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1 Reminder

1.1 Bug List

- 沒開 long long
- 陣列戳出界／陣列開不夠大
- 寫好的函式忘記呼叫
- 變數打錯
- 0-base / 1-base
- 忘記初始化
- == 打成 =
- <= 打成 <+
- dp[i] 從 dp[i-1] 轉移時忘記特判 i > 0
- std::sort 比較運算子寫成 < 或是讓 = 的情況為 true
- 漏 case
- 線段樹改值懶標初始值不能設為 0
- DFS 的時候不小心覆寫到全域變數
- 浮點數誤差
- unsigned int128
- 多筆測資不能沒讀完直接 return
- 記得刪 cerr

1.2 OwO

- 可以構造複雜點的測資幫助思考
- 真的卡太久請跳題
- Enjoy The Contest!

2 Basic

2.1 Default

```
#include <bits/stdc++.h>

using namespace std;
using ll = long long;
using pii = pair<int, int>;
using pll = pair<ll, ll>;

#define endl '\n'

#define F first
#define S second
#define ep emplace
#define pb push_back
#define eb emplace_back
#define ALL(x) x.begin(), x.end()
#define SZ(x) (int)x.size()

namespace{
const int INF = 0x3f3f3f3f;
const ll LINF = 0x3f3f3f3f3f3f3f3f;

template<typename T> using V=vector<T>;
template<typename T1,typename T2=T1> using P = pair<T1, T2>;

void _debug() {}
template<typename A,typename... B> void _debug(A a,B... b){
    cerr<<a<<' ',_debug(b...);
}

#define debug(...) cerr<<#__VA_ARGS__<<" ":_debug(
__VA_ARGS__),cerr<<endl;

template<typename T>
ostream& operator<<(ostream& os,const vector<T>& v){
    for(const auto& i:v)
        os<<i<<' ';
    return os;
}

}

/*-----*/

const ll MOD = 1e9 + 7;
const int maxn = 2e5 + 5;
```

```

42
43 void init() {
44     ;
45 }
46
47 void solve() {
48     ;
49 }
50
51 /*
52
53 */
54
55 signed main() {
56     cin.tie(0), ios::sync_with_stdio(0);
57
58 int T = 1;
59 // cin >> T;
60 while (T--) {
61     init();
62     solve();
63 }
64
65 return 0;
66 }
67

```

2.2 Vimrc

```

1 syn on
2 se si nu rnu ru cul mouse=a
3 se cin et ts=4 sw=4 sts=4 re=0
4 colo koehler # evening, wildcharm
5 set autochdir
6
7 no <F1> :!%:~/.out < %:~/.input > %:~/.output;
8     echo "-----"; cat %:~/.output<CR>
9 no <F2> :!g++ -std=c++20 -fsanitize=address,undefined -
10     g -Wextra -Wall %:~/.input > %:~/.output; echo "
11     -----"; cat %:~/.output<CR>

```

2.3 Stress

```

1 g++ gen.cpp -o gen.out
2 g++ ac.cpp -o ac.out
3 g++ wa.cpp -o wa.out
4 for ((i=0;;i++))
5 do
6     echo "$i"
7     ./gen.out > in.txt
8     ./ac.out < in.txt > ac.txt
9     ./wa.out < in.txt > wa.txt
10    diff ac.txt wa.txt || break
11 done

```

2.4 PBDS

```

1 #include <bits/extc++.h>
2 using namespace __gnu_pbds;
3
4 // map
5 tree<int, int, less<>, rb_tree_tag,
6     tree_order_statistics_node_update> tr;
7 tr.order_of_key(element);
8 tr.find_by_order(rank);
9
10 // set
11 tree<int, null_type, less<>, rb_tree_tag,
12     tree_order_statistics_node_update> tr;
13 tr.order_of_key(element);
14 tr.find_by_order(rank);
15
16 // priority queue
17 __gnu_pbds::priority_queue<int, less<int> > big_q; //
18     Big First
19 __gnu_pbds::priority_queue<int, greater<int> > small_q;
20 // Small First
21 q1.join(q2); // join

```

2.5 Random

```

1 mt19937 gen(chrono::steady_clock::now().
2     time_since_epoch().count());
3 uniform_int_distribution<int> dis(1, 100);
4 cout << dis(gen) << endl;
5 shuffle(v.begin(), v.end(), gen);

```

3 Python

3.1 I/O

```

1 import sys
2 input = sys.stdin.readline
3
4 # Input
5 def readInt():
6     return int(input())
7 def readList():
8     return list(map(int, input().split()))
9 def readStr():
10    s = input()
11    return list(s[:len(s) - 1])
12 def readVars():
13    return map(int, input().split())
14
15 # Output
16 sys.stdout.write(string)
17
18 # faster
19 def main():
20     pass
21 main()

```

3.2 Decimal

```

1 from decimal import *
2 getcontext().prec = 2500000
3 getcontext().Emax = 2500000
4 a,b = Decimal(input()), Decimal(input())
5 a*=b
6 print(a)

```

4 Data Structure

4.1 Heavy Light Decomposition

```

1 constexpr int maxn=2e5+5;
2 int arr[(maxn+1)<<2];
3 #define m ((l+r)>>1)
4 void build(V<int>& v, int l=1, int r=maxn){
5     if((int)v.size()<=1) return;
6     if(r-l==1){arr[l]=v[l];return;}
7     build(v, l<<1, l+m), build(v, l+m, r);
8     arr[l]=max(arr[l<<1], arr[l+m]);
9 }
10 void modify(int p, int k, int l=1, int r=maxn){
11     if(p<l||r<=p) return;
12     if(r-l==1){arr[l]=k;return;}
13     if(p<m) modify(p, k, l<<1, l+m);
14     else modify(p, k, l+m, r);
15     arr[l]=max(arr[l<<1], arr[l+m]);
16 }
17 int query(int ql, int qr, int l=1, int r=maxn){
18     if(ql<=l||r<=qr) return 0;
19     if(ql<=l&&r<=qr) return arr[l];
20     if(ql<=m) return query(ql, qr, l<<1, l+m);
21     if(m<=qr) return query(ql, qr, l+m, r);
22     return max(query(ql, qr, l<<1, l+m), query(ql, qr, l+m, r));
23 }
24 #undef m
25 inline void solve(){
26     int n,q;cin>>n>>q;
27     V<int> v(n);
28     for(auto& i:v)
29         cin>>i;

```

```

30 V<int>> e(n);
31 for(int i=1;i<n;i++){
32     int a,b;cin>>a>>b,a--,b--;
33     e[a].emplace_back(b);
34     e[b].emplace_back(a);
35 }
36 V<int> d(n,0),f(n,0),sz(n,1),son(n,-1);
37 F<void(int,int)> dfs1=
38 [&](int x,int pre){
39     for(auto i:e[x]) if(i!=pre){
40         d[i]=d[x]+1,f[i]=x;
41         dfs1(i,x),sz[x]+=sz[i];
42         if(!son[x]||sz[son[x]]<sz[i])
43             son[x]=i;
44     }
45 };dfs1(0,0);
46 V<int> top(n,0),dfn(n,-1),rnk(n,0);
47 F<void(int,int)> dfs2=
48 [&](int x,int t){
49     static int cnt=0;
50     dfn[x]=cnt++,rnk[dfn[x]]=x,top[x]=t;
51     if(!son[x]) return;
52     dfs2(son[x],t);
53     for(auto i:e[x])
54         if(!dfn[i]) dfs2(i,i);
55 };dfs2(0,0);
56 V<int> dfnv(n);
57 for(int i=0;i<n;i++)
58     dfnv[dfn[i]]=v[i];
59 build(dfnv);
60 while(q--){
61     int op,a,b;cin>>op>>a>>b;
62     switch(op){
63     case 1:{
64         modify(dfn[a-1],b);
65     }break;
66     case 2:{
67         a--,b--;
68         int ans=0;
69         while(top[a]!=top[b]){
70             if(d[top[a]]>d[top[b]]) swap(a,b);
71             ans=max(ans,query(dfn[top[b]],dfn[b]+1));
72             b=f[top[b]];
73         }
74         if(dfn[a]>dfn[b]) swap(a,b);
75         ans=max(ans,query(dfn[a],dfn[b]+1));
76         cout<<ans<<endl;
77     }break;
78     }
79 }
80 }

```

4.2 Skew Heap

```

1 struct node{
2     node *l,*r;
3     int v;
4     node(int x):v(x){
5         l=r=nullptr;
6     }
7 };
8 node* merge(node* a,node* b){
9     if(!a||!b) return a?:b;
10    // min heap
11    if(a->v>b->v) swap(a,b);
12    a->r=merge(a->r,b);
13    swap(a->l,a->r);
14    return a;
15 }

```

4.3 Leftist Heap

```

1 struct node{
2     node *l,*r;
3     int d, v;
4     node(int x):d(1),v(x){
5         l=r=nullptr;
6     }
7 };

```

```

8 static inline int d(node* x){return x?x->d:0;}
9 node* merge(node* a,node* b){
10     if(!a||!b) return a?:b;
11    // min heap
12    if(a->v>b->v) swap(a,b);
13    a->r=merge(a->r,b);
14    if(d(a->l)<d(a->r))
15        swap(a->l,a->r);
16    a->d=d(a->r)+1;
17    return a;
18 }

```

4.4 Persistent Treap

```

1 struct node {
2     node *l, *r;
3     char c; int v, sz;
4     node(char x = '$'): c(x), v(mt()), sz(1) {
5         l = r = nullptr;
6     }
7     node(node* p) { *this = *p; }
8     void pull() {
9         sz = 1;
10        for (auto i : {l, r})
11            if (i) sz += i->sz;
12    }
13 } arr[maxn], *ptr = arr;
14 inline int size(node* p) {return p ? p->sz : 0;}
15 node* merge(node* a, node* b) {
16     if (!a || !b) return a ? : b;
17     if (a->v < b->v) {
18         node* ret = new(ptr++) node(a);
19         ret->r = merge(ret->r, b), ret->pull();
20         return ret;
21     }
22     else {
23         node* ret = new(ptr++) node(b);
24         ret->l = merge(a, ret->l), ret->pull();
25         return ret;
26     }
27 }
28 P<node*> split(node* p, int k) {
29     if (!p) return {nullptr, nullptr};
30     if (k >= size(p->l) + 1) {
31         auto [a, b] = split(p->r, k - size(p->l) - 1);
32         node* ret = new(ptr++) node(p);
33         ret->r = a, ret->pull();
34         return {ret, b};
35     }
36     else {
37         auto [a, b] = split(p->l, k);
38         node* ret = new(ptr++) node(p);
39         ret->l = b, ret->pull();
40         return {a, ret};
41     }
42 }

```

4.5 Li Chao Tree

```

1 constexpr int maxn = 5e4 + 5;
2 struct line {
3     ld a, b;
4     ld operator()(ld x) {return a * x + b;}
5 } arr[(maxn + 1) << 2];
6 bool operator<(line a, line b) {return a.a < b.a;}
7 #define m ((l+r)>>1)
8 void insert(line x, int i = 1, int l = 0, int r = maxn)
9 {
10    if (r - l == 1) {
11        if (x(l) > arr[i](l))
12            arr[i] = x;
13        return;
14    }
15    line a = max(arr[i], x), b = min(arr[i], x);
16    if (a(m) > b(m))
17        arr[i] = a, insert(b, i << 1, l, m);
18    else
19        arr[i] = b, insert(a, i << 1 | 1, m, r);
20 }
21 ld query(int x, int i = 1, int l = 0, int r = maxn) {

