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0.1 OwO

- 可以構造複雜點的測資幫助思考
- 真的卡太久請跳題
- Enjoy The Contest!

1 Basic

1.1 Vimrc

```

1 set number relativenumber ai t_Co=256 tabstop=4
2 set mouse=a shiftwidth=4 encoding=utf8
3 set bs=2 ruler laststatus=2 cmdheight=2
4 set clipboard=unnamedplus showcmd autoread
5 set belloff=all
6 filetype indent on
7
8 inoremap ( ()<Esc>i
9 inoremap " ""<Esc>i
10 inoremap [ []<Esc>i
11 inoremap ' ''<Esc>i
12 inoremap { {<CR>}<Esc>ko
13
14 nnoremap <tab> gt
15 nnoremap <S-tab> gT
16 inoremap <C-n> <Esc>:tabnew<CR>
17 nnoremap <C-n> :tabnew<CR>
18
19 inoremap <F9> <Esc>:w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>
20 nnoremap <F9> :w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>
21
22 syntax on
23 colorscheme desert
24 set filetype=cpp
25 set background=dark

```

Codebook

1.2 Stress

```

g++ gen.cpp -o gen.out
g++ ac.cpp -o ac.out
g++ wa.cpp -o wa.out
for ((i=0;i++))
do
echo "$i"
./gen.out > in.txt
./ac.out < in.txt > ac.txt
./wa.out < in.txt > wa.txt
if [ "$?" -ne 0 ]; then
exit 1
fi
{ cat in.txt; echo; cat ac.txt; } > case_i.txt
diff ac.txt wa.txt || break
done

```

1.3 Run Sample

```

prog=$1
shift
g++ -O2 -std=c++20 -fsanitize=address -Wall -Wextra -
Wshadow ${prog}.cpp -o ${prog}.out
for f in "$@"; do
out=${prog}_${(basename "$f")}.out
echo "input: $f"
cat "$f"
echo "output: $out"
./${prog}.out < "$f" | tee "$out"
echo
done

```

1.4 Runcpp.sh

```

#!/bin/bash
clear
echo "Start compiling $1..."
echo
g++ -O2 -std=c++20 -fsanitize=address -Wall -Wextra -
Wshadow $2/$1 -o $2/out
if [ "$?" -ne 0 ]
then
exit 1
fi
echo
echo "Done compiling"
echo =====
echo
echo "Input file:"
echo
cat $2/in.txt
echo
echo =====
echo
declare startTime=`date +%s%N`
$2/out < $2/in.txt > $2/out.txt
declare endTime=`date +%s%N`
delta=`expr $endTime - $startTime`
delta=`expr $delta / 1000000`
cat $2/out.txt
echo
echo "time: $delta ms"

```

1.5 Others

```

1 #pragma GCC optimize("Ofast,unroll-loops,no-stack-
protector,fast-math")
2 #pragma GCC target("see,see2,see3,see4,avx2,bmi,bmi2,
3 lzcnt,popcnt,tune=native")
# pragma GCC optimize("trapv")
4 mt19937 gen(chrono::steady_clock::now().
5   time_since_epoch().count());
uniform_int_distribution<int> dis(1, 100);
cout << dis(gen) << endl;
shuffle(v.begin(), v.end(), gen);
6
7 struct edge {
8   int a, b, w;
9
10

```

```

11 friend istream& operator>>(istream& in, edge& x) {
12     in >> x.a >> x.b >> x.w; }
13 friend ostream& operator<<(ostream& out, const edge&
14     & x) {
15     out << "(" << x.a << ", " << x.b << ", " << x.w
16     << ")";
17     return out;
18 }
19 };
20 struct cmp {
21     bool operator()(const edge& x, const edge& y) const
22     { return x.w < y.w; }
23 };
24 set<edge, cmp> st; // 遞增
25 map<edge, long long, cmp> mp; // 遞增
26 priority_queue<edge, vector<edge>, cmp> pq; // 遞減
27
28 #include <bits/extc++.h>
29 #include <ext/pb_ds/assoc_container.hpp>
30 #include <ext/pb_ds/tree_policy.hpp>
31 using namespace __gnu_pbds;
32
33 // map
34 tree<int, int, less<>, rb_tree_tag,
35     tree_order_statistics_node_update> tr;
36 tr.order_of_key(element);
37 tr.find_by_order(rank);
38
39 gp_hash_table<int, int> ht;
40 ht.find(element);
41 ht.insert({key, value});
42 ht.erase(element);
43
44 // priority queue Big First
45 __gnu_pbds::priority_queue<int, less<int>> big_q;
46 __gnu_pbds::priority_queue<int, greater<int>> small_q;
47     // Small First
48 q1.join(q2); // join

```

2 Data Structure

2.1 BIT

```

1 struct BIT {
2     int n;
3     long long bit[N];
4
5     void init(int x, vector<long long> &a) {
6         n = x;
7         for (int i = 1, j; i <= n; i++) {
8             bit[i] += a[i - 1], j = i + (i & -i);
9             if (j <= n) bit[j] += bit[i];
10        }
11    }
12
13    void update(int x, long long dif) {
14        while (x <= n) bit[x] += dif, x += x & -x;
15    }
16
17    long long query(int l, int r) {
18        if (l != 1) return query(1, r) - query(1, l -
19            1);
20
21        long long ret = 0;
22        while (l <= r) ret += bit[r], r -= r & -r;
23        return ret;
24    }
25 } bm;

```

2.2 Lazy Propagation Segment Tree

```

1 struct lazy_propagation{
2     // 0-based, [l, r], tg[0]->add, tg[1]->set
3     ll seg[N * 4], tg[2][N*4];
4     void assign (bool op, ll val, int idx){

```

```

5         if (op == 0){
6             if (tg[1][idx]) tg[1][idx] += val;
7             else tg[0][idx] += val;
8         }
9         else seg[idx] = 0, tg[0][idx] = 0, tg[1][idx]
10            = val;
11     }
12     sum (int idx, int len){
13         if (tg[1][idx]) return tg[1][idx] * len;
14         return tg[0][idx] * len + seg[idx];
15     }
16     void pull (int idx, int len){
17         seg[idx] = sum(2*idx, (len+1)/2) + sum(2*idx+1,
18             len/2);
19     }
20     void push (int idx){
21         if (!tg[0][idx] && !tg[1][idx]) return ;
22         if (tg[0][idx]){
23             assign(0, tg[0][idx], 2*idx);
24             assign(0, tg[0][idx], 2*idx+1);
25             tg[0][idx] = 0;
26         }
27         else{
28             assign(1, tg[1][idx], 2*idx);
29             assign(1, tg[1][idx], 2*idx+1);
30             tg[1][idx] = 0;
31         }
32     }
33     void update (bool op, ll val, int gl, int gr, int l
34 , int r, int idx){
35         if (r < l || gr < l || r < gl) return ;
36         if (gl <= l && r <= gr){
37             assign(op, val, idx);
38             return ;
39         }
40
41         int mid = (l + r) / 2;
42         push(idx);
43         update(op, val, gl, gr, l, mid, 2*idx);
44         update(op, val, gl, gr, mid+1, r, 2*idx+1);
45         pull(idx, r-l+1);
46     }
47     int query (int gl, int gr, int l, int r, int idx){
48         if (r < l || gr < l || r < gl) return 0;
49         if (gl <= l && r <= gr) return sum(idx, r-l+1);
50
51         push(idx), pull(idx, r-l+1);
52         int mid = (l + r) / 2;
53         return query(gl, gr, l, mid, 2*idx) + query(gl,
54             gr, mid+1, r, 2*idx+1);
55     }
56 }bm;

```

2.3 Treap

```

1 mt19937 rng(random_device{}());
2 struct Treap {
3     Treap *l, *r;
4     int val, sum, real, tag, num, pri, rev;
5     Treap(int k) {
6         l = r = NULL;
7         val = sum = k;
8         num = 1;
9         real = -1;
10        tag = 0;
11        rev = 0;
12        pri = rng();
13    }
14    int siz(Treap *now) { return now ? now->num : 0ll; }
15    int sum(Treap *now) {
16        if (!now) return 0;
17        if (now->real != -1) return (now->real + now->tag)
18            * now->num;
19        return now->sum + now->tag * now->num;
20    }
21    void pull(Treap *&now) {
22        now->num = siz(now->l) + siz(now->r) + 1ll;
23        now->sum = sum(now->l) + sum(now->r) + now->val +
24            now->tag;
25    }
26    void push(Treap *&now) {

```

```

26     if (now->rev) {
27         swap(now->l, now->r);
28         now->l->rev ^= 1;
29         now->r->rev ^= 1;
30         now->rev = 0;
31     }
32     if (now->real != -1) {
33         now->real += now->tag;
34         if (now->l) {
35             now->l->tag = 0;
36             now->l->real = now->real;
37             now->l->val = now->real;
38         }
39         if (now->r) {
40             now->r->tag = 0;
41             now->r->real = now->real;
42             now->r->val = now->real;
43         }
44         now->val = now->real;
45         now->sum = now->real * now->num;
46         now->real = -1;
47         now->tag = 0;
48     } else {
49         if (now->l) now->l->tag += now->tag;
50         if (now->r) now->r->tag += now->tag;
51         now->sum += sum(now);
52         now->val += now->tag;
53         now->tag = 0;
54     }
55 }
56 Treap *merge(Treap *a, Treap *b) {
57     if (!a || !b) return a ? a : b;
58     else if (a->pri > b->pri) {
59         push(a);
60         a->r = merge(a->r, b);
61         pull(a);
62         return a;
63     } else {
64         push(b);
65         b->l = merge(a, b->l);
66         pull(b);
67         return b;
68     }
69 }
70 void split_size(Treap *rt, Treap *&a, Treap *&b, int
71 val) {
72     if (!rt) {
73         a = b = NULL;
74         return;
75     }
76     push(rt);
77     if (siz(rt->l) + 1 > val) {
78         b = rt;
79         split_size(rt->l, a, b->l, val);
80         pull(b);
81     } else {
82         a = rt;
83         split_size(rt->r, a->r, b, val - siz(a->l) - 1)
84         ;
85         pull(a);
86     }
87 }
88 void split_val(Treap *rt, Treap *&a, Treap *&b, int val
89 ) {
90     if (!rt) {
91         a = b = NULL;
92         return;
93     }
94     push(rt);
95     if (rt->val <= val) {
96         a = rt;
97         split_val(rt->r, a->r, b, val);
98         pull(a);
99     } else {
100        b = rt;
101        split_val(rt->l, a, b->l, val);
102        pull(b);
103    }
104 }
```

2.4 Li Chao Tree

```

1 const int eps=1e-9;
2 struct line
3 {
4     double m,k;
5 }p[N];
6 int seg[N<<2],cnt;
7 double cal(int id,int x){return 1.0*p[id].m*x+p[id].k;}
8 void add(int x0,int y0,int x1,int y1)
9 {
10     cnt++;
11     if(x0==x1)p[cnt].m=0,p[cnt].k=max(y0,y1);
12     else p[cnt].m=1.0*(y1-y0)/(x1-x0),p[cnt].k=y0-p[
13         cnt].m*x0;
14 }
15 void update(int x,int l,int r,int ql,int qr,int u)
16 {
17     int v=seg[x],mid=(l+r)>>1;
18     double resu=cal(u,mid),resv=cal(v,mid);
19     if(qr<l||r<ql) return;
20     if(ql<=l&&r<=qr)
21     {
22         if(l==r)
23         {
24             if(resu>resv)seg[x]=u;
25             return;
26         }
27         if(p[u].m-p[v].m>eps)
28         {
29             if(resu-resv>eps)
30             {
31                 seg[x]=u;
32                 update(x<<1,l,mid,ql,qr,v);
33             }
34             else update(x<<1|1,mid+1,r,ql,qr,u);
35         }
36         else if(p[v].m-p[u].m>eps)
37         {
38             if(resu-resv>eps)
39             {
40                 seg[x]=u;
41                 update(x<<1|1,mid+1,r,ql,qr,v);
42             }
43             else update(x<<1,l,mid,ql,qr,u);
44         }
45         else if(p[u].k-p[v].k>eps)seg[x]=u;
46         return;
47     }
48     update(x<<1,l,mid,ql,qr,u);
49     update(x<<1|1,mid+1,r,ql,qr,u);
50 }
51 double ask(int x,int l,int r,int qx)
52 {
53     if(r<qx||qx<l) return{0,0};
54     int mid=(l+r)>>1;
55     double res=cal(seg[x],qx);
56     if(l==r) return res;
57     return max({res,seg[x],ask(x<<1,l,mid,qx),ask(x
58         <<1|1,mid+1,r,qx)}); }
59 }

2.5 LineContainer
60 struct Line {
61     mutable ll k, m, p;
62     bool operator<(const Line& o) const { return k < o.
63         k; }
64     bool operator<=(ll x) const { return p < x; }
65 };
66
67 struct LineContainer : multiset<Line, less<>> {
68     // (for doubles, use inf = 1/.0, div(a,b) = a/b)
69     static const ll inf = LLONG_MAX;
70     ll div(ll a, ll b) { // floored division
71         return a / b - ((a ^ b) < 0 && a % b); }
72     bool intersect(iterator x, iterator y) {
73         if (y == end()) return x->p = inf, 0;
74         if (x->k == y->k) x->p = x->m > y->m ? inf : -
75             inf;
76         else x->p = div(y->m - x->m, x->k - y->k);
77         return x->p >= y->p;
78     }
79 }
```

```

19 void add(ll k, ll m) {
20     auto z = insert({k, m, 0}), y = z++, x = y;
21     while (isect(y, z)) z = erase(z);
22     if (x != begin() && isect(--x, y)) isect(x, y = 50)
23         erase(y));
24     while ((y = x) != begin() && (--x)->p >= y->p)
25         isect(x, erase(y));
26 }
27 ll query(ll x) {
28     assert(!empty());
29     auto l = *lower_bound(x);
30     return l.k * x + l.m;
31 }

```

2.6 Sparse Table

```

1 int a[N];
2 int st[N][30];
3 void pre(int n)
4 {
5     FOR(i, 1, n+1) st[i][0] = a[i];
6     for(int j=1; (1<<j) <= n+1; j++)
7         for(int i=0; i+(1<<j) <= n+1; i++)
8             st[i][j] = min(st[i][j-1], st[i+(1<<(j-1))][j-1]);
9 }
10 int ask(int l, int r)
11 {
12     int k = __lg(r-l+1);
13     return min(st[l][k], st[r-(1<<k)+1][k]);
14 }

