ^ true;

```



```

14     inodd[v] = inodd[v] ^ true;
15     g[u].emplace_back(v);
16     g[v].emplace_back(u);
17 } }
18 stack<int> stk;
19 void dfs(int u) {
20     while (!g[u].empty()) {
21         int v = g[u].back();
22         g[u].pop_back();
23         dfs(v);
24     }
25     stk.push(u);}

```

6.6 Eulerian Path - Dir

```

1 // from node 1 to node n
2 #define gg return cout << "IMPOSSIBLE\n", 0
3
4 int n, m;
5 vector<int> g[maxn];
6 stack<int> stk;
7 int in[maxn], out[maxn];
8
9 void init() {
10     cin >> n >> m;
11     for (int i = 0; i < m; i++) {
12         int u, v; cin >> u >> v;
13         g[u].emplace_back(v);
14         out[u]++, in[v]++;
15     }
16     for (int i = 1; i <= n; i++) {
17         if (i == 1 && out[i]-in[i] != 1) gg;
18         if (i == n && in[i]-out[i] != 1) gg;
19         if (i != 1 && i != n && in[i] != out[i]) gg;
20     } }
21 void dfs(int u) {
22     while (!g[u].empty()) {
23         int v = g[u].back();
24         g[u].pop_back();
25         dfs(v);
26     }
27     stk.push(u);
28 }
29 void solve() {
30     dfs(1)
31     for (int i = 1; i <= n; i++)
32         if ((int)g[i].size()) gg;
33     while (!stk.empty()) {
34         int u = stk.top();
35         stk.pop();
36         cout << u << ' ';
37     } }

```

6.7 Hamilton Path

```

1 // top down DP
2 // Be Aware Of Multiple Edges
3 int n, m;
4 ll dp[maxn][1<<maxn];
5 int adj[maxn][maxn];
6
7 void init() {
8     cin >> n >> m;
9     fill(dp[0], dp[maxn-1]+(1<<maxn), -1);
10 }
11
12 void DP(int i, int msk) {
13     if (dp[i][msk] != -1) return;
14     dp[i][msk] = 0;
15     REP(j, n) if (j != i && (msk & (1<<j)) && adj[j][i]) {
16         int sub = msk ^ (1<<i);
17         if (dp[j][sub] == -1) DP(j, sub);
18         dp[i][msk] += dp[j][sub] * adj[j][i];
19         if (dp[i][msk] >= MOD) dp[i][msk] %= MOD;
20     }
21 }
22
23
24 int main() {

```

```

25     WiwiHorz
26     init();
27
28     REP(i, m) {
29         int u, v;
30         cin >> u >> v;
31         if (u == v) continue;
32         adj[--u][--v]++;
33     }
34
35     dp[0][1] = 1;
36     FOR(i, 1, n, 1) {
37         dp[i][1] = 0;
38         dp[i][1|(1<<i)] = adj[0][i];
39     }
40     FOR(msk, 1, (1<<n), 1) {
41         if (msk == 1) continue;
42         dp[0][msk] = 0;
43     }
44
45     DP(n-1, (1<<n)-1);
46     cout << dp[n-1][(1<<n)-1] << endl;
47
48     return 0;
49 }
50

```

6.8 Kth Shortest Path

```

1 // time: O(|E| \lg |E|+|V| \lg |V|+K)
2 // memory: O(|E| \lg |E|+|V|)
3 struct KSP{ // 1-base
4     struct nd{
5         int u,v; ll d;
6         nd(int ui=0,int vi=0,ll di=INF){ u=ui; v=vi; d=di;
7         }
8     };
9     struct heap{ nd* edge; int dep; heap* chd[4]; };
10     static int cmp(heap* a,heap* b)
11     { return a->edge->d > b->edge->d; }
12     struct node{
13         int v; ll d; heap* H; nd* E;
14         node(ll _d,int _v,nd* _E){ d=_d; v=_v; E=_E; }
15         node(heap* _H,ll _d){ H=_H; d=_d; }
16         friend bool operator<(node a,node b)
17         { return a.d>b.d; }
18     };
19     int n,k,s,t,dst[N]; nd *nxt[N];
20     vector<nd*> g[N],rg[N]; heap *nullNd,*head[N];
21     void init(int _n,int _k,int _s,int _t){
22         n=_n; k=_k; s=_s; t=_t;
23         for(int i=1;i<=n;i++){
24             g[i].clear(); rg[i].clear();
25             nxt[i]=NULL; head[i]=NULL; dst[i]=-1;
26         }
27     }
28     void addEdge(int ui,int vi,ll di){
29         nd* e=new nd(ui,vi,di);
30         g[ui].push_back(e); rg[vi].push_back(e);
31     }
32     queue<int> dfsQ;
33     void dijkstra(){
34         while(dfsQ.size()) dfsQ.pop();
35         priority_queue<node> Q; Q.push(node(0,t,NULL));
36         while (!Q.empty()){
37             node p=Q.top(); Q.pop(); if(dst[p.v]!=-1)continue;
38             dst[p.v]=p.d; nxt[p.v]=p.E; dfsQ.push(p.v);
39             for(auto e:rg[p.v]) Q.push(node(p.d+e->d,e->u,e));
40         }
41     }
42     heap* merge(heap* curNd,heap* newNd){
43         if(curNd==nullNd) return newNd;
44         heap* root=new heap;memcpy(root,curNd,sizeof(heap));
45         if(newNd->edge->d<curNd->edge->d){
46             root->edge=newNd->edge;
47             root->chd[2]=newNd->chd[2];
48             root->chd[3]=newNd->chd[3];

```



```

49     newNd->edge=curNd->edge;
50     newNd->chd[2]=curNd->chd[2];
51     newNd->chd[3]=curNd->chd[3];
52 }
53 if(root->chd[0]->dep<root->chd[1]->dep)
54     root->chd[0]=merge(root->chd[0],newNd);
55 else root->chd[1]=merge(root->chd[1],newNd);
56 root->dep=max(root->chd[0]->dep,
57             root->chd[1]->dep)+1;
58 return root;
59 }
60 vector<heap*> V;
61 void build(){
62     nullNd=new heap; nullNd->dep=0; nullNd->edge=new nd
63     ;
64     fill(nullNd->chd,nullNd->chd+4,nullNd);
65     while(not dfsQ.empty()){
66         int u=dfsQ.front(); dfsQ.pop();
67         if(!nxt[u]) head[u]=nullNd;
68         else head[u]=head[nxt[u]->v];
69         V.clear();
70         for(auto&& e:g[u]){
71             int v=e->v;
72             if(dst[v]==-1) continue;
73             e->d+=dst[v]-dst[u];
74             if(nxt[u]!=e){
75                 heap* p=new heap; fill(p->chd,p->chd+4,nullNd)
76                 ;
77                 p->dep=1; p->edge=e; V.push_back(p);
78             }
79             if(V.empty()) continue;
80             make_heap(V.begin(),V.end(),cmp);
81 #define L(X) ((X<1)+1)
82 #define R(X) ((X<1)+2)
83             for(size_t i=0;i<V.size();i++){
84                 if(L(i)<V.size()) V[i]->chd[2]=V[L(i)];
85                 else V[i]->chd[2]=nullNd;
86                 if(R(i)<V.size()) V[i]->chd[3]=V[R(i)];
87                 else V[i]->chd[3]=nullNd;
88             }
89             head[u]=merge(head[u],V.front());
90         }
91     }
92     vector<ll> ans;
93     void first_K(){
94         ans.clear(); priority_queue<node> Q;
95         if(dst[s]==-1) return;
96         ans.push_back(dst[s]);
97         if(head[s]!=nullNd)
98             Q.push(node(head[s],dst[s]+head[s]->edge->d));
99         for(int _=1;_<k and not Q.empty();_++){
100             node p=Q.top(); Q.pop(); ans.push_back(p.d);
101             if(head[p.H->edge->v]!=nullNd){
102                 q.H=head[p.H->edge->v]; q.d=p.d+q.H->edge->d;
103                 Q.push(q);
104             }
105             for(int i=0;i<4;i++){
106                 if(p.H->chd[i]!=nullNd){
107                     q.H=p.H->chd[i];
108                     q.d=p.d-p.H->edge->d+p.H->chd[i]->edge->d;
109                     Q.push(q);
110                 }
111             }
112         }
113     }
114     void solve(){ // ans[i] stores the i-th shortest path
115         dijkstra(); build();
116         first_K(); // ans.size() might less than k
117     }
118 } solver;

```

6.9 System of Difference Constraints