```

```

21 if (x < 1 || r <= x) return -numeric_limits<ld>::max
    ();
22 if (r - 1 == 1) return arr[i](x);
23 return max({arr[i](x), query(x, i << 1, 1, m), query(
    x, i << 1 | 1, m, r)});
24 }
25 #undef m

```

4.6 Time Segment Tree

```

1 constexpr int maxn = 1e5 + 5;
2 V<P<int>> arr[(maxn + 1) << 2];
3 V<int> dsu, sz;
4 V<tuple<int, int, int>> his;
5 int cnt, q;
6 int find(int x) {
7     return x == dsu[x] ? x : find(dsu[x]);
8 };
9 inline bool merge(int x, int y) {
10     int a = find(x), b = find(y);
11     if (a == b) return false;
12     if (sz[a] > sz[b]) swap(a, b);
13     his.emplace_back(a, b, sz[b]), dsu[a] = b, sz[b] +=
        sz[a];
14     return true;
15 };
16 inline void undo() {
17     auto [a, b, s] = his.back(); his.pop_back();
18     dsu[a] = a, sz[b] = s;
19 }
20 #define m ((l + r) >> 1)
21 void insert(int ql, int qr, P<int> x, int i = 1, int l
    = 0, int r = q) {
22     // debug(ql, qr, x); return;
23     if (qr <= 1 || r <= ql) return;
24     if (ql <= 1 && r <= qr) {arr[i].push_back(x);
        return;}
25     if (qr <= m)
26         insert(ql, qr, x, i << 1, 1, m);
27     else if (m <= ql)
28         insert(ql, qr, x, i << 1 | 1, m, r);
29     else {
30         insert(ql, qr, x, i << 1, 1, m);
31         insert(ql, qr, x, i << 1 | 1, m, r);
32     }
33 }
34 void traversal(V<int>& ans, int i = 1, int l = 0, int r
    = q) {
35     int opcnt = 0;
36     // debug(i, l, r);
37     for (auto [a, b] : arr[i])
38         if (merge(a, b))
39             opcnt++, cnt--;
40     if (r - 1 == 1) ans[l] = cnt;
41     else {
42         traversal(ans, i << 1, 1, m);
43         traversal(ans, i << 1 | 1, m, r);
44     }
45     while (opcnt--)
46         undo(), cnt++;
47     arr[i].clear();
48 }
49 #undef m
50 inline void solve() {
51     int n, m; cin >> n >> m >> q, q++;
52     dsu.resize(cnt = n), sz.assign(n, 1);
53     iota(dsu.begin(), dsu.end(), 0);
54     // a, b, time, operation
55     unordered_map<ll, V<int>> s;
56     for (int i = 0; i < m; i++) {
57         int a, b; cin >> a >> b;
58         if (a > b) swap(a, b);
59         s[(((ll)a << 32) | b).emplace_back(0);
60     }
61     for (int i = 1; i < q; i++) {
62         int op, a, b;
63         cin >> op >> a >> b;
64         if (a > b) swap(a, b);
65         switch (op) {
66             case 1:
67                 s[(((ll)a << 32) | b).push_back(i);

```

```

        break;
        case 2:
            auto tmp = s[(((ll)a << 32) | b).back();
            s[(((ll)a << 32) | b).pop_back();
            insert(tmp, i, P<int> {a, b});
        }
    }
    for (auto [p, v] : s) {
        int a = p >> 32, b = p & -1;
        while (v.size()) {
            insert(v.back(), q, P<int> {a, b});
            v.pop_back();
        }
    }
    V<int> ans(q);
    traversal(ans);
    for (auto i : ans)
        cout << i << ' ';
    cout << endl;
}

```

5 DP

5.1 Aliens

```

1 int n; ll k;
2 vector<ll> a;
3 vector<pll> dp[2];
4 void init() {
5     cin >> n >> k;
6     Each(i, dp) i.clear(), i.resize(n);
7     a.clear(); a.resize(n);
8     Each(i, a) cin >> i;
9 }
10 pll calc(ll p) {
11     dp[0][0] = mp(0, 0);
12     dp[1][0] = mp(-a[0], 0);
13     FOR(i, 1, n, 1) {
14         if (dp[0][i-1].F > dp[1][i-1].F + a[i] - p) {
15             dp[0][i] = dp[0][i-1];
16         } else if (dp[0][i-1].F < dp[1][i-1].F + a[i] - p)
17             {
18                 dp[0][i] = mp(dp[1][i-1].F + a[i] - p, dp[1][i
19                     -1].S+1);
20             }
21         else {
22             dp[0][i] = mp(dp[0][i-1].F, min(dp[0][i-1].S, dp
23                 [1][i-1].S+1));
24         }
25         if (dp[0][i-1].F - a[i] > dp[1][i-1].F) {
26             dp[1][i] = mp(dp[0][i-1].F - a[i], dp[0][i-1].S);
27         } else if (dp[0][i-1].F - a[i] < dp[1][i-1].F) {
28             dp[1][i] = dp[1][i-1];
29         } else {
30             dp[1][i] = mp(dp[1][i-1].F, min(dp[0][i-1].S, dp
31                 [1][i-1].S));
32         }
33     }
34     return dp[0][n-1];
35 }
36 void solve() {
37     ll l = 0, r = 1e7;
38     pll res = calc(0);
39     if (res.S <= k) return cout << res.F << endl, void();
40     while (l < r) {
41         ll mid = (l+r) >> 1;
42         res = calc(mid);
43         if (res.S <= k) r = mid;
44         else l = mid+1;
45     }
46     res = calc(l);
47     cout << res.F + k*1 << endl;
48 }

```

6 Graph

6.1 Bellman-Ford + SPFA

```

1 int n, m;

```

```

2 // Graph
3 vector<vector<pair<int, ll> > > g;
4 vector<ll> dis;
5 vector<bool> negCycle;
6
7 // SPFA
8 vector<int> rlx;
9 queue<int> q;
10 vector<bool> inq;
11 vector<int> pa;
12 void SPFA(vector<int>& src) {
13     dis.assign(n+1, LINF);
14     negCycle.assign(n+1, false);
15     rlx.assign(n+1, 0);
16     while (!q.empty()) q.pop();
17     inq.assign(n+1, false);
18     pa.assign(n+1, -1);
19
20     for (auto& s : src) {
21         dis[s] = 0;
22         q.push(s); inq[s] = true;
23     }
24
25     while (!q.empty()) {
26         int u = q.front();
27         q.pop(); inq[u] = false;
28         if (rlx[u] >= n) {
29             negCycle[u] = true;
30         }
31         else for (auto& e : g[u]) {
32             int v = e.first;
33             ll w = e.second;
34             if (dis[v] > dis[u] + w) {
35                 dis[v] = dis[u] + w;
36                 rlx[v] = rlx[u] + 1;
37                 pa[v] = u;
38                 if (!inq[v]) {
39                     q.push(v);
40                     inq[v] = true;
41                 }
42             }
43         }
44     }
45
46 // Bellman-Ford
47 queue<int> q;
48 vector<int> pa;
49 void BellmanFord(vector<int>& src) {
50     dis.assign(n+1, LINF);
51     negCycle.assign(n+1, false);
52     pa.assign(n+1, -1);
53
54     for (auto& s : src) dis[s] = 0;
55
56     for (int rlx = 1; rlx <= n; rlx++) {
57         for (int u = 1; u <= n; u++) {
58             if (dis[u] == LINF) continue; // Important
59             !!
60             for (auto& e : g[u]) {
61                 int v = e.first; ll w = e.second;
62                 if (dis[v] > dis[u] + w) {
63                     dis[v] = dis[u] + w;
64                     pa[v] = u;
65                     if (rlx == n) negCycle[v] = true;
66                 }
67             }
68         }
69     }
70
71 // Negative Cycle Detection
72 void NegCycleDetect() {
73     /* No Neg Cycle: NO
74     Exist Any Neg Cycle:
75     YES
76     v0 v1 v2 ... vk v0 */
77
78     vector<int> src;
79     for (int i = 1; i <= n; i++)
80         src.emplace_back(i);
81
82     SPFA(src);
83     // BellmanFord(src);
84
85     int ptr = -1;
86     for (int i = 1; i <= n; i++) if (negCycle[i])

```

```

83     { ptr = i; break; }
84
85     if (ptr == -1) { return cout << "NO" << endl, void
86         (); }
87
88     cout << "YES\n";
89     vector<int> ans;
90     vector<bool> vis(n+1, false);
91
92     while (true) {
93         ans.emplace_back(ptr);
94         if (vis[ptr]) break;
95         vis[ptr] = true;
96         ptr = pa[ptr];
97     }
98     reverse(ans.begin(), ans.end());
99
100    vis.assign(n+1, false);
101    for (auto& x : ans) {
102        cout << x << ' ';
103        if (vis[x]) break;
104        vis[x] = true;
105    }
106    cout << endl;
107
108 // Distance Calculation
109 void calcDis(int s) {
110     vector<int> src;
111     src.emplace_back(s);
112     SPFA(src);
113     // BellmanFord(src);
114
115     while (!q.empty()) q.pop();
116     for (int i = 1; i <= n; i++)
117         if (negCycle[i]) q.push(i);
118
119     while (!q.empty()) {
120         int u = q.front(); q.pop();
121         for (auto& e : g[u]) {
122             int v = e.first;
123             if (!negCycle[v]) {
124                 q.push(v);
125                 negCycle[v] = true;
126             }
127         }
128     }

```

6.2 BCC - AP

```

1 int n, m;
2 int low[maxn], dfn[maxn], instp;
3 vector<int> E, g[maxn];
4 bitset<maxn> isap;
5 bitset<maxm> vis;
6 stack<int> stk;
7 int bccnt;
8 vector<int> bcc[maxn];
9 inline void popout(int u) {
10     bccnt++;
11     bcc[bccnt].emplace_back(u);
12     while (!stk.empty()) {
13         int v = stk.top();
14         if (u == v) break;
15         stk.pop();
16         bcc[bccnt].emplace_back(v);
17     }
18 }
19 void dfs(int u, bool rt = 0) {
20     stk.push(u);
21     low[u] = dfn[u] = ++instp;
22     int kid = 0;
23     Each(e, g[u]) {
24         if (vis[e]) continue;
25         vis[e] = true;
26         int v = E[e]^u;
27         if (!dfn[v]) {
28             // tree edge
29             kid++; dfs(v);
30             low[u] = min(low[u], low[v]);
31             if (!rt && low[v] >= dfn[u]) {
32                 // bcc found: u is ap
33                 isap[u] = true;

```

```

34     popout(u);
35 }
36 } else {
37     // back edge
38     low[u] = min(low[u], dfn[v]);
39 }
40 }
41 // special case: root
42 if (rt) {
43     if (kid > 1) isap[u] = true;
44     popout(u);
45 }
46 }
47 void init() {
48     cin >> n >> m;
49     fill(low, low+maxn, INF);
50     REP(i, m) {
51         int u, v;
52         cin >> u >> v;
53         g[u].emplace_back(i);
54         g[v].emplace_back(i);
55         E.emplace_back(u^v);
56     }
57 }
58 void solve() {
59     FOR(i, 1, n+1, 1) {
60         if (!dfn[i]) dfs(i, true);
61     }
62     vector<int> ans;
63     int cnt = 0;
64     FOR(i, 1, n+1, 1) {
65         if (isap[i]) cnt++, ans.emplace_back(i);
66     }
67     cout << cnt << endl;
68     Each(i, ans) cout << i << ' ';
69     cout << endl;
70 }