```

2.7 Dynamic Median

```

1 struct Dynamic_Median {
2     multiset<long long> lo, hi;
3     long long slo = 0, shi = 0;
4     void rebalance() {
5         // keep sz(lo) >= sz(hi) and sz(lo) - sz(hi) <= 1
6         while((int)lo.size() > (int)hi.size() + 1) {
7             auto it = prev(lo.end());
8             long long x = *it;
9             lo.erase(it); slo -= x;
10            hi.insert(x); shi += x;
11        }
12        while((int)lo.size() < (int)hi.size()) {
13            auto it = hi.begin();
14            long long x = *it;
15            hi.erase(it); shi -= x;
16            lo.insert(x); slo += x;
17        }
18    }
19    void add(long long x) {
20        if(lo.empty() || x <= *prev(lo.end())) {
21            lo.insert(x); slo += x;
22        } else {
23            hi.insert(x); shi += x;
24        }
25        rebalance();
26    }
27    void remove_one(long long x) {
28        if(!lo.empty() && x <= *prev(lo.end())) {
29            auto it = lo.find(x);
30            if(it != lo.end()) {
31                lo.erase(it); slo -= x;
32            } else {
33                auto it2 = hi.find(x);
34                hi.erase(it2); shi -= x;
35            }
36        } else {
37            auto it = hi.find(x);
38            if(it != hi.end()) {
39                hi.erase(it); shi -= x;
40            } else {
41                auto it2 = lo.find(x);
42                lo.erase(it2); slo -= x;
43            }
44        }
45    }
46 }

```

```

47         }
48     }
49     rebalance();
50 }
51 }

```

2.8 SOS DP

```

1 for (int mask = 0; mask < (1 << n); mask++) {
2     for (int submask = mask; submask != 0; submask = (submask - 1) & mask) {
3         int subset = mask ^ submask;
4     }
4 }

```

3 Flow / Matching

3.1 Dinic

```

1 using namespace std;
2 const int N = 2000 + 5;
3 int n, m, s, t, level[N], iter[N];
4 struct edge {int to, cap, rev;};
5 vector<edge> path[N];
6 void add(int a, int b, int c) {
7     path[a].pb({b, c, sz(path[b])});
8     path[b].pb({a, 0, sz(path[a]) - 1});
9 }
10 void bfs() {
11     memset(level, -1, sizeof(level));
12     level[s] = 0;
13     queue<int> q;
14     q.push(s);
15     while (q.size()) {
16         int now = q.front(); q.pop();
17         for (edge e : path[now]) if (e.cap > 0 && level[e.to] == -1) {
18             level[e.to] = level[now] + 1;
19             q.push(e.to);
20         }
21     }
22 }
23 int dfs(int now, int flow) {
24     if (now == t) return flow;
25     for (int &i = iter[now]; i < sz(path[now]); i++) {
26         edge &e = path[now][i];
27         if (e.cap > 0 && level[e.to] == level[now] + 1) {
28             int res = dfs(e.to, min(flow, e.cap));
29             if (res > 0) {
30                 e.cap -= res;
31                 path[e.to][e.rev].cap += res;
32                 return res;
33             }
34         }
35     }
36     return 0;
37 }
38 int dinic() {
39     int res = 0;
40     while (true) {
41         bfs();
42         if (level[t] == -1) break;
43         memset(iter, 0, sizeof(iter));
44         int now = 0;
45         while ((now = dfs(s, INF)) > 0) res += now;
46     }
47     return res;
48 }

```

3.2 MCMF

```

1 struct MCMF {
2     int n, s, t, par[N + 5], p_i[N + 5], dis[N + 5],
3         vis[N + 5];
4     struct edge {
5         int to, cap, rev, cost;
6     };
7     vector<edge> path[N];
8     void init(int _n, int _s, int _t) {
9         n = _n, s = _s, t = _t;
10        FOR(i, 0, 2 * n + 5)
11    }
12 }

```

```

par[i] = p_i[i] = vis[i] = 0;
}
void add(int a, int b, int c, int d) {
    path[a].pb({b, c, sz(path[b]), d});
    path[b].pb({a, 0, sz(path[a]) - 1, -d});
}
void spfa() {
    FOR(i, 0, n * 2 + 5)
        dis[i] = INF,
        vis[i] = 0;
    dis[s] = 0;
    queue<int> q;
    q.push(s);
    while (!q.empty()) {
        int now = q.front();
        q.pop();
        vis[now] = 0;
        for (int i = 0; i < sz(path[now]); i++) {
            edge e = path[now][i];
            if (e.cap > 0 && dis[e.to] > dis[now] + e.cost) {
                dis[e.to] = dis[now] + e.cost;
                par[e.to] = now;
                p_i[e.to] = i;
                if (vis[e.to] == 0) {
                    vis[e.to] = 1;
                    q.push(e.to);
                }
            }
        }
    }
}
pii flow() {
    int flow = 0, cost = 0;
    while (true) {
        spfa();
        if (dis[t] == INF)
            break;
        int mn = INF;
        for (int i = t; i != s; i = par[i])
            mn = min(mn, path[par[i]][p_i[i]].cap);
        flow += mn;
        cost += dis[t] * mn;
        for (int i = t; i != s; i = par[i]) {
            edge &now = path[par[i]][p_i[i]];
            now.cap -= mn;
            path[i][now.rev].cap += mn;
        }
    }
    return mp(flow, cost);
}
int t = lx[x] + ly[y] - g[x][y];
if (t == 0) {
    pa[y] = x;
    if (!my[y]) {
        augment(y);
        return;
    }
    vy[y] = 1, q.push(my[y]);
} else if (sy[y] > t)
    pa[y] = x, sy[y] = t;
}
int cut = INF;
FOR(y, 1, n + 1)
if (!vy[y] && cut > sy[y]) cut = sy[y];
FOR(j, 1, n + 1) {
    if (vx[j]) lx[j] -= cut;
    if (vy[j])
        ly[j] += cut;
    else
        sy[j] -= cut;
}
FOR(y, 1, n + 1) {
    if (!vy[y] && sy[y] == 0) {
        if (!my[y]) {
            augment(y);
            return;
        }
        vy[y] = 1;
        q.push(my[y]);
    }
}
int solve() {
    fill(mx, mx + n + 1, 0);
    fill(my, my + n + 1, 0);
    fill.ly, ly + n + 1, 0);
    fill(lx, lx + n + 1, 0);
    FOR(x, 1, n + 1)
    FOR(y, 1, n + 1)
        lx[x] = max(lx[x], g[x][y]);
    FOR(x, 1, n + 1)
        bfs(x);
    int ans = 0;
    FOR(y, 1, n + 1)
        ans += g[my[y]][y];
    return ans;
}

```

3.3 KM

```

1 struct KM {
2     int n, mx[1005], my[1005], pa[1005];
3     int g[1005][1005], lx[1005], ly[1005], sy[1005];
4     bool vx[1005], vy[1005];
5     void init(int _n) {
6         n = _n;
7         FOR(i, 1, n + 1)
8             fill(g[i], g[i] + 1 + n, 0);
9     }
10    void add(int a, int b, int c) { g[a][b] = c; }
11    void augment(int y) {
12        for (int x, z; y; y = z)
13            x = pa[y], z = mx[x], my[y] = x, mx[x] =
14    }
15    void bfs(int st) {
16        FOR(i, 1, n + 1)
17            sy[i] = INF,
18            vx[i] = vy[i] = 0;
19        queue<int> q;
20        q.push(st);
21        for (;;) {
22            while (!q.empty()) {
23                int x = q.front();
24                q.pop();
25                vx[x] = 1;
26                FOR(y, 1, n + 1)
27                    if (!vy[y]) {
28                        pa[y] = x;
29                        my[y] = x;
30                        sy[x] = y;
31                        vy[y] = 1;
32                        q.push(y);
33                    }
34            }
35        }
36    }
37    int minCost() {
38        int res = 0;
39        FOR(i, 1, n + 1)
40            res += sy[i] - my[i];
41        return res;
42    }
43 }

```

```

struct HopcroftKarp {
    // id: X = [1, nx], Y = [nx+1, nx+ny]
    int n, nx, ny, m, MXCNT;
    vector<vector<int>> g;
    vector<int> mx, my, dis, vis;
    void init(int nnx, int nny, int mm) {
        nx = nnx, ny = nny, m = mm;
        n = nx + ny + 1;
        g.clear();
        g.resize(n);
    }
    void add(int x, int y) {
        g[x].emplace_back(y);
        g[y].emplace_back(x);
    }
    bool dfs(int x) {
        vis[x] = true;
        Each(y, g[x]) {
            int px = my[y];
            if (px == -1 || (dis[px] == dis[x] + 1 &&
                !vis[px] && dfs(px))) {
                mx[x] = y;
                my[y] = x;
                return true;
            }
        }
        return false;
    }
}

```

```

30     void get() {
31         mx.clear();
32         mx.resize(n, -1);
33         my.clear();
34         my.resize(n, -1);
35
36         while (true) {
37             queue<int> q;
38             dis.clear();
39             dis.resize(n, -1);
40             for (int x = 1; x <= nx; x++) {
41                 if (mx[x] == -1) {
42                     dis[x] = 0;
43                     q.push(x);
44                 }
45             }
46             while (!q.empty()) {
47                 int x = q.front();
48                 q.pop();
49                 Each(y, g[x]) {
50                     if (my[y] != -1 && dis[my[y]] == -1) {
51                         dis[my[y]] = dis[x] + 1;
52                         q.push(my[y]);
53                     }
54                 }
55             }
56
57             bool brk = true;
58             vis.clear();
59             vis.resize(n, 0);
60             for (int x = 1; x <= nx; x++)
61                 if (mx[x] == -1 && dfs(x))
62                     brk = false;
63
64             if (brk) break;
65         }
66         MXCNT = 0;
67         for (int x = 1; x <= nx; x++)
68             if (mx[x] != -1) MXCNT++;
69     } hk;
```

3.5 Blossom

```

1 const int N=5e2+10;
2 struct Graph{
3     int to[N],bro[N],head[N],e;
4     int lnk[N],vis[N],stp,n;
5     void init(int _n){
6         stp=0;e=1;n=_n;
7         FOR(i,0,n+1)head[i]=lnk[i]=vis[i]=0;
8     }
9     void add(int u,int v){
10        to[e]=v,bro[e]=head[u],head[u]=e++;
11        to[e]=u,bro[e]=head[v],head[v]=e++;
12    }
13    bool dfs(int x){
14        vis[x]=stp;
15        for(int i=head[x];i;i=bro[i])
16        {
17            int v=to[i];
18            if(!lnk[v])
19            {
20                lnk[x]=v;lnk[v]=x;
21                return true;
22            }
23            else if(vis[lnk[v]]<stp)
24            {
25                int w=lnk[v];
26                lnk[x]=v,lnk[v]=x,lnk[w]=0;
27                if(dfs(w))return true;
28                lnk[w]=v,lnk[v]=w,lnk[x]=0;
29            }
30        }
31        return false;
32    }
33    int solve(){
34        int ans=0;
35        FOR(i,1,n+1){
36            if(!lnk[i]){
37                stp++;
```

```

38                ans+=dfs(i);
39            }
40        }
41        return ans;
42    }
43    void print_matching(){
44        FOR(i,1,n+1)
45            if(i<graph.lnk[i])
46                cout<<i<<" "<<graph.lnk[i]<<endl;
47    }
48};
```

3.6 Cover / Independent Set

```

1 V(E) Cover: choose some V(E) to cover all E(V)
2 V(E) Independ: set of V(E) not adj to each other
3
4 M = Max Matching
5 Cv = Min V Cover
6 Ce = Min E Cover
7 Iv = Max V Ind
8 Ie = Max E Ind (equiv to M)
9
10 M = Cv (Konig Theorem)
11 Iv = V \ Cv
12 Ce = V - M
13
14 Construct Cv:
15 1. Run Dinic
16 2. Find s-t min cut
17 3. Cv = {X in T} + {Y in S}
```

3.7 Hungarian Algorithm

```

1 const int N = 2e3;
2 int match[N];
3 bool vis[N];
4 int n;
5 vector<int> ed[N];
6 int match_cnt;
7 bool dfs(int u) {
8     vis[u] = 1;
9     for(int i : ed[u]) {
10         if(match[i] == 0 || !vis[match[i]] && dfs(match[i])) {
11             match[i] = u;
12             return true;
13         }
14     }
15     return false;
16 }
17 void hungary() {
18     memset(match, 0, sizeof(match));
19     match_cnt = 0;
20     for(int i = 1; i <= n; i++) {
21         memset(vis, 0, sizeof(vis));
22         if(dfs(i)) match_cnt++;
23     }
24 }
```

