Contents

1 Reminder

1.1 Bug List

- 沒開 long long
- 陣列戳出界／陣列開不夠大
- 寫好的函式忘記呼叫
- 變數打錯
- 0-base / 1-base
- 忘記初始化
- == 打成 =
- <= 打成 <+
- dp[i] 從 dp[i-1] 轉移時忘記特判 i > 0
- std::sort 比較運算子寫成 < 或是讓 = 的情況為 true
- 漏 case
- 線段樹改值懶標初始值不能設為 0
- DFS 的時候不小心覆寫到全域變數
- 浮點數誤差
- unsigned int128
- 多筆測資不能沒讀完直接 return
- 記得刪 cerr

1.2 OwO

- 可以構造複雜點的測資幫助思考
- 真的卡太久請跳題
- Enjoy The Contest!

2 Basic

2.1 Vimrc

```
set number relativenumber ai t_Co=256 tabstop=4
set mouse=a shiftwidth=4 encoding=utf8
set bs=2 ruler laststatus=2 cmdheight=2
set clipboard=unnamedplus showcmd autoread
set belloff=all
filetype indent on
"set guifont Hack:h16
":set guifont?

inoremap ( ()<Esc>i
inoremap " ""<Esc>i
inoremap [ []<Esc>i
inoremap ' ''<Esc>i
inoremap { {<CR>}<Esc>ko

vmap <C-c> "+y
inoremap <C-v> <Esc>p
nnoremap <C-v> p

nnoremap <tab> gt
nnoremap <S-tab> gT
inoremap <C-n> <Esc>:tabnew<CR>
nnoremap <C-n> :tabnew<CR>

inoremap <F9> <Esc>:w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>
nnoremap <F9> :w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>

syntax on
colorscheme desert
set filetype=cpp
set background=dark
hi Normal ctermfg=white ctermbg=black
```

2.2 Runcpp.sh

```
#! /bin/bash
clear
echo "Start compiling $1..."
echo
g++ -O2 -std=c++20 -Wall -Wextra -Wshadow $2/$1 -o $2/
    out
if [ "$?" -ne 0 ]
then
```

```
8       exit 1
9  fi
10 echo
11 echo "Done compiling"
12 echo "========================="
13 echo
14 echo "Input file:"
15 echo
16 cat $2/in.txt
17 echo
18 echo "========================="
19 echo
20 declare startTime=`date +%s%N`
21 $2/out < $2/in.txt > $2/out.txt
22 declare endTime=`date +%s%N`
23 delta=`expr $endTime - $startTime`
24 delta=`expr $delta / 1000000`
25 cat $2/out.txt
26 echo
27 echo "time: $delta ms"
```

2.3   Stress

```
1  g++ gen.cpp -o gen.out
2  g++ ac.cpp -o ac.out
3  g++ wa.cpp -o wa.out
4  for ((i=0;;i++))
5  do
6      echo "$i"
7      ./gen.out > in.txt
8      ./ac.out < in.txt > ac.txt
9      ./wa.out < in.txt > wa.txt
10     diff ac.txt wa.txt || break
11 done
```

2.4   PBDS

```
1  #include <bits/extc++.h>
2  using namespace __gnu_pbds;
3
4  // map
5  tree<int, int, less<>, rb_tree_tag,
       tree_order_statistics_node_update> tr;
6  tr.order_of_key(element);
7  tr.find_by_order(rank);
8
9  // set
10 tree<int, null_type, less<>, rb_tree_tag,
       tree_order_statistics_node_update> tr;
11 tr.order_of_key(element);
12 tr.find_by_order(rank);
13
14 // priority queue
15 __gnu_pbds::priority_queue<int, less<int> > big_q;  //
       Big First
16 __gnu_pbds::priority_queue<int, greater<int> > small_q;
         // Small First
17 q1.join(q2);  // join
```

2.5   Random

```
1  mt19937 gen(chrono::steady_clock::now().
       time_since_epoch().count());
2  uniform_int_distribution<int> dis(1, 100);
3  cout << dis(gen) << endl;
4  shuffle(v.begin(), v.end(), gen);
```

