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2 Basic

2.1 Vimrc

```
set number relativenumber ai t_Co=256 tabstop=4
set mouse=a shiftwidth=4 encoding=utf8
set bs=2 ruler laststatus=2 cmdheight=2
set clipboard=unnamedplus showcmd autoread
set belloff=all
filetype indent on

inoremap ( (<Esc>i
inoremap " "<Esc>i
inoremap [ [<Esc>i
inoremap ' '<Esc>i
inoremap { {<CR><Esc>ko

nnoremap <tab> gt
nnoremap <S-tab> gT
inoremap <C-n> <Esc>:tabnew<CR>
nnoremap <C-n> :tabnew<CR>

inoremap <F9> <Esc>:w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>
nnoremap <F9> :w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>

syntax on
colorscheme desert
set filetype=cpp
set background=dark
hi Normal ctermfg=white ctermbg=black
```

2.2 Runcpp.sh

```
#!/bin/bash
clear
echo "Start compiling $1..."
echo
g++ -O2 -std=c++20 -Wall -Wextra -Wshadow $2/$1 -o $2/
out
if [ "$?" -ne 0 ]
then
    exit 1
fi
echo
echo "Done compiling"
echo "===== "
echo
echo "Input file:"
echo
cat $2/in.txt
echo
echo "===== "
echo
declare startTime=`date +%s%N`
$2/out < $2/in.txt > $2/out.txt
declare endTime=`date +%s%N`
delta=`expr $endTime - $startTime`
delta=`expr $delta / 1000000`
cat $2/out.txt
echo
echo "time: $delta ms"
```

2.3 PBDS

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

// map
tree<int, int, less<>, rb_tree_tag,
    tree_order_statistics_node_update> tr;
tr.order_of_key(element);
tr.find_by_order(rank);

// set
tree<int, null_type, less<>, rb_tree_tag,
    tree_order_statistics_node_update> tr;
tr.order_of_key(element);
tr.find_by_order(rank);

// hash table
gp_hash_table<int, int> ht;
ht.find(element);
```

1 Reminder

1.1 Bug List

- 沒開 long long
- 陣列戳出界／開不夠大／開太大本地 compile 噴怪 error
- 傳之前先確定選對檔案
- 寫好的函式忘記呼叫
- 變數打錯
- 0-base / 1-base
- 忘記初始化
- == 打成 =
- <= 打成 <+
- dp[i] 從 dp[i-1] 轉移時忘記特判 i > 0
- std::sort 比較運算子寫成 < 或是讓 = 的情況為 true
- 漏 case / 分 case 要好好想
- 線段樹改值懶標初始值不能設為 0
- DFS 的時候不小心覆寫到全域變數
- 浮點數誤差
- 多筆測資不能沒讀完直接 return
- 記得刪 cerr

1.2 OwO

- 可以構造複雜點的測資幫助思考
- 真的卡太久請跳題
- Enjoy The Contest!

```

17 ht.insert({key, value});
18 ht.erase(element);
19
20 // priority queue
21 __gnu_pbds::priority_queue<int, less<int>> big_q;
22 // Big First
23 __gnu_pbds::priority_queue<int, greater<int>> small_q;
24 // Small First
25 q1.join(q2); // join

```

2.4 Random

```

1 mt19937 gen(chrono::steady_clock::now().
   time_since_epoch().count());
2 uniform_int_distribution<int> dis(1, 100);
3 cout << dis(gen) << endl;
4 shuffle(v.begin(), v.end(), gen);

```

3 Data Structure

3.1 BIT

```

1 struct BIT {
2     int n;
3     long long bit[N];
4
5     void init(int x, vector<long long> &a) {
6         n = x;
7         for (int i = 1, j; i <= n; i++) {
8             bit[i] += a[i - 1], j = i + (i & -i);
9             if (j <= n) bit[j] += bit[i];
10        }
11    }
12
13    void update(int x, long long dif) {
14        while (x <= n) bit[x] += dif, x += x & -x;
15    }
16
17    long long query(int l, int r) {
18        if (l != 1) return query(1, r) - query(1, l - 1);
19
20        long long ret = 0;
21        while (l <= r) ret += bit[r], r -= r & -r;
22        return ret;
23    }
24 } bm;

```

3.2 DSU

```

1 struct DSU {
2     int h[N], s[N];
3
4     void init(int n) { iota(h, h + n + 1, 0), fill(s, s + n + 1, 1); }
5
6     int fh(int x) { return (h[x] == x ? x : h[x] = fh(h[x])); }
7
8     bool mer(int x, int y) {
9         x = fh(x), y = fh(y);
10        if (x == y) return 0;
11        if (s[x] < s[y]) swap(x, y);
12        s[x] += s[y], s[y] = 0;
13        h[y] = x;
14        return 1;
15    }
16 } bm;

```

3.3 Segment Tree

```

1 struct segtree {
2     int n, seg[1 << 19];
3
4     void init(int x) {
5         n = 1 << (lg(x) + 1);
6         for (int i = 1; i < 2 * n; i++)
7             seg[i] = inf;
8     }
9
10    void update(int x, int val) {

```

```

11        x += n;
12        seg[x] = val, x /= 2;
13        while (x)
14            seg[x] = min(seg[2 * x], seg[2 * x + 1]), x /= 2;
15    }
16
17    int query(int l, int r) {
18        l += n, r += n;
19        int ret = inf;
20        while (l < r) {
21            if (l & 1)
22                ret = min(ret, seg[l++]);
23            if (r & 1)
24                ret = min(ret, seg[--r]);
25            l /= 2, r /= 2;
26        }
27        return ret;
28    }
29 } bm;

```

3.4 Treap

```

1 mt19937 rng(random_device{}());
2 struct Treap {
3     Treap *l, *r;
4     int val, num, pri;
5     Treap(int k) {
6         l = r = NULL;
7         val = k;
8         num = 1;
9         pri = rng();
10    }
11};
12 int siz(Treap *now) { return now ? now->num : 0; }
13 void pull(Treap *&now) {
14     now->num = siz(now->l) + siz(now->r) + 1;
15 }
16 Treap *merge(Treap *a, Treap *b) {
17     if (!a || !b)
18         return a ? a : b;
19     else if (a->pri > b->pri) {
20         a->r = merge(a->r, b);
21         pull(a);
22         return a;
23     } else {
24         b->l = merge(a, b->l);
25         pull(b);
26         return b;
27     }
28 }
29 void split_size(Treap *rt, Treap *&a, Treap *&b, int val) {
30     if (!rt) {
31         a = b = NULL;
32         return;
33     }
34     if (siz(rt->l) + 1 > val) {
35         b = rt;
36         split_size(rt->l, a, b->l, val);
37         pull(b);
38     } else {
39         a = rt;
40         split_size(rt->r, a->r, b, val - siz(a->l) - 1);
41         pull(a);
42     }
43 }
44 void split_val(Treap *rt, Treap *&a, Treap *&b, int val) {
45     if (!rt) {
46         a = b = NULL;
47         return;
48     }
49     if (rt->val <= val) {
50         a = rt;
51         split_val(rt->r, a->r, b, val);
52         pull(a);
53     } else {
54         b = rt;
55         split_val(rt->l, a, b->l, val);
56         pull(b);

```

```

57     }
58 }
59 void treap_dfs(Treap *now) {
60     if (!now) return;
61     treap_dfs(now->l);
62     cout << now->val << " ";
63     treap_dfs(now->r);
64 }

```

3.5 Persistent Treap

```

1 struct node {
2     node *l, *r;
3     char c;
4     int v, sz;
5     node(char x = '$') : c(x), v(mt()), sz(1) {
6         l = r = nullptr;
7     }
8     node(node* p) { *this = *p; }
9     void pull() {
10         sz = 1;
11         for (auto i : {l, r})
12             if (i) sz += i->sz;
13     }
14 } arr[maxn], *ptr = arr;
15 inline int size(node* p) { return p ? p->sz : 0; }
16 node* merge(node* a, node* b) {
17     if (!a || !b) return a ? b;
18     if (a->v < b->v) {
19         node* ret = new (ptr++) node(a);
20         ret->r = merge(ret->r, b); ret->pull();
21         return ret;
22     } else {
23         node* ret = new (ptr++) node(b);
24         ret->l = merge(a, ret->l); ret->pull();
25         return ret;
26     }
27 }
28 P<node*> split(node* p, int k) {
29     if (!p) return {nullptr, nullptr};
30     if (k >= size(p->l) + 1) {
31         auto [a, b] = split(p->r, k - size(p->l) - 1);
32         node* ret = new (ptr++) node(p);
33         ret->r = a; ret->pull();
34         return {ret, b};
35     } else {
36         auto [a, b] = split(p->l, k);
37         node* ret = new (ptr++) node(p);
38         ret->l = b; ret->pull();
39         return {a, ret};
40     }
41 }

```

3.6 Li Chao Tree

```

1 constexpr int maxn = 5e4 + 5;
2 struct line {
3     ld a, b;
4     ld operator()(ld x) { return a * x + b; }
5 } arr[(maxn + 1) << 2];
6 bool operator<(line a, line b) { return a.a < b.a; }
7 #define m ((l + r) >> 1)
8 void insert(line x, int i = 1, int l = 0, int r = maxn) {
9     if (r - l == 1) {
10         if (x(l) > arr[i](l))
11             arr[i] = x;
12         return;
13     }
14     line a = max(arr[i], x), b = min(arr[i], x);
15     if (a(m) > b(m))
16         arr[i] = a, insert(b, i << 1, l, m);
17     else
18         arr[i] = b, insert(a, i << 1 | 1, m, r);
19 }
20 ld query(int x, int i = 1, int l = 0, int r = maxn) {
21     if (x < l || r <= x) return -numeric_limits<ld>::
22         max();
23     if (r - l == 1) return arr[i](x);
24     return max({arr[i](x), query(x, i << 1, l, m),
25         query(x, i << 1 | 1, m, r)});

```

```

25 #undef m

```

3.7 Sparse Table

```

1 const int lgmx = 19;
2
3 int n, q;
4 int spt[lgmx][maxn];
5
6 void build() {
7     FOR(k, 1, lgmx, 1) {
8         for (int i = 0; i + (1 << k) - 1 < n; i++) {
9             spt[k][i] = min(spt[k - 1][i], spt[k - 1][i
10                 + (1 << (k - 1))]);
11         }
12     }
13 }
14 int query(int l, int r) {
15     int ln = len(l, r);
16     int lg = __lg(ln);
17     return min(spt[lg][l], spt[lg][r - (1 << lg) + 1]);
18 }

```

3.8 Time Segment Tree

```

1 constexpr int maxn = 1e5 + 5;
2 V<P<int>> arr[(maxn + 1) << 2];
3 V<int> dsu, sz;
4 V<tuple<int, int, int>> his;
5 int cnt, q;
6 int find(int x) {
7     return x == dsu[x] ? x : find(dsu[x]);
8 };
9 inline bool merge(int x, int y) {
10     int a = find(x), b = find(y);
11     if (a == b) return false;
12     if (sz[a] > sz[b]) swap(a, b);
13     his.emplace_back(a, b, sz[b]), dsu[a] = b, sz[b] +=
14         sz[a];
15     return true;
16 };
17 inline void undo() {
18     auto [a, b, s] = his.back();
19     his.pop_back();
20     dsu[a] = a, sz[b] = s;
21 }
22 #define m ((l + r) >> 1)
23 void insert(int ql, int qr, P<int> x, int i = 1, int l
24     = 0, int r = q) {
25     // debug(ql, qr, x); return;
26     if (qr <= l || r <= ql) return;
27     if (ql <= l && r <= qr) {
28         arr[i].push_back(x);
29         return;
30     }
31     if (qr <= m)
32         insert(ql, qr, x, i << 1, l, m);
33     else if (m <= ql)
34         insert(ql, qr, x, i << 1 | 1, m, r);
35     else {
36         insert(ql, qr, x, i << 1, l, m);
37         insert(ql, qr, x, i << 1 | 1, m, r);
38     }
39 }
40 void traversal(V<int>& ans, int i = 1, int l = 0, int r
41     = q) {
42     int opcnt = 0;
43     // debug(i, l, r);
44     for (auto [a, b] : arr[i])
45         if (merge(a, b))
46             opcnt++, cnt--;
47     if (r - l == 1)
48         ans[l] = cnt;
49     else {
50         traversal(ans, i << 1, l, m);
51         traversal(ans, i << 1 | 1, m, r);
52     }
53     while (opcnt--)
54         undo(), cnt++;
55     arr[i].clear();

```

```

54 #undef m
55 inline void solve() {
56     int n, m;
57     cin >> n >> m >> q, q++;
58     dsu.resize(cnt = n), sz.assign(n, 1);
59     iota(dsu.begin(), dsu.end(), 0);
60     // a, b, time, operation
61     unordered_map<ll, V<int>> s;
62     for (int i = 0; i < m; i++) {
63         int a, b;
64         cin >> a >> b;
65         if (a > b) swap(a, b);
66         s[((ll)a << 32) | b].emplace_back(0);
67     }
68     for (int i = 1; i < q; i++) {
69         int op, a, b;
70         cin >> op >> a >> b;
71         if (a > b) swap(a, b);
72         switch (op) {
73             case 1:
74                 s[((ll)a << 32) | b].push_back(i);
75                 break;
76             case 2:
77                 auto tmp = s[((ll)a << 32) | b].back();
78                 s[((ll)a << 32) | b].pop_back();
79                 insert(tmp, i, P<int>>{a, b});
80             }
81     }
82     for (auto [p, v] : s) {
83         int a = p >> 32, b = p & -1;
84         while (v.size()) {
85             insert(v.back(), q, P<int>>{a, b});
86             v.pop_back();
87         }
88     }
89     V<int> ans(q);
90     traversal(ans);
91     for (auto i : ans)
92         cout << i << ' ';
93     cout << endl;
94 }

```

3.9 Dynamic Median