4 Graph

4.1 Heavy-Light Decomposition

```

1 const int N = 2e5 + 5;
2 int n, dfn[N], son[N], top[N], num[N], dep[N], p[N];
3 vector<int> path[N];
4 struct node {
5     int mx, sum;
6 } seg[N << 2];
7 void update(int x, int l, int r, int qx, int val) {
8     if (l == r) {
9         seg[x].mx = seg[x].sum = val;
10        return;
11    }
12    int mid = (l + r) >> 1;
13    if (qx <= mid)update(x << 1, l, mid, qx, val);
14    else update(x << 1 | 1, mid + 1, r, qx, val);
15    seg[x].mx = max(seg[x << 1].mx, seg[x << 1 | 1].mx);
16    seg[x].sum = seg[x << 1].sum + seg[x << 1 | 1].sum;
```

```

17 }
18 int big(int x, int l, int r, int ql, int qr) {
19     if (ql <= l && r <= qr) return seg[x].mx;
20     int mid = (l + r) >> 1;
21     int res = -INF;
22     if (ql <= mid) res = max(res, big(x << 1, l, mid,
23         ql, qr));
24     if (mid < qr) res = max(res, big(x << 1 | 1, mid +
25         1, r, ql, qr));
26     return res;
27 }
28 int ask(int x, int l, int r, int ql, int qr) {
29     if (ql <= l && r <= qr) return seg[x].sum;
30     int mid = (l + r) >> 1;
31     int res = 0;
32     if (ql <= mid) res += ask(x << 1, l, mid, ql, qr);
33     if (mid < qr) res += ask(x << 1 | 1, mid + 1, r, ql
34         , qr);
35     return res;
36 }
37 void dfs1(int now) {
38     son[now] = -1;
39     num[now] = 1;
40     for (auto i : path[now]) {
41         if (!dep[i]) {
42             dep[i] = dep[now] + 1;
43             p[i] = now;
44             dfs1(i);
45             num[now] += num[i];
46             if (son[now] == -1 || num[i] > num[son[now]
47                 ]) son[now] = i;
48         }
49     }
50 }
51 int cnt;
52 void dfs2(int now, int t) {
53     top[now] = t;
54     cnt++;
55     dfn[now] = cnt;
56     if (son[now] == -1) return;
57     dfs2(son[now], t);
58     for (auto i : path[now])
59         if (i != p[now] && i != son[now]) dfs2(i, i);
60 }
61 int path_big(int x, int y) {
62     int res = -INF;
63     while (top[x] != top[y]) {
64         if (dep[top[x]] < dep[top[y]]) swap(x, y);
65         res = max(res, big(1, 1, n, dfn[top[x]], dfn[x
66             ]));
67         x = p[top[x]];
68     }
69     if (dfn[x] > dfn[y]) swap(x, y);
70     res = max(res, big(1, 1, n, dfn[x], dfn[y]));
71     return res;
72 }
73 int path_sum(int x, int y) {
74     int res = 0;
75     while (top[x] != top[y]) {
76         if (dep[top[x]] < dep[top[y]]) swap(x, y);
77         res += ask(1, 1, n, dfn[top[x]], dfn[x]);
78         x = p[top[x]];
79     }
80     if (dfn[x] > dfn[y]) swap(x, y);
81     res += ask(1, 1, n, dfn[x], dfn[y]);
82     return res;
83 }
84 void buildTree() {
85     FOR(i, 0, n - 1) {
86         int a, b;
87         cin >> a >> b;
88         path[a].pb(b);
89         path[b].pb(a);
90     }
91 }
92 void buildHLD(int root) {
93     dep[root] = 1;
94     dfs1(root);
95     dfs2(root, root);
96     FOR(i, 1, n + 1) {
97         int now;
98         cin >> now;
99     }
100 }
```

```

94     update(1, 1, n, dfn[i], now);
95 }
96 }
```

4.2 Centroid Decomposition

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const int N = 1e5 + 5;
4 vector<int> a[N];
5 int sz[N], lv[N];
6 bool used[N];
7 int f_sz(int x, int p) {
8     sz[x] = 1;
9     for (int i : a[x])
10         if (i != p && !used[i])
11             sz[x] += f_sz(i, x);
12     return sz[x];
13 }
14 int f_cen(int x, int p, int total) {
15     for (int i : a[x]) {
16         if (i != p && !used[i] && 2 * sz[i] > total)
17             return f_cen(i, x, total);
18     }
19     return x;
20 }
21 void cd(int x, int p) {
22     int total = f_sz(x, p);
23     int cen = f_cen(x, p, total);
24     lv[cen] = lv[p] + 1;
25     used[cen] = 1;
26     // cout << "cd: " << x << " " << p << " " << cen <<
27     "\n";
28     for (int i : a[cen]) {
29         if (!used[i])
30             cd(i, cen);
31     }
32 }
33 int main() {
34     ios_base::sync_with_stdio(0);
35     cin.tie(0);
36     int n;
37     cin >> n;
38     for (int i = 0, x, y; i < n - 1; i++) {
39         cin >> x >> y;
40         a[x].push_back(y);
41         a[y].push_back(x);
42     }
43     cd(1, 0);
44     for (int i = 1; i <= n; i++)
45         cout << (char)('A' + lv[i] - 1) << " ";
46     cout << "\n";
47 }
```

4.3 Bellman-Ford + SPFA

```

1 int n, m;
2
3 // Graph
4 vector<vector<pair<int, ll>>> g;
5 vector<ll> dis;
6 vector<bool> negCycle;
7
8 // SPFA
9 vector<int> rlx;
10 queue<int> q;
11 vector<bool> inq;
12 vector<int> pa;
13 void SPFA(vector<int>& src) {
14     dis.assign(n + 1, LINF);
15     negCycle.assign(n + 1, false);
16     rlx.assign(n + 1, 0);
17     while (!q.empty()) q.pop();
18     inq.assign(n + 1, false);
19     pa.assign(n + 1, -1);
20
21     for (auto& s : src) {
22         dis[s] = 0;
23         q.push(s);
24         inq[s] = true;
25     }
26 }
```

```

27     while (!q.empty()) {
28         int u = q.front();
29         q.pop();
30         inq[u] = false;
31         if (rlx[u] >= n) {
32             negCycle[u] = true;
33         } else {
34             for (auto& e : g[u]) {
35                 int v = e.first;
36                 ll w = e.second;
37                 if (dis[v] > dis[u] + w) {
38                     dis[v] = dis[u] + w;
39                     rlx[v] = rlx[u] + 1;
40                     pa[v] = u;
41                     if (!inq[v]) {
42                         q.push(v);
43                         inq[v] = true;
44                     }
45                 }
46             }
47         }
48     }
49
50 // Bellman-Ford
51 queue<int> q;
52 vector<int> pa;
53 void BellmanFord(vector<int>& src) {
54     dis.assign(n + 1, LINF);
55     negCycle.assign(n + 1, false);
56     pa.assign(n + 1, -1);
57
58     for (auto& s : src) dis[s] = 0;
59
60     for (int rlx = 1; rlx <= n; rlx++) {
61         for (int u = 1; u <= n; u++) {
62             if (dis[u] == LINF) continue; // Important
63             !
64             for (auto& e : g[u]) {
65                 int v = e.first;
66                 ll w = e.second;
67                 if (dis[v] > dis[u] + w) {
68                     dis[v] = dis[u] + w;
69                     pa[v] = u;
70                     if (rlx == n) negCycle[v] = true;
71                 }
72             }
73         }
74     }
75
76 // Negative Cycle Detection
77 void NegCycleDetect() {
78     /* No Neg Cycle: NO
79     Exist Any Neg Cycle:
80     YES
81     v0 v1 v2 ... vk v0 */
82
83     vector<int> src;
84     for (int i = 1; i <= n; i++)
85         src.emplace_back(i);
86
87     SPFA(src);
88     // BellmanFord(src);
89
90     int ptr = -1;
91     for (int i = 1; i <= n; i++)
92         if (negCycle[i]) {
93             ptr = i;
94             break;
95         }
96
97     if (ptr == -1) {
98         return cout << "NO" << endl, void();
99     }
100
101    cout << "YES\n";
102    vector<int> ans;
103    vector<bool> vis(n + 1, false);
104
105    while (true) {
106        ans.emplace_back(ptr);
107        if (vis[ptr]) break;
108
109        vis[ptr] = true;
110        ptr = pa[ptr];
111    }
112    reverse(ans.begin(), ans.end());
113
114    vis.assign(n + 1, false);
115    for (auto& x : ans) {
116        cout << x << ' ';
117        if (vis[x]) break;
118        vis[x] = true;
119    }
120    cout << endl;
121 }
122
123 // Distance Calculation
124 void calcDis(int s) {
125     vector<int> src;
126     src.emplace_back(s);
127     SPFA(src);
128     // BellmanFord(src);
129
130     while (!q.empty()) q.pop();
131     for (int i = 1; i <= n; i++)
132         if (negCycle[i]) q.push(i);
133
134     while (!q.empty()) {
135         int u = q.front();
136         q.pop();
137         for (auto& e : g[u]) {
138             int v = e.first;
139             if (!negCycle[v]) {
140                 q.push(v);
141                 negCycle[v] = true;
142             }
143         }
144     }
145 }

```

4.4 BCC - AP

```

1 int n, m;
2 int low[maxn], dfn[maxn], instp;
3 vector<int> E, g[maxn];
4 bitset<maxn> isap;
5 bitset<maxm> vis;
6 stack<int> stk;
7 int bccnt;
8 vector<int> bcc[maxn];
9 inline void popout(int u) {
10    bccnt++;
11    bcc[bccnt].emplace_back(u);
12    while (!stk.empty()) {
13        int v = stk.top();
14        if (u == v) break;
15        stk.pop();
16        bcc[bccnt].emplace_back(v);
17    }
18}
19 void dfs(int u, bool rt = 0) {
20    stk.push(u);
21    low[u] = dfn[u] = ++instp;
22    int kid = 0;
23    Each(e, g[u]) {
24        if (vis[e]) continue;
25        vis[e] = true;
26        int v = E[e] ^ u;
27        if (!dfn[v]) {
28            // tree edge
29            kid++;
30            dfs(v);
31            low[u] = min(low[u], low[v]);
32            if (!rt && low[v] >= dfn[u]) {
33                // bcc found: u is ap
34                isap[u] = true;
35                popout(u);
36            }
37        } else {
38            // back edge
39            low[u] = min(low[u], dfn[v]);
40        }
41    }
42 // special case: root

```

```

43     if (rt) {
44         if (kid > 1) isap[u] = true;
45         popout(u);
46     }
47 }
48 void init() {
49     cin >> n >> m;
50     fill(low, low + maxn, INF);
51     REP(i, m) {
52         int u, v;
53         cin >> u >> v;
54         g[u].emplace_back(i);
55         g[v].emplace_back(i);
56         E.emplace_back(u ^ v);
57     }
58 }
59 void solve() {
60     FOR(i, 1, n + 1, 1) {
61         if (!dfn[i]) dfs(i, true);
62     }
63     vector<int> ans;
64     int cnt = 0;
65     FOR(i, 1, n + 1, 1) {
66         if (isap[i]) cnt++, ans.emplace_back(i);
67     }
68     cout << cnt << endl;
69     Each(i, ans) cout << i << ' ';
70     cout << endl;
71 }

```

4.5 BCC - Bridge

```

1 int n, m;
2 vector<int> g[maxn], E;
3 int low[maxn], dfn[maxn], instp;
4 int bccnt, bccid[maxn];
5 stack<int> stk;
6 bitset<maxm> vis, isbрг;
7 void init() {
8     cin >> n >> m;
9     REP(i, m) {
10         int u, v;
11         cin >> u >> v;
12         E.emplace_back(u ^ v);
13         g[u].emplace_back(i);
14         g[v].emplace_back(i);
15     }
16     fill(low, low + maxn, INF);
17 }
18 void popout(int u) {
19     bccnt++;
20     while (!stk.empty()) {
21         int v = stk.top();
22         if (v == u) break;
23         stk.pop();
24         bccid[v] = bccnt;
25     }
26 }
27 void dfs(int u) {
28     stk.push(u);
29     low[u] = dfn[u] = ++instp;
30
31     Each(e, g[u]) {
32         if (vis[e]) continue;
33         vis[e] = true;
34
35         int v = E[e] ^ u;
36         if (dfn[v]) {
37             // back edge
38             low[u] = min(low[u], dfn[v]);
39         } else {
40             // tree edge
41             dfs(v);
42             low[u] = min(low[u], low[v]);
43             if (low[v] == dfn[v]) {
44                 isbрг[e] = true;
45                 popout(u);
46             }
47         }
48     }
49 }
50 void solve() {

```

```

51     FOR(i, 1, n + 1, 1) {
52         if (!dfn[i]) dfs(i);
53     }
54     vector<pii> ans;
55     vis.reset();
56     FOR(u, 1, n + 1, 1) {
57         Each(e, g[u]) {
58             if (!isbрг[e] || vis[e]) continue;
59             vis[e] = true;
60             int v = E[e] ^ u;
61             ans.emplace_back(mp(u, v));
62         }
63     }
64     cout << (int)ans.size() << endl;
65     Each(e, ans) cout << e.F << ' ' << e.S << endl;
66 }

```

4.6 SCC - Tarjan

```

1 // 2-SAT
2 vector<int> E, g[maxn]; // 1~n, n+1~2n
3 int low[maxn], in[maxn], instp;
4 int sccnt, sccid[maxn];
5 stack<int> stk;
6 bitset<maxn> ins, vis;
7 int n, m;
8 void init() {
9     cin >> m >> n;
10    E.clear();
11    fill(g, g + maxn, vector<int>());
12    fill(low, low + maxn, INF);
13    memset(in, 0, sizeof(in));
14    instp = 1;
15    sccnt = 0;
16    memset(sccid, 0, sizeof(sccid));
17    ins.reset();
18    vis.reset();
19 }
20 inline int no(int u) {
21     return (u > n ? u - n : u + n);
22 }
23 int ecnt = 0;
24 inline void clause(int u, int v) {
25     E.eb(no(u) ^ v);
26     g[no(u)].eb(ecnt++);
27     E.eb(no(v) ^ u);
28     g[no(v)].eb(ecnt++);
29 }
30 void dfs(int u) {
31     in[u] = instp++;
32     low[u] = in[u];
33     stk.push(u);
34     ins[u] = true;
35
36     Each(e, g[u]) {
37         if (vis[e]) continue;
38         vis[e] = true;
39
40         int v = E[e] ^ u;
41         if (ins[v])
42             low[u] = min(low[u], in[v]);
43         else if (!in[v]) {
44             dfs(v);
45             low[u] = min(low[u], low[v]);
46         }
47     }
48     if (low[u] == in[u]) {
49         sccnt++;
50         while (!stk.empty()) {
51             int v = stk.top();
52             stk.pop();
53             ins[v] = false;
54             sccid[v] = sccnt;
55             if (u == v) break;
56         }
57     }
58 }
59 int main() {
60     init();
61     REP(i, m) {
62         char su, sv;
63         int u, v;

```

```

64     cin >> su >> u >> sv >> v;
65     if (su == '-') u = no(u);
66     if (sv == '-') v = no(v);
67     clause(u, v);
68 }
69 FOR(i, 1, 2 * n + 1, 1) {
70     if (!in[i]) dfs(i);
71 }
72 FOR(u, 1, n + 1, 1) {
73     int du = no(u);
74     if (sccid[u] == sccid[du]) {
75         return cout << "IMPOSSIBLE\n", 0;
76     }
77 }
78 FOR(u, 1, n + 1, 1) {
79     int du = no(u);
80     cout << (sccid[u] < sccid[du] ? '+' : '-') << ' ';
81 }
82 cout << endl;
83 }
```

4.7 SCC - Kosaraju