```

1 vector<vector<pair<int, ll>>> G;
2 void add(int u, int v, ll w) {
3     G[u].emplace_back(make_pair(v, w));
4 }

```

- $x_u - x_v \leq c \Rightarrow \text{add}(v, u, c)$
- $x_u - x_v \geq c \Rightarrow \text{add}(u, v, -c)$

- $x_u - x_v = c \Rightarrow \text{add}(v, u, c), \text{add}(u, v, -c)$
- $x_u \geq c \Rightarrow \text{add super vertex } x_0 = 0, \text{ then } x_u - x_0 \geq c \Rightarrow \text{add}(u, 0, -c)$
- Don't forget non-negative constraints for every variable if specified implicitly.
- Interval sum \Rightarrow Use prefix sum to transform into differential constraints. Don't forget $S_{i+1} - S_i \geq 0$ if x_i needs to be non-negative.
- $\frac{x_u}{x_v} \leq c \Rightarrow \log x_u - \log x_v \leq \log c$

7 String

7.1 Rolling Hash

```

1 const ll C = 27;
2 inline int id(char c) {return c-'a'+1;}
3 struct RollingHash {
4     string s; int n; ll mod;
5     vector<ll> Cexp, hs;
6     RollingHash(string& _s, ll _mod):
7         s(_s), n((int)_s.size()), mod(_mod)
8     {
9         Cexp.assign(n, 0);
10        hs.assign(n, 0);
11        Cexp[0] = 1;
12        for (int i = 1; i < n; i++) {
13            Cexp[i] = Cexp[i-1] * C;
14            if (Cexp[i] >= mod) Cexp[i] %= mod;
15        }
16        hs[0] = id(s[0]);
17        for (int i = 1; i < n; i++) {
18            hs[i] = hs[i-1] * C + id(s[i]);
19            if (hs[i] >= mod) hs[i] %= mod;
20        }
21        inline ll query(int l, int r) {
22            ll res = hs[r] - (l ? hs[l-1] * Cexp[r-l+1] :
23                0);
24            res = (res % mod + mod) % mod;
25            return res; }
26 };

```

7.2 Trie

```

1 struct node {
2     int c[26]; ll cnt;
3     node(): cnt(0) {memset(c, 0, sizeof(c));}
4     node(ll x): cnt(x) {memset(c, 0, sizeof(c));}
5 };
6 struct Trie {
7     vector<node> t;
8     void init() {
9         t.clear();
10        t.emplace_back(node());
11    }
12    void insert(string s) { int ptr = 0;
13        for (auto& i : s) {
14            if (!t[ptr].c[i-'a']) {
15                t.emplace_back(node());
16                t[ptr].c[i-'a'] = (int)t.size()-1;
17                ptr = t[ptr].c[i-'a']; }
18            t[ptr].cnt++; }
19    } trie;

```

7.3 KMP

```

1 int n, m;
2 string s, p;
3 vector<int> f;
4 void build() {
5     f.clear(); f.resize(m, 0);
6     int ptr = 0; for (int i = 1; i < m; i++) {
7         while (ptr && p[i] != p[ptr]) ptr = f[ptr-1];
8         if (p[i] == p[ptr]) ptr++;

```

```

9         f[i] = ptr;
10    }}
11    void init() {
12        cin >> s >> p;
13        n = (int)s.size();
14        m = (int)p.size();
15        build(); }
16    void solve() {
17        int ans = 0, pi = 0;
18        for (int si = 0; si < n; si++) {
19            while (pi && s[si] != p[pi]) pi = f[pi-1];
20            if (s[si] == p[pi]) pi++;
21            if (pi == m) ans++, pi = f[pi-1];
22        }
23    cout << ans << endl; }

```

7.4 Z Value

```

1    string is, it, s;
2    int n; vector<int> z;
3    void init() {
4        cin >> is >> it;
5        s = it+'0'+is;
6        n = (int)s.size();
7        z.resize(n, 0); }
8    void solve() {
9        int ans = 0; z[0] = n;
10       for (int i = 1, l = 0, r = 0; i < n; i++) {
11           if (i <= r) z[i] = min(z[i-1], r-i+1);
12           while (i+z[i] < n && s[z[i]] == s[i+z[i]]) z[i]
13               ]++;
14           if (i+z[i]-1 > r) l = i, r = i+z[i]-1;
15           if (z[i] == (int)it.size()) ans++;
16       }
17       cout << ans << endl; }

```

7.5 Manacher

```

1    int n; string S, s;
2    vector<int> m;
3    void manacher() {
4        s.clear(); s.resize(2*n+1, '.');
5        for (int i = 0, j = 1; i < n; i++, j += 2) s[j] = S[i];
6        m.clear(); m.resize(2*n+1, 0);
7        // m[i] := max k such that s[i-k, i+k] is palindrome
8        int mx = 0, mxk = 0;
9        for (int i = 1; i < 2*n+1; i++) {
10           if (mx-(i-mx) >= 0) m[i] = min(m[mx-(i-mx)], mx+mxk
11               -i);
12           while (0 <= i-m[i]-1 && i+m[i]+1 < 2*n+1 &&
13               s[i-m[i]-1] == s[i+m[i]+1]) m[i]++;
14           if (i+m[i] > mx+mxk) mx = i, mxk = m[i];
15       } }
16    void init() { cin >> S; n = (int)S.size(); }
17    void solve() {
18        manacher();
19        int mx = 0, ptr = 0;
20        for (int i = 0; i < 2*n+1; i++) if (mx < m[i])
21            { mx = m[i]; ptr = i; }
22        for (int i = ptr-mx; i <= ptr+mx; i++)
23            if (s[i] != '.') cout << s[i];
24    cout << endl; }

```

7.6 Suffix Array

```

1    #define F first
2    #define S second
3    struct SuffixArray { // don't forget s += "$";
4        int n; string s;
5        vector<int> suf, lcp, rk;
6        vector<int> cnt, pos;
7        vector<pair<pii, int>> buc[2];
8        void init(string _s) {
9            s = _s; n = (int)s.size();
10           // resize(n): suf, rk, cnt, pos, lcp, buc[0~1]
11       }
12       void radix_sort() {
13           for (int t : {0, 1}) {
14               fill(cnt.begin(), cnt.end(), 0);

```

```

15           for (auto& i : buc[t]) cnt[ (t ? i.F.F : i.
16               F.S) ]++;
17           for (int i = 0; i < n; i++)
18               pos[i] = (!i ? 0 : pos[i-1] + cnt[i-1])
19               ;
20           for (auto& i : buc[t])
21               buc[t^1][pos[ (t ? i.F.F : i.F.S) ]++]
22               = i;
23       } }
24       bool fill_suf() {
25           bool end = true;
26           for (int i = 0; i < n; i++) suf[i] = buc[0][i].
27               S;
28           rk[suf[0]] = 0;
29           for (int i = 1; i < n; i++) {
30               int dif = (buc[0][i].F != buc[0][i-1].F);
31               end &= dif;
32               rk[suf[i]] = rk[suf[i-1]] + dif;
33           } return end;
34       }
35       void sa() {
36           for (int i = 0; i < n; i++)
37               buc[0][i] = make_pair(make_pair(s[i], s[i])
38               , i);
39           sort(buc[0].begin(), buc[0].end());
40           if (fill_suf()) return;
41           for (int k = 0; (1<<k) < n; k++) {
42               for (int i = 0; i < n; i++)
43                   buc[0][i] = make_pair(make_pair(rk[i],
44                       rk[(i + (1<<k)) % n]), i);
45               radix_sort();
46               if (fill_suf()) return;
47           } }
48       void LCP() { int k = 0;
49           for (int i = 0; i < n-1; i++) {
50               if (rk[i] == 0) continue;
51               int pi = rk[i];
52               int j = suf[pi-1];
53               while (i+k < n && j+k < n && s[i+k] == s[j+
54                   k]) k++;
55               lcp[pi] = k;
56               k = max(k-1, 0);
57           } }
58   };
59   SuffixArray suffixarray;

```

7.7 SA-IS