```

1 struct Dynamic_Median {
2     multiset<long long> lo, hi;
3     long long slo = 0, shi = 0;
4     void rebalance() {
5         // keep sz(lo) >= sz(hi) and sz(lo) - sz(hi) <= 1
6         while((int)lo.size() > (int)hi.size() + 1) {
7             auto it = prev(lo.end());
8             long long x = *it;
9             lo.erase(it); slo -= x;
10            hi.insert(x); shi += x;
11        }
12        while((int)lo.size() < (int)hi.size()) {
13            auto it = hi.begin();
14            long long x = *it;
15            hi.erase(it); shi -= x;
16            lo.insert(x); slo += x;
17        }
18    }
19    void add(long long x) {
20        if(lo.empty() || x <= *prev(lo.end())) {
21            lo.insert(x); slo += x;
22        }
23        else {
24            hi.insert(x); shi += x;
25        }
26        rebalance();
27    }
28    void remove_one(long long x) {
29        if(!lo.empty() && x <= *prev(lo.end())) {
30            auto it = lo.find(x);
31            if(it != lo.end()) {
32                lo.erase(it); slo -= x;
33            }
34            else {
35                auto it2 = hi.find(x);
36                hi.erase(it2); shi -= x;
37            }

```

```

38        }
39        else {
40            auto it = hi.find(x);
41            if(it != hi.end()) {
42                hi.erase(it); shi -= x;
43            }
44            else {
45                auto it2 = lo.find(x);
46                lo.erase(it2); slo -= x;
47            }
48        }
49        rebalance();
50    }
51 };

```

4 Flow / Matching

4.1 Dinic

```

1 struct Dinic {
2     struct Edge { int to, cap, rev; };
3     int n, s, t;
4     vector<vector<Edge>> g;
5     vector<int> level, it;
6
7     void init(int _n, int _s, int _t){
8         n=_n; s=_s; t=_t;
9         g.assign(n, {});
10        level.assign(n, 0);
11        it.assign(n, 0);
12    }
13    void add(int a, int b, int c){
14        Edge f{b, c, (int)g[b].size()};
15        Edge r{a, 0, (int)g[a].size()};
16        g[a].push_back(f);
17        g[b].push_back(r);
18    }
19    bool bfs(){
20        fill(level.begin(), level.end(), -1);
21        queue<int> q; level[s]=0; q.push(s);
22        while(!q.empty()){
23            int u=q.front(); q.pop();
24            for(const auto &e: g[u]){
25                if(e.cap>0 && level[e.to]==-1){
26                    level[e.to]=level[u]+1;
27                    q.push(e.to);
28                }
29            }
30        }
31        return level[t]!=-1;
32    }
33    int dfs(int u, int f){
34        if(!f || u==t) return f;
35        for(int &i=it[u]; i<(int)g[u].size(); ++i){
36            auto &e=g[u][i];
37            if(e.cap>0 && level[e.to]==level[u]+1){
38                int got=dfs(e.to, min(f, e.cap));
39                if(got){
40                    e.cap-=got;
41                    g[e.to][e.rev].cap+=got;
42                    return got;
43                }
44            }
45        }
46        return 0;
47    }
48    int maxflow(){
49        int flow=0, add;
50        while(bfs()){
51            fill(it.begin(), it.end(), 0);
52            while((add=dfs(s, INF))) flow+=add;
53        }
54        return flow;
55    }
56 };

```

4.2 MCMF

```

1 struct MCMF {
2     int n, s, t, par[N + 5], p_i[N + 5], dis[N + 5],
3         vis[N + 5];

```

```

3 struct edge {
4     int to, cap, rev, cost;
5 };
6 vector<edge> path[N];
7 void init(int _n, int _s, int _t) {
8     n = _n, s = _s, t = _t;
9     FOR(i, 0, 2 * n + 5)
10         par[i] = p_i[i] = vis[i] = 0;
11 }
12 void add(int a, int b, int c, int d) {
13     path[a].pb({b, c, sz(path[b]), d});
14     path[b].pb({a, 0, sz(path[a]) - 1, -d});
15 }
16 void spfa() {
17     FOR(i, 0, n * 2 + 5)
18         dis[i] = INF,
19         vis[i] = 0;
20     dis[s] = 0;
21     queue<int> q;
22     q.push(s);
23     while (!q.empty()) {
24         int now = q.front();
25         q.pop();
26         vis[now] = 0;
27         for (int i = 0; i < sz(path[now]); i++) {
28             edge e = path[now][i];
29             if (e.cap > 0 && dis[e.to] > dis[now] +
30                 e.cost) {
31                 dis[e.to] = dis[now] + e.cost;
32                 par[e.to] = now;
33                 p_i[e.to] = i;
34                 if (vis[e.to] == 0) {
35                     vis[e.to] = 1;
36                     q.push(e.to);
37                 }
38             }
39         }
40     }
41 }
42 pii flow() {
43     int flow = 0, cost = 0;
44     while (true) {
45         spfa();
46         if (dis[t] == INF)
47             break;
48         int mn = INF;
49         for (int i = t; i != s; i = par[i])
50             mn = min(mn, path[par[i]][p_i[i]].cap);
51         flow += mn;
52         cost += dis[t] * mn;
53         for (int i = t; i != s; i = par[i]) {
54             edge &now = path[par[i]][p_i[i]];
55             now.cap -= mn;
56             path[i][now.rev].cap += mn;
57         }
58         return mp(flow, cost);
59     }
60 };

```

4.3 KM

```

1 struct KM {
2     int n, mx[1005], my[1005], pa[1005];
3     int g[1005][1005], lx[1005], ly[1005], sy[1005];
4     bool vx[1005], vy[1005];
5     void init(int _n) {
6         n = _n;
7         FOR(i, 1, n + 1)
8             fill(g[i], g[i] + 1 + n, 0);
9     }
10    void add(int a, int b, int c) { g[a][b] = c; }
11    void augment(int y) {
12        for (int x, z; y; y = z)
13            x = pa[y], z = mx[x], my[y] = x, mx[x] = y;
14    }
15    void bfs(int st) {
16        FOR(i, 1, n + 1)
17            sy[i] = INF,
18            vx[i] = vy[i] = 0;
19        queue<int> q;
20        q.push(st);

```

```

21 for (;;) {
22     while (!q.empty()) {
23         int x = q.front();
24         q.pop();
25         vx[x] = 1;
26         FOR(y, 1, n + 1)
27             if (!vy[y]) {
28                 int t = lx[x] + ly[y] - g[x][y];
29                 if (t == 0) {
30                     pa[y] = x;
31                     if (!my[y]) {
32                         augment(y);
33                         return;
34                     }
35                     vy[y] = 1, q.push(my[y]);
36                 } else if (sy[y] > t)
37                     pa[y] = x, sy[y] = t;
38             }
39         }
40     int cut = INF;
41     FOR(y, 1, n + 1)
42         if (!vy[y] && cut > sy[y]) cut = sy[y];
43     FOR(j, 1, n + 1) {
44         if (vx[j]) lx[j] -= cut;
45         if (vy[j]) ly[j] += cut;
46         else sy[j] -= cut;
47     }
48     FOR(y, 1, n + 1) {
49         if (!vy[y] && sy[y] == 0) {
50             if (!my[y]) {
51                 augment(y);
52                 return;
53             }
54             vy[y] = 1;
55             q.push(my[y]);
56         }
57     }
58 }
59 }
60 }
61 }
62 int solve() {
63     fill(mx, mx + n + 1, 0);
64     fill(my, my + n + 1, 0);
65     fill(lx, lx + n + 1, 0);
66     fill(ly, ly + n + 1, 0);
67     FOR(x, 1, n + 1)
68         FOR(y, 1, n + 1)
69             lx[x] = max(lx[x], g[x][y]);
70     FOR(x, 1, n + 1)
71         bfs(x);
72     int ans = 0;
73     FOR(y, 1, n + 1)
74         ans += g[my[y]][y];
75     return ans;
76 }
77 };

```

4.4 Hopcroft-Karp

```

1 struct HopcroftKarp {
2     // id: X = [1, nx], Y = [nx+1, nx+ny]
3     int n, nx, ny, m, MXCNT;
4     vector<vector<int>> g;
5     vector<int> mx, my, dis, vis;
6     void init(int nnx, int nny, int mm) {
7         nx = nnx, ny = nny, m = mm;
8         n = nx + ny + 1;
9         g.clear();
10        g.resize(n);
11    }
12    void add(int x, int y) {
13        g[x].emplace_back(y);
14        g[y].emplace_back(x);
15    }
16    bool dfs(int x) {
17        vis[x] = true;
18        Each(y, g[x]) {
19            int px = my[y];
20            if (px == -1 ||
21                (dis[px] == dis[x] + 1 &&
22                 !vis[px] && dfs(px))) {

```

```

23         mx[x] = y;
24         my[y] = x;
25         return true;
26     }
27 }
28 return false;
29 }
30 void get() {
31     mx.clear();
32     mx.resize(n, -1);
33     my.clear();
34     my.resize(n, -1);
35
36     while (true) {
37         queue<int> q;
38         dis.clear();
39         dis.resize(n, -1);
40         for (int x = 1; x <= nx; x++) {
41             if (mx[x] == -1) {
42                 dis[x] = 0;
43                 q.push(x);
44             }
45         }
46         while (!q.empty()) {
47             int x = q.front();
48             q.pop();
49             Each(y, g[x]) {
50                 if (my[y] != -1 && dis[my[y]] ==
51                     -1) {
52                     dis[my[y]] = dis[x] + 1;
53                     q.push(my[y]);
54                 }
55             }
56
57             bool brk = true;
58             vis.clear();
59             vis.resize(n, 0);
60             for (int x = 1; x <= nx; x++)
61                 if (mx[x] == -1 && dfs(x))
62                     brk = false;
63
64             if (brk) break;
65         }
66         MXCNT = 0;
67         for (int x = 1; x <= nx; x++)
68             if (mx[x] != -1) MXCNT++;
69     }
70 } hk;

```

4.5 Blossom

```

1  const int N=5e2+10;
2  struct Graph{
3      int to[N],bro[N],head[N],e;
4      int lnk[N],vis[N],stp,n;
5      void init(int _n){
6          stp=0;e=1;n=_n;
7          FOR(i,0,n+1)head[i]=lnk[i]=vis[i]=0;
8      }
9      void add(int u,int v){
10         to[e]=v,bro[e]=head[u],head[u]=e++;
11         to[e]=u,bro[e]=head[v],head[v]=e++;
12     }
13     bool dfs(int x){
14         vis[x]=stp;
15         for(int i=head[x];i;i=bro[i])
16         {
17             int v=to[i];
18             if(!lnk[v])
19             {
20                 lnk[x]=v;lnk[v]=x;
21                 return true;
22             }
23             else if(vis[lnk[v]]<stp)
24             {
25                 int w=lnk[v];
26                 lnk[x]=v,lnk[v]=x,lnk[w]=0;
27                 if(dfs(w))return true;
28                 lnk[w]=v,lnk[v]=w,lnk[x]=0;
29             }
30         }

```

```

31         return false;
32     }
33     int solve(){
34         int ans=0;
35         FOR(i,1,n+1){
36             if(!lnk[i]){
37                 stp++;
38                 ans+=dfs(i);
39             }
40         }
41         return ans;
42     }
43     void print_matching(){
44         FOR(i,1,n+1)
45             if(i<graph.lnk[i])
46                 cout<<i<<" "<<graph.lnk[i]<<endl;
47     }
48 };

```

4.6 Weighted Blossom

```

1  struct WeightGraph { // 1-based
2      static const int inf = INT_MAX;
3      static const int maxn = 514;
4      struct edge {
5          int u, v, w;
6          edge() {}
7          edge(int u, int v, int w) : u(u), v(v), w(w) {}
8      };
9      int n, n_x;
10     edge g[maxn * 2][maxn * 2];
11     int lab[maxn * 2];
12     int match[maxn * 2], slack[maxn * 2], st[maxn * 2],
13         pa[maxn * 2];
14     int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
15         maxn * 2];
16     vector<int> flo[maxn * 2];
17     queue<int> q;
18     int e_delta(const edge &e) { return lab[e.u] + lab[
19         e.v] - g[e.u][e.v].w * 2; }
20     void update_slack(int u, int x) {
21         if (!slack[x] || e_delta(g[u][x]) < e_delta(g[
22             slack[x]][x])) slack[x] = u;
23     }
24     void set_slack(int x) {
25         slack[x] = 0;
26         for (int u = 1; u <= n; ++u)
27             if (g[u][x].w > 0 && st[u] != x && S[st[u]]
28                 == 0)
29                 update_slack(u, x);
30     }
31     void q_push(int x) {
32         if (x <= n)
33             q.push(x);
34         else
35             for (size_t i = 0; i < flo[x].size(); i++)
36                 q_push(flo[x][i]);
37     }
38     void set_st(int x, int b) {
39         st[x] = b;
40         if (x > n)
41             for (size_t i = 0; i < flo[x].size(); ++i)
42                 set_st(flo[x][i], b);
43     }
44     int get_pr(int b, int xr) {
45         int pr = find(flo[b].begin(), flo[b].end(), xr)
46             - flo[b].begin();
47         if (pr % 2 == 1) {
48             reverse(flo[b].begin() + 1, flo[b].end());
49             return (int)flo[b].size() - pr;
50         }
51         return pr;
52     }
53     void set_match(int u, int v) {
54         match[u] = g[u][v].v;
55         if (u <= n) return;
56         edge e = g[u][v];
57         int xr = flo_from[u][e.u], pr = get_pr(u, xr);
58         for (int i = 0; i < pr; ++i) set_match(flo[u][i]
59             ], flo[u][i ^ 1]);
60         set_match(xr, v);

```