```

1 const int N = 1e5 + 10;
2 vector<int> ed[N], ed_b[N]; // 反邊
3 vector<int> SCC(N); // 最後SCC的分組
4 bitset<N> vis;
5 int SCC_cnt;
6 int n, m;
7 vector<int> pre; // 後序遍歷
8
9 void dfs(int x) {
10    vis[x] = 1;
11    for (int i : ed[x]) {
12        if (vis[i]) continue;
13        dfs(i);
14    }
15    pre.push_back(x);
16 }
17
18 void dfs2(int x) {
19    vis[x] = 1;
20    SCC[x] = SCC_cnt;
21    for (int i : ed_b[x]) {
22        if (vis[i]) continue;
23        dfs2(i);
24    }
25 }
26
27 void kosaraju() {
28    for (int i = 1; i <= n; i++) {
29        if (!vis[i]) {
30            dfs(i);
31        }
32    }
33    SCC_cnt = 0;
34    vis = 0;
35    for (int i = n - 1; i >= 0; i--) {
36        if (!vis[pre[i]]) {
37            SCC_cnt++;
38            dfs2(pre[i]);
39        }
40    }
41 }
```

4.8 Eulerian Path - Undir

```

1 // from 1 to n
2 #define gg return cout << "IMPOSSIBLE\n", void();
3
4 int n, m;
5 vector<int> g[maxn];
6 bitset<maxn> inodd;
7
8 void init() {
9     cin >> n >> m;
10    inodd.reset();
11    for (int i = 0; i < m; i++) {
12        int u, v;
13        cin >> u >> v;
14    }
15 }
```

```

14     inodd[u] = inodd[u] ^ true;
15     inodd[v] = inodd[v] ^ true;
16     g[u].emplace_back(v);
17     g[v].emplace_back(u);
18 }
19 stack<int> stk;
20 void dfs(int u) {
21     while (!g[u].empty()) {
22         int v = g[u].back();
23         g[u].pop_back();
24         dfs(v);
25     }
26     stk.push(u);
27 }
```

4.9 Eulerian Path - Dir

```

1 // from node 1 to node n
2 #define gg return cout << "IMPOSSIBLE\n", 0
3
4 int n, m;
5 vector<int> g[maxn];
6 stack<int> stk;
7 int in[maxn], out[maxn];
8
9 void init() {
10    cin >> n >> m;
11    for (int i = 0; i < m; i++) {
12        int u, v;
13        cin >> u >> v;
14        g[u].emplace_back(v);
15        out[u]++;
16        in[v]++;
17    }
18    for (int i = 1; i <= n; i++) {
19        if (i == 1 && out[i] - in[i] != 1) gg;
20        if (i == n && in[i] - out[i] != 1) gg;
21        if (i != 1 && i != n && in[i] != out[i]) gg;
22    }
23 }
24 void dfs(int u) {
25     while (!g[u].empty()) {
26         int v = g[u].back();
27         g[u].pop_back();
28         dfs(v);
29     }
30     stk.push(u);
31 }
32 void solve() {
33    dfs(1) for (int i = 1; i <= n; i++) if ((int)g[i].size()) gg;
34    while (!stk.empty()) {
35        int u = stk.top();
36        stk.pop();
37        cout << u << ' ';
38    }
39 }
```

4.10 Hamilton Path

```

1 // top down DP
2 // Be Aware Of Multiple Edges
3 int n, m;
4 ll dp[maxn][1<<maxn];
5 int adj[maxn][maxn];
6
7 void init() {
8     cin >> n >> m;
9     fill(dp[0], dp[maxn-1]+(1<<maxn), -1);
10 }
11
12 void DP(int i, int msk) {
13     if (dp[i][msk] != -1) return;
14     dp[i][msk] = 0;
15     REP(j, n) if (j != i && (msk & (1<<j)) && adj[j][i]) {
16         int sub = msk ^ (1<<i);
17         if (dp[j][sub] == -1) DP(j, sub);
18         dp[i][msk] += dp[j][sub] * adj[j][i];
19         if (dp[i][msk] >= MOD) dp[i][msk] %= MOD;
20     }
21 }
```

```

22
23
24 int main() {
25     WiwiHorz
26     init();
27
28     REP(i, m) {
29         int u, v;
30         cin >> u >> v;
31         if (u == v) continue;
32         adj[--u][--v]++;
33     }
34
35     dp[0][1] = 1;
36     FOR(i, 1, n, 1) {
37         dp[i][1] = 0;
38         dp[i][1|(1<<i)] = adj[0][i];
39     }
40     FOR(msk, 1, (1<<n), 1) {
41         if (msk == 1) continue;
42         dp[0][msk] = 0;
43     }
44
45     DP(n-1, (1<<n)-1);
46     cout << dp[n-1][(1<<n)-1] << endl;
47
48     return 0;
49 }
```

4.11 Kth Shortest Path

```

1 // time: O(|E| \lg |E| + |V| \lg |V| + K)
2 // memory: O(|E| \lg |E| + |V| |V|)
3 struct KSP { // 1-base
4     struct nd {
5         int u, v;
6         ll d;
7         nd(int ui = 0, int vi = 0, ll di = INF) {
8             u = ui;
9             v = vi;
10            d = di;
11        }
12    };
13    struct heap {
14        nd* edge;
15        int dep;
16        heap* chd[4];
17    };
18    static int cmp(heap* a, heap* b) { return a->edge->d > b->edge->d; }
19    struct node {
20        int v;
21        ll d;
22        heap* H;
23        nd* E;
24        node() {}
25        node(ll _d, int _v, nd* _E) {
26            d = _d;
27            v = _v;
28            E = _E;
29        }
30        node(heap* _H, ll _d) {
31            H = _H;
32            d = _d;
33        }
34        friend bool operator<(node a, node b) { return a.d > b.d; }
35    };
36    int n, k, s, t, dst[N];
37    nd* nxt[N];
38    vector<nd*> g[N], rg[N];
39    heap *nullNd, *head[N];
40    void init(int _n, int _k, int _s, int _t) {
41        n = _n;
42        k = _k;
43        s = _s;
44        t = _t;
45        for (int i = 1; i <= n; i++) {
46            g[i].clear();
47            rg[i].clear();
48            nxt[i] = NULL;
49    }
50    head[i] = NULL;
51    dst[i] = -1;
52}
53 void addEdge(int ui, int vi, ll di) {
54    nd* e = new nd(ui, vi, di);
55    g[ui].push_back(e);
56    rg[vi].push_back(e);
57}
58 queue<int> dfsQ;
59 void dijkstra() {
60     while (dfsQ.size()) dfsQ.pop();
61     priority_queue<node> Q;
62     Q.push(node(0, t, NULL));
63     while (!Q.empty()) {
64         node p = Q.top();
65         Q.pop();
66         if (dst[p.v] != -1) continue;
67         dst[p.v] = p.d;
68         nxt[p.v] = p.E;
69         dfsQ.push(p.v);
70         for (auto e : rg[p.v]) Q.push(node(p.d + e->d, e->u, e));
71     }
72}
73 heap* merge(heap* curNd, heap* newNd) {
74    if (curNd == nullNd) return newNd;
75    heap* root = new heap;
76    memcpy(root, curNd, sizeof(heap));
77    if (newNd->edge->d < curNd->edge->d) {
78        root->edge = newNd->edge;
79        root->chd[2] = newNd->chd[2];
80        root->chd[3] = newNd->chd[3];
81        newNd->edge = curNd->edge;
82        newNd->chd[2] = curNd->chd[2];
83        newNd->chd[3] = curNd->chd[3];
84    }
85    if (root->chd[0]->dep < root->chd[1]->dep)
86        root->chd[0] = merge(root->chd[0], newNd);
87    else
88        root->chd[1] = merge(root->chd[1], newNd);
89    root->dep = max(root->chd[0]->dep,
90                      root->chd[1]->dep) +
91        1;
92    return root;
93}
94 vector<heap*> V;
95 void build() {
96    nullNd = new heap;
97    nullNd->dep = 0;
98    nullNd->edge = new nd;
99    fill(nullNd->chd, nullNd->chd + 4, nullNd);
100   while (not dfsQ.empty()) {
101       int u = dfsQ.front();
102       dfsQ.pop();
103       if (!nxt[u])
104           head[u] = nullNd;
105       else
106           head[u] = head[nxt[u]->v];
107       V.clear();
108       for (auto& e : g[u]) {
109           int v = e->v;
110           if (dst[v] == -1) continue;
111           e->d += dst[v] - dst[u];
112           if (nxt[u] != e) {
113               heap* p = new heap;
114               fill(p->chd, p->chd + 4, nullNd);
115               p->dep = 1;
116               p->edge = e;
117               V.push_back(p);
118           }
119       }
120       if (V.empty()) continue;
121       make_heap(V.begin(), V.end(), cmp);
122 #define L(X) ((X << 1) + 1)
123 #define R(X) ((X << 1) + 2)
124       for (size_t i = 0; i < V.size(); i++) {
125           if (L(i) < V.size())
126               V[i]->chd[2] = V[L(i)];
127           else
128               V[i]->chd[2] = nullNd;
129           if (R(i) < V.size())
130               V[R(i)]->chd[2] = nullNd;
131       }
132   }
133 }
```

```

130     V[i]->chd[3] = V[R(i)];
131     else
132         V[i]->chd[3] = nullNd;
133     }
134     head[u] = merge(head[u], V.front());
135   }
136   vector<ll> ans;
137   void first_K() {
138     ans.clear();
139     priority_queue<node> Q;
140     if (dst[s] == -1) return;
141     ans.push_back(dst[s]);
142     if (head[s] != nullNd)
143       Q.push(node(head[s], dst[s] + head[s]->edge->d));
144     for (int _ = 1; _ < k and not Q.empty(); _++) {
145       node p = Q.top(), q;
146       Q.pop();
147       ans.push_back(p.d);
148       if (head[p.H->edge->v] != nullNd) {
149         q.H = head[p.H->edge->v];
150         q.d = p.d + q.H->edge->d;
151         Q.push(q);
152       }
153       for (int i = 0; i < 4; i++)
154         if (p.H->chd[i] != nullNd) {
155           q.H = p.H->chd[i];
156           q.d = p.d - p.H->edge->d + p.H->chd[i]->edge->d;
157           Q.push(q);
158         }
159     }
160   }
161   void solve() { // ans[i] stores the i-th shortest
162     path
163     dijkstra();
164     build();
165     first_K(); // ans.size() might less than k
166   }
167 } solver;

```

4.12 System of Difference Constraints

```

1 vector<vector<pair<int, ll>>> G;
2 void add(int u, int v, ll w) {
3   G[u].emplace_back(make_pair(v, w));
4 }

```

- $x_u - x_v \leq c \Rightarrow \text{add}(v, u, c)$
- $x_u - x_v \geq c \Rightarrow \text{add}(u, v, -c)$
- $x_u - x_v = c \Rightarrow \text{add}(v, u, c), \text{add}(u, v, -c)$
- $x_u \geq c \Rightarrow \text{add super vertex } x_0 = 0, \text{then } x_u - x_0 \geq c \Rightarrow \text{add}(u, 0, -c)$
- Don't forget non-negative constraints for every variable if specified implicitly.
- Interval sum \Rightarrow Use prefix sum to transform into differential constraints. Don't forget $S_{i+1} - S_i \geq 0$ if x_i needs to be non-negative.
- $\frac{x_u}{x_v} \leq c \Rightarrow \log x_u - \log x_v \leq \log c$

5 String

5.1 Aho Corasick

```

1 struct ACautomata {
2   struct Node {
3     int cnt; // 停在此節點的數量
4     Node *go[26], *fail, *dic;
5     // 子節點 fail 指標 最近的模式結尾
6     Node() {
7       cnt = 0;

```

```

8       fail = 0;
9       dic = 0;
10      memset(go, 0, sizeof(go));
11    }
12  } pool[1048576], *root;
13  int nMem;
14  Node *new_Node() {
15    pool[nMem] = Node();
16    return &pool[nMem++];
17  }
18  void init() {
19    nMem = 0;
20    root = new_Node();
21  }
22  void add(const string &str) { insert(root, str, 0); }
23  void insert(Node *cur, const string &str, int pos)
24  {
25    for (int i = pos; i < str.size(); i++) {
26      if (!cur->go[str[i] - 'a'])
27        cur->go[str[i] - 'a'] = new_Node();
28      cur = cur->go[str[i] - 'a'];
29    }
30    cur->cnt++;
31  }
32  void make_fail() { // 全部 add 完做
33    queue<Node *> que;
34    que.push(root);
35    while (!que.empty()) {
36      Node *fr = que.front();
37      que.pop();
38      for (int i = 0; i < 26; i++) {
39        if (fr->go[i]) {
40          Node *ptr = fr->fail;
41          while (ptr && !ptr->go[i]) ptr =
42            ptr->fail;
43          fr->go[i]->fail = ptr = (ptr ? ptr-
44            >go[i] : root);
45          fr->go[i]->dic = (ptr->cnt ? ptr :
46            ptr->dic);
47          que.push(fr->go[i]);
48        }
49      }
50    }
51  }
52  // 出現過不同string的總數
53  int query_unique(const string& text) {
54    Node* p = root;
55    int ans = 0;
56    for(char ch : text) {
57      int i = ch - 'a';
58      while(p && !p->go[i]) p = p->fail;
59      p = p ? p->go[i] : root;
60      if(p->cnt) {ans += p->cnt, p->cnt = 0;}
61      for(Node* t = p->dic; t; t = t->dic) if(t->
62        cnt) {
63          ans += t->cnt; t->cnt = 0;
64        }
65    }
66    return ans;
67 } AC;

```

5.2 KMP

```

1 vector<int> f;
2 // 沒匹配到可以退回哪裡
3 void buildFailFunction(string &s) {
4   f.resize(s.size(), -1);
5   for (int i = 1; i < s.size(); i++) {
6     int now = f[i - 1];
7     while (now != -1 and s[now + 1] != s[i]) now =
8       f[now];
9     if (s[now + 1] == s[i]) f[i] = now + 1;
10  }
11
12 void KMPmatching(string &a, string &b) {
13   for (int i = 0, now = -1; i < a.size(); i++) {
14     while (a[i] != b[now + 1] and now != -1) now =
15       f[now];

```

```

15     if (a[i] == b[now + 1]) now++;
16     if (now + 1 == b.size()) {
17         cout << "found a match start at position "
18             << i - now << endl;
19         now = f[now];
20     }
21 }

```

5.3 Z Value