3   Python

3.1   I/O

```
1  import sys
2  input = sys.stdin.readline
3
4  # Input
5  def readInt():
6      return int(input())
```

```
7  def readList():
8      return list(map(int,input().split()))
9  def readStr():
10     s = input()
11     return list(s[:len(s) - 1])
12 def readVars():
13     return map(int,input().split())
14
15 # Output
16 sys.stdout.write(string)
17
18 # faster
19 def main():
20     pass
21 main()
```

3.2   Decimal

```
1  from decimal import *
2  getcontext().prec = 2500000
3  getcontext().Emax = 2500000
4  a,b = Decimal(input()),Decimal(input())
5  a*=b
6  print(a)
```

4   Data Structure

4.1   Heavy Light Decomposition

```
1  const int N=2e5+5;
2  int n,dfn[N],son[N],top[N],num[N],dep[N],p[N];
3  vector<int>path[N];
4  struct node
5  {
6      int mx,sum;
7  }seg[N<<2];
8  void update(int x,int l,int r,int qx,int val)
9  {
10     if(l==r)
11     {
12         seg[x].mx=seg[x].sum=val;
13         return;
14     }
15     int mid=(l+r)>>1;
16     if(qx<=mid)update(x<<1,l,mid,qx,val);
17     else update(x<<1|1,mid+1,r,qx,val);
18     seg[x].mx=max(seg[x<<1].mx,seg[x<<1|1].mx);
19     seg[x].sum=seg[x<<1].sum+seg[x<<1|1].sum;
20 }
21 int big(int x,int l,int r,int ql,int qr)
22 {
23     if(ql<=l&&r<=qr)return seg[x].mx;
24     int mid=(l+r)>>1;
25     int res=-INF;
26     if(ql<=mid)res=max(res,big(x<<1,l,mid,ql,qr));
27     if(mid<qr)res=max(res,big(x<<1|1,mid+1,r,ql,qr));
28     return res;
29 }
30 int ask(int x,int l,int r,int ql,int qr)
31 {
32     if(ql<=l&&r<=qr)return seg[x].sum;
33     int mid=(l+r)>>1;
34     int res=0;
35     if(ql<=mid)res+=ask(x<<1,l,mid,ql,qr);
36     if(mid<qr)res+=ask(x<<1|1,mid+1,r,ql,qr);
37     return res;
38 }
39 void dfs1(int now)
40 {
41     son[now]=-1;
42     num[now]=1;
43     for(auto i:path[now])
44     {
45         if(!dep[i])
46         {
47             dep[i]=dep[now]+1;
48             p[i]=now;
49             dfs1(i);
50             num[now]+=num[i];
```

```
51        if(son[now]==-1||num[i]>num[son[now]])son[
              now]=i;
52    }
53  }
54 }
55 int cnt;
56 void dfs2(int now,int t)
57 {
58     top[now]=t;
59     cnt++;
60     dfn[now]=cnt;
61     if(son[now]==-1)return;
62     dfs2(son[now],t);
63     for(auto i:path[now])
64         if(i!=p[now]&&i!=son[now])
65             dfs2(i,i);
66 }
67 int path_big(int x,int y)
68 {
69     int res=-INF;
70     while(top[x]!=top[y])
71     {
72         if(dep[top[x]]<dep[top[y]])swap(x,y);
73         res=max(res,big(1,1,n,dfn[top[x]],dfn[x]));
74         x=p[top[x]];
75     }
76     if(dfn[x]>dfn[y])swap(x,y);
77     res=max(res,big(1,1,n,dfn[x],dfn[y]));
78     return res;
79 }
80 int path_sum(int x,int y)
81 {
82     int res=0;
83     while(top[x]!=top[y])
84     {
85         if(dep[top[x]]<dep[top[y]])swap(x,y);
86         res+=ask(1,1,n,dfn[top[x]],dfn[x]);
87         x=p[top[x]];
88     }
89     if(dfn[x]>dfn[y])swap(x,y);
90     res+=ask(1,1,n,dfn[x],dfn[y]);
91     return res;
92 }
93 void buildTree()
94 {
95     FOR(i,0,n-1)
96     {
97         int a,b;cin>>a>>b;
98         path[a].pb(b);
99         path[b].pb(a);
100    }
101 }
102 void buildHLD(int root)
103 {
104     dep[root]=1;
105     dfs1(root);
106     dfs2(root,root);
107     FOR(i,1,n+1)
108     {
109         int now;cin>>now;
110         update(1,1,n,dfn[i],now);
111     }
112 }
```