```

52     rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].end());
53 }
54 void augment(int u, int v) {
55     for (;;) {
56         int xnv = st[match[u]];
57         set_match(u, v);
58         if (!xnv) return;
59         set_match(xnv, st[pa[xnv]]);
60         u = st[pa[xnv]], v = xnv;
61     }
62 }
63 int get_lca(int u, int v) {
64     static int t = 0;
65     for (++t; u || v; swap(u, v)) {
66         if (u == 0) continue;
67         if (vis[u] == t) return u;
68         vis[u] = t;
69         u = st[match[u]];
70         if (u) u = st[pa[u]];
71     }
72     return 0;
73 }
74 void add_blossom(int u, int lca, int v) {
75     int b = n + 1;
76     while (b <= n_x && st[b]) ++b;
77     if (b > n_x) ++n_x;
78     lab[b] = 0, S[b] = 0;
79     match[b] = match[lca];
80     flo[b].clear();
81     flo[b].push_back(lca);
82     for (int x = u, y; x != lca; x = st[pa[y]])
83         flo[b].push_back(x), flo[b].push_back(y = st[match[x]]), q_push(y);
84     reverse(flo[b].begin() + 1, flo[b].end());
85     for (int x = v, y; x != lca; x = st[pa[y]])
86         flo[b].push_back(x), flo[b].push_back(y = st[match[x]]), q_push(y);
87     set_st(b, b);
88     for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].w = 0;
89     for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
90     for (size_t i = 0; i < flo[b].size(); ++i) {
91         int xs = flo[b][i];
92         for (int x = 1; x <= n_x; ++x)
93             if (g[b][x].w == 0 || e_delta(g[xs][x]) < e_delta(g[b][x]))
94                 g[b][x] = g[xs][x], g[x][b] = g[x][xs];
95         for (int x = 1; x <= n; ++x)
96             if (flo_from[xs][x]) flo_from[b][x] = xs;
97     }
98     set_slack(b);
99 }
100 void expand_blossom(int b) {
101     for (size_t i = 0; i < flo[b].size(); ++i)
102         set_st(flo[b][i], flo[b][i]);
103     int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b, xr);
104     for (int i = 0; i < pr; i += 2) {
105         int xs = flo[b][i], xns = flo[b][i + 1];
106         pa[xs] = g[xns][xs].u;
107         S[xs] = 1, S[xns] = 0;
108         slack[xs] = 0, set_slack(xns);
109         q_push(xns);
110     }
111     S[xr] = 1, pa[xr] = pa[b];
112     for (size_t i = pr + 1; i < flo[b].size(); ++i) {
113         int xs = flo[b][i];
114         S[xs] = -1, set_slack(xs);
115     }
116     st[b] = 0;
117 }
118 bool on_found_edge(const edge &e) {
119     int u = st[e.u], v = st[e.v];
120     if (S[v] == -1) {
121         pa[v] = e.u, S[v] = 1;
122         int nu = st[match[v]];
123         slack[v] = slack[nu] = 0;
124
125         S[nu] = 0, q_push(nu);
126     } else if (S[v] == 0) {
127         int lca = get_lca(u, v);
128         if (!lca)
129             return augment(u, v), augment(v, u), true;
130         else
131             add_blossom(u, lca, v);
132     }
133     return false;
134 }
135 bool matching() {
136     memset(S + 1, -1, sizeof(int) * n_x);
137     memset(slack + 1, 0, sizeof(int) * n_x);
138     q = queue<int>();
139     for (int x = 1; x <= n_x; ++x)
140         if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0, q_push(x);
141     if (q.empty()) return false;
142     for (;;) {
143         while (q.size()) {
144             int u = q.front();
145             q.pop();
146             if (S[st[u]] == 1) continue;
147             for (int v = 1; v <= n; ++v)
148                 if (g[u][v].w > 0 && st[u] != st[v]) {
149                     if (e_delta(g[u][v]) == 0) {
150                         if (on_found_edge(g[u][v]))
151                             return true;
152                     } else
153                         update_slack(u, st[v]);
154                 }
155         }
156         int d = inf;
157         for (int b = n + 1; b <= n_x; ++b)
158             if (st[b] == b && S[b] == 1) d = min(d, lab[b] / 2);
159         for (int x = 1; x <= n_x; ++x)
160             if (st[x] == x && slack[x]) {
161                 if (S[x] == -1)
162                     d = min(d, e_delta(g[slack[x]][x]));
163                 else if (S[x] == 0)
164                     d = min(d, e_delta(g[slack[x]][x]) / 2);
165             }
166         for (int u = 1; u <= n; ++u) {
167             if (S[st[u]] == 0) {
168                 if (lab[u] <= d) return 0;
169                 lab[u] -= d;
170             } else if (S[st[u]] == 1)
171                 lab[u] += d;
172         }
173         for (int b = n + 1; b <= n_x; ++b)
174             if (st[b] == b) {
175                 if (S[st[b]] == 0)
176                     lab[b] += d * 2;
177                 else if (S[st[b]] == 1)
178                     lab[b] -= d * 2;
179             }
180         q = queue<int>();
181         for (int x = 1; x <= n_x; ++x)
182             if (st[x] == x && slack[x] && st[slack[x]] != x && e_delta(g[slack[x]][x]) == 0)
183                 if (on_found_edge(g[slack[x]][x]))
184                     return true;
185         for (int b = n + 1; b <= n_x; ++b)
186             if (st[b] == b && S[b] == 1 && lab[b] == 0)
187                 expand_blossom(b);
188     }
189     return false;
190 }
191 pair<long long, int> solve() {
192     memset(match + 1, 0, sizeof(int) * n);
193     n_x = n;
194     int n_matches = 0;
195     long long tot_weight = 0;
196     for (int u = 0; u <= n; ++u) st[u] = u, flo[u].clear();
197     int w_max = 0;

```

```

194     for (int u = 1; u <= n; ++u)
195         for (int v = 1; v <= n; ++v) {
196             flo_from[u][v] = (u == v ? u : 0);
197             w_max = max(w_max, g[u][v].w);
198         }
199     for (int u = 1; u <= n; ++u) lab[u] = w_max;
200     while (matching()) ++n_matches;
201     for (int u = 1; u <= n; ++u)
202         if (match[u] && match[u] < u)
203             tot_weight += g[u][match[u]].w;
204     return make_pair(tot_weight, n_matches);
205 }
206 void add_edge(int ui, int vi, int wi) { g[ui][vi].w
    = g[vi][ui].w = wi; }
207 void init(int _n) {
208     n = _n;
209     for (int u = 1; u <= n; ++u)
210         for (int v = 1; v <= n; ++v)
211             g[u][v] = edge(u, v, 0);
212 }
213 };

```

4.7 Cover / Independent Set

```

1 V(E) Cover: choose some V(E) to cover all E(V)
2 V(E) Independ: set of V(E) not adj to each other
3
4 M = Max Matching
5 Cv = Min V Cover
6 Ce = Min E Cover
7 Iv = Max V Ind
8 Ie = Max E Ind (equiv to M)
9
10 M = Cv (Konig Theorem)
11 Iv = V \ Cv
12 Ce = V - M
13
14 Construct Cv:
15 1. Run Dinic
16 2. Find s-t min cut
17 3. Cv = {X in T} + {Y in S}

```

5 Graph

5.1 Heavy-Light Decomposition

```

1 const int N = 2e5 + 5;
2 int n, dfn[N], son[N], top[N], num[N], dep[N], p[N];
3 vector<int> path[N];
4 struct node {
5     int mx, sum;
6 } seg[N << 2];
7 void update(int x, int l, int r, int qx, int val) {
8     if (l == r) {
9         seg[x].mx = seg[x].sum = val;
10        return;
11    }
12    int mid = (l + r) >> 1;
13    if (qx <= mid) update(x << 1, l, mid, qx, val);
14    else update(x << 1 | 1, mid + 1, r, qx, val);
15    seg[x].mx = max(seg[x << 1].mx, seg[x << 1 | 1].mx);
16    seg[x].sum = seg[x << 1].sum + seg[x << 1 | 1].sum;
17 }
18 int big(int x, int l, int r, int ql, int qr) {
19     if (ql <= l && r <= qr) return seg[x].mx;
20     int mid = (l + r) >> 1;
21     int res = -INF;
22     if (ql <= mid) res = max(res, big(x << 1, l, mid,
23         ql, qr));
24     if (mid < qr) res = max(res, big(x << 1 | 1, mid +
25         1, r, ql, qr));
26     return res;
27 }
28 int ask(int x, int l, int r, int ql, int qr) {
29     if (ql <= l && r <= qr) return seg[x].sum;
30     int mid = (l + r) >> 1;
31     int res = 0;
32     if (ql <= mid) res += ask(x << 1, l, mid, ql, qr);
33     if (mid < qr) res += ask(x << 1 | 1, mid + 1, r, ql
34         , qr);

```

```

32     return res;
33 }
34 void dfs1(int now) {
35     son[now] = -1;
36     num[now] = 1;
37     for (auto i : path[now]) {
38         if (!dep[i]) {
39             dep[i] = dep[now] + 1;
40             p[i] = now;
41             dfs1(i);
42             num[now] += num[i];
43             if (son[now] == -1 || num[i] > num[son[now]
44                 ]) son[now] = i;
45         }
46     }
47     int cnt;
48     void dfs2(int now, int t) {
49         top[now] = t;
50         cnt++;
51         dfn[now] = cnt;
52         if (son[now] == -1) return;
53         dfs2(son[now], t);
54         for (auto i : path[now])
55             if (i != p[now] && i != son[now]) dfs2(i, i);
56     }
57     int path_big(int x, int y) {
58         int res = -INF;
59         while (top[x] != top[y]) {
60             if (dep[top[x]] < dep[top[y]]) swap(x, y);
61             res = max(res, big(1, 1, n, dfn[top[x]], dfn[x
62                 ]));
63             x = p[top[x]];
64         }
65         if (dfn[x] > dfn[y]) swap(x, y);
66         res = max(res, big(1, 1, n, dfn[x], dfn[y]));
67         return res;
68     }
69     int path_sum(int x, int y) {
70         int res = 0;
71         while (top[x] != top[y]) {
72             if (dep[top[x]] < dep[top[y]]) swap(x, y);
73             res += ask(1, 1, n, dfn[top[x]], dfn[x]);
74             x = p[top[x]];
75         }
76         if (dfn[x] > dfn[y]) swap(x, y);
77         res += ask(1, 1, n, dfn[x], dfn[y]);
78         return res;
79     }
80     void buildTree() {
81         FOR(i, 0, n - 1) {
82             int a, b;
83             cin >> a >> b;
84             path[a].pb(b);
85             path[b].pb(a);
86         }
87     }
88     void buildHLD(int root) {
89         dep[root] = 1;
90         dfs1(root);
91         dfs2(root, root);
92         FOR(i, 1, n + 1) {
93             int now;
94             cin >> now;
95             update(1, 1, n, dfn[i], now);
96         }

```

5.2 Centroid Decomposition

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const int N = 1e5 + 5;
4 vector<int> a[N];
5 int sz[N], lv[N];
6 bool used[N];
7 int f_sz(int x, int p) {
8     sz[x] = 1;
9     for (int i : a[x])
10         if (i != p && !used[i])
11             sz[x] += f_sz(i, x);
12     return sz[x];

```



```

13 }
14 int f_cen(int x, int p, int total) {
15     for (int i : a[x]) {
16         if (i != p && !used[i] && 2 * sz[i] > total)
17             return f_cen(i, x, total);
18     }
19     return x;
20 }
21 void cd(int x, int p) {
22     int total = f_sz(x, p);
23     int cen = f_cen(x, p, total);
24     lv[cen] = lv[p] + 1;
25     used[cen] = 1;
26     // cout << "cd: " << x << " " << p << " " << cen <<
27     // "\n";
28     for (int i : a[cen]) {
29         if (!used[i])
30             cd(i, cen);
31     }
32 }
33 int main() {
34     ios_base::sync_with_stdio(0);
35     cin.tie(0);
36     int n;
37     cin >> n;
38     for (int i = 0, x, y; i < n - 1; i++) {
39         cin >> x >> y;
40         a[x].push_back(y);
41         a[y].push_back(x);
42     }
43     cd(1, 0);
44     for (int i = 1; i <= n; i++)
45         cout << (char)('A' + lv[i] - 1) << " ";
46     cout << "\n";

```

5.3 Bellman-Ford + SPFA

```

1 int n, m;
2
3 // Graph
4 vector<vector<pair<int, ll> > > g;
5 vector<ll> dis;
6 vector<bool> negCycle;
7
8 // SPFA
9 vector<int> rlx;
10 queue<int> q;
11 vector<bool> inq;
12 vector<int> pa;
13 void SPFA(vector<int>& src) {
14     dis.assign(n + 1, LINF);
15     negCycle.assign(n + 1, false);
16     rlx.assign(n + 1, 0);
17     while (!q.empty()) q.pop();
18     inq.assign(n + 1, false);
19     pa.assign(n + 1, -1);
20
21     for (auto& s : src) {
22         dis[s] = 0;
23         q.push(s);
24         inq[s] = true;
25     }
26
27     while (!q.empty()) {
28         int u = q.front();
29         q.pop();
30         inq[u] = false;
31         if (rlx[u] >= n) {
32             negCycle[u] = true;
33         } else
34             for (auto& e : g[u]) {
35                 int v = e.first;
36                 ll w = e.second;
37                 if (dis[v] > dis[u] + w) {
38                     dis[v] = dis[u] + w;
39                     rlx[v] = rlx[u] + 1;
40                     pa[v] = u;
41                     if (!inq[v]) {
42                         q.push(v);
43                         inq[v] = true;
44                     }

```

```

45     }
46     }
47 }
48
49 // Bellman-Ford
50 queue<int> q;
51 vector<int> pa;
52 void BellmanFord(vector<int>& src) {
53     dis.assign(n + 1, LINF);
54     negCycle.assign(n + 1, false);
55     pa.assign(n + 1, -1);
56
57     for (auto& s : src) dis[s] = 0;
58
59     for (int rlx = 1; rlx <= n; rlx++) {
60         for (int u = 1; u <= n; u++) {
61             if (dis[u] == LINF) continue; // Important
62             //
63             for (auto& e : g[u]) {
64                 int v = e.first;
65                 ll w = e.second;
66                 if (dis[v] > dis[u] + w) {
67                     dis[v] = dis[u] + w;
68                     pa[v] = u;
69                     if (rlx == n) negCycle[v] = true;
70                 }
71             }
72         }
73     }
74 }
75
76 // Negative Cycle Detection
77 void NegCycleDetect() {
78     /* No Neg Cycle: NO
79     Exist Any Neg Cycle:
80     YES
81     v0 v1 v2 ... vk v0 */
82
83     vector<int> src;
84     for (int i = 1; i <= n; i++)
85         src.emplace_back(i);
86
87     SPFA(src);
88     // BellmanFord(src);
89
90     int ptr = -1;
91     for (int i = 1; i <= n; i++)
92         if (negCycle[i]) {
93             ptr = i;
94             break;
95         }
96
97     if (ptr == -1) {
98         return cout << "NO" << endl, void();
99     }
100
101     cout << "YES\n";
102     vector<int> ans;
103     vector<bool> vis(n + 1, false);
104
105     while (true) {
106         ans.emplace_back(ptr);
107         if (vis[ptr]) break;
108         vis[ptr] = true;
109         ptr = pa[ptr];
110     }
111     reverse(ans.begin(), ans.end());
112
113     vis.assign(n + 1, false);
114     for (auto& x : ans) {
115         cout << x << ' ';
116         if (vis[x]) break;
117         vis[x] = true;
118     }
119     cout << endl;
120 }
121
122 // Distance Calculation
123 void calcDis(int s) {
124     vector<int> src;
125     src.emplace_back(s);