```

1 string is, it, s;
2 // is: 被搜尋 it: 要找的
3 int n;
4 vector<int> z;
5 // 計算每個位置 i 開始的字串，和 s 的共農前綴長度
6 void init() {
7     cin >> is >> it;
8     s = it + '0' + is;
9     n = (int)s.size();
10    z.resize(n, 0);
11 }
12 void solve() {
13     int ans = 0;
14     z[0] = n;
15     for (int i = 1, l = 0, r = 0; i < n; i++) {
16         if (i <= r) z[i] = min(z[i - 1], r - i + 1);
17         while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
18             z[i]++;
19             if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
20             if (z[i] == (int)it.size()) ans++;
21     }
22     cout << ans << endl;
}

```

5.4 Manacher

```

1 // 找最長回文
2 int n;
3 string S, s;
4 vector<int> m;
5 void manacher() {
6     s.clear();
7     s.resize(2 * n + 1, '.');
8     for (int i = 0, j = 1; i < n; i++, j += 2) s[j] = S[i];
9     m.clear();
10    m.resize(2 * n + 1, 0);
11    // m[i] := max k such that s[i-k, i+k] is
12        // palindrome
13    int mx = 0, mxk = 0;
14    for (int i = 1; i < 2 * n + 1; i++) {
15        if (mx - (i - mx) >= 0) m[i] = min(m[mx - (i - mx)], mx + mxk - i);
16        while (0 <= i - m[i] - 1 && i + m[i] + 1 < 2 * n + 1 &&
17            s[i - m[i] - 1] == s[i + m[i] + 1]) m[i]++;
18        if (i + m[i] > mx + mxk) mx = i, mxk = m[i];
19    }
20    void init() {
21        cin >> S;
22        n = (int)S.size();
23    }
24    void solve() {
25        manacher();
26        int mx = 0, ptr = 0;
27        for (int i = 0; i < 2 * n + 1; i++) {
28            if (mx < m[i]) {
29                mx = m[i];
30                ptr = i;
31            }
32            for (int i = ptr - mx; i <= ptr + mx; i++)
33                if (s[i] != '.') cout << s[i];
34        }
35    }
}

```

5.5 Suffix Array

```
#define F first
```

```

2 #define S second
3 struct SuffixArray { // don't forget s += "$";
4     int n;
5     string s;
6     vector<int> suf, lcp, rk;
7     // 後綴陣列: suf[i] = 第 i 小的後綴起點
8     // LCP 陣列: lcp[i] = suf[i] 與 suf[i-1] 的最長共同
9         前綴長度
10    // rank 陣列: rk[i] = 起點在 i 的後綴的名次
11    vector<int> cnt, pos;
12    vector<pair<pair<int, int>, int> > buc[2];
13    void init(string _s) {
14        s = _s;
15        n = (int)s.size();
16        // resize(n): suf, rk, cnt, pos, lcp, buc[0~1]
17        suf.assign(n, 0);
18        rk.assign(n, 0);
19        lcp.assign(n, 0);
20        cnt.assign(n, 0);
21        pos.assign(n, 0);
22        buc[0].assign(n, {{0, 0}, 0});
23        buc[1].assign(n, {{0, 0}, 0});
24    }
25    void radix_sort() {
26        for (int t : {0, 1}) {
27            fill(cnt.begin(), cnt.end(), 0);
28            for (auto& i : buc[t]) cnt[(t ? i.F.F : i.F.S)]++;
29            for (int i = 0; i < n; i++) {
30                pos[i] = (!i ? 0 : pos[i - 1] + cnt[i - 1]);
31            }
32            for (auto& i : buc[t])
33                buc[t ^ 1][pos[(t ? i.F.F : i.F.S)]++] = i;
34        }
35    }
36    bool fill_suf() {
37        bool end = true;
38        for (int i = 0; i < n; i++) suf[i] = buc[0][i].S;
39        rk[suf[0]] = 0;
40        for (int i = 1; i < n; i++) {
41            int dif = (buc[0][i].F != buc[0][i - 1].F);
42            end &= dif;
43            rk[suf[i]] = rk[suf[i - 1]] + dif;
44        }
45        return end;
46    }
47    void sa() {
48        for (int i = 0; i < n; i++)
49            buc[0][i] = make_pair(make_pair(s[i], s[i]), i);
50        sort(buc[0].begin(), buc[0].end());
51        if (fill_suf()) return;
52        for (int k = 0; (1 << k) < n; k++) {
53            for (int i = 0; i < n; i++)
54                buc[0][i] = make_pair(make_pair(rk[i], rk[(i + (1 << k)) % n]), i);
55            radix_sort();
56            if (fill_suf()) return;
57        }
58    }
59    void LCP() {
60        int k = 0;
61        for (int i = 0; i < n - 1; i++) {
62            if (rk[i] == 0) continue;
63            int pi = rk[i];
64            int j = suf[pi - 1];
65            while (i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
66            lcp[pi] = k;
67            k = max(k - 1, 0);
68        }
69    }
}
SuffixArray suffixarray;

```

5.6 Suffix Automaton

```

1 struct SAM {
2     struct State {

```

```

3   int next[26];
4   int link, len;
5   // suffix link, 指向最長真後綴所對應的狀態
6   // 該狀態代表的字串集合中的最長字串長度
7   State() : link(-1), len(0) { memset(next, -1,
8       sizeof next); }
9
10  vector<State> st;
11  int last;
12  vector<long long> occ; // 每個狀態的出現次數 (
13      endpos 個數)
14  vector<int> first_bkpos; // 出現在哪裡
15  SAM(int maxlen = 0) {
16      st.reserve(2 * maxlen + 5); st.push_back(State()
17          ());
18      last = 0;
19      occ.reserve(2 * maxlen + 5); occ.push_back(0);
20      first_bkpos.push_back(-1);
21  }
22  void extend(int c) {
23      int cur = (int)st.size();
24      st.push_back(State());
25      occ.push_back(0);
26      first_bkpos.push_back(0);
27      st[cur].len = st[last].len + 1;
28      first_bkpos[cur] = st[cur].len - 1;
29      int p = last;
30      while (p != -1 && st[p].next[c] == -1) {
31          st[p].next[c] = cur;
32          p = st[p].link;
33      }
34      if (p == -1) {
35          st[cur].link = 0;
36      } else {
37          int q = st[p].next[c];
38          if (st[p].len + 1 == st[q].len) {
39              st[cur].link = q;
40          } else {
41              int clone = (int)st.size();
42              st.push_back(st[q]);
43              first_bkpos.push_back(first_bkpos[q]);
44              occ.push_back(0);
45              st[clone].len = st[p].len + 1;
46              while (p != -1 && st[p].next[c] == q) {
47                  st[p].next[c] = clone;
48                  p = st[p].link;
49              }
50              st[q].link = st[cur].link = clone;
51          }
52      }
53      last = cur;
54      occ[cur] += 1;
55  }
56  void finalize_occ() {
57      int m = (int)st.size();
58      vector<int> order(m);
59      iota(order.begin(), order.end(), 0);
60      sort(order.begin(), order.end(), [&](int a, int
61          b){ return st[a].len > st[b].len; });
62      for (int v : order) {
63          int p = st[v].link;
64          if (p != -1) occ[p] += occ[v];
65      }
66  }
67 };

```

5.7 Minimum Rotation

```

1 // rotate(begin(s), begin(s)+minRotation(s), end(s))
2 // 找出字串的最小字典序旋轉
3 int minRotation(string s) {
4     int a = 0, n = s.size();
5     s += s;
6     for (int b = 0; b < n; b++)
7         for (int k = 0; k < n; k++) {
8             if (a + k == b || s[a + k] < s[b + k]) {
9                 b += max(0, k - 1);
10                break;
11            }
12            if (s[a + k] > s[b + k]) {
13                a = b;
14                break;
15            }
16        }
17    }
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```

6 Geometry

6.1 Basic Operations

```

1 // typedef long long T;
2 typedef long double T;
3 const long double eps = 1e-12;
4
5 short sgn(T x) {
6     if (abs(x) < eps) return 0;
7     return x < 0 ? -1 : 1;
8 }
9
10 struct Pt {
11     T x, y;
12     Pt(T _x = 0, T _y = 0) : x(_x), y(_y) {}
13     Pt operator+(Pt a) { return Pt(x + a.x, y + a.y); }
14     Pt operator-(Pt a) { return Pt(x - a.x, y - a.y); }
15     Pt operator*(T a) { return Pt(x * a, y * a); }
16     Pt operator/(T a) { return Pt(x / a, y / a); }
17     T operator*(Pt a) { return x * a.x + y * a.y; }
18     T operator^(Pt a) { return x * a.y - y * a.x; }
19     bool operator<(Pt a) { return x < a.x || (x == a.x
20         && y < a.y); }
21     // return sgn(x-a.x) < 0 || (sgn(x-a.x) == 0 && sgn
22     // (y-a.y) < 0); }
23     bool operator==(Pt a) { return sgn(x - a.x) == 0 &&
24         sgn(y - a.y) == 0; }
25
26     Pt mv(Pt a, Pt b) { return b - a; }
27     T len2(Pt a) { return a * a; }
28     T dis2(Pt a, Pt b) { return len2(b - a); }
29     Pt rotate(Pt u) { return {-u.y, u.x}; }
30     Pt unit(Pt x) { return x / sqrtl(x * x); }
31     short ori(Pt a, Pt b) { return ((a ^ b) > 0) - ((a ^ b)
32         < 0); }
33     bool onseg(Pt p, Pt l1, Pt l2) {
34         Pt a = mv(p, l1), b = mv(p, l2);
35         return ((a ^ b) == 0) && ((a * b) <= 0);
36     }
37
38     inline T cross(const Pt &a, const Pt &b, const Pt &c) {
39         return (b.x - a.x) * (c.y - a.y)
40             - (b.y - a.y) * (c.x - a.x);
41 }
42
43     long double polar_angle(Pt ori, Pt pt){
44         return atan2(pt.y - ori.y, pt.x - ori.x);
45 }
46     // slope to degree atan(Slope) * 180.0 / acos(-1.0);
47     bool argcmp(Pt u, Pt v) {
48         auto half = []()>>const Pt&p) {
49             return p.y > 0 || (p.y == 0 && p.x >= 0);
50         };
51         if (half(u) != half(v)) return half(u) < half(v);
52         return sgn(u ^ v) > 0;
53     }
54     int ori(Pt&o, Pt&a, Pt&b) {
55         return sgn((a - o) ^ (b - o));
56     }
57     struct Line {
58         Pt a, b;
59         Pt dir() { return b - a; }
60     };
61     int PtSide(Pt p, Line L) {
62         return sgn(ori(L.a, L.b, p)); // for int
63         return sgn(ori(L.a, L.b, p) / sqrt(len2(L.a - L.b))
64             );
65     }
66     bool PtOnSeg(Pt p, Line L) {
67         return PtSide(p, L) == 0 and sgn((p - L.a) * (p - L
68             .b)) <= 0;
69     }
70     Pt proj(Pt&p, Line&l) {
71         Pt d = l.b - l.a;
72         T d2 = len2(d);
73         if (sgn(d2) == 0) return l.a;
74         T t = ((p - l.a) * d) / d2;
75         return l.a + d * t;
76     }
77     struct Cir {
78         Pt o;
79     };

```

```

73     T r;
74 }
75     bool disjunct(Cir a, Cir b) {
76         return sgn(sqrtl(len2(a.o - b.o)) - a.r - b.r) >=
77             0;
78 }
79     bool contain(Cir a, Cir b) {
80         return sgn(a.r - b.r - sqrtl(len2(a.o - b.o))) >=
81             0;
82 }

```

6.2 Sort by Angle

```

1 int ud(Pt a) { // up or down half plane
2     if (a.y > 0) return 0;
3     if (a.y < 0) return 1;
4     return (a.x >= 0 ? 0 : 1);
5 }
6 sort(pts.begin(), pts.end(), [&](const Pt&a, const Pt&
7     b) {
8     if (ud(a) != ud(b)) return ud(a) < ud(b);
9     return (a ^ b) > 0;
10 });

```

6.3 Intersection

```

1 bool line_intersect_check(Pt p1, Pt p2, Pt q1, Pt q2) {
2     if (onseg(p1, q1, q2) || onseg(p2, q1, q2) || onseg
3         (q1, p1, p2) || onseg(q2, p1, p2)) return true;
4     Pt p = mv(p1, p2), q = mv(q1, q2);
5     return (ori(p, mv(p1, q1)) * ori(p, mv(p1, q2)) <
6         0) && (ori(q, mv(q1, p1)) * ori(q, mv(q1, p2))
7         < 0);
8 }
9 // long double
10 Pt line_intersect(Pt a1, Pt a2, Pt b1, Pt b2) {
11     Pt da = mv(a1, a2), db = mv(b1, b2);
12     T det = da ^ db;
13     if (sgn(det) == 0) { // parallel
14         // return Pt(NAN, NAN);
15     }
16     T t = ((b1 - a1) ^ db) / det;
17     return a1 + da * t;
18 }
19 vector<Pt> CircleInter(Cir a, Cir b) {
20     double d2 = len2(a.o - b.o), d = sqrt(d2);
21     if (d < max(a.r, b.r) - min(a.r, b.r) || d > a.r +
22         b.r) return {};
23     Pt u = (a.o + b.o) / 2 + (a.o - b.o) * ((b.r * b.r
24         - a.r * a.r) / (2 * d2));
25     double A = sqrt((a.r + b.r + d) * (a.r - b.r + d) *
26         (a.r + b.r - d) * (-a.r + b.r + d));
27     Pt v = rotate(b.o - a.o) * A / (2 * d2);
28     if (sgn(v.x) == 0 and sgn(v.y) == 0) return {u};
29     return {u - v, u + v}; // counter clockwise of a
30 }
31 vector<Pt> CircleLineInter(Cir c, Line l) {
32     Pt H = proj(c.o, l);
33     Pt dir = unit(l.b - l.a);
34     T h = sqrtl(len2(H - c.o));
35     if (sgn(h - c.r) > 0) return {};
36     T d = sqrtl(max((T)0, c.r * c.r - h * h));
37     if (sgn(d) == 0) return {H};
38     return {H - dir * d, H + dir * d};
39 }

```

6.4 Polygon Area

```

1 // 2 * area
2 T dbPoly_area(vector<Pt>& e) {
3     T res = 0;
4     int sz = e.size();
5     for (int i = 0; i < sz; i++) {
6         res += e[i] ^ e[(i + 1) % sz];
7     }
8     return abs(res);
9 }