```

1    const int N=300010;
2    struct SA{
3        #define REP(i,n) for(int i=0;i<int(n);i++)
4        #define REP1(i,a,b) for(int i=(a);i<=int(b);i++)
5        bool _t[N*2]; int _s[N*2],_sa[N*2];
6        int _c[N*2],x[N],_p[N],_q[N*2],hei[N],r[N];
7        int operator [](int i){ return _sa[i]; }
8        void build(int *s,int n,int m){
9            memcpy(_s,s,sizeof(int)*n);
10           sais(_s,_sa,_p,_q,_t,_c,n,m); mkhei(n);
11       }
12       void mkhei(int n){
13           REP(i,n) r[_sa[i]]=i;
14           hei[0]=0;
15           REP(i,n) if(r[i]) {
16               int ans=i>0?max(hei[r[i-1]]-1,0):0;
17               while(_s[i+ans]==_s[_sa[r[i]-1]+ans]) ans++;
18               hei[r[i]]=ans;
19           } }
20       }
21       void sais(int *s,int *sa,int *p,int *q,bool *t,int *c
22           ,int n,int z){
23           bool uniq=t[n-1]=true,neq;
24           int nn=0,nmxz=-1,*nsa=sa+n,*ns=s+n,lst=-1;
25           #define MS0(x,n) memset((x),0,n*sizeof(*(x)))
26           #define MAGIC(XD) MS0(sa,n);\
27               memcpy(x,c,sizeof(int)*z);\
28               XD;\
29               memcpy(x+1,c,sizeof(int)*(z-1));\
30           REP(i,n) if(sa[i]&&!t[sa[i]-1]) sa[x[sa[i]-1]]+=sa[
31               i]-1;\
32           memcpy(x,c,sizeof(int)*z);\
33           for(int i=n-1;i>=0;i--) if(sa[i]&&t[sa[i]-1]) sa[--x[
34               sa[i]-1]]=sa[i]-1;

```

```

31 MS0(c,z); REP(i,n) uniq&=++c[s[i]]<2;
32 REP(i,z-1) c[i+1]+=c[i];
33 if(uniq) { REP(i,n) sa[--c[s[i]]]=i; return; }
34 for(int i=n-2;i>=0;i--)
35     t[i]=(s[i]==s[i+1]?t[i+1]:s[i]<s[i+1]);
36 MAGIC(REP1(i,1,n-1) if(t[i]&&!t[i-1]) sa[--x[s[i]
37     ]]=p[q[i]=nn++]=i);
38 REP(i,n) if(sa[i]&&t[sa[i]]&&t[sa[i]-1]){
39     neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
40     [i])*sizeof(int));
41     ns[q[lst=sa[i]]]=nmxz+=neq;
42 }
43 sais(ns,nsa,p+nn,q+n,t+n,c+z,nn,nmxz+1);
44 MAGIC(for(int i=nn-1;i>=0;i--) sa[--x[s[p[nsa[i]
45     ]]]]=p[nsa[i]]);
46 }
47 }sa;
48 int H[N],SA[N],RA[N];
49 void suffix_array(int* ip,int len){
50     // should padding a zero in the back
51     // ip is int array, len is array length
52     // ip[0..n-1] != 0, and ip[len]=0
53     ip[len++]=0; sa.build(ip,len,128);
54     memcpy(H,sa.hei+1,len<2); memcpy(SA,sa._sa+1,len<2);
55     for(int i=0;i<len;i++) RA[i]=sa.r[i]-1;
56     // resulting height, sa array \in [0,len)
57 }

```

7.8 Minimum Rotation

```

1 //rotate(begin(s),begin(s)+minRotation(s), end(s))
2 int minRotation(string s) {
3     int a = 0, n = s.size(); s += s;
4     for(int b = 0; b < n; b++) for(int k = 0; k < n; k++) {
5         if(a + k == b || s[a + k] < s[b + k]) {
6             b += max(0, k - 1);
7             break; }
8         if(s[a + k] > s[b + k]) {
9             a = b;
10            break; }
11     } }
12 return a; }

```

7.9 Aho Corasick

```

1 struct AAutomata{
2     struct Node{
3         int cnt;
4         Node *go[26], *fail, *dic;
5         Node(){
6             cnt = 0; fail = 0; dic=0;
7             memset(go,0,sizeof(go));
8         }
9     }pool[1048576],*root;
10    int nMem;
11    Node* new_Node(){
12        pool[nMem] = Node();
13        return &pool[nMem++];
14    }
15    void init() { nMem = 0; root = new_Node(); }
16    void add(const string &str) { insert(root,str,0); }
17    void insert(Node *cur, const string &str, int pos){
18        for(int i=pos;i<str.size();i++){
19            if(!cur->go[str[i]-'a'])
20                cur->go[str[i]-'a'] = new_Node();
21            cur=cur->go[str[i]-'a'];
22        }
23        cur->cnt++;
24    }
25    void make_fail(){
26        queue<Node*> que;
27        que.push(root);
28        while(!que.empty()){
29            Node* fr=que.front(); que.pop();
30            for(int i=0;i<26;i++){
31                if(fr->go[i]){
32                    Node *ptr = fr->fail;
33                    while(ptr && !ptr->go[i]) ptr = ptr->fail;
34                    fr->go[i]->fail=ptr=(ptr?ptr->go[i]:root);

```

```

35         fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
36         que.push(fr->go[i]);
37     } } }
38 }AC;

```

8 Geometry

8.1 Basic Operations

```

1 typedef long long T;
2 // typedef long double T;
3 const long double eps = 1e-8;
4
5 short sgn(T x) {
6     if (abs(x) < eps) return 0;
7     return x < 0 ? -1 : 1;
8 }
9
10 struct Pt {
11     T x, y;
12     Pt(T _x=0, T _y=0):x(_x), y(_y) {}
13     Pt operator+(Pt a) { return Pt(x+a.x, y+a.y); }
14     Pt operator-(Pt a) { return Pt(x-a.x, y-a.y); }
15     Pt operator*(T a) { return Pt(x*a, y*a); }
16     Pt operator/(T a) { return Pt(x/a, y/a); }
17     T operator*(Pt a) { return x*a.x + y*a.y; }
18     T operator^(Pt a) { return x*a.y - y*a.x; }
19     bool operator<(Pt a)
20         { return x < a.x || (x == a.x && y < a.y); }
21     //return sgn(x-a.x) < 0 || (sgn(x-a.x) == 0 && sgn(y-a.
22         y) < 0); }
23     bool operator==(Pt a)
24         { return sgn(x-a.x) == 0 && sgn(y-a.y) == 0; }
25 };
26 Pt mv(Pt a, Pt b) { return b-a; }
27 T len2(Pt a) { return a*a; }
28 T dis2(Pt a, Pt b) { return len2(b-a); }
29
30 short ori(Pt a, Pt b) { return ((a^b)>0) - ((a^b)<0); }
31 bool onseg(Pt p, Pt l1, Pt l2) {
32     Pt a = mv(p, l1), b = mv(p, l2);
33     return ((a^b) == 0) && ((a*b) <= 0);
34 }

```

8.2 InPoly

```

1 short inPoly(Pt p) {
2     // 0=Bound 1=In -1=Out
3     REP(i, n) if (onseg(p, E[i], E[(i+1)%n])) return 0;
4     int cnt = 0;
5     REP(i, n) if (banana(p, Pt(p.x+1, p.y+2e9),
6         E[i], E[(i+1)%n])) cnt ^= 1;
7     return (cnt ? 1 : -1);
8 }

```

8.3 Sort by Angle

```

1 int ud(Pt a) { // up or down half plane
2     if (a.y > 0) return 0;
3     if (a.y < 0) return 1;
4     return (a.x >= 0 ? 0 : 1);
5 }
6 sort(ALL(E), [&](const Pt& a, const Pt& b){
7     if (ud(a) != ud(b)) return ud(a) < ud(b);
8     return (a^b) > 0;
9 });

```

8.4 Line Intersect Check