4.2 Skew Heap

```
1 struct node{
2     node *l,*r;
3     int v;
4     node(int x):v(x){
5         l=r=nullptr;
6     }
7 };
8 node* merge(node* a,node* b){
9     if(!a||!b) return a?:b;
10 // min heap
11     if(a->v>b->v) swap(a,b);
12     a->r=merge(a->r,b);
13     swap(a->l,a->r);
14     return a;
15 }
```

4.3 Leftist Heap

```
1 struct node{
2     node *l,*r;
3     int d, v;
4     node(int x):d(1),v(x){
5         l=r=nullptr;
6     }
7 };
8 static inline int d(node* x){return x?x->d:0;}
9 node* merge(node* a,node* b){
10     if(!a||!b) return a?:b;
11 // min heap
12     if(a->v>b->v) swap(a,b);
13     a->r=merge(a->r,b);
14     if(d(a->l)<d(a->r))
15         swap(a->l,a->r);
16     a->d=d(a->r)+1;
17     return a;
18 }
```

4.4 Treap

```
1 mt19937 rng(random_device{}());
2 struct Treap
3 {
4     Treap *l,*r;
5     int val,num,pri;
6     Treap(int k)
7     {
8         l=r=NULL;
9         val=k;
10         num=1;
11         pri=rng();
12     }
13 };
14 int siz(Treap *now){return now?now->num:0;}
15 void pull(Treap *&now)
16 {
17     now->num=siz(now->l)+siz(now->r)+1;
18 }
19 Treap* merge(Treap *a,Treap *b)
20 {
21     if(!a||!b)return a?a:b;
22     else if(a->pri>b->pri)
23     {
24         a->r=merge(a->r,b);
25         pull(a);
26         return a;
27     }
28     else
29     {
30         b->l=merge(a,b->l);
31         pull(b);
32         return b;
33     }
34 }
35 void split_size(Treap *rt,Treap *&a,Treap *&b,int val)
36 {
37     if(!rt)
38     {
39         a=b=NULL;
40         return;
41     }
42     if(siz(rt->l)+1>val)
43     {
44         b=rt;
45         split_size(rt->l,a,b->l,val);
46         pull(b);
47     }
48     else
49     {
50         a=rt;
51         split_size(rt->r,a->r,b,val-siz(a->l)-1);
52         pull(a);
53     }
54 }
55 void split_val(Treap *rt,Treap *&a,Treap *&b,int val)
56 {
57     if(!rt)
58     {
```

```
59        a=b=NULL;
60        return;
61    }
62    if(rt->val<=val)
63    {
64        a=rt;
65        split_val(rt->r,a->r,b,val);
66        pull(a);
67    }
68    else
69    {
70        b=rt;
71        split_val(rt->l,a,b->l,val);
72        pull(b);
73    }
74 }
75 void treap_dfs(Treap *now)
76 {
77    if(!now)return;
78    treap_dfs(now->l);
79    cout<<now->val<<" ";
80    treap_dfs(now->r);
81 }
```

4.5   Persistent Treap

```
1 struct node {
2    node *l, *r;
3    char c; int v, sz;
4    node(char x = '$'): c(x), v(mt()), sz(1) {
5        l = r = nullptr;
6    }
7    node(node* p) {*this = *p;}
8    void pull() {
9        sz = 1;
10        for (auto i : {l, r})
11            if (i) sz += i->sz;
12    }
13 } arr[maxn], *ptr = arr;
14 inline int size(node* p) {return p ? p->sz : 0;}
15 node* merge(node* a, node* b) {
16    if (!a || !b) return a ? : b;
17    if (a->v < b->v) {
18        node* ret = new(ptr++) node(a);
19        ret->r = merge(ret->r, b), ret->pull();
20        return ret;
21    }
22    else {
23        node* ret = new(ptr++) node(b);
24        ret->l = merge(a, ret->l), ret->pull();
25        return ret;
26    }
27 }
28 P<node*> split(node* p, int k) {
29    if (!p) return {nullptr, nullptr};
30    if (k >= size(p->l) + 1) {
31        auto [a, b] = split(p->r, k - size(p->l) - 1);
32        node* ret = new(ptr++) node(p);
33        ret->r = a, ret->pull();
34        return {ret, b};
35    }
36    else {
37        auto [a, b] = split(p->l, k);
38        node* ret = new(ptr++) node(p);
39        ret->l = b, ret->pull();
40        return {a, ret};
41    }
42 }
```