```

6.5 Convex Hull

```

1 vector<Pt> convexHull(vector<Pt> pts) {

```

```

2   vector<Pt> hull;
3   sort(pts.begin(), pts.end());
4   for (int i = 0; i < 2; i++) {
5     int b = hull.size();
6     for (auto ei : pts) {
7       while (hull.size() - b >= 2 && ori(mv(hull[7
8         .size() - 2], hull.back()), mv(hull[8
9         .size() - 2], ei)) == -1) {
10        hull.pop_back();
11      }
12      hull.emplace_back(ei);
13    }
14    hull.pop_back();
15    reverse(pts.begin(), pts.end());
16  }
17  return hull;
}

```

6.6 Point In Convex

```

1 bool point_in_convex(const vector<Pt> &C, Pt p, bool
2 strict = true) {
3   // only works when no three point are collinear
4   int n = C.size();
5   int a = 1, b = n - 1, r = !strict;
6   if (n == 0) return false;
7   if (n < 3) return r && onseg(p, C[0], C.back());
8   if (ori(mv(C[0], C[a]), mv(C[0], C[b])) > 0) swap(a
9     , b);
10  if (ori(mv(C[0], C[a]), mv(C[0], p)) >= r || ori(mv
11    (C[0], C[b]), mv(C[0], p)) <= -r) return false;
12  while (abs(a - b) > 1) {
13    int c = (a + b) / 2;
14    if (ori(mv(C[0], C[c]), mv(C[0], p)) > 0) b = c
15    ;
16    else a = c;
17  }
18  return ori(mv(C[a], C[b]), mv(C[a], p)) < r;
}

```

6.7 Point Segment Distance

```

1 double point_segment_dist(Pt q0, Pt q1, Pt p) {
2   if (q0 == q1) {
3     double dx = double(p.x - q0.x);
4     double dy = double(p.y - q0.y);
5     return sqrt(dx * dx + dy * dy);
6   }
7   T d1 = (q1 - q0) * (p - q0);
8   T d2 = (q0 - q1) * (p - q1);
9   if (d1 >= 0 && d2 >= 0) {
10    double area = fabs(double((q1 - q0) ^ (p - q0)))
11    );
12    double base = sqrt(double(dis2(q0, q1)));
13    return area / base;
14  }
15  double dx0 = double(p.x - q0.x), dy0 = double(p.y -
16    q0.y);
17  double dx1 = double(p.x - q1.x), dy1 = double(p.y -
18    q1.y);
19  return min(sqrt(dx0 * dx0 + dy0 * dy0), sqrt(dx1 *
20    dx1 + dy1 * dy1));
}

```

6.8 Point in Polygon

```

1 short inPoly(vector<Pt>& pts, Pt p) {
2   // 0=Bound 1=In -1=Out
3   int n = pts.size();
4   for (int i = 0; i < pts.size(); i++) if (onseg(p,
5     pts[i], pts[(i + 1) % n])) return 0;
6   int cnt = 0;
7   for (int i = 0; i < pts.size(); i++) if (
8     line_intersect_check(p, Pt(p.x + 1, p.y + 2e9),
9     pts[i], pts[(i + 1) % n])) cnt ^= 1;
10  return (cnt ? 1 : -1);
}

```

6.9 Minimum Euclidean Distance

```

1 long long Min_Euclidean_Dist(vector<Pt> &pts) {

```

```

2   sort(pts.begin(), pts.end());
3   set<pair<long long, long long>> s;
4   s.insert({pts[0].y, pts[0].x});
5   long long l = 0, best = LLONG_MAX;
6   for (int i = 1; i < (int)pts.size(); i++) {
7     Pt now = pts[i];
8     long long lim = (long long)ceil(sqrtl((long
9       double)best));
10    while (now.x - pts[1].x > lim) {
11      s.erase({pts[1].y, pts[1].x}); l++;
12    }
13    auto low = s.lower_bound({now.y - lim,
14      LLONG_MIN});
15    auto high = s.upper_bound({now.y + lim,
16      LLONG_MAX});
17    for (auto it = low; it != high; it++) {
18      long long dy = it->first - now.y;
19      long long dx = it->second - now.x;
20      best = min(best, dx * dx + dy * dy);
21    }
22    s.insert({now.y, now.x});
23  }
24  return best;
}

```

6.10 Minkowski Sum

```

1 void reorder(vector <Pt> &P) {
2   rotate(P.begin(), min_element(P.begin(), P.end()),
3     [&](Pt a, Pt b) { return make_pair(a.y, a.x) <
4       make_pair(b.y, b.x); }), P.end());
5 }
6 vector <Pt> Minkowski(vector <Pt> P, vector <Pt> Q) {
7   // P, Q: convex polygon
8   reorder(P), reorder(Q);
9   int n = P.size(), m = Q.size();
10  P.push_back(P[0]), P.push_back(P[1]), Q.push_back(Q
11    [0]), Q.push_back(Q[1]);
12  vector <Pt> ans;
13  for (int i = 0, j = 0; i < n || j < m; ) {
14    ans.push_back(P[i] + Q[j]);
15    auto val = (P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]);
16    if (val >= 0) i++;
17    if (val <= 0) j++;
18  }
19  return ans;
}

```

6.11 Lower Concave Hull

```

1 struct Line {
2   mutable ll m, b, p;
3   bool operator<(const Line& o) const { return m < o.m;
4     }
5   bool operator<(ll x) const { return p < x; }
6 };
7 struct LineContainer : multiset<Line, less<> {
8   // (for doubles, use inf = 1/.0, div(a,b) = a/b)
9   const ll inf = LLONG_MAX;
10  ll div(ll a, ll b) { // floored division
11    return a / b - ((a ^ b) < 0 && a % b); }
12  bool isect(iterator x, iterator y) {
13    if (y == end()) { x->p = inf; return false; }
14    if (x->m == y->m) x->p = x->b > y->b ? inf : -inf;
15    else x->p = div(y->b - x->b, x->m - y->m);
16    return x->p >= y->p;
17  }
18  void add(ll m, ll b) {
19    auto z = insert({m, b, 0}), y = z++, x = y;
20    while (isect(y, z)) z = erase(z);
21    if (x != begin() && isect(--x, y)) isect(x, y =
22      erase(y));
23    while ((y = y->p) != begin() && (--x)->p >= y->p)
24      isect(x, erase(y));
25  }
26  ll query(ll x) {
27    assert(!empty());
28    auto l = *lower_bound(x);
29    return l.m * x + l.b;
30  }
}

```

6.12 Pick's Theorem

Consider a polygon which vertices are all lattice points.

Let i = number of points inside the polygon.

Let b = number of points on the boundary of the polygon.

Then we have the following formula:

$$\text{Area} = i + \frac{b}{2} - 1$$

6.13 Rotating SweepLine

```

1 double cross(const Pt &a, const Pt &b) {
2     return a.x*b.y - a.y*b.x;
3 }
4 int rotatingCalipers(const vector<Pt>& hull) {
5     int m = hull.size();
6     if (m < 2) return 0;
7     int j = 1;
8     T maxd = 0;
9     for (int i = 0; i < m; ++i) {
10         int ni = (i + 1) % m;
11         while (abs(cross({hull[ni].x - hull[i].x, hull[ni].y - hull[i].y}, {hull[(j+1)%m].x - hull[i].x, hull[(j+1)%m].y - hull[i].y})) > abs(
12             (cross({hull[ni].x - hull[i].x, hull[ni].y - hull[i].y}, {hull[j].x - hull[i].x, hull[j].y - hull[i].y}) {
13                 j = (j + 1) % m;
14             }
15             maxd = max(maxd, dis2(hull[i], hull[j]));
16             maxd = max(maxd, dis2(hull[ni], hull[j]));
17         }
18     }
19     return maxd; // TODO
20 }
```

6.14 Half Plane Intersection

```

1 bool cover(Line& L, Line& P, Line& Q) {
2     long double u = (Q.a - P.a) ^ Q.dir();
3     long double v = P.dir() ^ Q.dir();
4     long double x = P.dir().x * u + (P.a - L.a).x * v;
5     long double y = P.dir().y * u + (P.a - L.a).y * v;
6     return sgn(x * L.dir().y - y * L.dir().x) * sgn(v)
7         >= 0;
8 }
9 vector<Line> HPI(vector<Line> P) {
10     sort(P.begin(), P.end(), [&](Line& l, Line& m) {
11         if (argcmp(l.dir(), m.dir())) return true;
12         if (argcmp(m.dir(), l.dir())) return false;
13         return ori(m.a, m.b, l.a) > 0;
14     });
15     int l = 0, r = -1;
16     for (size_t i = 0; i < P.size(); ++i) {
17         if (i && !argcmp(P[i - 1].dir(), P[i].dir()))
18             continue;
19         while (l < r && cover(P[i], P[r - 1], P[r])) --r;
20         while (l < r && cover(P[i], P[l], P[l + 1])) ++l;
21         P[++r] = P[i];
22     }
23     while (l < r && cover(P[1], P[r - 1], P[r])) --r;
24     while (l < r && cover(P[r], P[1], P[l + 1])) ++l;
25     if (r - 1 <= 1 || !argcmp(P[1].dir(), P[r].dir()))
26         return {};
27     if (cover(P[l + 1], P[1], P[r])) return {};
28     return vector<Line>(P.begin() + 1, P.begin() + r +
29         1);
30 }
```

6.15 Minimum Enclosing Circle

```

1 const int INF = 1e9;
2 Pt circumcenter(Pt A, Pt B, Pt C) {
3     // a1(x-A.x) + b1(y-A.y) = c1
4     // a2(x-A.x) + b2(y-A.y) = c2
5 }
```

```

5 // solve using Cramer's rule
6 T a1 = B.x - A.x, b1 = B.y - A.y, c1 = dis2(A, B) /
7     2.0;
8 T a2 = C.x - A.x, b2 = C.y - A.y, c2 = dis2(A, C) /
9     2.0;
10 T D = Pt(a1, b1) ^ Pt(a2, b2);
11 T Dx = Pt(c1, b1) ^ Pt(c2, b2);
12 T Dy = Pt(a1, c1) ^ Pt(a2, c2);
13 if (D == 0) return Pt(-INF, -INF);
14 return A + Pt(Dx / D, Dy / D);
15 }
16 Pt center;
17 T r2;
18 void minEncloseCircle(vector<Pt> pts) {
19     mt19937 gen(chrono::steady_clock::now());
20     time_since_epoch().count();
21     shuffle(pts.begin(), pts.end(), gen);
22     center = pts[0], r2 = 0;
23
24     for (int i = 0; i < pts.size(); i++) {
25         if (dis2(center, pts[i]) <= r2) continue;
26         center = pts[i], r2 = 0;
27         for (int j = 0; j < i; j++) {
28             if (dis2(center, pts[j]) <= r2) continue;
29             center = (pts[i] + pts[j]) / 2.0;
30             r2 = dis2(center, pts[i]);
31             for (int k = 0; k < j; k++) {
32                 if (dis2(center, pts[k]) <= r2)
33                     continue;
34                 center = circumcenter(pts[i], pts[j],
35                     pts[k]);
36                 r2 = dis2(center, pts[i]);
37             }
38         }
39     }
40 }
```

6.16 Union of Circles

```

1 // Area[i] : area covered by at least i circles
2 vector<T> CircleUnion(const vector<Cir> &C) {
3     const int n = C.size();
4     vector<T> Area(n + 1);
5     auto check = [&](int i, int j) {
6         if (!contain(C[i], C[j]))
7             return false;
8         return sgn(C[i].r - C[j].r) > 0 or (sgn(C[i].r
9             - C[j].r) == 0 and i < j);
10    };
11    struct Teve {
12        double ang; int add; Pt p;
13        bool operator<(const Teve &b) { return ang < b.
14            ang; }
15    };
16    auto ang = [&](Pt p) { return atan2(p.y, p.x); };
17    for (int i = 0; i < n; i++) {
18        int cov = 1;
19        vector<Teve> event;
20        for (int j = 0; j < n; j++) if (i != j) {
21            if (check(j, i)) cov++;
22            else if (!check(i, j) and !disjunct(C[i], C
23                [j])) {
24                auto I = CircleInter(C[i], C[j]);
25                assert(I.size() == 2);
26                double a1 = ang(I[0] - C[i].o), a2 =
27                    ang(I[1] - C[i].o);
28                event.push_back({a1, 1, I[0]});
29                event.push_back({a2, -1, I[1]});
30                if (a1 > a2) cov++;
31            }
32        }
33        if (event.empty()) {
34            Area[cov] += acos(-1) * C[i].r * C[i].r;
35            continue;
36        }
37        sort(event.begin(), event.end());
38        event.push_back(event[0]);
39        for (int j = 0; j + 1 < event.size(); j++) {
40            cov += event[j].add;
41            Area[cov] += (event[j].p ^ event[j + 1].p) /
42                2.;
```

```

38     double theta = event[j + 1].ang - event[j].ang;           41
39     if (theta < 0) theta += 2 * acos(-1);                     42
40     Area[cov] += (theta - sin(theta)) * C[i].r * C[i].r / 2.; 43
41   }
42 }
43 return Area;
44 }
```

6.17 Area Of Circle Polygon

```

1 double AreaOfCirclePoly(Cir C, vector<Pt> &P) {
2   auto arg = [&](Pt p, Pt q) { return atan2l(p ^ q, p
3     * q); };
4   double r2 = (double)(C.r * C.r / 2);
5   auto tri = [&](Pt p, Pt q) {
6     Pt d = q - p;
7     T a = (d * p) / (d * d);
8     T b = ((p * p) - C.r * C.r) / (d * d);
9     T det = a * a - b;
10    if (det <= 0) return (double)(arg(p, q) * r2);
11    T s = max((T)0.0L, -a - sqrtl(det));
12    T t = min((T)1.0L, -a + sqrtl(det));
13    if (t < 0 || 1 <= s) return (double)(arg(p, q)
14      * r2);
15    Pt u = p + d * s, v = p + d * t;
16    return (double)(arg(p, u) * r2 + (u ^ v) / 2 +
17      arg(v, q) * r2);
18  };
19  long double sum = 0.0L;
20  for (int i = 0; i < (int)P.size(); i++)
21    sum += tri(P[i] - C.o, P[(i + 1) % P.size()] -
22      C.o);
23  return (double)fabsl(sum);
24 }
```