```

1 inline bool banana(Pt p1, Pt p2, Pt q1, Pt q2) {
2     if (onseg(p1, q1, q2) || onseg(p2, q1, q2) ||
3         onseg(q1, p1, p2) || onseg(q2, p1, p2)) {
4         return true;
5     }
6     Pt p = mv(p1, p2), q = mv(q1, q2);
7     return (ori(p, mv(p1, q1)) * ori(p, mv(p1, q2))) < 0 &&

```

```

8 |         ori(q, mv(q1, p1)) * ori(q, mv(q1, p2)) < 0);
9 | }

```

8.5 Line Intersection

```

1 | // T: long double
2 | Pt bananaPoint(Pt p1, Pt p2, Pt q1, Pt q2) {
3 |     if (onseg(q1, p1, p2)) return q1;
4 |     if (onseg(q2, p1, p2)) return q2;
5 |     if (onseg(p1, q1, q2)) return p1;
6 |     if (onseg(p2, q1, q2)) return p2;
7 |     double s = abs(mv(p1, p2) ^ mv(p1, q1));
8 |     double t = abs(mv(p1, p2) ^ mv(p1, q2));
9 |     return q2 * (s/(s+t)) + q1 * (t/(s+t));
10 | }

```

8.6 Convex Hull

```

1 | vector<Pt> hull;
2 | void convexHull() {
3 |     hull.clear(); sort(ALL(E));
4 |     REP(t, 2) {
5 |         int b = SZ(hull);
6 |         Each(ei, E) {
7 |             while (SZ(hull) - b >= 2 &&
8 |                 ori(mv(hull[SZ(hull)-2], hull.back()),
9 |                 mv(hull[SZ(hull)-2], ei)) == -1) {
10 |                 hull.pop_back();
11 |             }
12 |             hull.eb(ei);
13 |         }
14 |         hull.pop_back();
15 |         reverse(ALL(E));
16 |     } }

```

8.7 Lower Concave Hull

```

1 | struct Line {
2 |     mutable ll m, b, p;
3 |     bool operator<(const Line& o) const { return m < o.m; }
4 |     bool operator<(ll x) const { return p < x; }
5 | };
6 |
7 | struct LineContainer : multiset<Line, less<>> {
8 |     // (for doubles, use inf = 1/.0, div(a,b) = a/b)
9 |     const ll inf = LLONG_MAX;
10 |     ll div(ll a, ll b) { // floored division
11 |         return a / b - ((a ^ b) < 0 && a % b); }
12 |     bool isect(iterator x, iterator y) {
13 |         if (y == end()) { x->p = inf; return false; }
14 |         if (x->m == y->m) x->p = x->b > y->b ? inf : -inf;
15 |         else x->p = div(y->b - x->b, x->m - y->m);
16 |         return x->p >= y->p;
17 |     }
18 |     void add(ll m, ll b) {
19 |         auto z = insert({m, b, 0}), y = z++, x = y;
20 |         while (isect(y, z)) z = erase(z);
21 |         if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
22 |         while ((y = x) != begin() && (--x)->p >= y->p)
23 |             isect(x, erase(y));
24 |     }
25 |     ll query(ll x) {
26 |         assert(!empty());
27 |         auto l = *lower_bound(x);
28 |         return l.m * x + l.b;
29 |     }
30 | };

```

8.8 Polygon Area

```

1 | T dbarea(vector<Pt>& e) {
2 |     ll res = 0;
3 |     REP(i, SZ(e)) res += e[i]^e[(i+1)%SZ(e)];
4 |     return abs(res);
5 | }

```

8.9 Pick's Theorem

Consider a polygon which vertices are all lattice points.

Let i = number of points inside the polygon.

Let b = number of points on the boundary of the polygon.

Then we have the following formula:

$$Area = i + \frac{b}{2} - 1$$

8.10 Minimum Enclosing Circle

```

1 | Pt circumcenter(Pt A, Pt B, Pt C) {
2 |     // a1(x-A.x) + b1(y-A.y) = c1
3 |     // a2(x-A.x) + b2(y-A.y) = c2
4 |     // solve using Cramer's rule
5 |     T a1 = B.x-A.x, b1 = B.y-A.y, c1 = dis2(A, B)/2.0;
6 |     T a2 = C.x-A.x, b2 = C.y-A.y, c2 = dis2(A, C)/2.0;
7 |     T D = Pt(a1, b1) ^ Pt(a2, b2);
8 |     T Dx = Pt(c1, b1) ^ Pt(c2, b2);
9 |     T Dy = Pt(a1, c1) ^ Pt(a2, c2);
10 |    if (D == 0) return Pt(-INF, -INF);
11 |    return A + Pt(Dx/D, Dy/D);
12 | }
13 | Pt center; T r2;
14 | void minEncloseCircle() {
15 |     mt19937 gen(chrono::steady_clock::now().
16 |         time_since_epoch().count());
17 |     shuffle(ALL(E), gen);
18 |     center = E[0], r2 = 0;
19 |     for (int i = 0; i < n; i++) {
20 |         if (dis2(center, E[i]) <= r2) continue;
21 |         center = E[i], r2 = 0;
22 |         for (int j = 0; j < i; j++) {
23 |             if (dis2(center, E[j]) <= r2) continue;
24 |             center = (E[i] + E[j]) / 2.0;
25 |             r2 = dis2(center, E[i]);
26 |             for (int k = 0; k < j; k++) {
27 |                 if (dis2(center, E[k]) <= r2) continue;
28 |                 center = circumcenter(E[i], E[j], E[k]);
29 |                 r2 = dis2(center, E[i]);
30 |             }
31 |         }
32 |     } }

```

8.11 PolyUnion

```

1 | struct PY{
2 |     int n; Pt pt[5]; double area;
3 |     Pt& operator[](const int x){ return pt[x]; }
4 |     void init(){ //n,pt[0~n-1] must be filled
5 |         area=pt[n-1]^pt[0];
6 |         for(int i=0;i<n-1;i++) area+=pt[i]^pt[i+1];
7 |         if((area/=2)<0)reverse(pt,pt+n),area=-area;
8 |     }
9 | };
10 | PY py[500]; pair<double,int> c[5000];
11 | inline double segP(Pt &p,Pt &p1,Pt &p2){
12 |     if(dcmp(p1.x-p2.x)==0) return (p.y-p1.y)/(p2.y-p1.y);
13 |     return (p.x-p1.x)/(p2.x-p1.x);
14 | }
15 | double polyUnion(int n){ //py[0~n-1] must be filled
16 |     int i,j,ii,jj,ta,tb,r,d; double z,w,s,sum=0,tc,td;
17 |     for(i=0;i<n;i++) py[i][py[i].n]=py[i][0];
18 |     for(i=0;i<n;i++){
19 |         for(ii=0;ii<py[i].n;ii++){
20 |             r=0;
21 |             c[r++]=make_pair(0.0,0); c[r++]=make_pair(1.0,0);
22 |             for(j=0;j<n;j++){
23 |                 if(i==j) continue;
24 |                 for(jj=0;jj<py[j].n;jj++){
25 |                     ta=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj]))
26 |                         ;
27 |                     tb=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj
28 |                         +1]));
29 |                     if(ta==0 && tb==0){
30 |                         if((py[j][jj+1]-py[j][jj])*(py[i][ii+1]-py[i][ii])>0&&j<i){

```

```

29         c[r++]=make_pair(segP(py[j][jj],py[i][ii
        ],py[i][ii+1]),1);
30         c[r++]=make_pair(segP(py[j][jj+1],py[i][
        ii],py[i][ii+1]),-1);
31     }
32     }else if(ta>=0 && tb<0){
33         tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
34         td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
35         c[r++]=make_pair(tc/(tc-td),1);
36     }else if(ta<0 && tb>=0){
37         tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
38         td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
39         c[r++]=make_pair(tc/(tc-td),-1);
40     } }
41     sort(c,c+r);
42     z=min(max(c[0].first,0.0),1.0); d=c[0].second; s
    =0;
43     for(j=1;j<r;j++){
44         w=min(max(c[j].first,0.0),1.0);
45         if(!d) s+=w-z;
46         d+=c[j].second; z=w;
47     }
48     sum+=(py[i][ii]^py[i][ii+1])*s;
49 }
50 }
51 return sum/2;
52 }

```

8.12 Minkowski Sum