4.6   Li Chao Tree

```
1 constexpr int maxn = 5e4 + 5;
2 struct line {
3    ld a, b;
4    ld operator()(ld x) {return a * x + b;}
5 } arr[(maxn + 1) << 2];
6 bool operator<(line a, line b) {return a.a < b.a;}
7 #define m ((l+r)>>1)
8 void insert(line x, int i = 1, int l = 0, int r = maxn)
     {
```

```
9        if (r - l == 1) {
10            if (x(l) > arr[i](l))
11                arr[i] = x;
12            return;
13        }
14        line a = max(arr[i], x), b = min(arr[i], x);
15        if (a(m) > b(m))
16            arr[i] = a, insert(b, i << 1, l, m);
17        else
18            arr[i] = b, insert(a, i << 1 | 1, m, r);
19 }
20 ld query(int x, int i = 1, int l = 0, int r = maxn) {
21    if (x < l || r <= x) return -numeric_limits<ld>::
        max();
22    if (r - l == 1) return arr[i](x);
23    return max({arr[i](x), query(x, i << 1, l, m),
        query(x, i << 1 | 1, m, r)});
24 }
25 #undef m
```

4.7   Time Segment Tree

```
1 constexpr int maxn = 1e5 + 5;
2 V<P<int>> arr[(maxn + 1) << 2];
3 V<int> dsu, sz;
4 V<tuple<int, int, int>> his;
5 int cnt, q;
6 int find(int x) {
7    return x == dsu[x] ? x : find(dsu[x]);
8 };
9 inline bool merge(int x, int y) {
10    int a = find(x), b = find(y);
11    if (a == b) return false;
12    if (sz[a] > sz[b]) swap(a, b);
13    his.emplace_back(a, b, sz[b]), dsu[a] = b, sz[b] +=
        sz[a];
14    return true;
15 };
16 inline void undo() {
17    auto [a, b, s] = his.back(); his.pop_back();
18    dsu[a] = a, sz[b] = s;
19 }
20 #define m ((l + r) >> 1)
21 void insert(int ql, int qr, P<int> x, int i = 1, int l
     = 0, int r = q) {
22    // debug(ql, qr, x); return;
23    if (qr <= l || r <= ql) return;
24    if (ql <= l && r <= qr) {arr[i].push_back(x);
        return;}
25    if (qr <= m)
26        insert(ql, qr, x, i << 1, l, m);
27    else if (m <= ql)
28        insert(ql, qr, x, i << 1 | 1, m, r);
29    else {
30        insert(ql, qr, x, i << 1, l, m);
31        insert(ql, qr, x, i << 1 | 1, m, r);
32    }
33 }
34 void traversal(V<int>& ans, int i = 1, int l = 0, int r
     = q) {
35    int opcnt = 0;
36    // debug(i, l, r);
37    for (auto [a, b] : arr[i])
38        if (merge(a, b))
39            opcnt++, cnt--;
40    if (r - l == 1) ans[l] = cnt;
41    else {
42        traversal(ans, i << 1, l, m);
43        traversal(ans, i << 1 | 1, m, r);
44    }
45    while (opcnt--)
46        undo(), cnt++;
47    arr[i].clear();
48 }
49 #undef m
50 inline void solve() {
51    int n, m; cin>>n>>m>>q,q++;
52    dsu.resize(cnt = n), sz.assign(n, 1);
53    iota(dsu.begin(), dsu.end(), 0);
54    // a, b, time, operation
55    unordered_map<ll, V<int>> s;
```

```
56         for (int i = 0; i < m; i++) {
57             int a, b; cin>>a>>b;
58             if (a > b) swap(a, b);
59             s[((ll)a << 32) | b].emplace_back(0);
60         }
61         for (int i = 1; i < q; i++) {
62             int op,a, b;
63             cin>>op>>a>>b;
64             if (a > b) swap(a, b);
65             switch (op) {
66             case 1:
67                 s[((ll)a << 32) | b].push_back(i);
68                 break;
69             case 2:
70                 auto tmp = s[((ll)a << 32) | b].back();
71                 s[((ll)a << 32) | b].pop_back();
72                 insert(tmp, i, P<int> {a, b});
73             }
74         }
75         for (auto [p, v] : s) {
76             int a = p >> 32, b = p & -1;
77             while (v.size()) {
78                 insert(v.back(), q, P<int> {a, b});
79                 v.pop_back();
80             }
81         }
82         V<int> ans(q);
83         traversal(ans);
84         for (auto i : ans)
85             cout<<i<<' ';
86         cout<<endl;
87 }
```