6.18 3D Point

```

1 struct Pt {
2   double x, y, z;
3   Pt(double _x = 0, double _y = 0, double _z = 0): x(_x
4     ), y(_y), z(_z){}
5   Pt operator + (const Pt &o) const
6   { return Pt(x + o.x, y + o.y, z + o.z); }
7   Pt operator - (const Pt &o) const
8   { return Pt(x - o.x, y - o.y, z - o.z); }
9   Pt operator * (const double &k) const
10  { return Pt(x * k, y * k, z * k); }
11  Pt operator / (const double &k) const
12  { return Pt(x / k, y / k, z / k); }
13  double operator * (const Pt &o) const
14  { return x * o.x + y * o.y + z * o.z; }
15  Pt operator ^ (const Pt &o) const
16  { return {Pt(y * o.z - z * o.y, z * o.x - x * o.z, x
17    * o.y - y * o.x)}; }
18  double abs2(Pt o) { return o * o; }
19  double abs(Pt o) { return sqrt(abs2(o)); }
20  Pt cross3(Pt a, Pt b, Pt c)
21  { return (b - a) ^ (c - a); }
22  double area(Pt a, Pt b, Pt c)
23  { return abs(cross3(a, b, c)); }
24  double volume(Pt a, Pt b, Pt c, Pt d)
25  { return cross3(a, b, c) * (d - a); }
26  bool coplaner(Pt a, Pt b, Pt c, Pt d)
27  { return sign(volume(a, b, c, d)) == 0; }
28  Pt proj(Pt o, Pt a, Pt b, Pt c) // o proj to plane abc
29  { Pt n = cross3(a, b, c);
30  return o - n * ((o - a) * (n / abs2(n))); }
31  Pt line_plane_intersect(Pt u, Pt v, Pt a, Pt b, Pt c) {
32  // intersection of line uv and plane abc
33  Pt n = cross3(a, b, c);
34  double s = n * (u - v);
35  if (sign(s) == 0) return {-1, -1, -1}; // not found
36  return v + (u - v) * ((n * (a - v)) / s); }
37  Pt rotateAroundAxis(Pt v, Pt axis, double theta) {
38    axis = axis / abs(axis); // axis must be unit
39    vector
40    double cosT = cos(theta);
41    double sinT = sin(theta);
42    Pt term1 = v * cosT;
43 }
```

```

      Pt term2 = (axis ^ v) * sinT;
      Pt term3 = axis * ((axis * v) * (1 - cosT));
      return term1 + term2 + term3;
}
```

7 Number Theory

7.1 FFT

```

1 1 typedef complex<double> cp;
2 2
3 3 const double pi = acos(-1);
4 4 const int NN = 131072;
5 5
6 6 struct FastFourierTransform {
7 7
8 8   /*
9 9     Iterative Fast Fourier Transform
10 10    How this works? Look at this
11 11    0th recursion 0(000) 1(001) 2(010)
12 12    3(011) 4(100) 5(101) 6(110)
13 13    7(111)
14 14    1th recursion 0(000) 2(010) 4(100)
15 15    6(110) | 1(011) 3(011) 5(101)
16 16    7(111)
17 17    2th recursion 0(000) 4(100) / 2(010)
18 18    6(110) / 1(011) 5(101) / 3(011)
19 19    7(111)
20 20    3th recursion 0(000) / 4(100) / 2(010) /
21 21    6(110) / 1(011) / 5(101) / 3(011) /
22 22    7(111)
23 23    All the bits are reversed => We can save
24 24    the reverse of the numbers in an array!
25 25 */
26 26
27 27 int n, rev[NN];
28 28 cp omega[NN], iomega[NN];
29 29 void init(int n_) {
30 30   n = n_;
31 31   for (int i = 0; i < n_; i++) {
32 32     // Calculate the nth roots of unity
33 33     omega[i] = cp(cos(2 * pi * i / n_), sin(2 *
34 34     pi * i / n_));
35 35     iomega[i] = conj(omega[i]);
36 36   }
37 37   int k = __lg(n_);
38 38   for (int i = 0; i < n_; i++) {
39 39     int t = 0;
40 40     for (int j = 0; j < k; j++) {
41 41       if (i & (1 << j)) t |= (1 << (k - j -
42 42       1));
43 43     }
44 44     rev[i] = t;
45 45   }
46 46
47 47   void transform(vector<cp> &a, cp *xomega) {
48 48     for (int i = 0; i < n; i++)
49 49       if (i < rev[i]) swap(a[i], a[rev[i]]);
50 50     for (int len = 2; len <= n; len <= 1) {
51 51       int mid = len >> 1;
52 52       int r = n / len;
53 53       for (int j = 0; j < n; j += len)
54 54         for (int i = 0; i < mid; i++)
55 55           cp tmp = xomega[r * i] * a[j + mid
56 56           + i];
57 57           a[j + mid + i] = a[j + i] - tmp;
58 58           a[j + i] = a[j + i] + tmp;
59 59     }
60 60   }
61 61
62 62   void fft(vector<cp> &a) { transform(a, omega); }
63 63   void ifft(vector<cp> &a) {
64 64     transform(a, iomega);
65 65     for (int i = 0; i < n; i++) a[i] /= n;
66 66   }
67 67 } FFT;
68 68
69 69 const int MAXN = 262144;
70 70 // (must be 2^k)
71 71 // 262144, 524288, 1048576, 2097152, 4194304
72 72 // before any usage, run pre_fft() first
73 73 }
```

```

61 typedef long double ld;
62 typedef complex<ld> cplx; // real(), imag()
63 const ld PI = acosl(-1);
64 const cplx I(0, 1);
65 cplx omega[MAXN + 1];
66 void pre_fft() {
67     for (int i = 0; i <= MAXN; i++) {
68         omega[i] = exp(i * 2 * PI / MAXN * I);
69     }
70 }
71 // n must be 2^k
72 void fft(int n, cplx a[], bool inv = false) {
73     int basic = MAXN / n;
74     int theta = basic;
75     for (int m = n; m >= 2; m >>= 1) {
76         int mh = m >> 1;
77         for (int i = 0; i < mh; i++) {
78             cplx w = omega[inv ? MAXN - (i * theta % MAXN) : i * theta % MAXN];
79             for (int j = i; j < n; j += m) {
80                 int k = j + mh;
81                 cplx x = a[j] - a[k];
82                 a[j] += a[k];
83                 a[k] = w * x;
84             }
85         }
86         theta = (theta * 2) % MAXN;
87     }
88     int i = 0;
89     for (int j = 1; j < n - 1; j++) {
90         for (int k = n >> 1; k > (i ^= k); k >>= 1);
91         if (j < i) swap(a[i], a[j]);
92     }
93     if (inv) {
94         for (int i = 0; i < n; i++) a[i] /= n;
95     }
96 }
97 cplx arr[MAXN + 1];
98 inline void mul(int _n, long long a[], int _m, long
99     long b[], long long ans[]) {
100    int n = 1, sum = _n + _m - 1;
101    while (n < sum) n <= 1;
102    for (int i = 0; i < n; i++) {
103        double x = (i < _n ? a[i] : 0), y = (i < _m ? b[i] : 0);
104        arr[i] = complex<double>(x + y, x - y);
105    }
106    fft(n, arr);
107    for (int i = 0; i < n; i++) arr[i] = arr[i] * arr[i];
108    fft(n, arr, true);
109    for (int i = 0; i < sum; i++) ans[i] = (long long
110        int)(arr[i].real() / 4 + 0.5);
111 }
112 long long a[MAXN];
113 long long b[MAXN];
114 long long ans[MAXN];
115 int a_length;
116 int b_length;

```

7.2 Pollard's rho

```

1  ll add(ll x, ll y, ll p) {
2     return (x + y) % p;
3 }
4  ll qMul(ll x, ll y, ll mod) {
5     ll ret = x * y - ((long double)x / mod * y) *
6         mod;
7     return ret < 0 ? ret + mod : ret;
8 }
9  ll f(ll x, ll mod) { return add(qMul(x, x, mod), 1, mod);
10    );
11 }
11 pollard_rho(ll n) {
12     if (!(n & 1)) return 2;
13     while (true) {
14         ll y = 2, x = rand() % (n - 1) + 1, res = 1;
15         for (int sz = 2; res == 1; sz *= 2) {
16             for (int i = 0; i < sz && res <= 1; i++) {
17                 x = f(x, n);
18                 res = __gcd(labs(x - y), n);
19             }
20         }
21     }
22 }

```

```

18         y = x;
19     }
20     if (res != 0 && res != n) return res;
21 }
22 }
23 vector<ll> ret;
24 void fact(ll x) {
25     if (miller_rabin(x)) {
26         ret.push_back(x);
27         return;
28     }
29     ll f = pollard_rho(x);
30     fact(f);
31     fact(x / f);
32 }

```

7.3 Miller Rabin

```

1 // n < 4,759,123,141      3 : 2, 7, 61
2 // n < 1,122,004,669,633   4 : 2, 13, 23, 1662803
3 // n < 3,474,749,660,383   6 : pirmes <= 13
4 // n < 2^64                  7 :
5 // 2, 325, 9375, 28178, 450775, 9780504, 1795265022
6 bool witness(ll a, ll n, ll u, int t) {
7     if (!(a % n)) return 0;
8     ll x = mypow(a, u, n);
9     for (int i = 0; i < t; i++) {
10        ll nx = mul(x, x, n);
11        if (nx == 1 && x != 1 && x != n - 1) return 1;
12        x = nx;
13    }
14    return x != 1;
15 }
16 bool miller_rabin(ll n, int s = 100) {
17     // iterate s times of witness on n
18     // return 1 if prime, 0 otherwise
19     if (n < 2) return 0;
20     if (!(n & 1)) return n == 2;
21     ll u = n - 1;
22     int t = 0;
23     while (!(u & 1)) u >>= 1, t++;
24     while (s--) {
25         ll a = randll() % (n - 1) + 1;
26         if (witness(a, n, u, t)) return 0;
27     }
28     return 1;
29 }

```

7.4 Fast Power

Note: $a^n \equiv a^{(n \bmod (p-1))} \pmod{p}$

7.5 Extend GCD

```

1 ll GCD;
2 pll extgcd(ll a, ll b) {
3     if (b == 0) {
4         GCD = a;
5         return pll{1, 0};
6     }
7     pll ans = extgcd(b, a % b);
8     return pll{ans.S, ans.F - a / b * ans.S};
9 }
10 pll bezout(ll a, ll b, ll c) {
11     bool negx = (a < 0), negy = (b < 0);
12     pll ans = extgcd(abs(a), abs(b));
13     if (c % GCD != 0) return pll{-LLINF, -LLINF};
14     return pll{ans.F * c / GCD * (negx ? -1 : 1),
15                ans.S * c / GCD * (negy ? -1 : 1)};
16 }
17 ll inv(ll a, ll p) {
18     if (p == 1) return -1;
19     pll ans = bezout(a % p, -p, 1);
20     if (ans == pll{-LLINF, -LLINF}) return -1;
21     return (ans.F % p + p) % p;
22 }

```

7.6 Mu + Phi

```

1 const int maxn = 1e6 + 5;
2 ll f[maxn];
3 vector<int> lpf, prime;

```

```
4 void build() {
5     lpf.clear();
6     lpf.resize(maxn, 1);
7     prime.clear();
8     f[1] = ...; /* mu[1] = 1, phi[1] = 1 */
9     for (int i = 2; i < maxn; i++) {
10         if (lpf[i] == 1) {
11             lpf[i] = i;
12             prime.emplace_back(i);
13             f[i] = ...; /* mu[i] = 1, phi[i] = i-1 */
14         }
15         for (auto& j : prime) {
16             if (i * j >= maxn) break;
17             lpf[i * j] = j;
18             if (i % j == 0)
19                 f[i * j] = ...; /* θ, phi[i]*j */
20             else
21                 f[i * j] = ...; /* -mu[i], phi[i]*phi[i] */
22             if (j >= lpf[i]) break;
23         }
24     }
25 }
```

```

if ((r & 1) && ((m + 2) & 4)) s = -s;
a >= r;
if (a & m & 2) s = -s;
swap(a, m);
}
return s;

solve x^2 = a (mod p)
0: a == 0
-1: a isn't a quad res of p
else: return X with X^2 % p == a
doesn't work with long long
QuadraticResidue(int a, int p) {
if (p == 2) return a & 1;
if (int jc = Jacobi(a, p); jc <= 0) return jc;
int b, d;
for (; ; ) {
    b = rand() % p;
    d = (1LL * b * b + p - a) % p;
    if (Jacobi(d, p) == -1) break;
}
int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
for (int e = (1LL + p) >> 1; e; e >>= 1) {
    if (e & 1) {
        tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1
            * f1 % p)) % p;
        g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
        g0 = tmp;
    }
    tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1
        % p)) % p;
    f1 = (2LL * f0 * f1) % p;
    f0 = tmp;
}
return g0;
}

```

7.7 Discrete Log

```

1 long long mod_pow(long long a, long long e, long long p)
2     {
3         long long r = 1 % p;
4         while(e){
5             if(e & 1) r = (__int128)r * a % p;
6             a = (__int128)a * a % p;
7             e >>= 1;
8         }
9         return r;
10    }
11    long long mod_inv(long long a, long long p){
12        return mod_pow((a%p+p)%p, p-2, p);
13    }
14 // BSGS: solve a^x = y (mod p), gcd(a,p)=1, p prime,
15 // return minimal x>=0, or -1 if no solution
16 long long bsgs(long long a, long long y, long long p){
17     a%=p; y%=p;
18     if(y==1%p) return 0; // x=0
19     long long m = (long long)ceil(sqrt((long double)p));
20     ;
21     // baby steps: a^j
22     unordered_map<long long, long long> table;
23     table.reserve(m*2);
24     long long cur = 1%p;
25     for(long long j=0;j<m;++j){
26         if(!table.count(cur)) table[cur]=j;
27         cur = (__int128)cur * a % p;
28     }
29     long long am = mod_pow(a, m, p);
30     long long am_inv = mod_inv(am, p);
31     long long gamma = y % p;
32     for(long long i=0;i<=m;++i){
33         auto it = table.find(gamma);
34         if(it != table.end()){
35             long long x = i*m + it->second;
36             return x;
37         }
38         gamma = (__int128)gamma * am_inv % p;
39     }
40     return -1;
41 }
```

7.8 sqrt mod

```
1 // the Jacobi symbol is a generalization of the
2 // Legendre symbol,
3 // such that the bottom doesn't need to be prime.
4 // (n/p) -> same as legendre
5 // (n/ab) = (n/a)(n/b)
6 // work with long long
7 int Jacobi(int a, int m) {
8     int s = 1;
9     for ( ; m > 1; ) {
10         a %= m;
11         if (a == 0) return 0;
12         const int r = builtin_ctz(a);
```

7.9 Primitive Root

```
1 unsigned long long primitiveRoot(ull p) {
2     auto fac = factor(p - 1);
3     sort(all(fac));
4     fac.erase(unique(all(fac)), fac.end());
5     auto test = [p, fac](ull x) {
6         for(ull d : fac)
7             if (modpow(x, (p - 1) / d, p) == 1)
8                 return false;
9         return true;
10    };
11    uniform_int_distribution<unsigned long long> unif
12        (1, p - 1);
13    unsigned long long root;
14    while(!test(root = unif(rng)));
15    return root;
}
```

7.10 LinearSieve

```
1 const int C = 1e7 + 2;
2 int mo[C], lp[C], phi[C], isp[C];
3 vector<int> prime;
4 void sieve() {
5     mo[1] = phi[1] = 1;
6     for(int i = 1; i < C; i++) lp[i] = 1;
7     for(int i = 2; i < C; i++) {
8         if(lp[i] == 1) {
9             lp[i] = i;
10            prime.push_back(i);
11            isp[i] = 1;
12            mo[i] = -1;
13            phi[i] = i - 1;
14        }
15        for(int p : prime) {
16            if(i * p >= C) break;
17            lp[i * p] = p;
18            if(i % p == 0) {
19                phi[p * i] = phi[i] * p;
20                break;
21            }
22            phi[i * p] = phi[i] * (p - 1);
23            mo[i * p] = mo[i] * mo[p];
24        }
25    }
26}
```

```
25 }  
26 }
```

7.11 Other Formulas

- Inversion:
 $aa^{-1} \equiv 1 \pmod{m}$. a^{-1} exists iff $\gcd(a, m) = 1$.