```

1  /* convex hull Minkowski Sum*/
2  #define INF 10000000000000LL
3  int pos( const Pt& tp ){
4      if( tp.Y == 0 ) return tp.X > 0 ? 0 : 1;
5      return tp.Y > 0 ? 0 : 1;
6  }
7  #define N 300030
8  Pt pt[ N ], qt[ N ], rt[ N ];
9  LL Lx,Rx;
10 int dn,un;
11 inline bool cmp( Pt a, Pt b ){
12     int pa=pos( a ),pb=pos( b );
13     if(pa==pb) return (a^b)>0;
14     return pa<pb;
15 }
16 int minkowskiSum(int n,int m){
17     int i,j,r,p,q,fi,fj;
18     for(i=1,p=0;i<n;i++){
19         if( pt[i].Y<pt[p].Y ||
20             (pt[i].Y==pt[p].Y && pt[i].X<pt[p].X) ) p=i; }
21     for(i=1,q=0;i<m;i++){
22         if( qt[i].Y<qt[q].Y ||
23             (qt[i].Y==qt[q].Y && qt[i].X<qt[q].X) ) q=i; }
24     rt[0]=pt[p]+qt[q];
25     r=1; i=p; j=q; fi=fj=0;
26     while(1){
27         if((fj&&j==q) ||
28             ( (!fi||i==p) &&
29                 cmp(pt[(p+1)%n]-pt[p],qt[(q+1)%n]-qt[q]) ) ){
30             rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
31             p=(p+1)%n;
32             fi=1;
33         }else{
34             rt[r]=rt[r-1]+qt[(q+1)%n]-qt[q];
35             q=(q+1)%m;
36             fj=1;
37         }
38         if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))
39             !=0) r++;
40         else rt[r-1]=rt[r];
41         if(i==p && j==q) break;
42     }
43     return r-1;
44 }
45 void initInConvex(int n){
46     int i,p,q;
47     LL Ly,Ry;
48     Lx=INF; Rx=-INF;
49     for(i=0;i<n;i++){
50         if(pt[i].X<Lx) Lx=pt[i].X;
51         if(pt[i].X>Rx) Rx=pt[i].X;
52         Ly=Ry=INF;
53         for(i=0;i<n;i++){
54             if(pt[i].X==Lx && pt[i].Y<Ly){ Ly=pt[i].Y; p=i;
55             }
56             if(pt[i].X==Rx && pt[i].Y<Ry){ Ry=pt[i].Y; q=i;
57             }
58         }
59         for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
60         qt[dn]=pt[q]; Ly=Ry=-INF;
61         for(i=0;i<n;i++){
62             if(pt[i].X==Lx && pt[i].Y>Ly){ Ly=pt[i].Y; p=i;
63             }
64             if(pt[i].X==Rx && pt[i].Y>Ry){ Ry=pt[i].Y; q=i;
65             }
66         }
67         for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
68         rt[un]=pt[q];
69     }
70     inline int inConvex(Pt p){
71         int L,R,M;
72         if(p.X<Lx || p.X>Rx) return 0;
73         L=0;R=dn;
74         while(L<R-1){ M=(L+R)/2;
75             if(p.X<qt[M].X) R=M; else L=M; }
76         if(tri(qt[L],qt[R],p)<0) return 0;
77         L=0;R=un;
78         while(L<R-1){ M=(L+R)/2;
79             if(p.X<rt[M].X) R=M; else L=M; }
80         if(tri(rt[L],rt[R],p)>0) return 0;
81         return 1;
82     }
83     int main(){
84         int n,m,i;
85         Pt p;
86         scanf("%d",&n);
87         for(i=0;i<n;i++) scanf("%lld%lld",&pt[i].X,&pt[i].Y
88             );
89         scanf("%d",&m);
90         for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y
91             );
92         n=minkowskiSum(n,m);
93         for(i=0;i<n;i++) pt[i]=rt[i];
94         scanf("%d",&m);
95         for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y
96             );
97         n=minkowskiSum(n,m);
98         for(i=0;i<n;i++) pt[i]=rt[i];
99         initInConvex(n);
100        scanf("%d",&m);
101        for(i=0;i<m;i++){
102            scanf("%lld %lld",&p.X,&p.Y);
103            p.X*=3; p.Y*=3;
104            puts(inConvex(p)?"YES":"NO");
105        }
106    }

```

9 Number Theory

9.1 Pollard's rho

```

1  from itertools import count
2  from math import gcd
3  from sys import stdin
4
5  for s in stdin:
6      number, x = int(s), 2
7      break2 = False
8      for cycle in count(1):
9          y = x
10         if break2:
11             break
12         for i in range(1 << cycle):
13             x = (x * x + 1) % number
14             factor = gcd(x - y, number)
15             if factor > 1:
16                 print(factor)
17                 break2 = True
18                 break

```

9.2 Miller Rabin

```

1 // n < 4,759,123,141      3 : 2, 7, 61
2 // n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
3 // n < 3,474,749,660,383  6 : pirmses <= 13
4 // n < 2^64              7 :
5 // 2, 325, 9375, 28178, 450775, 9780504, 1795265022
6 bool witness(ll a, ll n, ll u, int t){
7     if(!(a%n)) return 0;
8     ll x=myspow(a,u,n);
9     for(int i=0; i<t; i++) {
10         ll nx=mul(x,x,n);
11         if(nx==1&&x!=1&&x!=n-1) return 1;
12         x=nx;
13     }
14     return x!=1;
15 }
16 bool miller_rabin(ll n, int s=100) {
17     // iterate s times of witness on n
18     // return 1 if prime, 0 otherwise
19     if(n<2) return 0;
20     if(!(n&1)) return n == 2;
21     ll u=n-1; int t=0;
22     while(!(u&1)) u>>=1, t++;
23     while(s--){
24         ll a=randll()%n-1+1;
25         if(witness(a,n,u,t)) return 0;
26     }
27     return 1;
28 }

```

9.3 Fast Power

Note: $a^n \equiv a^{(n \bmod (p-1))} \pmod{p}$

9.4 Extend GCD

```

1 ll GCD;
2 pll extgcd(ll a, ll b) {
3     if (b == 0) {
4         GCD = a;
5         return pll{1, 0};
6     }
7     pll ans = extgcd(b, a % b);
8     return pll{ans.S, ans.F - a/b * ans.S};
9 }
10 pll bezout(ll a, ll b, ll c) {
11     bool negx = (a < 0), negy = (b < 0);
12     pll ans = extgcd(abs(a), abs(b));
13     if (c % GCD != 0) return pll{-LLINF, -LLINF};
14     return pll{ans.F * c/GCD * (negx ? -1 : 1),
15               ans.S * c/GCD * (negy ? -1 : 1)};
16 }
17 ll inv(ll a, ll p) {
18     if (p == 1) return -1;
19     pll ans = bezout(a % p, -p, 1);
20     if (ans == pll{-LLINF, -LLINF}) return -1;
21     return (ans.F % p + p) % p;
22 }

```

9.5 Mu + Phi

```

1 const int maxn = 1e6 + 5;
2 ll f[maxn];
3 vector<int> lpf, prime;
4 void build() {
5     lpf.clear(); lpf.resize(maxn, 1);
6     prime.clear();
7     f[1] = ...; /* mu[1] = 1, phi[1] = 1 */
8     for (int i = 2; i < maxn; i++) {
9         if (lpf[i] == 1) {
10             lpf[i] = i; prime.emplace_back(i);
11             f[i] = ...; /* mu[i] = 1, phi[i] = i-1 */
12         }
13         for (auto& j : prime) {
14             if (i*j >= maxn) break;
15             lpf[i*j] = j;
16             if (i % j == 0) f[i*j] = ...; /* 0, phi[i]*j */
17             else f[i*j] = ...; /* -mu[i], phi[i]*phi[j] */
18         }
19     }
20 }

```