```

6.3 BCC - Bridge

```

1 int n, m;
2 vector<int> g[maxn], E;
3 int low[maxn], dfn[maxn], instp;
4 int bccnt, bccid[maxn];
5 stack<int> stk;
6 bitset<maxn> vis, isbrg;
7 void init() {
8     cin >> n >> m;
9     REP(i, m) {
10         int u, v;
11         cin >> u >> v;
12         E.emplace_back(u^v);
13         g[u].emplace_back(i);
14         g[v].emplace_back(i);
15     }
16     fill(low, low+maxn, INF);
17 }
18 void popout(int u) {
19     bccnt++;
20     while (!stk.empty()) {
21         int v = stk.top();
22         if (v == u) break;
23         stk.pop();
24         bccid[v] = bccnt;
25     }
26 }
27 void dfs(int u) {
28     stk.push(u);
29     low[u] = dfn[u] = ++instp;
30
31     Each(e, g[u]) {
32         if (vis[e]) continue;
33         vis[e] = true;
34
35         int v = E[e]^u;
36         if (dfn[v]) {
37             // back edge
38             low[u] = min(low[u], dfn[v]);
39         } else {
40             // tree edge
41             dfs(v);

```

```

42         low[u] = min(low[u], low[v]);
43         if (low[v] == dfn[v]) {
44             isbrg[e] = true;
45             popout(u);
46         }
47     }
48 }
49 }
50 void solve() {
51     FOR(i, 1, n+1, 1) {
52         if (!dfn[i]) dfs(i);
53     }
54     vector<pii> ans;
55     vis.reset();
56     FOR(u, 1, n+1, 1) {
57         Each(e, g[u]) {
58             if (!isbrg[e] || vis[e]) continue;
59             vis[e] = true;
60             int v = E[e]^u;
61             ans.emplace_back(mp(u, v));
62         }
63     }
64     cout << (int)ans.size() << endl;
65     Each(e, ans) cout << e.F << ' ' << e.S << endl;
66 }

```

6.4 SCC - Tarjan

```

1 // 2-SAT
2 vector<int> E, g[maxn]; // 1~n, n+1~2n
3 int low[maxn], in[maxn], instp;
4 int sccnt, sccid[maxn];
5
6 stack<int> stk;
7 bitset<maxn> ins, vis;
8
9 int n, m;
10
11 void init() {
12     cin >> m >> n;
13     E.clear();
14     fill(g, g+maxn, vector<int>());
15     fill(low, low+maxn, INF);
16     memset(in, 0, sizeof(in));
17     instp = 1;
18     sccnt = 0;
19     memset(sccid, 0, sizeof(sccid));
20     ins.reset();
21     vis.reset();
22 }
23
24 inline int no(int u) {
25     return (u > n ? u-n : u+n);
26 }
27
28 int ecnt = 0;
29 inline void clause(int u, int v) {
30     E.eb(no(u)^v);
31     g[no(u)].eb(ecnt++);
32     E.eb(no(v)^u);
33     g[no(v)].eb(ecnt++);
34 }
35
36 void dfs(int u) {
37     in[u] = instp++;
38     low[u] = in[u];
39     stk.push(u);
40     ins[u] = true;
41
42     Each(e, g[u]) {
43         if (vis[e]) continue;
44         vis[e] = true;
45
46         int v = E[e]^u;
47         if (ins[v]) low[u] = min(low[u], in[v]);
48         else if (!in[v]) {
49             dfs(v);
50             low[u] = min(low[u], low[v]);
51         }
52     }
53 }

```

```

54     if (low[u] == in[u]) {
55         sccnt++;
56         while (!stk.empty()) {
57             int v = stk.top();
58             stk.pop();
59             ins[v] = false;
60             sccid[v] = sccnt;
61             if (u == v) break;
62         }
63     }
64 }
65
66 int main() {
67     WiWiHorz
68     init();
69
70     REP(i, m) {
71         char su, sv;
72         int u, v;
73         cin >> su >> u >> sv >> v;
74         if (su == '-') u = no(u);
75         if (sv == '-') v = no(v);
76         clause(u, v);
77     }
78
79     FOR(i, 1, 2*n+1, 1) {
80         if (!in[i]) dfs(i);
81     }
82
83     FOR(u, 1, n+1, 1) {
84         int du = no(u);
85         if (sccid[u] == sccid[du]) {
86             return cout << "IMPOSSIBLE\n", 0;
87         }
88     }
89
90     FOR(u, 1, n+1, 1) {
91         int du = no(u);
92         cout << (sccid[u] < sccid[du] ? '+' : '-') << ' ';
93     }
94     cout << endl;
95
96     return 0;
97 }

```

6.5 Eulerian Path - Undir

```

1 // from 1 to n
2 #define gg return cout << "IMPOSSIBLE\n", void();
3
4 int n, m;
5 vector<int> g[maxn];
6 bitset<maxn> inodd;
7
8 void init() {
9     cin >> n >> m;
10    inodd.reset();
11    for (int i = 0; i < m; i++) {
12        int u, v; cin >> u >> v;
13        inodd[u] = inodd[u] ^ true;
14        inodd[v] = inodd[v] ^ true;
15        g[u].emplace_back(v);
16        g[v].emplace_back(u);
17    }
18    stack<int> stk;
19    void dfs(int u) {
20        while (!g[u].empty()) {
21            int v = g[u].back();
22            g[u].pop_back();
23            dfs(v);
24        }
25        stk.push(u);

```

6.6 Eulerian Path - Dir

```

1 // from node 1 to node n
2 #define gg return cout << "IMPOSSIBLE\n", 0
3

```

```

4 int n, m;
5 vector<int> g[maxn];
6 stack<int> stk;
7 int in[maxn], out[maxn];
8
9 void init() {
10    cin >> n >> m;
11    for (int i = 0; i < m; i++) {
12        int u, v; cin >> u >> v;
13        g[u].emplace_back(v);
14        out[u]++, in[v]++;
15    }
16    for (int i = 1; i <= n; i++) {
17        if (i == 1 && out[i]-in[i] != 1) gg;
18        if (i == n && in[i]-out[i] != 1) gg;
19        if (i != 1 && i != n && in[i] != out[i]) gg;
20    }
21    void dfs(int u) {
22        while (!g[u].empty()) {
23            int v = g[u].back();
24            g[u].pop_back();
25            dfs(v);
26        }
27        stk.push(u);
28    }
29    void solve() {
30        dfs(1)
31        for (int i = 1; i <= n; i++)
32            if ((int)g[i].size()) gg;
33        while (!stk.empty()) {
34            int u = stk.top();
35            stk.pop();
36            cout << u << ' ';
37        }

```

6.7 Hamilton Path

```

1 // top down DP
2 // Be Aware Of Multiple Edges
3 int n, m;
4 ll dp[maxn][1<<maxn];
5 int adj[maxn][maxn];
6
7 void init() {
8     cin >> n >> m;
9     fill(dp[0], dp[maxn-1]+(1<<maxn), -1);
10 }
11
12 void DP(int i, int msk) {
13     if (dp[i][msk] != -1) return;
14     dp[i][msk] = 0;
15     REP(j, n) if (j != i && (msk & (1<<j)) && adj[j][i]) {
16         int sub = msk ^ (1<<i);
17         if (dp[j][sub] == -1) DP(j, sub);
18         dp[i][msk] += dp[j][sub] * adj[j][i];
19         if (dp[i][msk] >= MOD) dp[i][msk] %= MOD;
20     }
21 }
22
23 int main() {
24     WiWiHorz
25     init();
26
27     REP(i, m) {
28         int u, v;
29         cin >> u >> v;
30         if (u == v) continue;
31         adj[--u][--v]++;
32     }
33
34     dp[0][1] = 1;
35     FOR(i, 1, n, 1) {
36         dp[i][1] = 0;
37         dp[i][1|(1<<i)] = adj[0][i];
38     }
39     FOR(msk, 1, (1<<n), 1) {
40         if (msk == 1) continue;
41         dp[0][msk] = 0;
42     }
43 }

```



```

44
45
46     DP(n-1, (1<<n)-1);
47     cout << dp[n-1][(1<<n)-1] << endl;
48
49     return 0;
50 }

```

6.8 Kth Shortest Path

```

1 // time: O(|E| \lg |E| + |V| \lg |V| + K)
2 // memory: O(|E| \lg |E| + |V|)
3 struct KSP{ // 1-base
4     struct nd{
5         int u,v; ll d;
6         nd(int ui=0,int vi=0,ll di=INF){ u=ui; v=vi; d=di;
7         }
8     };
9     struct heap{ nd* edge; int dep; heap* chd[4]; };
10    static int cmp(heap* a,heap* b)
11    { return a->edge->d > b->edge->d; }
12    struct node{
13        int v; ll d; heap* H; nd* E;
14        node(){
15            node(ll _d,int _v,nd* _E){ d=_d; v=_v; E=_E; }
16            node(heap* _H,ll _d){ H=_H; d=_d; }
17            friend bool operator<(node a,node b)
18            { return a.d>b.d; }
19        };
20    int n,k,s,t,dst[N]; nd *nxt[N];
21    vector<nd*> g[N],rg[N]; heap *nullNd,*head[N];
22    void init(int _n,int _k,int _s,int _t){
23        n=_n; k=_k; s=_s; t=_t;
24        for(int i=1;i<=n;i++){
25            g[i].clear(); rg[i].clear();
26            nxt[i]=NULL; head[i]=NULL; dst[i]=-1;
27        }
28    }
29    void addEdge(int ui,int vi,ll di){
30        nd* e=new nd(ui,vi,di);
31        g[ui].push_back(e); rg[vi].push_back(e);
32    }
33    queue<int> dfsQ;
34    void dijkstra(){
35        while(dfsQ.size()) dfsQ.pop();
36        priority_queue<node> Q; Q.push(node(0,t,NULL));
37        while (!Q.empty()){
38            node p=Q.top(); Q.pop(); if(dst[p.v]!=-1) continue;
39            dst[p.v]=p.d; nxt[p.v]=p.E; dfsQ.push(p.v);
40            for(auto e:rg[p.v]) Q.push(node(p.d+e->d,e->u,e));
41        }
42    }
43    heap* merge(heap* curNd,heap* newNd){
44        if(curNd==nullNd) return newNd;
45        heap* root=new heap; memcpy(root,curNd,sizeof(heap));
46        if(newNd->edge->d<curNd->edge->d){
47            root->edge=newNd->edge;
48            root->chd[2]=newNd->chd[2];
49            root->chd[3]=newNd->chd[3];
50            newNd->edge=curNd->edge;
51            newNd->chd[2]=curNd->chd[2];
52            newNd->chd[3]=curNd->chd[3];
53        }
54        if(root->chd[0]->dep<root->chd[1]->dep)
55            root->chd[0]=merge(root->chd[0],newNd);
56        else root->chd[1]=merge(root->chd[1],newNd);
57        root->dep=max(root->chd[0]->dep,
58            root->chd[1]->dep)+1;
59        return root;
60    }
61    vector<heap*> V;
62    void build(){
63        nullNd=new heap; nullNd->dep=0; nullNd->edge=new nd
64        ;
65        fill(nullNd->chd,nullNd->chd+4,nullNd);
66        while(not dfsQ.empty()){
67            int u=dfsQ.front(); dfsQ.pop();
68            if(!nxt[u]) head[u]=nullNd;