- Linear inversion:
 $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod{m}$

- Fermat's little theorem:
 $a^p \equiv a \pmod{p}$ if p is prime.

- Euler function:
 $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$

- Euler theorem:
 $a^{\phi(n)} \equiv 1 \pmod{n}$ if $\gcd(a, n) = 1$.

- Extended Euclidean algorithm:

$$ax + by = \gcd(a, b) = \gcd(b, a \bmod b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$$

- Divisor function:

$$\sigma_x(n) = \sum_{d|n} d^x. n = \prod_{i=1}^r p_i^{a_i}.$$

$$\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x}-1}{p_i^x-1} \text{ if } x \neq 0. \sigma_0(n) = \prod_{i=1}^r (a_i + 1).$$

- Chinese remainder theorem (Coprime Moduli):

$$x \equiv a_i \pmod{m_i}.$$

$$M = \prod m_i. M_i = M/m_i. t_i = M_i^{-1}.$$

$$x = kM + \sum a_i t_i M_i, k \in \mathbb{Z}.$$

- Chinese remainder theorem:

$$x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2} \Rightarrow x = m_1 p + a_1 = m_2 q + a_2 \Rightarrow m_1 p - m_2 q = a_2 - a_1$$

Solve for (p, q) using ExtGCD.

$$x \equiv m_1 p + a_1 \equiv m_2 q + a_2 \pmod{\text{lcm}(m_1, m_2)}$$

- Avoiding Overflow: $ca \bmod cb = c(a \bmod b)$

- Dirichlet Convolution: $(f * g)(n) = \sum_{d|n} f(n)g(n/d)$

- Important Multiplicative Functions + Properties:

1. $\epsilon(n) = [n = 1]$
2. $1(n) = 1$
3. $id(n) = n$
4. $\mu(n) = 0$ if n has squared prime factor
5. $\mu(n) = (-1)^k$ if $n = p_1 p_2 \cdots p_k$
6. $\epsilon = \mu * 1$
7. $\phi = \mu * id$
8. $[n = 1] = \sum_{d|n} \mu(d)$
9. $[\gcd = 1] = \sum_{d|\gcd} \mu(d)$

- Möbius inversion: $f = g * 1 \Leftrightarrow g = f * \mu$

7.12 Polynomial

```
1 const int maxk = 20;  
2 const int maxn = 1<<maxk;  
3 const ll LINF = 1e18;  
4  
5 /* P = r*2^k + 1  
6 P r k g  
7 998244353 119 23 3  
8 1004535809 479 21 3  
9  
10 P r k g  
11 3 1 1 2  
12 5 1 2 2  
13 17 1 4 3  
14 97 3 5 5
```

15	193	3	6	5
16	257	1	8	3
17	7681	15	9	17
18	12289	3	12	11
19	40961	5	13	3
20	65537	1	16	3
21	786433	3	18	10
22	5767169	11	19	3
23	7340033	7	20	3
24	23068673	11	21	3
25	104857601	25	22	3
26	167772161	5	25	3
27	469762049	7	26	3
28	1004535809	479	21	3
29	2013265921	15	27	31
30	2281701377	17	27	3
31	3221225473	3	30	5
32	75161927681	35	31	3
33	77309411329	9	33	7
34	206158430209	3	36	22
35	2061584302081	15	37	7
36	2748779069441	5	39	3
37	6597069766657	3	41	5
38	39582418599937	9	42	5
39	79164837199873	9	43	5
40	263882790666241	15	44	7
41	1231453023109121	35	45	3
42	1337006139375617	19	46	3
43	3799912185593857	27	47	5
44	4222124650659841	15	48	19
45	7881299347898369	7	50	6
46	31525197391593473	7	52	3
47	180143985094819841	5	55	6
48	194555039024054273	27	56	5
49	4179340454199820289	29	57	3
50	9097271247288401921	505	54	6 */
51	const int g = 3;			
52	const ll MOD = 998244353;			
53	ll pw(ll a, ll n) { /* fast pow */ }			
54	#define siz(x) (int)x.size()			
55	template<typename T>			
56	vector<T>& operator+=(vector<T>& a, const vector<T>& b)			
57	{			
58	if (siz(a) < siz(b)) a.resize(siz(b));			
59	for (int i = 0; i < min(siz(a), siz(b)); i++) {			
60	a[i] += b[i];			
61	a[i] -= a[i] >= MOD ? MOD : 0;			
62	}			
63	return a;			
64	}			
65	template<typename T>			
66	vector<T>& operator-=(vector<T>& a, const vector<T>& b)			
67	{			
68	if (siz(a) < siz(b)) a.resize(siz(b));			
69	for (int i = 0; i < min(siz(a), siz(b)); i++) {			
70	a[i] -= b[i];			
71	a[i] += a[i] < 0 ? MOD : 0;			
72	}			
73	return a;			
74	}			
75	template<typename T>			
76	vector<T> operator-(const vector<T>& a) {			
77	vector<T> ret(siz(a));			
78	for (int i = 0; i < siz(a); i++) {			
79	ret[i] = -a[i] < 0 ? -a[i] + MOD : -a[i];			
80	}			
81	return ret;			
82	}			
83	vector<ll> X, ix;			
84	vector<int> rev;			
85	void init_ntt() {			
86	X.clear(); X.resize(maxn, 1); // x1 = g^((p-1)/n)			
87	ix.clear(); ix.resize(maxn, 1);			
88	91			
89	92			
90	93			
91	94			

```

11 u = pw(g, (MOD-1)/maxn);
11 iu = pw(u, MOD-2);

for (int i = 1; i < maxn; i++) {
    X[i] = X[i-1] * u;
    iX[i] = iX[i-1] * iu;
    if (X[i] >= MOD) X[i] %= MOD;
    if (iX[i] >= MOD) iX[i] %= MOD;
}

rev.clear(); rev.resize(maxn, 0);
for (int i = 1, hb = -1; i < maxn; i++) {
    if (!(i & (i-1))) hb++;
    rev[i] = rev[i ^ (1<<hb)] | (1<<(maxk-hb-1));
} }

template<typename T>
void NTT(vector<T>& a, bool inv=false) {
    int _n = (int)a.size();
    int k = __lg(_n) + ((1<<__lg(_n)) != _n);
    int n = 1<<k;
    a.resize(n, 0);

    short shift = maxk-k;
    for (int i = 0; i < n; i++)
        if (i > (rev[i]>>shift))
            swap(a[i], a[rev[i]>>shift]);

    for (int len = 2, half = 1, div = maxn>>1; len <= n;
         ; len<<=1, half<<=1, div>>=1) {
        for (int i = 0; i < n; i += len) {
            for (int j = 0; j < half; j++) {
                T u = a[i+j];
                T v = a[i+j+half] * (inv ? iX[j*div] : 1)
                    X[j*div]) % MOD;
                a[i+j] = (u+v >= MOD ? u+v-MOD : u+v);
                a[i+j+half] = (u-v < 0 ? u-v+MOD : u-v);
            }
        }
    }

    if (inv) {
        T dn = pw(n, MOD-2);
        for (auto& x : a) {
            x *= dn;
            if (x >= MOD) x %= MOD;
        }
    }
}

template<typename T>
inline void resize(vector<T>& a) {
    int cnt = (int)a.size();
    for (; cnt > 0; cnt--) if (a[cnt-1]) break;
    a.resize(max(cnt, 1));
}

template<typename T>
vector<T>& operator*=(vector<T>& a, vector<T> b) {
    int na = (int)a.size();
    int nb = (int)b.size();
    a.resize(na + nb - 1, 0);
    b.resize(na + nb - 1, 0);

    NTT(a); NTT(b);
    for (int i = 0; i < (int)a.size(); i++) {
        a[i] *= b[i];
        if (a[i] >= MOD) a[i] %= MOD;
    }
    NTT(a, true);

    resize(a);
    return a;
}

template<typename T>
void inv(vector<T>& ia, int N) {
    vector<T> _a(move(ia));
    ia.resize(1, pw(_a[0], MOD-2));
    vector<T> a(1, -_a[0] + (-_a[0] < 0 ? MOD : 0));

    for (int n = 1; n < N; n<<=1) {
        // n -> 2*n
        // ia' = ia(2-a*ia);
        for (int i = 1; i < min(siz(_a), (n<<1)); i++)
            a.emplace_back(-_a[i] + (-_a[i] < 0 ? MOD : 0));

        vector<T> tmp = ia;
        ia *= a;
        ia.resize(n<<1);
        ia[0] = ia[0] + 2 >= MOD ? ia[0] + 2 - MOD : ia[0] + 2;
        ia *= tmp;
        ia.resize(n<<1);
    }
    ia.resize(N);
}

template<typename T>
void mod(vector<T>& a, vector<T>& b) {
    int n = (int)a.size()-1, m = (int)b.size()-1;
    if (n < m) return;

    vector<T> ra = a, rb = b;
    reverse(ra.begin(), ra.end()); ra.resize(min(n+1, n-m+1));
    reverse(rb.begin(), rb.end()); rb.resize(min(m+1, n-m+1));

    inv(rb, n-m+1);

    vector<T> q = move(ra);
    q *= rb;
    q.resize(n-m+1);
    reverse(q.begin(), q.end());

    q *= b;
    a -= q;
    resize(a);

    /* Kitamasa Method (Fast Linear Recurrence):
     * Find a[K] (Given a[j] = c[0]a[j-N] + ... + c[N-1]a[j-1])
     * Let B(x) = x^N - c[N-1]x^(N-1) - ... - c[1]x^1 - c[0]
     * Let R(x) = x^K mod B(x) (get x^K using fast pow and
     * use poly mod to get R(x))
     * Let r[i] = the coefficient of x^i in R(x)
     * => a[K] = a[0]r[0] + a[1]r[1] + ... + a[N-1]r[N-1] */
}

8 Linear Algebra
  8.1 Gaussian-Jordan Elimination

int n;
vector<vector<ll>> v;
void gauss(vector<vector<ll>>& v) {
    int r = 0;
    for (int i = 0; i < n; i++) {
        bool ok = false;
        for (int j = r; j < n; j++) {
            if (v[j][i] == 0) continue;
            swap(v[j], v[r]);
            ok = true;
            break;
        }
        if (!ok) continue;
        ll div = inv(v[r][i]);
        for (int j = 0; j < n + 1; j++) {
            v[r][j] *= div;
            if (v[r][j] >= MOD) v[r][j] %= MOD;
        }
        for (int j = 0; j < n; j++) {
            if (j == r) continue;
            ll t = v[j][i];
            for (int k = 0; k < n + 1; k++) {
                v[j][k] -= v[r][k] * t % MOD;
                if (v[j][k] < 0) v[j][k] += MOD;
            }
        }
        r++;
    }
}

```

8.2 Determinant

1. Use GJ Elimination, if there's any row consists of only 0, then $\det = 0$, otherwise $\det = \text{product of diagonal elements}$.

2. Properties of \det :

- Transpose: Unchanged
- Row Operation 1 - Swap 2 rows: $-\det$
- Row Operation 2 - $k\vec{r}_i$: $k \times \det$
- Row Operation 3 - $k\vec{r}_i$ add to \vec{r}_j : Unchanged

9 Combinatorics

9.1 Catalan Number

$$C_0 = 1, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}, C_n = C_n^{2n} - C_{n-1}^{2n}$$

0	1	1	2	5
4	14	42	132	429
8	1430	4862	16796	58786
12	208012	742900	2674440	9694845

9.2 Burnside's Lemma

Let X be the original set.

Let G be the group of operations acting on X .

Let X^g be the set of x not affected by g .

Let X/G be the set of orbits.

Then the following equation holds:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

10 Reminder

10.1 Bug List

- 沒開 long long
- 本地編譯請開-Wall -Wextra -Wshadow -fsanitize=address
- 陣列越出界 / 開不夠大 / 開太大本地 compile 噴怪 error
- 傳之前先確定選對檔案
- 變數打錯
- 0-base / 1-base
- 忘記初始化
- == 打成 =
- dp[i] 從 dp[i-1] 轉移時忘記特判 $i > 0$
- std::sort 比較運算子寫成 < 或是讓 = 的情況為 true
- 漏 case / 分 case 要好好想
- 線段樹改值懶標初始值不能設為 0
- 少碰動態開點，能離散化就離散化
- 能不用浮點數運算就不用
- DFS 的時候不小心覆寫到全域變數
- 記得刪 cerr
- 注意 n m 有沒有亂用
- 1 不是質數
- map TLE 用 gp_hash_table
- 雙指針沒好好處理另一個指針
- 取% 注意會不會有負數情況