```

```

126 SPFA(src);
127 // BellmanFord(src);
128
129 while (!q.empty()) q.pop();
130 for (int i = 1; i <= n; i++)
131     if (negCycle[i]) q.push(i);
132
133 while (!q.empty()) {
134     int u = q.front();
135     q.pop();
136     for (auto& e : g[u]) {
137         int v = e.first;
138         if (!negCycle[v]) {
139             q.push(v);
140             negCycle[v] = true;
141         }
142     }
143 }
144 }

```

5.4 BCC - AP

```

1 int n, m;
2 int low[maxn], dfn[maxn], instp;
3 vector<int> E, g[maxn];
4 bitset<maxn> isap;
5 bitset<maxn> vis;
6 stack<int> stk;
7 int bccnt;
8 vector<int> bcc[maxn];
9 inline void popout(int u) {
10     bccnt++;
11     bcc[bccnt].emplace_back(u);
12     while (!stk.empty()) {
13         int v = stk.top();
14         if (u == v) break;
15         stk.pop();
16         bcc[bccnt].emplace_back(v);
17     }
18 }
19 void dfs(int u, bool rt = 0) {
20     stk.push(u);
21     low[u] = dfn[u] = ++instp;
22     int kid = 0;
23     Each(e, g[u]) {
24         if (vis[e]) continue;
25         vis[e] = true;
26         int v = E[e] ^ u;
27         if (!dfn[v]) {
28             // tree edge
29             kid++;
30             dfs(v);
31             low[u] = min(low[u], low[v]);
32             if (!rt && low[v] >= dfn[u]) {
33                 // bcc found: u is ap
34                 isap[u] = true;
35                 popout(u);
36             }
37         } else {
38             // back edge
39             low[u] = min(low[u], dfn[v]);
40         }
41     }
42     // special case: root
43     if (rt) {
44         if (kid > 1) isap[u] = true;
45         popout(u);
46     }
47 }
48 void init() {
49     cin >> n >> m;
50     fill(low, low + maxn, INF);
51     REP(i, m) {
52         int u, v;
53         cin >> u >> v;
54         g[u].emplace_back(i);
55         g[v].emplace_back(i);
56         E.emplace_back(u ^ v);
57     }
58 }
59 void solve() {
60     FOR(i, 1, n + 1, 1) {

```

```

61         if (!dfn[i]) dfs(i, true);
62     }
63     vector<int> ans;
64     int cnt = 0;
65     FOR(i, 1, n + 1, 1) {
66         if (isap[i]) cnt++, ans.emplace_back(i);
67     }
68     cout << cnt << endl;
69     Each(i, ans) cout << i << ' ';
70     cout << endl;
71 }

```

5.5 BCC - Bridge

```

1 int n, m;
2 vector<int> g[maxn], E;
3 int low[maxn], dfn[maxn], instp;
4 int bccnt, bccid[maxn];
5 stack<int> stk;
6 bitset<maxn> vis, isbrg;
7 void init() {
8     cin >> n >> m;
9     REP(i, m) {
10         int u, v;
11         cin >> u >> v;
12         E.emplace_back(u ^ v);
13         g[u].emplace_back(i);
14         g[v].emplace_back(i);
15     }
16     fill(low, low + maxn, INF);
17 }
18 void popout(int u) {
19     bccnt++;
20     while (!stk.empty()) {
21         int v = stk.top();
22         if (v == u) break;
23         stk.pop();
24         bccid[v] = bccnt;
25     }
26 }
27 void dfs(int u) {
28     stk.push(u);
29     low[u] = dfn[u] = ++instp;
30
31     Each(e, g[u]) {
32         if (vis[e]) continue;
33         vis[e] = true;
34
35         int v = E[e] ^ u;
36         if (dfn[v]) {
37             // back edge
38             low[u] = min(low[u], dfn[v]);
39         } else {
40             // tree edge
41             dfs(v);
42             low[u] = min(low[u], low[v]);
43             if (low[v] == dfn[v]) {
44                 isbrg[e] = true;
45                 popout(u);
46             }
47         }
48     }
49 }
50 void solve() {
51     FOR(i, 1, n + 1, 1) {
52         if (!dfn[i]) dfs(i);
53     }
54     vector<pii> ans;
55     vis.reset();
56     FOR(u, 1, n + 1, 1) {
57         Each(e, g[u]) {
58             if (!isbrg[e] || vis[e]) continue;
59             vis[e] = true;
60             int v = E[e] ^ u;
61             ans.emplace_back(mp(u, v));
62         }
63     }
64     cout << (int)ans.size() << endl;
65     Each(e, ans) cout << e.F << ' ' << e.S << endl;
66 }

```

5.6 SCC - Tarjan

```

1 // 2-SAT
2 vector<int> E, g[maxn]; // 1~n, n+1~2n
3 int low[maxn], in[maxn], instp;
4 int sccnt, sccid[maxn];
5 stack<int> stk;
6 bitset<maxn> ins, vis;
7 int n, m;
8 void init() {
9     cin >> m >> n;
10    E.clear();
11    fill(g, g + maxn, vector<int>());
12    fill(low, low + maxn, INF);
13    memset(in, 0, sizeof(in));
14    instp = 1;
15    sccnt = 0;
16    memset(sccid, 0, sizeof(sccid));
17    ins.reset();
18    vis.reset();
19 }
20 inline int no(int u) {
21     return (u > n ? u - n : u + n);
22 }
23 int ecnt = 0;
24 inline void clause(int u, int v) {
25     E.pb(no(u) ^ v);
26     g[no(u)].pb(ecnt++);
27     E.pb(no(v) ^ u);
28     g[no(v)].pb(ecnt++);
29 }
30 void dfs(int u) {
31     in[u] = instp++;
32     low[u] = in[u];
33     stk.push(u);
34     ins[u] = true;
35
36     Each(e, g[u]) {
37         if (vis[e]) continue;
38         vis[e] = true;
39
40         int v = E[e] ^ u;
41         if (ins[v])
42             low[u] = min(low[u], in[v]);
43         else if (!in[v]) {
44             dfs(v);
45             low[u] = min(low[u], low[v]);
46         }
47     }
48     if (low[u] == in[u]) {
49         sccnt++;
50         while (!stk.empty()) {
51             int v = stk.top();
52             stk.pop();
53             ins[v] = false;
54             sccid[v] = sccnt;
55             if (u == v) break;
56         }
57     }
58 }
59 int main() {
60     init();
61     REP(i, m) {
62         char su, sv;
63         int u, v;
64         cin >> su >> u >> sv >> v;
65         if (su == '-') u = no(u);
66         if (sv == '-') v = no(v);
67         clause(u, v);
68     }
69     FOR(i, 1, 2 * n + 1, 1) {
70         if (!in[i]) dfs(i);
71     }
72     FOR(u, 1, n + 1, 1) {
73         int du = no(u);
74         if (sccid[u] == sccid[du]) {
75             return cout << "IMPOSSIBLE\n", 0;
76         }
77     }
78     FOR(u, 1, n + 1, 1) {
79         int du = no(u);

```

```

80         cout << (sccid[u] < sccid[du] ? '+' : '-') << '
81         '
82     }
83     cout << endl;

```

5.7 SCC - Kosaraju

```

1 const int N = 1e5 + 10;
2 vector<int> ed[N], ed_b[N]; // 反邊
3 vector<int> SCC(N); // 最後SCC的分組
4 bitset<N> vis;
5 int SCC_cnt;
6 int n, m;
7 vector<int> pre; // 後序遍歷
8
9 void dfs(int x) {
10     vis[x] = 1;
11     for (int i : ed[x]) {
12         if (vis[i]) continue;
13         dfs(i);
14     }
15     pre.push_back(x);
16 }
17
18 void dfs2(int x) {
19     vis[x] = 1;
20     SCC[x] = SCC_cnt;
21     for (int i : ed_b[x]) {
22         if (vis[i]) continue;
23         dfs2(i);
24     }
25 }
26
27 void kosaraju() {
28     for (int i = 1; i <= n; i++) {
29         if (!vis[i]) {
30             dfs(i);
31         }
32     }
33     SCC_cnt = 0;
34     vis = 0;
35     for (int i = n - 1; i >= 0; i--) {
36         if (!vis[pre[i]]) {
37             SCC_cnt++;
38             dfs2(pre[i]);
39         }
40     }
41 }

```

5.8 Eulerian Path - Undir

```

1 // from 1 to n
2 #define gg return cout << "IMPOSSIBLE\n", void();
3
4 int n, m;
5 vector<int> g[maxn];
6 bitset<maxn> inodd;
7
8 void init() {
9     cin >> n >> m;
10    inodd.reset();
11    for (int i = 0; i < m; i++) {
12        int u, v;
13        cin >> u >> v;
14        inodd[u] = inodd[u] ^ true;
15        inodd[v] = inodd[v] ^ true;
16        g[u].emplace_back(v);
17        g[v].emplace_back(u);
18    }
19 }
20 stack<int> stk;
21 void dfs(int u) {
22     while (!g[u].empty()) {
23         int v = g[u].back();
24         g[u].pop_back();
25         dfs(v);
26     }
27     stk.push(u);
28 }

```

5.9 Eulerian Path - Dir

```

1 // from node 1 to node n
2 #define gg return cout << "IMPOSSIBLE\n", 0
3
4 int n, m;
5 vector<int> g[maxn];
6 stack<int> stk;
7 int in[maxn], out[maxn];
8
9 void init() {
10     cin >> n >> m;
11     for (int i = 0; i < m; i++) {
12         int u, v;
13         cin >> u >> v;
14         g[u].emplace_back(v);
15         out[u]++, in[v]++;
16     }
17     for (int i = 1; i <= n; i++) {
18         if (i == 1 && out[i] - in[i] != 1) gg;
19         if (i == n && in[i] - out[i] != 1) gg;
20         if (i != 1 && i != n && in[i] != out[i]) gg;
21     }
22 }
23 void dfs(int u) {
24     while (!g[u].empty()) {
25         int v = g[u].back();
26         g[u].pop_back();
27         dfs(v);
28     }
29     stk.push(u);
30 }
31 void solve() {
32     dfs(1) for (int i = 1; i <= n; i++) if ((int)g[i].
33         size()) gg;
34     while (!stk.empty()) {
35         int u = stk.top();
36         stk.pop();
37         cout << u << ' ';
38     }
39 }

```

5.10 Hamilton Path

```

1 // top down DP
2 // Be Aware Of Multiple Edges
3 int n, m;
4 ll dp[maxn][1<<maxn];
5 int adj[maxn][maxn];
6
7 void init() {
8     cin >> n >> m;
9     fill(dp[0], dp[maxn-1]+(1<<maxn), -1);
10 }
11
12 void DP(int i, int msk) {
13     if (dp[i][msk] != -1) return;
14     dp[i][msk] = 0;
15     REP(j, n) if (j != i && (msk & (1<<j)) && adj[j][i]) {
16         int sub = msk ^ (1<<i);
17         if (dp[j][sub] == -1) DP(j, sub);
18         dp[i][msk] += dp[j][sub] * adj[j][i];
19         if (dp[i][msk] >= MOD) dp[i][msk] %= MOD;
20     }
21 }
22
23 int main() {
24     WiWiHorz
25     init();
26
27     REP(i, m) {
28         int u, v;
29         cin >> u >> v;
30         if (u == v) continue;
31         adj[--u][--v]++;
32     }
33
34     dp[0][1] = 1;
35     FOR(i, 1, n, 1) {
36         dp[i][1] = 0;

```

```

38         dp[i][1|(1<<i)] = adj[0][i];
39     }
40     FOR(msk, 1, (1<<n), 1) {
41         if (msk == 1) continue;
42         dp[0][msk] = 0;
43     }
44
45     DP(n-1, (1<<n)-1);
46     cout << dp[n-1][(1<<n)-1] << endl;
47
48     return 0;
49 }
50 }

```

5.11 Kth Shortest Path

```

1 // time: O(|E| \lg |E| + |V| \lg |V| + K)
2 // memory: O(|E| \lg |E| + |V|)
3 struct KSP { // 1-base
4     struct nd {
5         int u, v;
6         ll d;
7         nd(int ui = 0, int vi = 0, ll di = INF) {
8             u = ui;
9             v = vi;
10            d = di;
11        }
12    };
13    struct heap {
14        nd* edge;
15        int dep;
16        heap* chd[4];
17    };
18    static int cmp(heap* a, heap* b) { return a->edge->
19        d > b->edge->d; }
20    struct node {
21        int v;
22        ll d;
23        heap* H;
24        nd* E;
25        node() {}
26        node(ll _d, int _v, nd* _E) {
27            d = _d;
28            v = _v;
29            E = _E;
30        }
31        node(heap* _H, ll _d) {
32            H = _H;
33            d = _d;
34        }
35        friend bool operator<(node a, node b) { return
36            a.d > b.d; }
37    };
38    int n, k, s, t, dst[N];
39    nd* nxt[N];
40    vector<nd*> g[N], rg[N];
41    heap *nullNd, *head[N];
42    void init(int _n, int _k, int _s, int _t) {
43        n = _n;
44        k = _k;
45        s = _s;
46        t = _t;
47        for (int i = 1; i <= n; i++) {
48            g[i].clear();
49            rg[i].clear();
50            nxt[i] = NULL;
51            head[i] = NULL;
52            dst[i] = -1;
53        }
54    }
55    void addEdge(int ui, int vi, ll di) {
56        nd* e = new nd(ui, vi, di);
57        g[ui].push_back(e);
58        rg[vi].push_back(e);
59    }
60    queue<int> dfsQ;
61    void dijkstra() {
62        while (dfsQ.size()) dfsQ.pop();
63        priority_queue<node> Q;
64        Q.push(node(0, t, NULL));
65        while (!Q.empty()) {
66            node p = Q.top();