```

```

67 else head[u]=head[nxt[u]->v];
68 V.clear();
69 for(auto&& e:g[u]){
70     int v=e->v;
71     if(dst[v]==-1) continue;
72     e->d+=dst[v]-dst[u];
73     if(nxt[u]!=e){
74         heap* p=new heap; fill(p->chd,p->chd+4,nullNd)
75         ;
76         p->dep=1; p->edge=e; V.push_back(p);
77     }
78 }
79 if(V.empty()) continue;
80 make_heap(V.begin(),V.end(),cmp);
81 #define L(X) ((X<<1)+1)
82 #define R(X) ((X<<1)+2)
83 for(size_t i=0;i<V.size();i++){
84     if(L(i)<V.size()) V[i]->chd[2]=V[L(i)];
85     else V[i]->chd[2]=nullNd;
86     if(R(i)<V.size()) V[i]->chd[3]=V[R(i)];
87     else V[i]->chd[3]=nullNd;
88 }
89 head[u]=merge(head[u],V.front());
90 }
91 }
92 vector<ll> ans;
93 void first_K(){
94     ans.clear(); priority_queue<node> Q;
95     if(dst[s]==-1) return;
96     ans.push_back(dst[s]);
97     if(head[s]!=nullNd)
98         Q.push(node(head[s],dst[s]+head[s]->edge->d));
99     for(int _=1;_<k and not Q.empty();_++){
100         node p=Q.top(); Q.pop(); ans.push_back(p.d);
101         if(head[p.H->edge->v]!=nullNd){
102             q.H=head[p.H->edge->v]; q.d=p.d+q.H->edge->d;
103             Q.push(q);
104         }
105         for(int i=0;i<4;i++){
106             if(p.H->chd[i]!=nullNd){
107                 q.H=p.H->chd[i];
108                 q.d=p.d+p.H->edge->d+p.H->chd[i]->edge->d;
109                 Q.push(q);
110             }
111         }
112     }
113     void solve(){ // ans[i] stores the i-th shortest path
114         dijkstra(); build();
115         first_K(); // ans.size() might less than k
116     }
117 } solver;

```

6.9 System of Difference Constraints

```

1 vector<vector<pair<int, ll>>> G;
2 void add(int u, int v, ll w) {
3     G[u].emplace_back(make_pair(v, w));
4 }

```

- $x_u - x_v \leq c \Rightarrow \text{add}(v, u, c)$
- $x_u - x_v \geq c \Rightarrow \text{add}(u, v, -c)$
- $x_u - x_v = c \Rightarrow \text{add}(v, u, c), \text{add}(u, v, -c)$
- $x_u \geq c \Rightarrow \text{add super vertex } x_0 = 0, \text{ then } x_u - x_0 \geq c \Rightarrow \text{add}(u, 0, -c)$
- Don't forget non-negative constraints for every variable if specified implicitly.
- Interval sum \Rightarrow Use prefix sum to transform into differential constraints. Don't forget $S_{i+1} - S_i \geq 0$ if x_i needs to be non-negative.
- $\frac{x_u}{x_v} \leq c \Rightarrow \log x_u - \log x_v \leq \log c$

7 String

7.1 Rolling Hash

```

1 const ll C = 27;
2 inline int id(char c) {return c-'a'+1;}
3 struct RollingHash {
4     string s; int n; ll mod;
5     vector<ll> Cexp, hs;
6     RollingHash(string& _s, ll _mod):
7         s(_s), n((int)_s.size()), mod(_mod)
8     {
9         Cexp.assign(n, 0);
10        hs.assign(n, 0);
11        Cexp[0] = 1;
12        for (int i = 1; i < n; i++) {
13            Cexp[i] = Cexp[i-1] * C;
14            if (Cexp[i] >= mod) Cexp[i] %= mod;
15        }
16        hs[0] = id(s[0]);
17        for (int i = 1; i < n; i++) {
18            hs[i] = hs[i-1] * C + id(s[i]);
19            if (hs[i] >= mod) hs[i] %= mod;
20        }
21        inline ll query(int l, int r) {
22            ll res = hs[r] - (l ? hs[l-1] * Cexp[r-l+1] : 0);
23            res = (res % mod + mod) % mod;
24            return res;
25        };
26    };

```

7.2 Trie

```

1 struct node {
2     int c[26]; ll cnt;
3     node(): cnt(0) {memset(c, 0, sizeof(c));}
4     node(ll x): cnt(x) {memset(c, 0, sizeof(c));}
5 };
6 struct Trie {
7     vector<node> t;
8     void init() {
9         t.clear();
10        t.emplace_back(node());
11    }
12    void insert(string s) { int ptr = 0;
13        for (auto& i : s) {
14            if (!t[ptr].c[i-'a']) {
15                t.emplace_back(node());
16                t[ptr].c[i-'a'] = (int)t.size()-1;
17                ptr = t[ptr].c[i-'a'];
18            }
19        }
20    };
21    };

```

7.3 KMP

```

1 int n, m;
2 string s, p;
3 vector<int> f;
4 void build() {
5     f.clear(); f.resize(m, 0);
6     int ptr = 0; for (int i = 1; i < m; i++) {
7         while (ptr && p[i] != p[ptr]) ptr = f[ptr-1];
8         if (p[i] == p[ptr]) ptr++;
9         f[i] = ptr;
10    }
11    void init() {
12        cin >> s >> p;
13        n = (int)s.size();
14        m = (int)p.size();
15        build();
16    }
17    void solve() {
18        int ans = 0, pi = 0;
19        for (int si = 0; si < n; si++) {
20            while (pi && s[si] != p[pi]) pi = f[pi-1];
21            if (s[si] == p[pi]) pi++;
22            if (pi == m) ans++, pi = f[pi-1];
23        }
24        cout << ans << endl;
25    }

```

7.4 Z Value

```

1 string is, it, s;
2 int n; vector<int> z;
3 void init() {
4     cin >> is >> it;
5     s = it+'0'+is;
6     n = (int)s.size();
7     z.resize(n, 0);
8 }
9 void solve() {
10    int ans = 0; z[0] = n;
11    for (int i = 1, l = 0, r = 0; i < n; i++) {
12        if (i <= r) z[i] = min(z[i-1], r-i+1);
13        while (i+z[i] < n && s[z[i]] == s[i+z[i]]) z[i]++;
14        if (i+z[i]-1 > r) l = i, r = i+z[i]-1;
15        if (z[i] == (int)it.size()) ans++;
16    }
17    cout << ans << endl;
18 }

```

7.5 Manacher

```

1 int n; string S, s;
2 vector<int> m;
3 void manacher() {
4     s.clear(); s.resize(2*n+1, '.');
5     for (int i = 0, j = 1; i < n; i++, j += 2) s[j] = S[i];
6     m.clear(); m.resize(2*n+1, 0);
7     // m[i] := max k such that s[i-k, i+k] is palindrome
8     int mx = 0, mxk = 0;
9     for (int i = 1; i < 2*n+1; i++) {
10        if (mx-(i-mx) >= 0) m[i] = min(m[mx-(i-mx)], mx+mxk-i);
11        while (0 <= i-m[i]-1 && i+m[i]+1 < 2*n+1 &&
12            s[i-m[i]-1] == s[i+m[i]+1]) m[i]++;
13        if (i+m[i] > mx+mxk) mx = i, mxk = m[i];
14    }
15    void init() { cin >> S; n = (int)S.size(); }
16    void solve() {
17        manacher();
18        int mx = 0, ptr = 0;
19        for (int i = 0; i < 2*n+1; i++) if (mx < m[i]) {
20            mx = m[i]; ptr = i;
21        }
22        for (int i = ptr-mx; i <= ptr+mx; i++)
23            if (s[i] != '.') cout << s[i];
24        cout << endl;
25    }

```

7.6 Suffix Array

```

1 #define F first
2 #define S second
3 struct SuffixArray { // don't forget s += "$";
4     int n; string s;
5     vector<int> suf, lcp, rk;
6     vector<int> cnt, pos;
7     vector<pair<pii, int>> buc[2];
8     void init(string _s) {
9         s = _s; n = (int)s.size();
10        // resize(n): suf, rk, cnt, pos, lcp, buc[0~1]
11    }
12    void radix_sort() {
13        for (int t : {0, 1}) {
14            fill(cnt.begin(), cnt.end(), 0);
15            for (auto& i : buc[t]) cnt[(t ? i.F : i.S)]++;
16            for (int i = 0; i < n; i++)
17                pos[i] = (i ? 0 : pos[i-1] + cnt[i-1]);
18            for (auto& i : buc[t])
19                buc[t^1][pos[(t ? i.F : i.S)]] = i;
20        }
21    }
22    bool fill_suf() {
23        bool end = true;
24        for (int i = 0; i < n; i++) suf[i] = buc[0][i].S;
25        rk[suf[0]] = 0;
26        for (int i = 1; i < n; i++) {
27            int dif = (buc[0][i].F != buc[0][i-1].F);
28            end &= dif;
29        }
30    }

```

```

28     rk[suf[i]] = rk[suf[i-1]] + dif;
29     } return end;
30 }
31 void sa() {
32     for (int i = 0; i < n; i++)
33         buc[0][i] = make_pair(make_pair(s[i], s[i]), i);
34     sort(buc[0].begin(), buc[0].end());
35     if (fill_suf()) return;
36     for (int k = 0; (1<<k) < n; k++) {
37         for (int i = 0; i < n; i++)
38             buc[0][i] = make_pair(make_pair(rk[i],
39                 rk[(i + (1<<k)) % n]), i);
40         radix_sort();
41         if (fill_suf()) return;
42     }
43     void LCP() { int k = 0;
44         for (int i = 0; i < n-1; i++) {
45             if (rk[i] == 0) continue;
46             int pi = rk[i];
47             int j = suf[pi-1];
48             while (i+k < n && j+k < n && s[i+k] == s[j+k]) k++;
49             lcp[pi] = k;
50             k = max(k-1, 0);
51         }
52     };
53     SuffixArray suffixarray;

```

7.7 SA-IS

```

1  const int N=300010;
2  struct SA{
3      #define REP(i,n) for(int i=0;i<int(n);i++)
4      #define REP1(i,a,b) for(int i=(a);i<=int(b);i++)
5      bool _t[N*2]; int _s[N*2],_sa[N*2];
6      int _c[N*2],x[N],_p[N],_q[N*2],hei[N],r[N];
7      int operator [](int i){ return _sa[i]; }
8      void build(int *s,int n,int m){
9          memcpy(_s,s,sizeof(int)*n);
10         sais(_s,_sa,_p,_q,_t,_c,n,m); mkhei(n);
11     }
12     void mkhei(int n){
13         REP(i,n) r[_sa[i]]=i;
14         hei[0]=0;
15         REP(i,n) if(r[i]) {
16             int ans=i>0?max(hei[r[i-1]]-1,0):0;
17             while(_s[i+ans]==_s[_sa[r[i]-1]+ans]) ans++;
18             hei[r[i]]=ans;
19         }
20     }
21     void sais(int *s,int *sa,int *p,int *q,bool *t,int *c,
22         int n,int z){
23         bool uniq=t[n-1]=true,neq;
24         int nn=0,nmxz=-1,*nsa=s+n,*ns=s+n,lst=-1;
25         #define MS0(x,n) memset((x),0,n*sizeof(*(x)))
26         #define MAGIC(XD) MS0(sa,n);\
27         memcpy(x,c,sizeof(int)*z); XD;\
28         memcpy(x+1,c,sizeof(int)*(z-1));\
29         REP(i,n) if(sa[i]&&!t[sa[i]-1]) sa[x[s[sa[i]-1]]++]=sa[i]-1;\
30         memcpy(x,c,sizeof(int)*z);\
31         for(int i=n-1;i>=0;i--) if(sa[i]&&t[sa[i]-1]) sa[--x[s[sa[i]-1]]]=sa[i]-1;
32         MS0(c,z); REP(i,n) uniq&=++c[s[i]]<2;
33         REP(i,z-1) c[i+1]+=c[i];
34         if(uniq) { REP(i,n) sa[--c[s[i]]]=i; return; }
35         for(int i=n-2;i>=0;i--)
36             t[i]=(s[i]==s[i+1]?t[i+1]:s[i]<s[i+1]);
37         MAGIC(REP1(i,1,n-1) if(t[i]&&!t[i-1]) sa[--x[s[i]]]=p[q[i]=nn++]=i);
38         REP(i,n) if(sa[i]&&t[sa[i]]&&t[sa[i]-1]){
39             neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa[i])*sizeof(int));
40             ns[q[lst=sa[i]]]=nmxz+=neq;
41         }
42         sais(ns,nsa,p+nn,q+n,t+n,c+z,nn,nmxz+1);
43         MAGIC(for(int i=nn-1;i>=0;i--) sa[--x[s[p[nsa[i]]]]]=p[nsa[i]]);
44     }