```

18     if (j >= lpf[i]) break;
19 } } }

```

9.6 Other Formulas

- Inversion:**
 $aa^{-1} \equiv 1 \pmod{m}$. a^{-1} exists iff $\gcd(a, m) = 1$.
- Linear inversion:**
 $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod{m}$
- Fermat's little theorem:**
 $a^p \equiv a \pmod{p}$ if p is prime.
- Euler function:**
 $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$
- Euler theorem:**
 $a^{\phi(n)} \equiv 1 \pmod{n}$ if $\gcd(a, n) = 1$.
- Extended Euclidean algorithm:**
 $ax + by = \gcd(a, b) = \gcd(b, a \bmod b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$
- Divisor function:**
 $\sigma_x(n) = \sum_{d|n} d^x$. $n = \prod_{i=1}^r p_i^{a_i}$.
 $\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$ if $x \neq 0$. $\sigma_0(n) = \prod_{i=1}^r (a_i + 1)$.
- Chinese remainder theorem (Coprime Moduli):**
 $x \equiv a_i \pmod{m_i}$.
 $M = \prod m_i$. $M_i = M/m_i$. $t_i = M_i^{-1}$.
 $x = kM + \sum a_i t_i M_i$, $k \in \mathbb{Z}$.
- Chinese remainder theorem:**
 $x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2} \Rightarrow x = m_1 p + a_1 = m_2 q + a_2 \Rightarrow m_1 p - m_2 q = a_2 - a_1$
Solve for (p, q) using ExtGCD.
 $x \equiv m_1 p + a_1 \equiv m_2 q + a_2 \pmod{\text{lcm}(m_1, m_2)}$
- Avoiding Overflow:** $ca \bmod cb = c(a \bmod b)$
- Dirichlet Convolution:** $(f * g)(n) = \sum_{d|n} f(n)g(n/d)$
- Important Multiplicative Functions + Properties:**
 - $\epsilon(n) = [n = 1]$
 - $1(n) = 1$
 - $id(n) = n$
 - $\mu(n) = 0$ if n has squared prime factor
 - $\mu(n) = (-1)^k$ if $n = p_1 p_2 \cdots p_k$
 - $\epsilon = \mu * 1$
 - $\phi = \mu * id$
 - $[n = 1] = \sum_{d|n} \mu(d)$
 - $[gcd = 1] = \sum_{d|gcd} \mu(d)$
- Möbius inversion:** $f = g * 1 \Leftrightarrow g = f * \mu$

9.7 Polynomial

```

1 const int maxk = 20;
2 const int maxn = 1<<maxk;
3 const ll LINF = 1e18;
4
5 /* P = r*2^k + 1
6 P      r      k      g
7 998244353 119 23 3
8 1004535809 479 21 3
9
10 P      r      k      g
11 3      1      1      2
12 5      1      2      2
13 17     1      4      3
14 97     3      5      5

```



```

15 193          3 6 5
16 257          1 8 3
17 7681         15 9 17
18 12289        3 12 11
19 40961        5 13 3
20 65537        1 16 3
21 786433       3 18 10
22 5767169      11 19 3
23 7340033      7 20 3
24 23068673     11 21 3
25 104857601    25 22 3
26 167772161    5 25 3
27 469762049    7 26 3
28 1004535809   479 21 3
29 2013265921   15 27 31
30 2281701377   17 27 3
31 3221225473   3 30 5
32 75161927681  35 31 3
33 77309411329  9 33 7
34 206158430209 3 36 22
35 2061584302081 15 37 7
36 2748779069441 5 39 3
37 6597069766657 3 41 5
38 3958241859937 9 42 5
39 79164837199873 9 43 5
40 263882790666241 15 44 7
41 1231453023109121 35 45 3
42 1337006139375617 19 46 3
43 37999912185593857 27 47 5
44 4222124650659841 15 48 19
45 7881299347898369 7 50 6
46 31525197391593473 7 52 3
47 180143985094819841 5 55 6
48 1945555039024054273 27 56 5
49 4179340454199820289 29 57 3
50 9097271247288401921 505 54 6 */

51 const int g = 3;
52 const ll MOD = 998244353;

53 ll pw(ll a, ll n) { /* fast pow */ }

54 #define siz(x) (int)x.size()
55
56 template<typename T>
57 vector<T>& operator+=(vector<T>& a, const vector<T>& b)
58 {
59     if (siz(a) < siz(b)) a.resize(siz(b));
60     for (int i = 0; i < min(siz(a), siz(b)); i++) {
61         a[i] += b[i];
62         a[i] -= a[i] >= MOD ? MOD : 0;
63     }
64     return a;
65 }
66
67 template<typename T>
68 vector<T>& operator-=(vector<T>& a, const vector<T>& b)
69 {
70     if (siz(a) < siz(b)) a.resize(siz(b));
71     for (int i = 0; i < min(siz(a), siz(b)); i++) {
72         a[i] -= b[i];
73         a[i] += a[i] < 0 ? MOD : 0;
74     }
75     return a;
76 }
77
78 template<typename T>
79 vector<T> operator-(const vector<T>& a) {
80     vector<T> ret(siz(a));
81     for (int i = 0; i < siz(a); i++) {
82         ret[i] = -a[i] < 0 ? -a[i] + MOD : -a[i];
83     }
84     return ret;
85 }
86
87 vector<ll> X, iX;
88 vector<int> rev;
89
90 void init_ntt() {
91     X.clear(); X.resize(maxn, 1); // x1 = g^((p-1)/n)
92     iX.clear(); iX.resize(maxn, 1);
93
94     ll u = pw(g, (MOD-1)/maxn);
95     ll iu = pw(u, MOD-2);
96
97     for (int i = 1; i < maxn; i++) {
98         X[i] = X[i-1] * u;
99         iX[i] = iX[i-1] * iu;
100         if (X[i] >= MOD) X[i] %= MOD;
101         if (iX[i] >= MOD) iX[i] %= MOD;
102     }
103
104     rev.clear(); rev.resize(maxn, 0);
105     for (int i = 1, hb = -1; i < maxn; i++) {
106         if (!(i & (i-1))) hb++;
107         rev[i] = rev[i ^ (1<<hb)] | (1<<(maxk-hb-1));
108     }
109
110 template<typename T>
111 void NTT(vector<T>& a, bool inv=false) {
112
113     int _n = (int)a.size();
114     int k = __lg(_n) + ((1<<__lg(_n)) != _n);
115     int n = 1<<k;
116     a.resize(n, 0);
117
118     short shift = maxk-k;
119     for (int i = 0; i < n; i++)
120         if (i > (rev[i]>>shift))
121             swap(a[i], a[rev[i]>>shift]);
122
123     for (int len = 2, half = 1, div = maxn>>1; len <= n
124         ; len<=1, half<=1, div>=1) {
125         for (int i = 0; i < n; i += len) {
126             for (int j = 0; j < half; j++) {
127                 T u = a[i+j];
128                 T v = a[i+j+half] * (inv ? iX[j*div] :
129                     X[j*div]) % MOD;
130                 a[i+j] = (u+v >= MOD ? u+v-MOD : u+v);
131                 a[i+j+half] = (u-v < 0 ? u-v+MOD : u-v);
132             }
133         }
134     }
135
136     if (inv) {
137         T dn = pw(n, MOD-2);
138         for (auto& x : a) {
139             x *= dn;
140             if (x >= MOD) x %= MOD;
141         }
142     }
143
144 template<typename T>
145 inline void resize(vector<T>& a) {
146     int cnt = (int)a.size();
147     for (; cnt > 0; cnt--) if (a[cnt-1]) break;
148     a.resize(max(cnt, 1));
149 }
150
151 template<typename T>
152 vector<T>& operator*(vector<T>& a, vector<T> b) {
153     int na = (int)a.size();
154     int nb = (int)b.size();
155     a.resize(na + nb - 1, 0);
156     b.resize(na + nb - 1, 0);
157
158     NTT(a); NTT(b);
159     for (int i = 0; i < (int)a.size(); i++) {
160         a[i] *= b[i];
161         if (a[i] >= MOD) a[i] %= MOD;
162     }
163     NTT(a, true);
164
165     resize(a);
166     return a;
167 }
168
169 template<typename T>
170 void inv(vector<T>& ia, int N) {
171     vector<T> _a(move(ia));
172     ia.resize(1, pw(_a[0], MOD-2));
173     vector<T> a(1, -_a[0] + (-_a[0] < 0 ? MOD : 0));
174
175     for (int n = 1; n < N; n<=1) {
176         // n -> 2*n
177         // ia' = ia(2-a*ia);
178     }
179 }

```