```

```

65     Q.pop();
66     if (dst[p.v] != -1) continue;
67     dst[p.v] = p.d;
68     nxt[p.v] = p.E;
69     dfsQ.push(p.v);
70     for (auto e : rg[p.v]) Q.push(node(p.d + e
71         ->d, e->u, e));
72 }
73 heap* merge(heap* curNd, heap* newNd) {
74     if (curNd == nullNd) return newNd;
75     heap* root = new heap;
76     memcpy(root, curNd, sizeof(heap));
77     if (newNd->edge->d < curNd->edge->d) {
78         root->edge = newNd->edge;
79         root->chd[2] = newNd->chd[2];
80         root->chd[3] = newNd->chd[3];
81         newNd->edge = curNd->edge;
82         newNd->chd[2] = curNd->chd[2];
83         newNd->chd[3] = curNd->chd[3];
84     }
85     if (root->chd[0]->dep < root->chd[1]->dep)
86         root->chd[0] = merge(root->chd[0], newNd);
87     else
88         root->chd[1] = merge(root->chd[1], newNd);
89     root->dep = max(root->chd[0]->dep,
90         root->chd[1]->dep) +
91         1;
92     return root;
93 }
94 vector<heap*> V;
95 void build() {
96     nullNd = new heap;
97     nullNd->dep = 0;
98     nullNd->edge = new nd;
99     fill(nullNd->chd, nullNd->chd + 4, nullNd);
100    while (not dfsQ.empty()) {
101        int u = dfsQ.front();
102        dfsQ.pop();
103        if (!nxt[u])
104            head[u] = nullNd;
105        else
106            head[u] = head[nxt[u]->v];
107        V.clear();
108        for (auto& e : g[u]) {
109            int v = e->v;
110            if (dst[v] == -1) continue;
111            e->d += dst[v] - dst[u];
112            if (nxt[u] != e) {
113                heap* p = new heap;
114                fill(p->chd, p->chd + 4, nullNd);
115                p->dep = 1;
116                p->edge = e;
117                V.push_back(p);
118            }
119        }
120        if (V.empty()) continue;
121        make_heap(V.begin(), V.end(), cmp);
122    #define L(X) ((X << 1) + 1)
123    #define R(X) ((X << 1) + 2)
124    for (size_t i = 0; i < V.size(); i++) {
125        if (L(i) < V.size())
126            V[i]->chd[2] = V[L(i)];
127        else
128            V[i]->chd[2] = nullNd;
129        if (R(i) < V.size())
130            V[i]->chd[3] = V[R(i)];
131        else
132            V[i]->chd[3] = nullNd;
133    }
134    head[u] = merge(head[u], V.front());
135 }
136 }
137 vector<ll> ans;
138 void first_K() {
139     ans.clear();
140     priority_queue<node> Q;
141     if (dst[s] == -1) return;
142     ans.push_back(dst[s]);
143     if (head[s] != nullNd)
144         Q.push(node(head[s], dst[s] + head[s]->edge

```

```

145     for (int _ = 1; _ < k and not Q.empty(); _++) {
146         node p = Q.top(), q;
147         Q.pop();
148         ans.push_back(p.d);
149         if (head[p.H->edge->v] != nullNd) {
150             q.H = head[p.H->edge->v];
151             q.d = p.d + q.H->edge->d;
152             Q.push(q);
153         }
154         for (int i = 0; i < 4; i++)
155             if (p.H->chd[i] != nullNd) {
156                 q.H = p.H->chd[i];
157                 q.d = p.d - p.H->edge->d + p.H->chd
158                     [i]->edge->d;
159                 Q.push(q);
160             }
161     }
162     void solve() { // ans[i] stores the i-th shortest
163         path
164         dijkstra();
165         build();
166         first_K(); // ans.size() might less than k
167     }
168 } solver;

```

5.12 Hungarian Algorithm

```

1 const int N = 2e3;
2 int match[N];
3 bool vis[N];
4 int n;
5 vector<int> ed[N];
6 int match_cnt;
7 bool dfs(int u) {
8     vis[u] = 1;
9     for(int i : ed[u]) {
10         if(match[i] == 0 || !vis[match[i]] && dfs(match
11             [i])) {
12             match[i] = u;
13             return true;
14         }
15     }
16     return false;
17 }
18 void hungary() {
19     memset(match, 0, sizeof(match));
20     match_cnt = 0;
21     for(int i = 1; i <= n; i++) {
22         memset(vis, 0, sizeof(vis));
23         if(dfs(i)) match_cnt++;
24     }
25 }

```

5.13 System of Difference Constraints

```

1 vector<vector<pair<int, ll>>> G;
2 void add(int u, int v, ll w) {
3     G[u].emplace_back(make_pair(v, w));
4 }

```

- $x_u - x_v \leq c \Rightarrow \text{add}(v, u, c)$
- $x_u - x_v \geq c \Rightarrow \text{add}(u, v, -c)$
- $x_u - x_v = c \Rightarrow \text{add}(v, u, c), \text{add}(u, v, -c)$
- $x_u \geq c \Rightarrow \text{add super vertex } x_0 = 0, \text{ then } x_u - x_0 \geq c \Rightarrow \text{add}(u, 0, -c)$
- Don't forget non-negative constraints for every variable if specified implicitly.
- Interval sum \Rightarrow Use prefix sum to transform into differential constraints. Don't forget $S_{i+1} - S_i \geq 0$ if x_i needs to be non-negative.
- $\frac{x_u}{x_v} \leq c \Rightarrow \log x_u - \log x_v \leq \log c$

6 String

6.1 Aho Corasick

```

1 struct ACautomata {
2     struct Node {
3         int cnt;
4         Node *go[26], *fail, *dic;
5         Node() {
6             cnt = 0;
7             fail = 0;
8             dic = 0;
9             memset(go, 0, sizeof(go));
10        }
11    } pool[1048576], *root;
12    int nMem;
13    Node *new_Node() {
14        pool[nMem] = Node();
15        return &pool[nMem++];
16    }
17    void init() {
18        nMem = 0;
19        root = new_Node();
20    }
21    void add(const string &str) { insert(root, str, 0); }
22    void insert(Node *cur, const string &str, int pos) {
23        for (int i = pos; i < str.size(); i++) {
24            if (!cur->go[str[i] - 'a'])
25                cur->go[str[i] - 'a'] = new_Node();
26            cur = cur->go[str[i] - 'a'];
27        }
28        cur->cnt++;
29    }
30    void make_fail() {
31        queue<Node*> que;
32        que.push(root);
33        while (!que.empty()) {
34            Node *fr = que.front();
35            que.pop();
36            for (int i = 0; i < 26; i++) {
37                if (fr->go[i]) {
38                    Node *ptr = fr->fail;
39                    while (ptr && !ptr->go[i]) ptr = ptr->fail;
40                    fr->go[i]->fail = ptr = (ptr ? ptr->go[i] : root);
41                    fr->go[i]->dic = (ptr->cnt ? ptr : ptr->dic);
42                    que.push(fr->go[i]);
43                }
44            }
45        }
46    }
47 } AC;

```

6.2 KMP

```

1 vector<int> f;
2 void buildFailFunction(string &s) {
3     f.resize(s.size(), -1);
4     for (int i = 1; i < s.size(); i++) {
5         int now = f[i - 1];
6         while (now != -1 and s[now + 1] != s[i]) now = f[now];
7         if (s[now + 1] == s[i]) f[i] = now + 1;
8     }
9 }
10
11 void KMPmatching(string &a, string &b) {
12     for (int i = 0, now = -1; i < a.size(); i++) {
13         while (a[i] != b[now + 1] and now != -1) now = f[now];
14         if (a[i] == b[now + 1]) now++;
15         if (now + 1 == b.size()) {
16             cout << "found a match start at position "
17                  << i - now << endl;
18             now = f[now];
19         }
20     }
21 }

```

6.3 Z Value

```

1 string is, it, s;
2 int n;
3 vector<int> z;
4 void init() {
5     cin >> is >> it;
6     s = it + '0' + is;
7     n = (int)s.size();
8     z.resize(n, 0);
9 }
10 void solve() {
11     int ans = 0;
12     z[0] = n;
13     for (int i = 1, l = 0, r = 0; i < n; i++) {
14         if (i <= r) z[i] = min(z[i - l], r - i + 1);
15         while (i + z[i] < n && s[z[i]] == s[i + z[i]])
16             z[i]++;
17         if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
18         if (z[i] == (int)it.size()) ans++;
19     }
20     cout << ans << endl;
21 }

```

6.4 Manacher

```

1 int n;
2 string S, s;
3 vector<int> m;
4 void manacher() {
5     s.clear();
6     s.resize(2 * n + 1, '.');
7     for (int i = 0, j = 1; i < n; i++, j += 2) s[j] = S[i];
8     m.clear();
9     m.resize(2 * n + 1, 0);
10    // m[i] := max k such that s[i-k, i+k] is
11    // palindrome
12    int mx = 0, mxk = 0;
13    for (int i = 1; i < 2 * n + 1; i++) {
14        if (mx - (i - mx) >= 0) m[i] = min(m[mx - (i - mx)], mx + mxk - i);
15        while (0 <= i - m[i] - 1 && i + m[i] + 1 < 2 * n + 1 && s[i - m[i] - 1] == s[i + m[i] + 1]) m[i]++;
16        if (i + m[i] > mx + mxk) mx = i, mxk = m[i];
17    }
18 }
19 void init() {
20     cin >> S;
21     n = (int)S.size();
22 }
23 void solve() {
24     manacher();
25     int mx = 0, ptr = 0;
26     for (int i = 0; i < 2 * n + 1; i++)
27         if (mx < m[i]) {
28             mx = m[i];
29             ptr = i;
30         }
31     for (int i = ptr - mx; i <= ptr + mx; i++)
32         if (s[i] != '.') cout << s[i];
33     cout << endl;
34 }

```

6.5 Suffix Array

```

1 #define F first
2 #define S second
3 struct SuffixArray { // don't forget s += "$";
4     int n;
5     string s;
6     vector<int> suf, lcp, rk;
7     vector<int> cnt, pos;
8     vector<pair<pii, int>> buc[2];
9     void init(string _s) {
10         s = _s;
11         n = (int)s.size();
12         // resize(n): suf, rk, cnt, pos, lcp, buc[0~1]
13     }
14     void radix_sort() {

```



```

15     for (int t : {0, 1}) {
16         fill(cnt.begin(), cnt.end(), 0);
17         for (auto& i : buc[t]) cnt[(t ? i.F.F : i.F.S)]++;
18         for (int i = 0; i < n; i++)
19             pos[i] = (!i ? 0 : pos[i - 1] + cnt[i - 1]);
20         for (auto& i : buc[t])
21             buc[t ^ 1][pos[(t ? i.F.F : i.F.S)]]++
22             = i;
23     }
24     bool fill_suf() {
25         bool end = true;
26         for (int i = 0; i < n; i++) suf[i] = buc[0][i].S;
27         rk[suf[0]] = 0;
28         for (int i = 1; i < n; i++) {
29             int dif = (buc[0][i].F != buc[0][i - 1].F);
30             end &= dif;
31             rk[suf[i]] = rk[suf[i - 1]] + dif;
32         }
33         return end;
34     }
35     void sa() {
36         for (int i = 0; i < n; i++)
37             buc[0][i] = make_pair(make_pair(s[i], s[i]), i);
38         sort(buc[0].begin(), buc[0].end());
39         if (fill_suf()) return;
40         for (int k = 0; (1 << k) < n; k++) {
41             for (int i = 0; i < n; i++)
42                 buc[0][i] = make_pair(make_pair(rk[i], rk[(i + (1 << k)) % n]), i);
43             radix_sort();
44             if (fill_suf()) return;
45         }
46     }
47     void LCP() {
48         int k = 0;
49         for (int i = 0; i < n - 1; i++) {
50             if (rk[i] == 0) continue;
51             int pi = rk[i];
52             int j = suf[pi - 1];
53             while (i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
54             lcp[pi] = k;
55             k = max(k - 1, 0);
56         }
57     }
58 };
59 SuffixArray suffixarray;

```

6.6 Minimum Rotation

```

1 // rotate(begin(s), begin(s)+minRotation(s), end(s))
2 int minRotation(string s) {
3     int a = 0, n = s.size();
4     s += s;
5     for (int b = 0; b < n; b++)
6         for (int k = 0; k < n; k++) {
7             if (a + k == b || s[a + k] < s[b + k]) {
8                 b += max(0, k - 1);
9                 break;
10            }
11            if (s[a + k] > s[b + k]) {
12                a = b;
13                break;
14            }
15        }
16     return a;
17 }

```

6.7 Lyndon Factorization

```

1 vector<string> duval(string const& s) {
2     int n = s.size();
3     int i = 0;
4     vector<string> factorization;
5     while (i < n) {
6         int j = i + 1, k = i;
7         while (j < n && s[k] <= s[j]) {

```

```

8             if (s[k] < s[j])
9                 k = i;
10            else
11                k++;
12            j++;
13        }
14        while (i <= k) {
15            factorization.push_back(s.substr(i, j - k));
16            i += j - k;
17        }
18    }
19    return factorization; // O(n)
20 }

```

6.8 Rolling Hash

```

1 const ll C = 27;
2 inline int id(char c) { return c - 'a' + 1; }
3 struct RollingHash {
4     string s;
5     int n;
6     ll mod;
7     vector<ll> Cexp, hs;
8     RollingHash(string& _s, ll _mod) : s(_s), n((int)_s.size()), mod(_mod) {
9         Cexp.assign(n, 0);
10        hs.assign(n, 0);
11        Cexp[0] = 1;
12        for (int i = 1; i < n; i++) {
13            Cexp[i] = Cexp[i - 1] * C;
14            if (Cexp[i] >= mod) Cexp[i] %= mod;
15        }
16        hs[0] = id(s[0]);
17        for (int i = 1; i < n; i++) {
18            hs[i] = hs[i - 1] * C + id(s[i]);
19            if (hs[i] >= mod) hs[i] %= mod;
20        }
21    }
22    inline ll query(int l, int r) {
23        ll res = hs[r] - (l ? hs[l - 1] * Cexp[r - l + 1] : 0);
24        res = (res % mod + mod) % mod;
25        return res;
26    }
27 };