```

```

45 int H[N],SA[N],RA[N];
46 void suffix_array(int* ip,int len){
47     // should padding a zero in the back
48     // ip is int array, len is array length
49     // ip[0..n-1] != 0, and ip[len]=0
50     ip[len++]=0; sa.build(ip,len,128);
51     memcpy(H,sa.hei+1,len<<2); memcpy(SA,sa._sa+1,len<<2);
52     ;
53     for(int i=0;i<len;i++) RA[i]=sa.r[i]-1;
54     // resulting height, sa array \in [0,len)

```

7.8 Minimum Rotation

```

1 //rotate(begin(s), begin(s)+minRotation(s), end(s))
2 int minRotation(string s) {
3     int a = 0, n = s.size(); s += s;
4     for(int b = 0; b < n; b++) for(int k = 0; k < n; k++) {
5         if(a + k == b || s[a + k] < s[b + k]) {
6             b += max(0, k - 1);
7             break; }
8         if(s[a + k] > s[b + k]) {
9             a = b;
10            break;
11        } }
12     return a; }

```

7.9 Aho Corasick

```

1 struct ACautomata{
2     struct Node{
3         int cnt;
4         Node *go[26], *fail, *dic;
5         Node(){
6             cnt = 0; fail = 0; dic=0;
7             memset(go,0,sizeof(go));
8         }
9     }pool[1048576],*root;
10    int nMem;
11    Node* new_Node(){
12        pool[nMem] = Node();
13        return &pool[nMem++];
14    }
15    void init() { nMem = 0; root = new_Node(); }
16    void add(const string &str) { insert(root,str,0); }
17    void insert(Node *cur, const string &str, int pos){
18        for(int i=pos;i<str.size();i++){
19            if(!cur->go[str[i]-'a'])
20                cur->go[str[i]-'a'] = new_Node();
21            cur=cur->go[str[i]-'a'];
22        }
23        cur->cnt++;
24    }
25    void make_fail(){
26        queue<Node*> que;
27        que.push(root);
28        while (!que.empty()){
29            Node* fr=que.front(); que.pop();
30            for (int i=0; i<26; i++){
31                if (fr->go[i]){
32                    Node *ptr = fr->fail;
33                    while (ptr && !ptr->go[i]) ptr = ptr->fail;
34                    fr->go[i]->fail=ptr=(ptr?ptr->go[i]:root);
35                    fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
36                    que.push(fr->go[i]);
37                } } }
38    }AC;

```

8 Geometry

8.1 Basic Operations

```

1 typedef long long T;
2 // typedef long double T;
3 const long double eps = 1e-8;
4
5 short sgn(T x) {
6     if (abs(x) < eps) return 0;

```

```

7   return x < 0 ? -1 : 1;
8 }
9
10 struct Pt {
11     T x, y;
12     Pt(T _x=0, T _y=0):x(_x), y(_y) {}
13     Pt operator+(Pt a) { return Pt(x+a.x, y+a.y); }
14     Pt operator-(Pt a) { return Pt(x-a.x, y-a.y); }
15     Pt operator*(T a) { return Pt(x*a, y*a); }
16     Pt operator/(T a) { return Pt(x/a, y/a); }
17     T operator*(Pt a) { return x*a.x + y*a.y; }
18     T operator^(Pt a) { return x*a.y - y*a.x; }
19     bool operator<(Pt a)
20     { return x < a.x || (x == a.x && y < a.y); }
21     //return sgn(x-a.x) < 0 || (sgn(x-a.x) == 0 && sgn(y-a.
22     y) < 0); }
23     bool operator==(Pt a)
24     { return sgn(x-a.x) == 0 && sgn(y-a.y) == 0; }
25 };
26
27 Pt mv(Pt a, Pt b) { return b-a; }
28 T len2(Pt a) { return a*a; }
29 T dis2(Pt a, Pt b) { return len2(b-a); }
30
31 short ori(Pt a, Pt b) { return ((a^b)>0) - ((a^b)<0); }
32 bool onseg(Pt p, Pt l1, Pt l2) {
33     Pt a = mv(p, l1), b = mv(p, l2);
34     return ((a^b) == 0) && ((a^b) <= 0);
35 }

```

8.2 InPoly

```

1 short inPoly(Pt p) {
2     // 0=Bound 1=In -1=Out
3     REP(i, n) if (onseg(p, E[i], E[(i+1)%n])) return 0;
4     int cnt = 0;
5     REP(i, n) if (banana(p, Pt(p.x+1, p.y+2e9),
6         E[i], E[(i+1)%n])) cnt ^= 1;
7     return (cnt ? 1 : -1);
8 }

```

8.3 Sort by Angle

```

1 int ud(Pt a) { // up or down half plane
2     if (a.y > 0) return 0;
3     if (a.y < 0) return 1;
4     return (a.x >= 0 ? 0 : 1);
5 }
6 sort(ALL(E), [&](const Pt& a, const Pt& b){
7     if (ud(a) != ud(b)) return ud(a) < ud(b);
8     return (a^b) > 0;
9 });

```

8.4 Line Intersect Check

```

1 inline bool banana(Pt p1, Pt p2, Pt q1, Pt q2) {
2     if (onseg(p1, q1, q2) || onseg(p2, q1, q2) ||
3         onseg(q1, p1, p2) || onseg(q2, p1, p2)) {
4         return true;
5     }
6     Pt p = mv(p1, p2), q = mv(q1, q2);
7     return (ori(p, mv(p1, q1)) * ori(p, mv(p1, q2)) < 0 &&
8         ori(q, mv(q1, p1)) * ori(q, mv(q1, p2)) < 0);
9 }

```

8.5 Line Intersection

```

1 // T: long double
2 Pt bananaPoint(Pt p1, Pt p2, Pt q1, Pt q2) {
3     if (onseg(q1, p1, p2)) return q1;
4     if (onseg(q2, p1, p2)) return q2;
5     if (onseg(p1, q1, q2)) return p1;
6     if (onseg(p2, q1, q2)) return p2;
7     double s = abs(mv(p1, p2) ^ mv(p1, q1));
8     double t = abs(mv(p1, p2) ^ mv(p1, q2));
9     return q2 * (s/(s+t)) + q1 * (t/(s+t));
10 }

```

8.6 Convex Hull

```

1 vector<Pt> hull;
2 void convexHull() {
3     hull.clear(); sort(ALL(E));
4     REP(t, 2) {
5         int b = SZ(hull);
6         Each(ei, E) {
7             while (SZ(hull) - b >= 2 &&
8                 ori(mv(hull[SZ(hull)-2], hull.back()),
9                     mv(hull[SZ(hull)-2], ei)) == -1) {
10                 hull.pop_back();
11             }
12             hull.pb(ei);
13         }
14         hull.pop_back();
15         reverse(ALL(E));
16     }
17 }

```

8.7 Lower Concave Hull

```

1 struct Line {
2     mutable ll m, b, p;
3     bool operator<(const Line& o) const { return m < o.m; }
4     bool operator<(ll x) const { return p < x; }
5 };
6
7 struct LineContainer : multiset<Line, less<>> {
8     // (for doubles, use inf = 1/.0, div(a,b) = a/b)
9     const ll inf = LLONG_MAX;
10    ll div(ll a, ll b) { // floored division
11        return a / b - ((a ^ b) < 0 && a % b); }
12    bool isect(iterator x, iterator y) {
13        if (y == end()) { x->p = inf; return false; }
14        if (x->m == y->m) x->p = x->b > y->b ? inf : -inf;
15        else x->p = div(y->b - x->b, x->m - y->m);
16        return x->p >= y->p;
17    }
18    void add(ll m, ll b) {
19        auto z = insert({m, b, 0}); y = z++, x = y;
20        while (isect(y, z)) z = erase(z);
21        if (x != begin() && isect(--x, y)) isect(x, y =
22            erase(y));
23        while ((y = x) != begin() && (--x)->p >= y->p)
24            isect(x, erase(y));
25    }
26    ll query(ll x) {
27        assert(!empty());
28        auto l = *lower_bound(x);
29        return l.m * x + l.b;
30    }
31 };

```

8.8 Polygon Area

```

1 T dbarea(vector<Pt>& e) {
2     ll res = 0;
3     REP(i, SZ(e)) res += e[i]^e[(i+1)%SZ(e)];
4     return abs(res);
5 }

```

8.9 Pick's Theorem

Consider a polygon which vertices are all lattice points.
 Let i = number of points inside the polygon.
 Let b = number of points on the boundary of the polygon.
 Then we have the following formula:

$$Area = i + \frac{b}{2} - 1$$

8.10 Minimum Enclosing Circle

```

1 Pt circumcenter(Pt A, Pt B, Pt C) {
2     // a1(x-A.x) + b1(y-A.y) = c1
3     // a2(x-A.x) + b2(y-A.y) = c2

```

```

4 // solve using Cramer's rule
5 T a1 = B.x-A.x, b1 = B.y-A.y, c1 = dis2(A, B)/2.0;
6 T a2 = C.x-A.x, b2 = C.y-A.y, c2 = dis2(A, C)/2.0;
7 T D = Pt(a1, b1) ^ Pt(a2, b2);
8 T Dx = Pt(c1, b1) ^ Pt(c2, b2);
9 T Dy = Pt(a1, c1) ^ Pt(a2, c2);
10 if (D == 0) return Pt(-INF, -INF);
11 return A + Pt(Dx/D, Dy/D);
12 }
13 Pt center; T r2;
14 void minEncloseCircle() {
15 mt19937 gen(chrono::steady_clock::now().
16   time_since_epoch().count());
17 shuffle(ALL(E), gen);
18 center = E[0], r2 = 0;
19 for (int i = 0; i < n; i++) {
20   if (dis2(center, E[i]) <= r2) continue;
21   center = E[i], r2 = 0;
22   for (int j = 0; j < i; j++) {
23     if (dis2(center, E[j]) <= r2) continue;
24     center = (E[i] + E[j]) / 2.0;
25     r2 = dis2(center, E[i]);
26     for (int k = 0; k < j; k++) {
27       if (dis2(center, E[k]) <= r2) continue;
28       center = circumcenter(E[i], E[j], E[k]);
29       r2 = dis2(center, E[i]);
30     }
31   }
32 } }