```

174     for (int i = n; i < min(siz(_a), (n<<1)); i++)
175         a.emplace_back(-_a[i] + (-_a[i] < 0 ? MOD :
176             0));
177
178     vector<T> tmp = ia;
179     ia *= a;
180     ia.resize(n<<1);
181     ia[0] = ia[0] + 2 >= MOD ? ia[0] + 2 - MOD : ia
182         [0] + 2;
183     ia *= tmp;
184     ia.resize(n<<1);
185 }
186 }
187
188 template<typename T>
189 void mod(vector<T>& a, vector<T>& b) {
190     int n = (int)a.size()-1, m = (int)b.size()-1;
191     if (n < m) return;
192
193     vector<T> ra = a, rb = b;
194     reverse(ra.begin(), ra.end()); ra.resize(min(n+1, n
195         -m+1));
196     reverse(rb.begin(), rb.end()); rb.resize(min(m+1, n
197         -m+1));
198
199     inv(rb, n-m+1);
200
201     vector<T> q = move(ra);
202     q *= rb;
203     q.resize(n-m+1);
204     reverse(q.begin(), q.end());
205
206     q *= b;
207     a -= q;
208     resize(a);
209 }
210
211 /* Kitamasa Method (Fast Linear Recurrence):
212 Find a[K] (Given a[j] = c[0]a[j-N] + ... + c[N-1]a[j
213     -1])
214 Let B(x) = x^N - c[N-1]x^(N-1) - ... - c[1]x^1 - c[0]
215 Let R(x) = x^K mod B(x) (get x^K using fast pow and
216     use poly mod to get R(x))
217 Let r[i] = the coefficient of x^i in R(x)
218 => a[K] = a[0]r[0] + a[1]r[1] + ... + a[N-1]r[N-1] */

```

10 Linear Algebra

10.1 Gaussian-Jordan Elimination

```

1 int n; vector<vector<ll>> > v;
2 void gauss(vector<vector<ll>>& v) {
3     int r = 0;
4     for (int i = 0; i < n; i++) {
5         bool ok = false;
6         for (int j = r; j < n; j++) {
7             if (v[j][i] == 0) continue;
8             swap(v[j], v[r]);
9             ok = true; break;
10        }
11        if (!ok) continue;
12        ll div = inv(v[r][i]);
13        for (int j = 0; j < n+1; j++) {
14            v[r][j] *= div;
15            if (v[r][j] >= MOD) v[r][j] %= MOD;
16        }
17        for (int j = 0; j < n; j++) {
18            if (j == r) continue;
19            ll t = v[j][i];
20            for (int k = 0; k < n+1; k++) {
21                v[j][k] -= v[r][k] * t % MOD;
22                if (v[j][k] < 0) v[j][k] += MOD;
23            }
24            r++;
25        }
26    }
27 }

```

10.2 Determinant

1. Use GJ Elimination, if there's any row consists of only 0, then $\det = 0$, otherwise $\det = \text{product of diagonal elements}$.

2. Properties of \det :

- Transpose: Unchanged
- Row Operation 1 - Swap 2 rows: $-\det$
- Row Operation 2 - $k\vec{r}_i$: $k \times \det$
- Row Operation 3 - $k\vec{r}_i$ add to \vec{r}_j : Unchanged

11 Flow / Matching

11.1 Dinic

```

1 struct Dinic
2 {
3     int n,s,t,level[N],iter[N];
4     struct edge{int to,cap,rev;};
5     vector<edge>path[N];
6     void init(int _n,int _s,int _t)
7     {
8         n=_n,s=_s,t=_t;
9         FOR(i,0,n+1)path[i].clear();
10    }
11    void add(int a,int b,int c)
12    {
13        edge now;
14        now.to=b,now.cap=c,now.rev=sz(path[b]);
15        path[a].pb(now);
16        now.to=a,now.cap=0,now.rev=sz(path[a])-1;
17        path[b].pb(now);
18    }
19    void bfs()
20    {
21        memset(level,-1,sizeof(level));
22        level[s]=0;
23        queue<int>q;q.push(s);
24        while(q.size())
25        {
26            int now=q.front();q.pop();
27            for(edge e:path[now])
28            {
29                if(e.cap>0&&level[e.to]==-1)
30                {
31                    level[e.to]=level[now]+1;
32                    q.push(e.to);
33                }
34            }
35        }
36    }
37    int dfs(int now,int flow)
38    {
39        if(now==t)return flow;
40        for(int &i=iter[now];i<sz(path[now]);i++)
41        {
42            edge &e=path[now][i];
43            if(e.cap>0&&level[e.to]==level[now]+1)
44            {
45                int res=dfs(e.to,min(flow,e.cap));
46                if(res>0)
47                {
48                    e.cap-=res;
49                    path[e.to][e.rev].cap+=res;
50                    return res;
51                }
52            }
53        }
54        return 0;
55    }
56    int dinic()
57    {
58        int res=0;
59        while(true)
60        {
61            bfs();
62            if(level[t]==-1)break;
63            memset(iter,0,sizeof(iter));

```

```

64         int now=0;
65         while((now=dfs(s, INF))>0)res+=now;
66     }
67     return res;
68 }
69 };

```

11.2 ISAP

```

1 #define SZ(c) ((int)(c).size())
2 struct Maxflow{
3     static const int MAXV=50010;
4     static const int INF =1000000;
5     struct Edge{
6         int v,c,r;
7         Edge(int _v,int _c,int _r):v(_v),c(_c),r(_r){}
8     };
9     int s,t; vector<Edge> G[MAXV];
10    int iter[MAXV],d[MAXV],gap[MAXV],tot;
11    void init(int n,int _s,int _t){
12        tot=n,s=_s,t=_t;
13        for(int i=0;i<=tot;i++){
14            G[i].clear(); iter[i]=d[i]=gap[i]=0;
15        }
16    }
17    void addEdge(int u,int v,int c){
18        G[u].push_back(Edge(v,c,SZ(G[v])));
19        G[v].push_back(Edge(u,0,SZ(G[u])-1));
20    }
21    int DFS(int p,int flow){
22        if(p==t) return flow;
23        for(int &i=iter[p];i<SZ(G[p]);i++){
24            Edge &e=G[p][i];
25            if(e.c>0&&d[p]==d[e.v]+1){
26                int f=DFS(e.v,min(flow,e.c));
27                if(f){ e.c-=f; G[e.v][e.r].c+=f; return f; }
28            }
29        }
30        if(--gap[d[p]]==0) d[s]=tot;
31        else{ d[p]++; iter[p]=0; ++gap[d[p]]; }
32        return 0;
33    }
34    int flow(){
35        int res=0;
36        for(res=0,gap[0]=tot;d[s]<tot;res+=DFS(s,INF));
37        return res;
38    } // reset: set iter,d,gap to 0
39 } flow;

```

11.3 MCMF

```

1 struct MCMF
2 {
3     int n,s,t,par[N+5],p_i[N+5],dis[N+5],vis[N+5];
4     struct edge{int to,cap,rev,cost;};
5     vector<edge>path[N];
6     void init(int _n,int _s,int _t)
7     {
8         n=_n,s=_s,t=_t;
9         FOR(i,0,2*n+5)par[i]=p_i[i]=vis[i]=0;
10    }
11    void add(int a,int b,int c,int d)
12    {
13        path[a].pb({b,c,sz(path[b]),d});
14        path[b].pb({a,0,sz(path[a])-1,-d});
15    }
16    void spfa()
17    {
18        FOR(i,0,n*2+5)dis[i]=INF,vis[i]=0;
19        dis[s]=0;
20        queue<int>q;q.push(s);
21        while(!q.empty())
22        {
23            int now=q.front();
24            q.pop();
25            vis[now]=0;
26            for(int i=0;i<sz(path[now]);i++)
27            {
28                edge e=path[now][i];
29                if(e.cap>0&&dis[e.to]>dis[now]+e.cost)

```