```

6.9 Trie

```

1 pii a[N][26];
2
3 void build(string &s) {
4     static int idx = 0;
5     int n = s.size();
6     for (int i = 0, v = 0; i < n; i++) {
7         pii &now = a[v][s[i] - 'a'];
8         if (now.first != -1)
9             v = now.first;
10        else
11            v = now.first = ++idx;
12        if (i == n - 1)
13            now.second++;
14    }
15 }

```

7 Geometry

7.1 Basic Operations

```

1 // typedef long long T;
2 typedef long double T;
3 const long double eps = 1e-12;
4
5 short sgn(T x) {
6     if (abs(x) < eps) return 0;
7     return x < 0 ? -1 : 1;
8 }
9
10 struct Pt {
11     T x, y;
12     Pt(T _x = 0, T _y = 0) : x(_x), y(_y) {}

```

```

13 Pt operator+(Pt a) { return Pt(x + a.x, y + a.y); } 2
14 Pt operator-(Pt a) { return Pt(x - a.x, y - a.y); }
15 Pt operator*(T a) { return Pt(x * a, y * a); } 3
16 Pt operator/(T a) { return Pt(x / a, y / a); } 4
17 T operator*(Pt a) { return x * a.x + y * a.y; }
18 T operator^(Pt a) { return x * a.y - y * a.x; }
19 bool operator<(Pt a) { return x < a.x || (x == a.x 5
    && y < a.y); } 6 // long double
20 // return sgn(x-a.x) < 0 || (sgn(x-a.x) == 0 && sgn 7
    (y-a.y) < 0); } 8
21 bool operator==(Pt a) { return sgn(x - a.x) == 0 && 9
    sgn(y - a.y) == 0; } 10
22 }; 11
23 Pt mv(Pt a, Pt b) { return b - a; } 12
24 T len2(Pt a) { return a * a; } 13
25 T dis2(Pt a, Pt b) { return len2(b - a); } 14
26 Pt rotate(Pt u) { return {-u.y, u.x}; } 15
27 short ori(Pt a, Pt b) { return ((a ^ b) > 0) - ((a ^ b) 16
    < 0); } 17
28 bool onseg(Pt p, Pt l1, Pt l2) { 18
    Pt a = mv(p, l1), b = mv(p, l2); 19
    return ((a ^ b) == 0) && ((a * b) <= 0); 20
29 } 21
30 inline int cross(const Pt &a, const Pt &b, const Pt &c) 22
    { 23
    return (b.x - a.x) * (c.y - a.y) 24
        - (b.y - a.y) * (c.x - a.x); 25
    } 26
31 double polar_angle(Pt ori, Pt pt){ 27
    return atan2(pt.y - ori.y, pt.x - ori.x); 28
32 } 29
33 // slope to degree atan(Slope) * 180.0 / acos(-1.0); 30
34 bool argcmp(Pt u, Pt v) { 31
    auto half = [](const Pt& p) { 32
        return p.y > 0 || (p.y == 0 && p.x >= 0); 33
    }; 34
    if (half(u) != half(v)) return half(u) < half(v); 35
    return sgn(u ^ v) > 0; 36
37 } 37
38 struct Line { 38
    Pt a, b; 39
    Line() {} 40
    Line(Pt _a, Pt _b) : a(_a), b(_b) {} 41
    Pt dir() { return b - a; } 42
39 }; 43
40 int ori(Pt& o, Pt& a, Pt& b) { 44
    return sgn((a - o) ^ (b - o)); 45
41 } 46
42 struct Cir { 47
    Pt o; 48
    T r; 49
43 }; 50
44 bool disjunct(Cir a, Cir b) { 51
    return sgn(sqrtl(len2(a.o - b.o)) - a.r - b.r) >= 52
        0; 53
45 } 54
46 bool contain(Cir a, Cir b) { 55
    return sgn(a.r - b.r - sqrtl(len2(a.o - b.o))) >= 56
        0; 57
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```

7.2 SVG Writer

7.3 Sort by Angle

```

1 int ud(Pt a) { // up or down half plane
2     if (a.y > 0) return 0;
3     if (a.y < 0) return 1;
4     return (a.x >= 0 ? 0 : 1);
5 }
6 sort(pts.begin(), pts.end(), [&](const Pt& a, const Pt& b) {
7     if (ud(a) != ud(b)) return ud(a) < ud(b);
8     return (a ^ b) > 0;
9 });

```

7.4 Line Intersection

```

1 bool line_intersect_check(Pt p1, Pt p2, Pt q1, Pt q2) {

```

7.5 Polygon Area

```

1 // 2 * area
2 T dbPoly_area(vector<Pt>& e) {
3     T res = 0;
4     int sz = e.size();
5     for (int i = 0; i < sz; i++) {
6         res += e[i] ^ e[(i + 1) % sz];
7     }
8     return abs(res);
9 }

```

7.6 Convex Hull

```

1 vector<Pt> convexHull(vector<Pt> pts) {
2     vector<Pt> hull;
3     sort(pts.begin(), pts.end());
4     for (int i = 0; i < pts.size(); i++) {
5         int b = hull.size();
6         for (auto ei : pts) {
7             while (hull.size() - b >= 2 && ori(mv(hull[
                hull.size() - 2], hull.back()), mv(hull[
                hull.size() - 2], ei)) == -1) {
8                 hull.pop_back();
9             }
10            hull.emplace_back(ei);
11        }
12        hull.pop_back();
13        reverse(pts.begin(), pts.end());
14    }
15    return hull;
16 }

```

7.7 Point In Convex

```

1 bool point_in_convex(const vector<Pt> &C, Pt p, bool
    strict = true) {
2     // only works when no three point are collinear
3     int n = C.size();
4     int a = 1, b = n - 1, r = !strict;
5     if (n == 0) return false;
6     if (n < 3) return r && onseg(p, C[0], C.back());
7     if (ori(mv(C[0], C[a]), mv(C[0], C[b])) > 0) swap(a, b);
8     if (ori(mv(C[0], C[a]), mv(C[0], p)) >= r || ori(mv(
        C[0], C[b]), mv(C[0], p)) <= -r) return false;
9     while (abs(a - b) > 1) {
10        int c = (a + b) / 2;
11        if (ori(mv(C[0], C[c]), mv(C[0], p)) > 0) b = c;
        else a = c;

```

```

13     }
14     return ori(mv(C[a], C[b]), mv(C[a], p)) < r;
15 }

```

7.8 Point Segment Distance

```

1 double point_segment_dist(Pt q0, Pt q1, Pt p) {
2     if (q0 == q1) {
3         double dx = double(p.x - q0.x);
4         double dy = double(p.y - q0.y);
5         return sqrt(dx * dx + dy * dy);
6     }
7     T d1 = (q1 - q0) * (p - q0);
8     T d2 = (q0 - q1) * (p - q1);
9     if (d1 >= 0 && d2 >= 0) {
10        double area = fabs(double((q1 - q0) ^ (p - q0))
11                               );
12        double base = sqrt(double(dis2(q0, q1)));
13        return area / base;
14    }
15    double dx0 = double(p.x - q0.x), dy0 = double(p.y -
16    q0.y);
17    double dx1 = double(p.x - q1.x), dy1 = double(p.y -
18    q1.y);
19    return min(sqrt(dx0 * dx0 + dy0 * dy0), sqrt(dx1 *
20    dx1 + dy1 * dy1));
21 }

```

7.9 Point in Polygon

```

1 short inPoly(vector<Pt>& pts, Pt p) {
2     // 0=Bound 1=In -1=Out
3     int n = pts.size();
4     for (int i = 0; i < pts.size(); i++) if (onseg(p,
5     pts[i], pts[(i + 1) % n])) return 0;
6     int cnt = 0;
7     for (int i = 0; i < pts.size(); i++) if (
8     line_intersect_check(p, Pt(p.x + 1, p.y + 2e9),
9     pts[i], pts[(i + 1) % n])) cnt ^= 1;
10    return (cnt ? 1 : -1);
11 }

```

7.10 Minimum Euclidean Distance

```

1 long long Min_Euclidean_Dist(vector<Pt> &pts) {
2     sort(pts.begin(), pts.end());
3     set<pair<long long, long long>> s;
4     s.insert({pts[0].y, pts[0].x});
5     long long l = 0, best = LLONG_MAX;
6     for (int i = 1; i < (int)pts.size(); i++) {
7         Pt now = pts[i];
8         long long lim = (long long)ceil(sqrt1l((long
9         double)best));
10        while (now.x - pts[l].x > lim) {
11            s.erase({pts[l].y, pts[l].x}); l++;
12        }
13        auto low = s.lower_bound({now.y - lim,
14        LLONG_MIN});
15        auto high = s.upper_bound({now.y + lim,
16        LLONG_MAX});
17        for (auto it = low; it != high; it++) {
18            long long dy = it->first - now.y;
19            long long dx = it->second - now.x;
20            best = min(best, dx * dx + dy * dy);
21        }
22        s.insert({now.y, now.x});
23    }
24    return best;
25 }

```

7.11 Minkowski Sum

```

1 void reorder(vector<Pt> &P) {
2     rotate(P.begin(), min_element(P.begin(), P.end(),
3     [&](Pt a, Pt b) { return make_pair(a.y, a.x) <
4     make_pair(b.y, b.x); }), P.end());
5 }
6 vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
7     // P, Q: convex polygon
8     reorder(P), reorder(Q);
9     int n = P.size(), m = Q.size();

```

```

8     P.push_back(P[0]), P.push_back(P[1]), Q.push_back(Q
9     [0]), Q.push_back(Q[1]);
10    vector<Pt> ans;
11    for (int i = 0, j = 0; i < n || j < m; ) {
12        ans.push_back(P[i] + Q[j]);
13        auto val = (P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]);
14        if (val >= 0) i++;
15        if (val <= 0) j++;
16    }
17    return ans;

```

7.12 Lower Concave Hull

```

1 struct Line {
2     mutable ll m, b, p;
3     bool operator<(const Line& o) const { return m < o.m;
4     }
5     bool operator<(ll x) const { return p < x; }
6 };
7 struct LineContainer : multiset<Line, less<>> {
8     // (for doubles, use inf = 1/.0, div(a,b) = a/b)
9     const ll inf = LLONG_MAX;
10    ll div(ll a, ll b) { // floored division
11        return a / b - ((a ^ b) < 0 && a % b); }
12    bool isect(iterator x, iterator y) {
13        if (y == end()) { x->p = inf; return false; }
14        if (x->m == y->m) x->p = x->b > y->b ? inf : -inf;
15        else x->p = div(y->b - x->b, x->m - y->m);
16        return x->p >= y->p;
17    }
18    void add(ll m, ll b) {
19        auto z = insert({m, b, 0}), y = z++, x = y;
20        while (isect(y, z)) z = erase(z);
21        if (x != begin() && isect(--x, y)) isect(x, y =
22        erase(y));
23        while ((y = x) != begin() && (--x)->p >= y->p)
24            isect(x, erase(y));
25    }
26    ll query(ll x) {
27        assert(!empty());
28        auto l = *lower_bound(x);
29        return l.m * x + l.b;
30    }

```

7.13 Pick's Theorem

Consider a polygon which vertices are all lattice points.
Let i = number of points inside the polygon.
Let b = number of points on the boundary of the polygon.

Then we have the following formula:

$$Area = i + \frac{b}{2} - 1$$

7.14 Rotating SweepLine

```

1 double cross(const Pt &a, const Pt &b) {
2     return a.x*b.y - a.y*b.x;
3 }
4 int rotatingCalipers(const vector<Pt>& hull) {
5     int m = hull.size();
6     if (m < 2) return 0;
7     int j = 1;
8     T maxd = 0;
9     for (int i = 0; i < m; ++i) {
10        int ni = (i + 1) % m;
11        while (abs(cross({hull[ni].x - hull[i].x, hull[
12        ni].y - hull[i].y}, {hull[(j+1)%m].x - hull
13        [i].x, hull[(j+1)%m].y - hull[i].y})) > abs
14        (cross({hull[ni].x - hull[i].x, hull[ni].y
15        - hull[i].y}, {hull[j].x - hull[i].x,
16        hull[j].y - hull[i].y}))) {
17            j = (j + 1) % m;
18        }
19        maxd = max(maxd, dis2(hull[i], hull[j]));
20        maxd = max(maxd, dis2(hull[ni], hull[j]));
21    }

```

```

17 |     return maxd; // TODO
18 | }
35 | }

```

7.15 Half Plane Intersection

```

1 | bool cover(Line& L, Line& P, Line& Q) {
2 |     long double u = (Q.a - P.a) ^ Q.dir();
3 |     long double v = P.dir() ^ Q.dir();
4 |     long double x = P.dir().x * u + (P.a - L.a).x * v;
5 |     long double y = P.dir().y * u + (P.a - L.a).y * v;
6 |     return sgn(x * L.dir().y - y * L.dir().x) * sgn(v)
7 |     >= 0;
8 | }
9 | vector<Line> HPI(vector<Line> P) {
10 |     sort(P.begin(), P.end(), [&](Line& l, Line& m) {
11 |         if (argcmp(l.dir(), m.dir()) return true;
12 |         if (argcmp(m.dir(), l.dir()) return false;
13 |         return ori(m.a, m.b, l.a) > 0;
14 |     });
15 |     int l = 0, r = -1;
16 |     for (size_t i = 0; i < P.size(); ++i) {
17 |         if (i && !argcmp(P[i - 1].dir(), P[i].dir()))
18 |             continue;
19 |         while (l < r && cover(P[i], P[r - 1], P[r])) --r;
20 |         while (l < r && cover(P[i], P[l], P[l + 1])) ++l;
21 |         P[++r] = P[i];
22 |     }
23 |     while (l < r && cover(P[l], P[r - 1], P[r])) --r;
24 |     while (l < r && cover(P[r], P[l], P[l + 1])) ++l;
25 |     if (r - l <= 1 || !argcmp(P[l].dir(), P[r].dir()))
26 |         return {};
27 |     if (cover(P[l + 1], P[l], P[r])) return {};
28 |     return vector<Line>(P.begin() + l, P.begin() + r +
29 |     1);

```

7.16 Minimum Enclosing Circle