```

8.11 PolyUnion

```

1 struct PY{
2   int n; Pt pt[5]; double area;
3   Pt& operator[](const int x){ return pt[x]; }
4   void init(){ //n,pt[0~n-1] must be filled
5     area=pt[n-1]^pt[0];
6     for(int i=0;i<n-1;i++) area+=pt[i]^pt[i+1];
7     if((area/=2)<0)reverse(pt,pt+n),area=-area;
8   }
9 };
10 PY py[500]; pair<double,int> c[5000];
11 inline double segP(Pt &p,Pt &p1,Pt &p2){
12   if(dcmp(p1.x-p2.x)==0) return (p.y-p1.y)/(p2.y-p1.y);
13   return (p.x-p1.x)/(p2.x-p1.x);
14 }
15 double polyUnion(int n){ //py[0~n-1] must be filled
16   int i,j,ii,jj,ta,tb,r,d; double z,w,s,sum=0,tc,td;
17   for(i=0;i<n;i++) py[i][py[i].n]=py[i][0];
18   for(i=0;i<n;i++){
19     for(ii=0;ii<py[i].n;ii++){
20       r=0;
21       c[r++]=make_pair(0.0,0); c[r++]=make_pair(1.0,0);
22       for(j=0;j<n;j++){
23         if(i==j) continue;
24         for(jj=0;jj<py[j].n;jj++){
25           ta=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj]))
26             ;
27           tb=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj
28             +1]));
29           if(ta==0 && tb==0){
30             if((py[j][jj+1]-py[j][jj])*(py[i][ii+1]-py[
31               i][ii])>0&&j<i){
32               c[r++]=make_pair(segP(py[j][jj],py[i][ii
33                 ],py[i][ii+1]),1);
34               c[r++]=make_pair(segP(py[j][jj+1],py[i][
35                 ii],py[i][ii+1]),-1);
36             }
37             else if(ta>0 && tb<0){
38               tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
39               td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
40               c[r++]=make_pair(tc/(tc+td),1);
41             }
42             else if(ta<0 && tb>0){
43               tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
44               td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
45               c[r++]=make_pair(tc/(tc+td),-1);
46             }
47           }
48         }
49       }
50       sort(c,c+r);
51       z=min(max(c[0].first,0.0),1.0); d=c[0].second; s
52       =0;

```

```

43   for(j=1;j<r;j++){
44     w=min(max(c[j].first,0.0),1.0);
45     if(!d) s+=w-z;
46     d+=c[j].second; z=w;
47   }
48   sum+=(py[i][ii]^py[i][ii+1])*s;
49 }
50 }
51 return sum/2;
52 }

```

8.12 Minkowski Sum

```

1 /* convex hull Minkowski Sum*/
2 #define INF 100000000000000LL
3 int pos(const Pt& tp){
4   if( tp.Y == 0 ) return tp.X > 0 ? 0 : 1;
5   return tp.Y > 0 ? 0 : 1;
6 }
7 #define N 300030
8 Pt pt[ N ], qt[ N ], rt[ N ];
9 LL Lx,Rx;
10 int dn,un;
11 inline bool cmp( Pt a, Pt b ){
12   int pa=pos( a ),pb=pos( b );
13   if(pa==pb) return (a^b)>0;
14   return pa<pb;
15 }
16 int minkowskiSum(int n,int m){
17   int i,j,r,p,q,fi,fj;
18   for(i=1,p=0;i<n;i++){
19     if( pt[i].Y<pt[p].Y ||
20       (pt[i].Y==pt[p].Y && pt[i].X<pt[p].X) ) p=i; }
21   for(i=1,q=0;i<m;i++){
22     if( qt[i].Y<qt[q].Y ||
23       (qt[i].Y==qt[q].Y && qt[i].X<qt[q].X) ) q=i; }
24   rt[0]=pt[p]+qt[q];
25   r=1; i=p; j=q; fi=fj=0;
26   while(1){
27     if((fj&&j==q) ||
28       ( (!fi||i!=p) &&
29         cmp(pt[(p+1)%n]-pt[p],qt[(q+1)%m]-qt[q]) ) ){
30       rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
31       p=(p+1)%n;
32       fi=1;
33     }else{
34       rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
35       q=(q+1)%m;
36       fj=1;
37     }
38     if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)
39       r++;
40     else rt[r-1]=rt[r];
41     if(i==p && j==q) break;
42   }
43   return r-1;
44 }
45 void initInConvex(int n){
46   int i,p,q;
47   LL Ly,Ry;
48   Lx=INF; Rx=-INF;
49   for(i=0;i<n;i++){
50     if(pt[i].X<Lx) Lx=pt[i].X;
51     if(pt[i].X>Rx) Rx=pt[i].X;
52   }
53   Ly=Ry=INF;
54   for(i=0;i<n;i++){
55     if(pt[i].X==Lx && pt[i].Y<Ly){ Ly=pt[i].Y; p=i; }
56     if(pt[i].X==Rx && pt[i].Y>Ry){ Ry=pt[i].Y; q=i; }
57   }
58   for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
59   qt[dn]=pt[q]; Ly=Ry=-INF;
60   for(i=0;i<n;i++){
61     if(pt[i].X==Lx && pt[i].Y>Ly){ Ly=pt[i].Y; p=i; }
62     if(pt[i].X==Rx && pt[i].Y<Ry){ Ry=pt[i].Y; q=i; }
63   }
64   for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
65   rt[un]=pt[q];
66 }
67 inline int inConvex(Pt p){
68   int L,R,M;

```

```

68 if(p.X<Lx || p.X>Rx) return 0;
69 L=0;R=dn;
70 while(L<R-1){ M=(L+R)/2;
71   if(p.X<qt[M].X) R=M; else L=M; }
72   if(tri(qt[L],qt[R],p)<0) return 0;
73   L=0;R=un;
74   while(L<R-1){ M=(L+R)/2;
75     if(p.X<rt[M].X) R=M; else L=M; }
76     if(tri(rt[L],rt[R],p)>0) return 0;
77     return 1;
78 }
79 int main(){
80   int n,m,i;
81   Pt p;
82   scanf("%d",&n);
83   for(i=0;i<n;i++) scanf("%lld%lld",&pt[i].X,&pt[i].Y);
84   scanf("%d",&m);
85   for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);
86   n=minkowskiSum(n,m);
87   for(i=0;i<n;i++) pt[i]=rt[i];
88   scanf("%d",&m);
89   for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);
90   n=minkowskiSum(n,m);
91   for(i=0;i<n;i++) pt[i]=rt[i];
92   initInConvex(n);
93   scanf("%d",&m);
94   for(i=0;i<m;i++){
95     scanf("%lld %lld",&p.X,&p.Y);
96     p.X*=3; p.Y*=3;
97     puts(inConvex(p)? "YES": "NO");
98   }
99 }

```

9 Number Theory

9.1 Pollard's rho

```

1 from itertools import count
2 from math import gcd
3 from sys import stdin
4
5 for s in stdin:
6   number, x = int(s), 2
7   break2 = False
8   for cycle in count(1):
9     y = x
10    if break2:
11      break
12    for i in range(1 << cycle):
13      x = (x * x + 1) % number
14      factor = gcd(x - y, number)
15      if factor > 1:
16        print(factor)
17        break2 = True
18        break

```

9.2 Miller Rabin

```

1 // n < 4,759,123,141      3 : 2, 7, 61
2 // n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
3 // n < 3,474,749,660,383  6 : pimes <= 13
4 // n < 2^64              7 :
5 // 2, 325, 9375, 28178, 450775, 9780504, 1795265022
6 bool witness(ll a,ll n,ll u,int t){
7   if(!(a%n)) return 0;
8   ll x=mpow(a,u,n);
9   for(int i=0;i<t;i++) {
10     ll nx=mul(x,x,n);
11     if(nx==1&&x!=1&&x!=n-1) return 1;
12     x=nx;
13   }
14   return x!=1;
15 }
16 bool miller_rabin(ll n,int s=100) {
17   // iterate s times of witness on n
18   // return 1 if prime, 0 otherwise
19   if(n<2) return 0;
20   if(!(n&1)) return n == 2;
21   ll u=n-1; int t=0;

```

```

22 while(!(u&1)) u>>=1, t++;
23 while(s--){
24   ll a=randll()%(n-1)+1;
25   if(witness(a,n,u,t)) return 0;
26 }
27 return 1;
28 }

```

9.3 Fast Power

Note: $a^n \equiv a^{(n \bmod (p-1))} \pmod{p}$

9.4 Extend GCD

```

1 ll gcd;
2 pll extgcd(ll a, ll b) {
3   if (b == 0) {
4     gcd = a;
5     return pll{1, 0};
6   }
7   pll ans = extgcd(b, a % b);
8   return pll{ans.S, ans.F - a/b * ans.S};
9 }
10 pll bezout(ll a, ll b, ll c) {
11   bool negx = (a < 0), negy = (b < 0);
12   pll ans = extgcd(abs(a), abs(b));
13   if (c % gcd != 0) return pll{-LLINF, -LLINF};
14   return pll{ans.F * c/gcd * (negx ? -1 : 1),
15             ans.S * c/gcd * (negy ? -1 : 1)};
16 }
17 ll inv(ll a, ll p) {
18   if (p == 1) return -1;
19   pll ans = bezout(a % p, -p, 1);
20   if (ans == pll{-LLINF, -LLINF}) return -1;
21   return (ans.F % p + p) % p;
22 }

```

9.5 Mu + Phi

```

1 const int maxn = 1e6 + 5;
2 ll f[maxn];
3 vector<int> lpf, prime;
4 void build() {
5   lpf.clear(); lpf.resize(maxn, 1);
6   prime.clear();
7   f[1] = ...; /* mu[1] = 1, phi[1] = 1 */
8   for (int i = 2; i < maxn; i++) {
9     if (lpf[i] == 1) {
10       lpf[i] = i; prime.emplace_back(i);
11       f[i] = ...; /* mu[i] = 1, phi[i] = i-1 */
12     }
13     for (auto& j : prime) {
14       if (i*j >= maxn) break;
15       lpf[i*j] = j;
16       if (i % j == 0) f[i*j] = ...; /* 0, phi[i]*j */
17       else f[i*j] = ...; /* -mu[i], phi[i]*phi[j] */
18       if (j >= lpf[i]) break;
19     }
20   }
21 }

```

9.6 Other Formulas

- Inversion:
 $aa^{-1} \equiv 1 \pmod{m}$. a^{-1} exists iff $\gcd(a, m) = 1$.
- Linear inversion:
 $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod{m}$
- Fermat's little theorem:
 $a^p \equiv a \pmod{p}$ if p is prime.
- Euler function:
 $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$
- Euler theorem:
 $a^{\phi(n)} \equiv 1 \pmod{n}$ if $\gcd(a, n) = 1$.

- Extended Euclidean algorithm:

$$ax + by = \gcd(a, b) = \gcd(b, a \bmod b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$$

- Divisor function:

$$\sigma_x(n) = \sum_{d|n} d^x. n = \prod_{i=1}^r p_i^{a_i}.$$

$$\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1} \text{ if } x \neq 0. \sigma_0(n) = \prod_{i=1}^r (a_i + 1).$$