5 DP

5.1 Aliens

```
1  int n; ll k;
2  vector<ll> a;
3  vector<pll> dp[2];
4  void init() {
5      cin >> n >> k;
6      Each(i, dp) i.clear(), i.resize(n);
7      a.clear(); a.resize(n);
8      Each(i, a) cin >> i;
9  }
10 pll calc(ll p) {
11     dp[0][0] = mp(0, 0);
12     dp[1][0] = mp(-a[0], 0);
13     FOR(i, 1, n, 1) {
14         if (dp[0][i-1].F > dp[1][i-1].F + a[i] - p) {
15             dp[0][i] = dp[0][i-1];
16         } else if (dp[0][i-1].F < dp[1][i-1].F + a[i] -
                  p) {
17             dp[0][i] = mp(dp[1][i-1].F + a[i] - p, dp
                  [1][i-1].S+1);
18         } else {
19             dp[0][i] = mp(dp[0][i-1].F, min(dp[0][i-1].
                  S, dp[1][i-1].S+1));
20         }
21         if (dp[0][i-1].F - a[i] > dp[1][i-1].F) {
22             dp[1][i] = mp(dp[0][i-1].F - a[i], dp[0][i
                  -1].S);
23         } else if (dp[0][i-1].F - a[i] < dp[1][i-1].F)
                  {
24             dp[1][i] = dp[1][i-1];
25         } else {
26             dp[1][i] = mp(dp[1][i-1].F, min(dp[0][i-1].
                  S, dp[1][i-1].S));
27         }
28     }
29     return dp[0][n-1];
30 }
31 void solve() {
32     ll l = 0, r = 1e7;
33     pll res = calc(0);
34     if (res.S <= k) return cout << res.F << endl, void
                  ();
35     while (l < r) {
36         ll mid = (l+r)>>1;
```

```
37             res = calc(mid);
38             if (res.S <= k) r = mid;
39             else l = mid+1;
40         }
41         res = calc(l);
42         cout << res.F + k*l << endl;
43 }
```