```

1 | const int INF = 1e9;
2 | Pt circumcenter(Pt A, Pt B, Pt C) {
3 |     // a1(x-A.x) + b1(y-A.y) = c1
4 |     // a2(x-A.x) + b2(y-A.y) = c2
5 |     // solve using Cramer's rule
6 |     T a1 = B.x - A.x, b1 = B.y - A.y, c1 = dis2(A, B) /
7 |     2.0;
8 |     T a2 = C.x - A.x, b2 = C.y - A.y, c2 = dis2(A, C) /
9 |     2.0;
10 |     T D = Pt(a1, b1) ^ Pt(a2, b2);
11 |     T Dx = Pt(c1, b1) ^ Pt(c2, b2);
12 |     T Dy = Pt(a1, c1) ^ Pt(a2, c2);
13 |     if (D == 0) return Pt(-INF, -INF);
14 |     return A + Pt(Dx / D, Dy / D);
15 | }
16 | Pt center;
17 | T r2;
18 | void minEncloseCircle(vector<Pt> pts) {
19 |     mt19937 gen(chrono::steady_clock::now().
20 |     time_since_epoch().count());
21 |     shuffle(pts.begin(), pts.end(), gen);
22 |     center = pts[0], r2 = 0;
23 |     for (int i = 0; i < pts.size(); ++i) {
24 |         if (dis2(center, pts[i]) <= r2) continue;
25 |         center = pts[i], r2 = 0;
26 |         for (int j = 0; j < i; ++j) {
27 |             if (dis2(center, pts[j]) <= r2) continue;
28 |             center = (pts[i] + pts[j]) / 2.0;
29 |             r2 = dis2(center, pts[i]);
30 |             for (int k = 0; k < j; ++k) {
31 |                 if (dis2(center, pts[k]) <= r2)
32 |                     continue;
33 |                 center = circumcenter(pts[i], pts[j],
34 |                 pts[k]);
35 |                 r2 = dis2(center, pts[i]);
36 |             }
37 |         }
38 |     }
39 | }

```

7.17 Union of Circles

```

1 | // Area[i] : area covered by at least i circle
2 | vector<T> CircleUnion(const vector<Cir> &C) {
3 |     const int n = C.size();
4 |     vector<T> Area(n + 1);
5 |     auto check = [&](int i, int j) {
6 |         if (!contain(C[i], C[j]))
7 |             return false;
8 |         return sgn(C[i].r - C[j].r) > 0 or (sgn(C[i].r
9 |         - C[j].r) == 0 and i < j);
10 |     };
11 |     struct Teve {
12 |         double ang; int add; Pt p;
13 |         bool operator<(const Teve &b) { return ang < b.
14 |         ang; }
15 |     };
16 |     auto ang = [&](Pt p) { return atan2(p.y, p.x); };
17 |     for (int i = 0; i < n; ++i) {
18 |         int cov = 1;
19 |         vector<Teve> event;
20 |         for (int j = 0; j < n; ++j) if (i != j) {
21 |             if (check(j, i)) cov++;
22 |             else if (!check(i, j) and !disjunct(C[i], C
23 |             [j])) {
24 |                 auto I = CircleInter(C[i], C[j]);
25 |                 assert(I.size() == 2);
26 |                 double a1 = ang(I[0] - C[i].o), a2 =
27 |                 ang(I[1] - C[i].o);
28 |                 event.push_back({a1, 1, I[0]});
29 |                 event.push_back({a2, -1, I[1]});
30 |                 if (a1 > a2) cov++;
31 |             }
32 |         }
33 |         if (event.empty()) {
34 |             Area[cov] += acos(-1) * C[i].r * C[i].r;
35 |             continue;
36 |         }
37 |         sort(event.begin(), event.end());
38 |         event.push_back(event[0]);
39 |         for (int j = 0; j + 1 < event.size(); ++j) {
40 |             cov += event[j].add;
41 |             Area[cov] += (event[j].p ^ event[j + 1].p)
42 |             / 2.;
43 |             double theta = event[j + 1].ang - event[j].
44 |             ang;
45 |             if (theta < 0) theta += 2 * acos(-1);
46 |             Area[cov] += (theta - sin(theta)) * C[i].r
47 |             * C[i].r / 2.;
48 |         }
49 |     }
50 |     return Area;

```

7.18 Union of Polygons

7.19 3D Point

7.20 3D Convex Hull

8 Number Theory

8.1 FFT

```

1 | typedef complex<double> cp;
2 |
3 | const double pi = acos(-1);
4 | const int NN = 131072;
5 |
6 | struct FastFourierTransform {
7 |     /*
8 |         Iterative Fast Fourier Transform
9 |         How this works? Look at this
10 |         0th recursion 0(000) 1(001) 2(010)
11 |         3(011) 4(100) 5(101) 6(110)
12 |         7(111)
13 |         1th recursion 0(000) 2(010) 4(100)
14 |         6(110) / 1(011) 3(011) 5(101)
15 |         7(111)

```

```

12      2th recursion 0(000) 4(100) / 2(010)
13      6(110) / 1(011) 5(101) / 3(011)
14      7(111)
15      3th recursion 0(000) / 4(100) / 2(010) /
16      6(110) / 1(011) / 5(101) / 3(011) /
17      7(111)
18      All the bits are reversed => We can save
19      the reverse of the numbers in an array!
20
21  */
22  int n, rev[NN];
23  cp omega[NN], iomega[NN];
24  void init(int n_) {
25      n = n_;
26      for (int i = 0; i < n; i++) {
27          // Calculate the nth roots of unity
28          omega[i] = cp(cos(2 * pi * i / n), sin(2 *
29              pi * i / n));
30          iomega[i] = conj(omega[i]);
31      }
32      int k = __lg(n);
33      for (int i = 0; i < n; i++) {
34          int t = 0;
35          for (int j = 0; j < k; j++) {
36              if (i & (1 << j)) t |= (1 << (k - j -
37                  1));
38          }
39          rev[i] = t;
40      }
41
42  void transform(vector<cp> &a, cp *xomega) {
43      for (int i = 0; i < n; i++)
44          if (i < rev[i]) swap(a[i], a[rev[i]]);
45      for (int len = 2; len <= n; len <<= 1) {
46          int mid = len >> 1;
47          int r = n / len;
48          for (int j = 0; j < n; j += len)
49              for (int i = 0; i < mid; i++) {
50                  cp tmp = xomega[r * i] * a[j + mid
51                      + i];
52                  a[j + mid + i] = a[j + i] - tmp;
53                  a[j + i] = a[j + i] + tmp;
54              }
55      }
56
57  void fft(vector<cp> &a) { transform(a, omega); }
58  void ifft(vector<cp> &a) {
59      transform(a, iomega);
60      for (int i = 0; i < n; i++) a[i] /= n;
61  }
62  } FFT;
63
64  const int MAXN = 262144;
65  // (must be 2^k)
66  // 262144, 524288, 1048576, 2097152, 4194304
67  // before any usage, run pre_fft() first
68  typedef long double ld;
69  typedef complex<ld> cplx; // real(), imag()
70  const ld PI = acos(-1);
71  const cplx I(0, 1);
72  cplx omega[MAXN + 1];
73  void pre_fft() {
74      for (int i = 0; i <= MAXN; i++) {
75          omega[i] = exp(i * 2 * PI / MAXN * I);
76      }
77  }
78  // n must be 2^k
79  void fft(int n, cplx a[], bool inv = false) {
80      int basic = MAXN / n;
81      int theta = basic;
82      for (int m = n; m >= 2; m >>= 1) {
83          int mh = m >> 1;
84          for (int i = 0; i < mh; i++) {
85              cplx w = omega[inv ? MAXN - (i * theta %
86                  MAXN) : i * theta % MAXN];
87              for (int j = i; j < n; j += m) {
88                  int k = j + mh;
89                  cplx x = a[j] - a[k];
90                  a[j] += a[k];
91                  a[k] = w * x;
92              }
93          }
94      }
95      if (inv)
96          for (int i = 0; i < n; i++) a[i] /= n;
97  }
98
99  inline void mul(int _n, long long a[], int _m, long
100      long b[], long long ans[]) {
101      int n = 1, sum = _n + _m - 1;
102      while (n < sum) n <<= 1;
103      for (int i = 0; i < n; i++) {
104          double x = (i < _n ? a[i] : 0), y = (i < _m ? b
105              [i] : 0);
106          arr[i] = complex<double>(x + y, x - y);
107      }
108      fft(n, arr);
109      for (int i = 0; i < n; i++) arr[i] = arr[i] * arr[i]
110          ];
111      fft(n, arr, true);
112      for (int i = 0; i < sum; i++) ans[i] = (long long
113          int)(arr[i].real() / 4 + 0.5);
114  }
115
116  long long a[MAXN];
117  long long b[MAXN];
118  long long ans[MAXN];
119  int a_length;
120  int b_length;

```

8.2 Pollard's rho

```

1  ll add(ll x, ll y, ll p) {
2      return (x + y) % p;
3  }
4  ll qMul(ll x, ll y, ll mod) {
5      ll ret = x * y - (ll)((long double)x / mod * y) *
6          mod;
7      return ret < 0 ? ret + mod : ret;
8  }
9  ll f(ll x, ll mod) { return add(qMul(x, x, mod), 1, mod
10      ); }
11  ll pollard_rho(ll n) {
12      if (!(n & 1)) return 2;
13      while (true) {
14          ll y = 2, x = rand() % (n - 1) + 1, res = 1;
15          for (int sz = 2; res == 1; sz *= 2) {
16              for (int i = 0; i < sz && res <= 1; i++) {
17                  x = f(x, n);
18                  res = __gcd(llabs(x - y), n);
19              }
20              y = x;
21          }
22          if (res != 0 && res != n) return res;
23      }
24  }
25  vector<ll> ret;
26  void fact(ll x) {
27      if (miller_rabin(x)) {
28          ret.push_back(x);
29          return;
30      }
31      ll f = pollard_rho(x);
32      fact(f);
33      fact(x / f);
34  }

```

8.3 Miller Rabin

```

1  // n < 4,759,123,141      3 : 2, 7, 61
2  // n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
3  // n < 3,474,749,660,383  6 : pimes <= 13
4  // n < 2^64              7 :
5  // 2, 325, 9375, 28178, 450775, 9780504, 1795265022
6  bool witness(ll a, ll n, ll u, int t) {
7      if (!(a % n)) return 0;

```

```

8   ll x = mypow(a, u, n);
9   for (int i = 0; i < t; i++) {
10      ll nx = mul(x, x, n);
11      if (nx == 1 && x != 1 && x != n - 1) return 1;
12      x = nx;
13   }
14   return x != 1;
15 }
16 bool miller_rabin(ll n, int s = 100) {
17     // iterate s times of witness on n
18     // return 1 if prime, 0 otherwise
19     if (n < 2) return 0;
20     if (!(n & 1)) return n == 2;
21     ll u = n - 1;
22     int t = 0;
23     while (!(u & 1)) u >>= 1, t++;
24     while (s--) {
25         ll a = randll() % (n - 1) + 1;
26         if (witness(a, n, u, t)) return 0;
27     }
28     return 1;
29 }

```

8.4 Fast Power

Note: $a^n \equiv a^{(n \bmod (p-1))} \pmod{p}$

8.5 Extend GCD

```

1  ll GCD;
2  pll extgcd(ll a, ll b) {
3      if (b == 0) {
4          GCD = a;
5          return pll{1, 0};
6      }
7      pll ans = extgcd(b, a % b);
8      return pll{ans.S, ans.F - a / b * ans.S};
9  }
10 pll bezout(ll a, ll b, ll c) {
11     bool negx = (a < 0), negy = (b < 0);
12     pll ans = extgcd(abs(a), abs(b));
13     if (c % GCD != 0) return pll{-LLINF, -LLINF};
14     return pll{ans.F * c / GCD * (negx ? -1 : 1),
15                ans.S * c / GCD * (negy ? -1 : 1)};
16 }
17 ll inv(ll a, ll p) {
18     if (p == 1) return -1;
19     pll ans = bezout(a % p, -p, 1);
20     if (ans == pll{-LLINF, -LLINF}) return -1;
21     return (ans.F % p + p) % p;
22 }

```

8.6 Mu + Phi

```

1  const int maxn = 1e6 + 5;
2  ll f[maxn];
3  vector<int> lpf, prime;
4  void build() {
5      lpf.clear();
6      lpf.resize(maxn, 1);
7      prime.clear();
8      f[1] = ...; /* mu[1] = 1, phi[1] = 1 */
9      for (int i = 2; i < maxn; i++) {
10         if (lpf[i] == 1) {
11             lpf[i] = i;
12             prime.emplace_back(i);
13             f[i] = ...; /* mu[i] = 1, phi[i] = i-1 */
14         }
15         for (auto& j : prime) {
16             if (i * j >= maxn) break;
17             lpf[i * j] = j;
18             if (i % j == 0)
19                 f[i * j] = ...; /* 0, phi[i]*j */
20             else
21                 f[i * j] = ...; /* -mu[i], phi[i]*phi[j] */
22             if (j >= lpf[i]) break;
23         }
24     }
25 }

```

8.7 Discrete Log