- Chinese remainder theorem (Coprime Moduli):

$$x \equiv a_i \pmod{m_i}.$$

$$M = \prod m_i. M_i = M/m_i. t_i = M_i^{-1}.$$

$$x = kM + \sum a_i t_i M_i, k \in \mathbb{Z}.$$

- Chinese remainder theorem:

$$x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2} \Rightarrow x = m_1 p + a_1 = m_2 q + a_2 \Rightarrow m_1 p - m_2 q = a_2 - a_1$$

$$\text{Solve for } (p, q) \text{ using ExtGCD.}$$

$$x \equiv m_1 p + a_1 \equiv m_2 q + a_2 \pmod{\text{lcm}(m_1, m_2)}$$

- Avoiding Overflow: $ca \bmod cb = c(a \bmod b)$

- Dirichlet Convolution: $(f * g)(n) = \sum_{d|n} f(d)g(n/d)$

- Important Multiplicative Functions + Properties:

$$1. \epsilon(n) = [n = 1]$$

$$2. 1(n) = 1$$

$$3. id(n) = n$$

$$4. \mu(n) = 0 \text{ if } n \text{ has squared prime factor}$$

$$5. \mu(n) = (-1)^k \text{ if } n = p_1 p_2 \cdots p_k$$

$$6. \epsilon = \mu * 1$$

$$7. \phi = \mu * id$$

$$8. [n = 1] = \sum_{d|n} \mu(d)$$

$$9. [gcd = 1] = \sum_{d|gcd} \mu(d)$$

- Möbius inversion: $f = g * 1 \Leftrightarrow g = f * \mu$

9.7 Polynomial

```
1 const int maxk = 20;
2 const int maxn = 1<<maxk;
3 const ll LINF = 1e18;
```

```
4
5 /* P = r*2^k + 1
6 P          r    k    g
7 998244353  119  23    3
8 1004535809 479  21    3
```

```
9
10 P          r    k    g
11 3          1    1    2
12 5          1    2    2
13 17         1    4    3
14 97         3    5    5
15 193        3    6    5
16 257        1    8    3
17 7681       15    9   17
18 12289      3   12   11
19 40961      5   13    3
20 65537      1   16    3
21 786433     3   18   10
22 5767169   11   19    3
23 7340033    7   20    3
24 23068673   11  21    3
25 104857601  25  22    3
26 167772161   5  25    3
27 469762049   7  26    3
28 1004535809 479 21    3
29 2013265921 15 27   31
30 2281701377 17 27    3
31 3221225473  3  30    5
32 75161927681 35 31    3
33 77309411329  9  33    7
34 206158430209 3  36  22
35 2061584302081 15 37    7
36 2748779069441 5  39    3
```

```
37 6597069766657      3   41    5
38 39582418599937     9   42    5
39 79164837199873     9   43    5
40 263882790666241   15   44    7
41 1231453023109121  35   45    3
42 1337006139375617  19   46    3
43 3799912185593857  27   47    5
44 4222124650659841  15   48   19
45 7881299347898369   7   50    6
46 31525197391593473  7   52    3
47 180143985094819841 5   55    6
48 1945555039024054273 27  56    5
49 4179340454199820289 29  57    3
50 9097271247288401921 505 54    6 */
51
52 const int g = 3;
53 const ll MOD = 998244353;
54
55 ll pw(ll a, ll n) { /* fast pow */ }
56
57 #define siz(x) (int)x.size()
58
59 template<typename T>
60 vector<T>& operator+=(vector<T>& a, const vector<T>& b)
61 {
62     if (siz(a) < siz(b)) a.resize(siz(b));
63     for (int i = 0; i < min(siz(a), siz(b)); i++) {
64         a[i] += b[i];
65         a[i] -= a[i] >= MOD ? MOD : 0;
66     }
67     return a;
68 }
69
70 template<typename T>
71 vector<T>& operator-=(vector<T>& a, const vector<T>& b)
72 {
73     if (siz(a) < siz(b)) a.resize(siz(b));
74     for (int i = 0; i < min(siz(a), siz(b)); i++) {
75         a[i] -= b[i];
76         a[i] += a[i] < 0 ? MOD : 0;
77     }
78     return a;
79 }
80
81 template<typename T>
82 vector<T> operator-(const vector<T>& a) {
83     vector<T> ret(siz(a));
84     for (int i = 0; i < siz(a); i++) {
85         ret[i] = -a[i] < 0 ? -a[i] + MOD : -a[i];
86     }
87     return ret;
88 }
89
90 vector<ll> X, iX;
91 vector<int> rev;
92
93 void init_ntt() {
94     X.clear(); X.resize(maxn, 1); // x1 = g^((p-1)/n)
95     iX.clear(); iX.resize(maxn, 1);
96
97     ll u = pw(g, (MOD-1)/maxn);
98     ll iu = pw(u, MOD-2);
99
100     for (int i = 1; i < maxn; i++) {
101         X[i] = X[i-1] * u;
102         iX[i] = iX[i-1] * iu;
103         if (X[i] >= MOD) X[i] %= MOD;
104         if (iX[i] >= MOD) iX[i] %= MOD;
105     }
106
107     rev.clear(); rev.resize(maxn, 0);
108     for (int i = 1, hb = -1; i < maxn; i++) {
109         if (!(i & (i-1))) hb++;
110         rev[i] = rev[i ^ (1<<hb)] | (1<<(maxk-hb-1));
111     }
112 }
113
114 template<typename T>
115 void NTT(vector<T>& a, bool inv=false) {
116     int _n = (int)a.size();
117     int k = __lg(_n) + ((1<<__lg(_n)) != _n);
118     int n = 1<<k;
```



```

117     a.resize(n, 0);
118
119     short shift = maxk-k;
120     for (int i = 0; i < n; i++)
121         if (i > (rev[i]>>shift))
122             swap(a[i], a[rev[i]>>shift]);
123
124     for (int len = 2, half = 1, div = maxn>>1; len <= n; len<=1, half<=1, div>>=1) {
125         for (int i = 0; i < n; i += len) {
126             for (int j = 0; j < half; j++) {
127                 T u = a[i+j];
128                 T v = a[i+j+half] * (inv ? ix[j*div] :
129                     X[j*div] % MOD;
130                 a[i+j] = (u+v >= MOD ? u+v-MOD : u+v);
131                 a[i+j+half] = (u-v < 0 ? u-v+MOD : u-v);
132             }
133         }
134         if (inv) {
135             T dn = pw(n, MOD-2);
136             for (auto& x : a) {
137                 x *= dn;
138                 if (x >= MOD) x %= MOD;
139             }
140         }
141     }
142
143     template<typename T>
144     inline void resize(vector<T>& a) {
145         int cnt = (int)a.size();
146         for (; cnt > 0; cnt--) if (a[cnt-1]) break;
147         a.resize(max(cnt, 1));
148     }
149
150     template<typename T>
151     vector<T>& operator*=(vector<T>& a, vector<T> b) {
152         int na = (int)a.size();
153         int nb = (int)b.size();
154         a.resize(na + nb - 1, 0);
155         b.resize(na + nb - 1, 0);
156
157         NTT(a); NTT(b);
158         for (int i = 0; i < (int)a.size(); i++) {
159             a[i] *= b[i];
160             if (a[i] >= MOD) a[i] %= MOD;
161         }
162         NTT(a, true);
163
164         resize(a);
165         return a;
166     }
167
168     template<typename T>
169     void inv(vector<T>& ia, int N) {
170         vector<T> _a(move(ia));
171         ia.resize(1, pw(_a[0], MOD-2));
172         vector<T> a(1, _a[0] + (-_a[0] < 0 ? MOD : 0));
173
174         for (int n = 1; n < N; n<=1) {
175             // n -> 2*n
176             // ia' = ia(2-a*ia);
177
178             for (int i = n; i < min(siz(_a), (n<1)); i++)
179                 a.emplace_back(-_a[i] + (-_a[i] < 0 ? MOD : 0));
180
181             vector<T> tmp = ia;
182             ia *= a;
183             ia.resize(n<1);
184             ia[0] = ia[0] + 2 >= MOD ? ia[0] + 2 - MOD : ia[0] + 2;
185             ia *= tmp;
186             ia.resize(n<1);
187         }
188         ia.resize(N);
189     }
190
191     template<typename T>
192     void mod(vector<T>& a, vector<T>& b) {
193         int n = (int)a.size()-1, m = (int)b.size()-1;
194         if (n < m) return;
195
196         vector<T> ra = a, rb = b;
197
198         reverse(ra.begin(), ra.end()); ra.resize(min(n+1, n-m+1));
199         reverse(rb.begin(), rb.end()); rb.resize(min(m+1, n-m+1));
200
201         inv(rb, n-m+1);
202
203         vector<T> q = move(ra);
204         q *= rb;
205         q.resize(n-m+1);
206         reverse(q.begin(), q.end());
207
208         q *= b;
209         a -= q;
210         resize(a);
211     }
212
213     /* Kitamasa Method (Fast Linear Recurrence):
214     Find a[K] (Given a[j] = c[0]a[j-N] + ... + c[N-1]a[j-1])
215     Let B(x) = x^N - c[N-1]x^(N-1) - ... - c[1]x^1 - c[0]
216     Let R(x) = x^K mod B(x) (get x^K using fast pow and use poly mod to get R(x))
217     Let r[i] = the coefficient of x^i in R(x)
218     => a[K] = a[0]r[0] + a[1]r[1] + ... + a[N-1]r[N-1] */

```

10 Linear Algebra

10.1 Gaussian-Jordan Elimination

```

1 int n; vector<vector<ll>> v;
2 void gauss(vector<vector<ll>>& v) {
3     int r = 0;
4     for (int i = 0; i < n; i++) {
5         bool ok = false;
6         for (int j = r; j < n; j++) {
7             if (v[j][i] == 0) continue;
8             swap(v[j], v[r]);
9             ok = true; break;
10        }
11        if (!ok) continue;
12        ll div = inv(v[r][i]);
13        for (int j = 0; j < n; j++) {
14            v[r][j] *= div;
15            if (v[r][j] >= MOD) v[r][j] %= MOD;
16        }
17        for (int j = 0; j < n; j++) {
18            if (j == r) continue;
19            ll t = v[j][i];
20            for (int k = 0; k < n; k++) {
21                v[j][k] -= v[r][k] * t % MOD;
22                if (v[j][k] < 0) v[j][k] += MOD;
23            }
24        }
25        r++;
26    }

```

10.2 Determinant

1. Use GJ Elimination, if there's any row consists of only 0, then $\det = 0$, otherwise $\det = \text{product of diagonal elements}$.
2. Properties of \det :
 - Transpose: Unchanged
 - Row Operation 1 - Swap 2 rows: $-\det$
 - Row Operation 2 - $k\vec{r}_i$: $k \times \det$
 - Row Operation 3 - $k\vec{r}_i$ add to \vec{r}_j : Unchanged

11 Flow / Matching

11.1 Dinic


```

1 struct Dinic {
2     struct Edge {
3         int t, c, r;
4         Edge() {}
5         Edge(int _t, int _c, int _r):
6             t(_t), c(_c), r(_r) {}
7     };
8     vector<vector<Edge>> G;
9     vector<int> dis, iter;
10    int s, t;
11    void init(int n) {
12        G.resize(n), dis.resize(n), iter.resize(n);
13        for(int i = 0; i < n; ++i)
14            G[i].clear();
15    }
16    void add(int a, int b, int c) {
17        G[a].eb(b, c, G[b].size());
18        G[b].eb(a, 0, G[a].size() - 1);
19    }
20    bool bfs() {
21        fill(ALL(dis), -1);
22        dis[s] = 0;
23        queue<int> que;
24        que.push(s);
25        while(!que.empty()) {
26            int u = que.front(); que.pop();
27            for(auto& e : G[u]) {
28                if(e.c > 0 && dis[e.t] == -1) {
29                    dis[e.t] = dis[u] + 1;
30                    que.push(e.t);
31                }
32            }
33        }
34        return dis[t] != -1;
35    }
36    int dfs(int u, int cur) {
37        if(u == t) return cur;
38        for(int &i = iter[u]; i < (int)G[u].size(); ++i) {
39            auto& e = G[u][i];
40            if(e.c > 0 && dis[u] + 1 == dis[e.t]) {
41                int ans = dfs(e.t, min(cur, e.c));
42                if(ans > 0) {
43                    G[e.t][e.r].c += ans;
44                    e.c -= ans;
45                    return ans;
46                }
47            }
48        }
49        return 0;
50    }
51    int flow(int a, int b) {
52        s = a, t = b;
53        int ans = 0;
54        while(bfs()) {
55            fill(ALL(iter), 0);
56            int tmp;
57            while((tmp = dfs(s, INF)) > 0)
58                ans += tmp;
59        }
60        return ans;
61    }
62 }
63 };