```

30        {
31            dis[e.to]=dis[now]+e.cost;
32            par[e.to]=now;
33            p_i[e.to]=i;
34            if(vis[e.to]==0)
35            {
36                vis[e.to]=1;
37                q.push(e.to);
38            }
39        }
40    }
41 }
42
43 pii flow()
44 {
45     int flow=0,cost=0;
46     while(true)
47     {
48         spfa();
49         if(dis[t]==INF)break;
50         int mn=INF;
51         for(int i=t;i!=s;i=par[i])
52             mn=min(mn,path[par[i]][p_i[i]].cap);
53         flow+=mn;cost+=dis[t]*mn;
54         for(int i=t;i!=s;i=par[i])
55         {
56             edge &now=path[par[i]][p_i[i]];
57             now.cap-=mn;
58             path[i][now.rev].cap+=mn;
59         }
60     }
61     return mp(flow,cost);
62 }
63 };

```

11.4 Hopcroft-Karp

```

1 struct HopcroftKarp {
2     // id: X = [1, nx], Y = [nx+1, nx+ny]
3     int n, nx, ny, m, MXCNT;
4     vector<vector<int>> g;
5     vector<int> mx, my, dis, vis;
6     void init(int nnx, int nny, int mm) {
7         nx = nnx, ny = nny, m = mm;
8         n = nx + ny + 1;
9         g.clear(); g.resize(n);
10    }
11    void add(int x, int y) {
12        g[x].emplace_back(y);
13        g[y].emplace_back(x);
14    }
15    bool dfs(int x) {
16        vis[x] = true;
17        Each(y, g[x]) {
18            int px = my[y];
19            if (px == -1 ||
20                (dis[px] == dis[x]+1 &&
21                 !vis[px] && dfs(px))) {
22                mx[x] = y;
23                my[y] = x;
24                return true;
25            }
26        }
27        return false;
28    }
29    void get() {
30        mx.clear(); mx.resize(n, -1);
31        my.clear(); my.resize(n, -1);
32    }
33    while (true) {
34        queue<int> q;
35        dis.clear(); dis.resize(n, -1);
36        for (int x = 1; x <= nx; x++){
37            if (mx[x] == -1) {
38                dis[x] = 0;
39                q.push(x);
40            }
41        }
42        while (!q.empty()) {
43            int x = q.front(); q.pop();
44            Each(y, g[x]) {

```

```

45         if (my[y] != -1 && dis[my[y]] ==
46             -1) {
47             dis[my[y]] = dis[x] + 1;
48             q.push(my[y]);
49         }
50     }
51
52     bool brk = true;
53     vis.clear(); vis.resize(n, 0);
54     for (int x = 1; x <= nx; x++)
55         if (mx[x] == -1 && dfs(x))
56             brk = false;
57
58     if (brk) break;
59 }
60 MXCNT = 0;
61 for (int x = 1; x <= nx; x++) if (mx[x] != -1)
62     MXCNT++;
63 } hk;

```

11.5 Cover / Independent Set

```

1 V(E) Cover: choose some V(E) to cover all E(V)
2 V(E) Independ: set of V(E) not adj to each other
3
4 M = Max Matching
5 Cv = Min V Cover
6 Ce = Min E Cover
7 Iv = Max V Ind
8 Ie = Max E Ind (equiv to M)
9
10 M = Cv (Konig Theorem)
11 Iv = V \ Cv
12 Ce = V - M
13
14 Construct Cv:
15 1. Run Dinic
16 2. Find s-t min cut
17 3. Cv = {X in T} + {Y in S}

```

11.6 KM

```

1 struct KM
2 {
3     int n, mx[1005], my[1005], pa[1005];
4     int g[1005][1005], lx[1005], ly[1005], sy[1005];
5     bool vx[1005], vy[1005];
6     void init(int _n)
7     {
8         n=_n;
9         FOR(i,1,n+1) fill(g[i],g[i]+1+n,0);
10    }
11    void add(int a,int b,int c){g[a][b]=c;}
12    void augment(int y)
13    {
14        for(int x,z;y;y=z)
15            x=pa[y],z=mx[x],my[y]=x,mx[x]=y;
16    }
17    void bfs(int st)
18    {
19        FOR(i,1,n+1) sy[i]=INF, vx[i]=vy[i]=0;
20        queue<int> q; q.push(st);
21        for(;;)
22        {
23            while(!q.empty())
24            {
25                int x=q.front(); q.pop();
26                vx[x]=1;
27                FOR(y,1,n+1) if(!vy[y])
28                {
29                    int t=lx[x]+ly[y]-g[x][y];
30                    if(t==0)
31                    {
32                        pa[y]=x;
33                        if(!my[y]){augment(y);return;}
34                        vy[y]=1, q.push(my[y]);
35                    }
36                    else if(sy[y]>t) pa[y]=x, sy[y]=t;

```

```

37        }
38    }
39    int cut=INF;
40    FOR(y,1,n+1) if(!vy[y]&&cut>sy[y]) cut=sy[y];
41    FOR(j,1,n+1)
42    {
43        if(vx[j]) lx[j]-=cut;
44        if(vy[j]) ly[j]+=cut;
45        else sy[j]-=cut;
46    }
47    FOR(y,1,n+1)
48    {
49        if(!vy[y]&&sy[y]==0)
50        {
51            if(!my[y]){augment(y);return;}
52            vy[y]=1; q.push(my[y]);
53        }
54    }
55    }
56 }
57 int solve()
58 {
59     fill(mx,mx+n+1,0); fill(my,my+n+1,0);
60     fill(ly,ly+n+1,0); fill(lx,lx+n+1,0);
61     FOR(x,1,n+1) FOR(y,1,n+1)
62         lx[x]=max(lx[x],g[x][y]);
63     FOR(x,1,n+1) bfs(x);
64     int ans=0;
65     FOR(y,1,n+1) ans+=g[my[y]][y];
66     return ans;
67 }
68 };

```

12 Combinatorics

12.1 Catalan Number

$$C_0 = 1, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}, C_n = C_n^{2n} - C_{n-1}^{2n}$$

0	1	1	2	5
4	14	42	132	429
8	1430	4862	16796	58786
12	208012	742900	2674440	9694845

12.2 Burnside's Lemma

Let X be the original set.

Let G be the group of operations acting on X .

Let X^g be the set of x not affected by g .

Let X/G be the set of orbits.

Then the following equation holds:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

13 Special Numbers

13.1 Fibonacci Series

1	1	1	2	3
5	5	8	13	21
9	34	55	89	144
13	233	377	610	987
17	1597	2584	4181	6765
21	10946	17711	28657	46368
25	75025	121393	196418	317811
29	514229	832040	1346269	2178309
33	3524578	5702887	9227465	14930352

$$f(45) \approx 10^9, f(88) \approx 10^{18}$$

13.2 Prime Numbers

- First 50 prime numbers:

1	2	3	5	7	11
6	13	17	19	23	29
11	31	37	41	43	47
16	53	59	61	67	71
21	73	79	83	89	97
26	101	103	107	109	113
31	127	131	137	139	149
36	151	157	163	167	173
41	179	181	191	193	197
46	199	211	223	227	229

- Very large prime numbers:

1000001333 1000500889 2500001909
 2000000659 900004151 850001359

- $\pi(n) \equiv$ Number of primes $\leq n \approx n/((\ln n) - 1)$

$$\pi(100) = 25, \pi(200) = 46$$

$$\pi(500) = 95, \pi(1000) = 168$$

$$\pi(2000) = 303, \pi(4000) = 550$$

$$\pi(10^4) = 1229, \pi(10^5) = 9592$$

$$\pi(10^6) = 78498, \pi(10^7) = 664579$$