```

1  long long mod_pow(long long a, long long e, long long p)
2  ){
3      long long r = 1 % p;
4      while(e){
5          if(e & 1) r = (__int128)r * a % p;
6          a = (__int128)a * a % p;
7          e >>= 1;
8      }
9      return r;
10 }
11 long long mod_inv(long long a, long long p){
12     return mod_pow((a%p+p)%p, p-2, p);
13 }
14 // BSGS: solve a^x = y (mod p), gcd(a,p)=1, p prime,
15 // return minimal x>=0, or -1 if no solution
16 long long bsgs(long long a, long long y, long long p){
17     a%=p; y%=p;
18     if(y==1%p) return 0; // x=0
19     long long m = (long long)ceil(sqrt((long double)p));
20     ;
21     // baby steps: a^j
22     unordered_map<long long, long long> table;
23     table.reserve(m*2);
24     long long cur = 1%p;
25     for(long long j=0; j<m; ++j){
26         if(!table.count(cur)) table[cur]=j;
27         cur = (__int128)cur * a % p;
28     }
29     long long am = mod_pow(a, m, p);
30     long long am_inv = mod_inv(am, p);
31     long long gamma = y % p;
32     for(long long i=0; i<=m; ++i){
33         auto it = table.find(gamma);
34         if(it != table.end()){
35             long long x = i*m + it->second;
36             return x;
37         }
38         gamma = (__int128)gamma * am_inv % p;
39     }
40     return -1;
41 }

```

8.8 Other Formulas

- Inversion: $aa^{-1} \equiv 1 \pmod{m}$. a^{-1} exists iff $\gcd(a, m) = 1$.
- Linear inversion: $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod{m}$
- Fermat's little theorem: $a^p \equiv a \pmod{p}$ if p is prime.
- Euler function: $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$
- Euler theorem: $a^{\phi(n)} \equiv 1 \pmod{n}$ if $\gcd(a, n) = 1$.
- Extended Euclidean algorithm: $ax + by = \gcd(a, b) = \gcd(b, a \bmod b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$
- Divisor function: $\sigma_x(n) = \sum_{d|n} d^x$. $n = \prod_{i=1}^r p_i^{a_i}$.
 $\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$ if $x \neq 0$. $\sigma_0(n) = \prod_{i=1}^r (a_i + 1)$.
- Chinese remainder theorem (Coprime Moduli):
 $x \equiv a_i \pmod{m_i}$.
 $M = \prod m_i$. $M_i = M / m_i$. $t_i = M_i^{-1}$.
 $x = kM + \sum a_i t_i M_i$, $k \in \mathbb{Z}$.
- Chinese remainder theorem:
 $x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2} \Rightarrow x = m_1 p + a_1 = m_2 q + a_2 \Rightarrow m_1 p - m_2 q = a_2 - a_1$

Solve for (p, q) using ExtGCD.

$$x \equiv m_1 p + a_1 \equiv m_2 q + a_2 \pmod{\text{lcm}(m_1, m_2)}$$

- Avoiding Overflow: $ca \bmod cb = c(a \bmod b)$
- Dirichlet Convolution: $(f * g)(n) = \sum_{d|n} f(n)g(n/d)$
- Important Multiplicative Functions + Properties:
 1. $\epsilon(n) = [n = 1]$
 2. $1(n) = 1$
 3. $id(n) = n$
 4. $\mu(n) = 0$ if n has squared prime factor
 5. $\mu(n) = (-1)^k$ if $n = p_1 p_2 \cdots p_k$
 6. $\epsilon = \mu * 1$
 7. $\phi = \mu * id$
 8. $[n = 1] = \sum_{d|n} \mu(d)$
 9. $[gcd = 1] = \sum_{d|gcd} \mu(d)$
- Möbius inversion: $f = g * 1 \Leftrightarrow g = f * \mu$

8.9 Polynomial

```

1 const int maxk = 20;
2 const int maxn = 1<<maxk;
3 const ll LINF = 1e18;
4
5 /* P = r*2^k + 1
6 P      r      k      g
7 998244353      119 23   3
8 1004535809     479 21   3
9
10 P      r      k      g
11 3      1      1      2
12 5      1      2      2
13 17     1      4      3
14 97     3      5      5
15 193    3      6      5
16 257    1      8      3
17 7681   15     9      17
18 12289  3     12     11
19 40961  5     13      3
20 65537  1     16      3
21 786433 3     18     10
22 5767169 11    19      3
23 7340033 7     20      3
24 23068673 11   21      3
25 104857601 25  22      3
26 167772161 5   25      3
27 469762049 7   26      3
28 1004535809 479 21      3
29 2013265921 15  27     31
30 2281701377 17  27      3
31 3221225473 3   30      5
32 75161927681 35  31      3
33 77309411329 9   33      7
34 206158430209 3  36     22
35 2061584302081 15 37      7
36 2748779069441 5  39      3
37 6597069766657 3  41      5
38 39582418599937 9  42      5
39 79164837199873 9  43      5
40 263882790666241 15 44      7
41 1231453023109121 35 45      3
42 1337006139375617 19 46      3
43 3799912185593857 27 47      5
44 4222124650659841 15 48     19
45 7881299347898369 7  50      6
46 31525197391593473 7  52      3
47 180143985094819841 5  55      6
48 1945555039024054273 27 56      5
49 4179340454199820289 29 57      3
50 9097271247288401921 505 54      6 */
51
52 const int g = 3;
53 const ll MOD = 998244353;
54
55 ll pw(ll a, ll n) { /* fast pow */ }
56
```

```

57 #define siz(x) (int)x.size()
58
59 template<typename T>
60 vector<T>& operator+=(vector<T>& a, const vector<T>& b)
61 {
62     if (siz(a) < siz(b)) a.resize(siz(b));
63     for (int i = 0; i < min(siz(a), siz(b)); i++) {
64         a[i] += b[i];
65         a[i] -= a[i] >= MOD ? MOD : 0;
66     }
67     return a;
68 }
69
70 template<typename T>
71 vector<T>& operator-=(vector<T>& a, const vector<T>& b)
72 {
73     if (siz(a) < siz(b)) a.resize(siz(b));
74     for (int i = 0; i < min(siz(a), siz(b)); i++) {
75         a[i] -= b[i];
76         a[i] += a[i] < 0 ? MOD : 0;
77     }
78     return a;
79 }
80
81 template<typename T>
82 vector<T> operator-(const vector<T>& a) {
83     vector<T> ret(siz(a));
84     for (int i = 0; i < siz(a); i++) {
85         ret[i] = -a[i] < 0 ? -a[i] + MOD : -a[i];
86     }
87     return ret;
88 }
89
90 vector<ll> X, iX;
91 vector<int> rev;
92
93 void init_ntt() {
94     X.clear(); X.resize(maxn, 1); // x1 = g^((p-1)/n)
95     iX.clear(); iX.resize(maxn, 1);
96
97     ll u = pw(g, (MOD-1)/maxn);
98     ll iu = pw(u, MOD-2);
99
100     for (int i = 1; i < maxn; i++) {
101         X[i] = X[i-1] * u;
102         iX[i] = iX[i-1] * iu;
103         if (X[i] >= MOD) X[i] %= MOD;
104         if (iX[i] >= MOD) iX[i] %= MOD;
105     }
106
107     rev.clear(); rev.resize(maxn, 0);
108     for (int i = 1, hb = -1; i < maxn; i++) {
109         if (!(i & (i-1))) hb++;
110         rev[i] = rev[i ^ (1<<hb)] | (1<<(maxk-hb-1));
111     }
112 }
113
114 template<typename T>
115 void NTT(vector<T>& a, bool inv=false) {
116
117     int _n = (int)a.size();
118     int k = __lg(_n) + ((1<<__lg(_n)) != _n);
119     int n = 1<<k;
120     a.resize(n, 0);
121
122     short shift = maxk-k;
123     for (int i = 0; i < n; i++)
124         if (i > (rev[i]>>shift))
125             swap(a[i], a[rev[i]>>shift]);
126
127     for (int len = 2, half = 1, div = maxn>>1; len <= n; len<<=1, half<<=1, div>>=1) {
128         for (int i = 0; i < n; i += len) {
129             for (int j = 0; j < half; j++) {
130                 T u = a[i+j];
131                 T v = a[i+j+half] * (inv ? iX[j*div] : X[j*div]) % MOD;
132                 a[i+j] = (u+v >= MOD ? u+v-MOD : u+v);
133                 a[i+j+half] = (u-v < 0 ? u-v+MOD : u-v);
134             }
135         }
136     }
137
138     if (inv) {

```

```

134     T dn = pw(n, MOD-2);
135     for (auto& x : a) {
136         x *= dn;
137         if (x >= MOD) x %= MOD;
138     } }
139
140 template<typename T>
141 inline void resize(vector<T>& a) {
142     int cnt = (int)a.size();
143     for (; cnt > 0; cnt--) if (a[cnt-1]) break;
144     a.resize(max(cnt, 1));
145 }
146
147 template<typename T>
148 vector<T>& operator*=(vector<T>& a, vector<T> b) {
149     int na = (int)a.size();
150     int nb = (int)b.size();
151     a.resize(na + nb - 1, 0);
152     b.resize(na + nb - 1, 0);
153
154     NTT(a); NTT(b);
155     for (int i = 0; i < (int)a.size(); i++) {
156         a[i] *= b[i];
157         if (a[i] >= MOD) a[i] %= MOD;
158     }
159     NTT(a, true);
160
161     resize(a);
162     return a;
163 }
164
165 template<typename T>
166 void inv(vector<T>& ia, int N) {
167     vector<T> _a(move(ia));
168     ia.resize(1, pw(_a[0], MOD-2));
169     vector<T> a(1, _a[0] + (-_a[0] < 0 ? MOD : 0));
170
171     for (int n = 1; n < N; n<=1) {
172         // n -> 2*n
173         // ia' = ia(2-a*ia);
174
175         for (int i = n; i < min(siz(_a), (n<1)); i++)
176             a.emplace_back(-_a[i] + (-_a[i] < 0 ? MOD : 0));
177
178         vector<T> tmp = ia;
179         ia *= a;
180         ia.resize(n<1);
181         ia[0] = ia[0] + 2 >= MOD ? ia[0] + 2 - MOD : ia[0] + 2;
182         ia *= tmp;
183         ia.resize(n<1);
184     }
185     ia.resize(N);
186 }
187
188 template<typename T>
189 void mod(vector<T>& a, vector<T>& b) {
190     int n = (int)a.size()-1, m = (int)b.size()-1;
191     if (n < m) return;
192
193     vector<T> ra = a, rb = b;
194     reverse(ra.begin(), ra.end()); ra.resize(min(n+1, n-m+1));
195     reverse(rb.begin(), rb.end()); rb.resize(min(m+1, n-m+1));
196
197     inv(rb, n-m+1);
198
199     vector<T> q = move(ra);
200     q *= rb;
201     q.resize(n-m+1);
202     reverse(q.begin(), q.end());
203
204     q *= b;
205     a -= q;
206     resize(a);
207 }
208
209 /* Kitamasa Method (Fast Linear Recurrence):
210 Find a[K] (Given a[j] = c[0]a[j-N] + ... + c[N-1]a[j-1])

```

```

211 Let B(x) = x^N - c[N-1]x^(N-1) - ... - c[1]x^1 - c[0]
212 Let R(x) = x^K mod B(x) (get x^K using fast pow and
213 use poly mod to get R(x))
214 Let r[i] = the coefficient of x^i in R(x)
215 => a[K] = a[0]r[0] + a[1]r[1] + ... + a[N-1]r[N-1] */

```

9 Linear Algebra

9.1 Gaussian-Jordan Elimination

```

1 int n;
2 vector<vector<ll>>> v;
3 void gauss(vector<vector<ll>>& v) {
4     int r = 0;
5     for (int i = 0; i < n; i++) {
6         bool ok = false;
7         for (int j = r; j < n; j++) {
8             if (v[j][i] != 0) continue;
9             swap(v[j], v[r]);
10            ok = true;
11            break;
12        }
13        if (!ok) continue;
14        ll div = inv(v[r][i]);
15        for (int j = 0; j < n + 1; j++) {
16            v[r][j] *= div;
17            if (v[r][j] >= MOD) v[r][j] %= MOD;
18        }
19        for (int j = 0; j < n; j++) {
20            if (j == r) continue;
21            ll t = v[j][i];
22            for (int k = 0; k < n + 1; k++) {
23                v[j][k] -= v[r][k] * t % MOD;
24                if (v[j][k] < 0) v[j][k] += MOD;
25            }
26        }
27        r++;
28    }
29 }

```

9.2 Determinant

1. Use GJ Elimination, if there's any row consists of only 0, then det = 0, otherwise det = product of diagonal elements.
2. Properties of det:
 - Transpose: Unchanged
 - Row Operation 1 - Swap 2 rows: $-det$
 - Row Operation 2 - $k\vec{r}_i$: $k \times det$
 - Row Operation 3 - $k\vec{r}_i$ add to \vec{r}_j : Unchanged

10 Combinatorics

10.1 Catalan Number

$$C_0 = 1, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}, C_n = C_n^{2n} - C_{n-1}^{2n}$$

0	1	1	2	5
4	14	42	132	429
8	1430	4862	16796	58786
12	208012	742900	2674440	9694845

10.2 Burnside's Lemma

Let X be the original set.

Let G be the group of operations acting on X .

Let X^g be the set of x not affected by g .

Let X/G be the set of orbits.

Then the following equation holds:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

11 Special Numbers

11.1 Fibonacci Series

1	1	1	2	3
5	5	8	13	21
9	34	55	89	144
13	233	377	610	987
17	1597	2584	4181	6765
21	10946	17711	28657	46368
25	75025	121393	196418	317811
29	514229	832040	1346269	2178309
33	3524578	5702887	9227465	14930352

$$f(45) \approx 10^9, f(88) \approx 10^{18}$$

11.2 Prime Numbers

- First 50 prime numbers:

1	2	3	5	7	11
6	13	17	19	23	29
11	31	37	41	43	47
16	53	59	61	67	71
21	73	79	83	89	97
26	101	103	107	109	113
31	127	131	137	139	149
36	151	157	163	167	173
41	179	181	191	193	197
46	199	211	223	227	229

- Very large prime numbers:

1000001333 1000500889 2500001909
 2000000659 900004151 850001359

- $\pi(n) \equiv \text{Number of primes} \leq n \approx n/((\ln n) - 1)$

$$\pi(100) = 25, \pi(200) = 46$$

$$\pi(500) = 95, \pi(1000) = 168$$

$$\pi(2000) = 303, \pi(4000) = 550$$

$$\pi(10^4) = 1229, \pi(10^5) = 9592$$

$$\pi(10^6) = 78498, \pi(10^7) = 664579$$