```

11.2 ISAP

```

1 #define SZ(c) ((int)(c).size())
2 struct Maxflow{
3     static const int MAXV=50010;
4     static const int INF =1000000;
5     struct Edge{
6         int v,c,r;
7         Edge(int _v,int _c,int _r):v(_v),c(_c),r(_r){}
8     };
9     int s,t; vector<Edge> G[MAXV];
10    int iter[MAXV],d[MAXV],gap[MAXV],tot;
11    void init(int n,int _s,int _t){
12        tot=n,s=_s,t=_t;
13        for(int i=0;i<=tot;i++){
14            G[i].clear(); iter[i]=d[i]=gap[i]=0;

```

```

15    }
16    }
17    void addEdge(int u,int v,int c){
18        G[u].push_back(Edge(v,c,SZ(G[v])));
19        G[v].push_back(Edge(u,0,SZ(G[u])-1));
20    }
21    int DFS(int p,int flow){
22        if(p==t) return flow;
23        for(int &i=iter[p];i<SZ(G[p]);i++){
24            Edge &e=G[p][i];
25            if(e.c>0&&d[p]==d[e.v]+1){
26                int f=DFS(e.v,min(flow,e.c));
27                if(f){ e.c-=f; G[e.v][e.r].c+=f; return f; }
28            }
29        }
30        if(--gap[d[p]]==0) d[s]=tot;
31        else{ d[p]++; iter[p]=0; ++gap[d[p]]; }
32        return 0;
33    }
34    int flow(){
35        int res=0;
36        for(res=0,gap[0]=tot;d[s]<tot;res+=DFS(s,INF));
37        return res;
38    } // reset: set iter,d,gap to 0
39 } flow;

```

11.3 MCMF

```

1 struct MCMF {
2     struct Edge {
3         int to, cap, rev;
4         ll cost;
5         Edge() {}
6         Edge(int _to, int _cap, int _rev, ll _cost) :
7             to(_to), cap(_cap), rev(_rev), cost(_cost) {}
8     };
9     static const int N = 2000;
10    vector<Edge> G[N];
11    int n, s, t;
12    void init(int _n, int _s, int _t) {
13        n = _n, s = _s, t = _t;
14        for(int i = 0; i <= n; ++i)
15            G[i].clear();
16    }
17    void add_edge(int from, int to, int cap, ll cost) {
18        G[from].eb(to, cap, (int)G[to].size(), cost);
19        G[to].eb(from, 0, (int)G[from].size() - 1, -cost);
20    }
21    bool vis[N];
22    int iter[N];
23    ll dis[N];
24    bool SPFA() {
25        for(int i = 0; i <= n; ++i)
26            vis[i] = 0, dis[i] = LINF;
27
28        dis[s] = 0; vis[s] = 1;
29        queue<int> que; que.push(s);
30        while(!que.empty()) {
31            int u = que.front(); que.pop();
32            vis[u] = 0;
33            for(auto& e : G[u]) if(e.cap > 0 && dis[e.to] > dis[u] + e.cost) {
34                dis[e.to] = dis[u] + e.cost;
35                if(!vis[e.to]) {
36                    que.push(e.to);
37                    vis[e.to] = 1;
38                }
39            }
40        }
41        return dis[t] != LINF;
42    }
43    int dfs(int u, int cur) {
44        if(u == t) return cur;
45        int ret = 0; vis[u] = 1;
46        for(int &i = iter[u]; i < (int)G[u].size(); ++i) {
47            auto &e = G[u][i];

```

```

50     if(e.cap > 0 && dis[e.to] == dis[u] + e.
51         cost && !vis[e.to]) {
52         int tmp = dfs(e.to, min(cur, e.cap));
53         e.cap -= tmp;
54         G[e.to][e.rev].cap += tmp;
55         cur -= tmp;
56         ret += tmp;
57         if(cur == 0) {
58             vis[u] = 0;
59             return ret;
60         }
61     }
62     vis[u] = 0;
63     return ret;
64 }
65 pair<int, ll> flow() {
66     int flow = 0; ll cost = 0;
67     while(SPFA()) {
68         memset(iter, 0, sizeof(iter));
69         int tmp = dfs(s, INF);
70         flow += tmp, cost += tmp * dis[t];
71     }
72     return {flow, cost};
73 }
74 };

```

11.4 Hopcroft-Karp

```

1 struct HopcroftKarp {
2     // id: X = [1, nx], Y = [nx+1, nx+ny]
3     int n, nx, ny, m, MXCNT;
4     vector<vector<int>> g;
5     vector<int> mx, my, dis, vis;
6     void init(int nnx, int nny, int mm) {
7         nx = nnx, ny = nny, m = mm;
8         n = nx + ny + 1;
9         g.clear(); g.resize(n);
10    }
11    void add(int x, int y) {
12        g[x].emplace_back(y);
13        g[y].emplace_back(x);
14    }
15    bool dfs(int x) {
16        vis[x] = true;
17        Each(y, g[x]) {
18            int px = my[y];
19            if (px == -1 ||
20                (dis[px] == dis[x]+1 &&
21                 !vis[px] && dfs(px))) {
22                mx[x] = y;
23                my[y] = x;
24                return true;
25            }
26        }
27        return false;
28    }
29    void get() {
30        mx.clear(); mx.resize(n, -1);
31        my.clear(); my.resize(n, -1);
32
33        while (true) {
34            queue<int> q;
35            dis.clear(); dis.resize(n, -1);
36            for (int x = 1; x <= nx; x++){
37                if (mx[x] == -1) {
38                    dis[x] = 0;
39                    q.push(x);
40                }
41            }
42            while (!q.empty()) {
43                int x = q.front(); q.pop();
44                Each(y, g[x]) {
45                    if (my[y] != -1 && dis[my[y]] ==
46                        -1) {
47                        dis[my[y]] = dis[x] + 1;
48                        q.push(my[y]);
49                    }
50                }
51            }

```

```

52        bool brk = true;
53        vis.clear(); vis.resize(n, 0);
54        for (int x = 1; x <= nx; x++)
55            if (mx[x] == -1 && dfs(x))
56                brk = false;
57
58        if (brk) break;
59    }
60    MXCNT = 0;
61    for (int x = 1; x <= nx; x++) if (mx[x] != -1)
62        MXCNT++;
63 } hk;

```

11.5 Cover / Independent Set

1 V(E) Cover: choose some V(E) to cover all E(V)
2 V(E) Independ: set of V(E) not adj to each other

3
4 M = Max Matching
5 Cv = Min V Cover
6 Ce = Min E Cover
7 Iv = Max V Ind
8 Ie = Max E Ind (equiv to M)

9
10 M = Cv (Konig Theorem)
11 Iv = V \ Cv
12 Ce = V - M

13
14 Construct Cv:
15 1. Run Dinic
16 2. Find s-t min cut
17 3. Cv = {X in T} + {Y in S}

11.6 KM

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4 const int inf = 1e9;
5
6 struct KuhnMunkres {
7     int n;
8     vector<vector<int>> g;
9     vector<int> lx, ly, slack;
10    vector<int> match, visx, visy;
11    KuhnMunkres(int n) : n(n), g(n, vector<int>(n)),
12        lx(n), ly(n), slack(n), match(n), visx(n), visy
13        (n)) {}
14    vector<int> & operator[](int i) { return g[i]; }
15    bool dfs(int i, bool aug) { // aug = true 表示要更
16        新 match
17        if(visx[i]) return false;
18        visx[i] = true;
19        for(int j = 0; j < n; j++) {
20            if(visy[j]) continue;
21            // 一邊擴增交錯樹、尋找增廣路徑
22            // 一邊更新slack: 樹上的點跟樹外的點所造成
23            的最小權重
24            int d = lx[i] + ly[j] - g[i][j];
25            if(d == 0) {
26                visy[j] = true;
27                if(match[j] == -1 || dfs(match[j], aug))
28                    {
29                        if(aug)
30                            match[j] = i;
31                        return true;
32                    }
33            } else {
34                slack[j] = min(slack[j], d);
35            }
36        }
37        return false;
38    }
39    bool augment() { // 回傳是否有增廣路
40        for(int j = 0; j < n; j++) if(!visy[j] && slack
41            [j] == 0) {
42                visy[j] = true;
43                if(match[j] == -1 || dfs(match[j], false))
44                    {

```

```

39         return true;
40     }
41 }
42 return false;
43 }
44 void relabel() {
45     int delta = inf;
46     for(int j = 0; j < n; j++) if(!visy[j]) delta =
47         min(delta, slack[j]);
48     for(int i = 0; i < n; i++) if(visx[i]) lx[i] -=
49         delta;
50     for(int j = 0; j < n; j++) {
51         if(visy[j]) ly[j] += delta;
52         else slack[j] -= delta;
53     }
54 }
55 int solve() {
56     for(int i = 0; i < n; i++) {
57         lx[i] = 0;
58         for(int j = 0; j < n; j++) lx[i] = max(lx[i]
59             , g[i][j]);
60     }
61     fill(ly.begin(), ly.end(), 0);
62     fill(match.begin(), match.end(), -1);
63     for(int i = 0; i < n; i++) {
64         // slack 在每一輪都要初始化
65         fill(slack.begin(), slack.end(), inf);
66         fill(visx.begin(), visx.end(), false);
67         fill(visy.begin(), visy.end(), false);
68         if(dfs(i, true)) continue;
69         // 重複調整頂標直到找到增廣路徑
70         while(!augment()) relabel();
71         fill(visx.begin(), visx.end(), false);
72         fill(visy.begin(), visy.end(), false);
73         dfs(i, true);
74     }
75     int ans = 0;
76     for(int j = 0; j < n; j++) if(match[j] != -1)
77         ans += g[match[j]][j];
78     return ans;
79 }
80 };
81 signed main() {
82     ios_base::sync_with_stdio(0), cin.tie(0);
83     int n;
84     while(cin >> n && n) {
85         KuhnMunkres KM(n);
86         for(int i = 0; i < n; i++) {
87             for(int j = 0; j < n; j++) {
88                 int c;
89                 cin >> c;
90                 if(c > 0)
91                     KM[i][j] = c;
92             }
93         }
94         cout << KM.solve() << '\n';
95     }
96 }

```

Let X/G be the set of orbits.

Then the following equation holds:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

13 Special Numbers

13.1 Fibonacci Series

1	1	1	2	3
5	5	8	13	21
9	34	55	89	144
13	233	377	610	987
17	1597	2584	4181	6765
21	10946	17711	28657	46368
25	75025	121393	196418	317811
29	514229	832040	1346269	2178309
33	3524578	5702887	9227465	14930352

$$f(45) \approx 10^9, f(88) \approx 10^{18}$$

13.2 Prime Numbers

- First 50 prime numbers:

1	2	3	5	7	11
6	13	17	19	23	29
11	31	37	41	43	47
16	53	59	61	67	71
21	73	79	83	89	97
26	101	103	107	109	113
31	127	131	137	139	149
36	151	157	163	167	173
41	179	181	191	193	197
46	199	211	223	227	229

- Very large prime numbers:

1000001333 1000500889 2500001909
2000000659 900004151 850001359

- $\pi(n) \equiv$ Number of primes $\leq n \approx n/((\ln n) - 1)$
 $\pi(100) = 25, \pi(200) = 46$
 $\pi(500) = 95, \pi(1000) = 168$
 $\pi(2000) = 303, \pi(4000) = 550$
 $\pi(10^4) = 1229, \pi(10^5) = 9592$
 $\pi(10^6) = 78498, \pi(10^7) = 664579$

12 Combinatorics

12.1 Catalan Number

$$C_0 = 1, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}, C_n = C_n^{2n} - C_{n-1}^{2n}$$

0	1	1	2	5
4	14	42	132	429
8	1430	4862	16796	58786
12	208012	742900	2674440	9694845

12.2 Burnside's Lemma

Let X be the original set.

Let G be the group of operations acting on X .

Let X^g be the set of x not affected by g .