6 Graph

6.1 Bellman-Ford + SPFA

```
1  int n, m;
2
3  // Graph
4  vector<vector<pair<int, ll> > > g;
5  vector<ll> dis;
6  vector<bool> negCycle;
7
8  // SPFA
9  vector<int> rlx;
10 queue<int> q;
11 vector<bool> inq;
12 vector<int> pa;
13 void SPFA(vector<int>& src) {
14     dis.assign(n+1, LINF);
15     negCycle.assign(n+1, false);
16     rlx.assign(n+1, 0);
17     while (!q.empty()) q.pop();
18     inq.assign(n+1, false);
19     pa.assign(n+1, -1);
20
21     for (auto& s : src) {
22         dis[s] = 0;
23         q.push(s); inq[s] = true;
24     }
25
26     while (!q.empty()) {
27         int u = q.front();
28         q.pop(); inq[u] = false;
29         if (rlx[u] >= n) {
30             negCycle[u] = true;
31         }
32         else for (auto& e : g[u]) {
33             int v = e.first;
34             ll w = e.second;
35             if (dis[v] > dis[u] + w) {
36                 dis[v] = dis[u] + w;
37                 rlx[v] = rlx[u] + 1;
38                 pa[v] = u;
39                 if (!inq[v]) {
40                     q.push(v);
41                     inq[v] = true;
42 } } } } }
43
44
45 // Bellman-Ford
46 queue<int> q;
47 vector<int> pa;
48 void BellmanFord(vector<int>& src) {
49     dis.assign(n+1, LINF);
50     negCycle.assign(n+1, false);
51     pa.assign(n+1, -1);
52
53     for (auto& s : src) dis[s] = 0;
54
55     for (int rlx = 1; rlx <= n; rlx++) {
56         for (int u = 1; u <= n; u++) {
57             if (dis[u] == LINF) continue;  // Important
                  !!
58             for (auto& e : g[u]) {
59                 int v = e.first; ll w = e.second;
60                 if (dis[v] > dis[u] + w) {
61                     dis[v] = dis[u] + w;
62                     pa[v] = u;
63                     if (rlx == n) negCycle[v] = true;
64 } } } } }
65
66
67 // Negative Cycle Detection
```

```
68  void NegCycleDetect() {
69  /* No Neg Cycle: NO
70  Exist Any Neg Cycle:
71  YES
72  v0 v1 v2 ... vk v0 */
73
74      vector<int> src;
75      for (int i = 1; i <= n; i++)
76          src.emplace_back(i);
77
78      SPFA(src);
79      // BellmanFord(src);
80
81      int ptr = -1;
82      for (int i = 1; i <= n; i++) if (negCycle[i])
83          { ptr = i; break; }
84
85      if (ptr == -1) { return cout << "NO" << endl, void
            (); }
86
87      cout << "YES\n";
88      vector<int> ans;
89      vector<bool> vis(n+1, false);
90
91      while (true) {
92          ans.emplace_back(ptr);
93          if (vis[ptr]) break;
94          vis[ptr] = true;
95          ptr = pa[ptr];
96      }
97      reverse(ans.begin(), ans.end());
98
99      vis.assign(n+1, false);
100     for (auto& x : ans) {
101         cout << x << ' ';
102         if (vis[x]) break;
103         vis[x] = true;
104     }
105     cout << endl;
106 }
107
108 // Distance Calculation
109 void calcDis(int s) {
110     vector<int> src;
111     src.emplace_back(s);
112     SPFA(src);
113     // BellmanFord(src);
114
115     while (!q.empty()) q.pop();
116     for (int i = 1; i <= n; i++)
117         if (negCycle[i]) q.push(i);
118
119     while (!q.empty()) {
120         int u = q.front(); q.pop();
121         for (auto& e : g[u]) {
122             int v = e.first;
123             if (!negCycle[v]) {
124                 q.push(v);
125                 negCycle[v] = true;
126 } } } }
```

6.2 BCC - AP

```
1   int n, m;
2   int low[maxn], dfn[maxn], instp;
3   vector<int> E, g[maxn];
4   bitset<maxn> isap;
5   bitset<maxm> vis;
6   stack<int> stk;
7   int bccnt;
8   vector<int> bcc[maxn];
9   inline void popout(int u) {
10      bccnt++;
11      bcc[bccnt].emplace_back(u);
12      while (!stk.empty()) {
13          int v = stk.top();
14          if (u == v) break;
15          stk.pop();
16          bcc[bccnt].emplace_back(v);
17      }
18  }
```

```
19  void dfs(int u, bool rt = 0) {
20      stk.push(u);
21      low[u] = dfn[u] = ++instp;
22      int kid = 0;
23      Each(e, g[u]) {
24          if (vis[e]) continue;
25          vis[e] = true;
26          int v = E[e]^u;
27          if (!dfn[v]) {
28              // tree edge
29              kid++; dfs(v);
30              low[u] = min(low[u], low[v]);
31              if (!rt && low[v] >= dfn[u]) {
32                  // bcc found: u is ap
33                  isap[u] = true;
34                  popout(u);
35              }
36          } else {
37              // back edge
38              low[u] = min(low[u], dfn[v]);
39          }
40      }
41      // special case: root
42      if (rt) {
43          if (kid > 1) isap[u] = true;
44          popout(u);
45      }
46  }
47  void init() {
48      cin >> n >> m;
49      fill(low, low+maxn, INF);
50      REP(i, m) {
51          int u, v;
52          cin >> u >> v;
53          g[u].emplace_back(i);
54          g[v].emplace_back(i);
55          E.emplace_back(u^v);
56      }
57  }
58  void solve() {
59      FOR(i, 1, n+1, 1) {
60          if (!dfn[i]) dfs(i, true);
61      }
62      vector<int> ans;
63      int cnt = 0;
64      FOR(i, 1, n+1, 1) {
65          if (isap[i]) cnt++, ans.emplace_back(i);
66      }
67      cout << cnt << endl;
68      Each(i, ans) cout << i << ' ';
69      cout << endl;
70  }
```

6.3 BCC - Bridge

```
1   int n, m;
2   vector<int> g[maxn], E;
3   int low[maxn], dfn[maxn], instp;
4   int bccnt, bccid[maxn];
5   stack<int> stk;
6   bitset<maxn> vis, isbrg;
7   void init() {
8       cin >> n >> m;
9       REP(i, m) {
10          int u, v;
11          cin >> u >> v;
12          E.emplace_back(u^v);
13          g[u].emplace_back(i);
14          g[v].emplace_back(i);
15      }
16      fill(low, low+maxn, INF);
17  }
18  void popout(int u) {
19      bccnt++;
20      while (!stk.empty()) {
21          int v = stk.top();
22          if (v == u) break;
23          stk.pop();
24          bccid[v] = bccnt;
25      }
26  }
```

```cpp
27  void dfs(int u) {
28      stk.push(u);
29      low[u] = dfn[u] = ++instp;
30
31      Each(e, g[u]) {
32          if (vis[e]) continue;
33          vis[e] = true;
34
35          int v = E[e]^u;
36          if (dfn[v]) {
37              // back edge
38              low[u] = min(low[u], dfn[v]);
39          } else {
40              // tree edge
41              dfs(v);
42              low[u] = min(low[u], low[v]);
43              if (low[v] == dfn[v]) {
44                  isbrg[e] = true;
45                  popout(u);
46              }
47          }
48      }
49  }
50  void solve() {
51      FOR(i, 1, n+1, 1) {
52          if (!dfn[i]) dfs(i);
53      }
54      vector<pii> ans;
55      vis.reset();
56      FOR(u, 1, n+1, 1) {
57          Each(e, g[u]) {
58              if (!isbrg[e] || vis[e]) continue;
59              vis[e] = true;
60              int v = E[e]^u;
61              ans.emplace_back(mp(u, v));
62          }
63      }
64      cout << (int)ans.size() << endl;
65      Each(e, ans) cout << e.F << ' ' << e.S << endl;
66  }
```

6.4   SCC - Tarjan

```cpp
1   // 2-SAT
2   vector<int> E, g[maxn];   // 1~n, n+1~2n
3   int low[maxn], in[maxn], instp;
4   int sccnt, sccid[maxn];
5
6   stack<int> stk;
7   bitset<maxn> ins, vis;
8
9   int n, m;
10
11  void init() {
12      cin >> m >> n;
13      E.clear();
14      fill(g, g+maxn, vector<int>());
15      fill(low, low+maxn, INF);
16      memset(in, 0, sizeof(in));
17      instp = 1;
18      sccnt = 0;
19      memset(sccid, 0, sizeof(sccid));
20      ins.reset();
21      vis.reset();
22  }
23
24  inline int no(int u) {
25      return (u > n ? u-n : u+n);
26  }
27
28  int ecnt = 0;
29  inline void clause(int u, int v) {
30      E.eb(no(u)^v);
31      g[no(u)].eb(ecnt++);
32      E.eb(no(v)^u);
33      g[no(v)].eb(ecnt++);
34  }
35
36  void dfs(int u) {
37      in[u] = instp++;
38      low[u] = in[u];
```

```cpp
39      stk.push(u);
40      ins[u] = true;
41
42      Each(e, g[u]) {
43          if (vis[e]) continue;
44          vis[e] = true;
45
46          int v = E[e]^u;
47          if (ins[v]) low[u] = min(low[u], in[v]);
48          else if (!in[v]) {
49              dfs(v);
50              low[u] = min(low[u], low[v]);
51          }
52      }
53
54      if (low[u] == in[u]) {
55          sccnt++;
56          while (!stk.empty()) {
57              int v = stk.top();
58              stk.pop();
59              ins[v] = false;
60              sccid[v] = sccnt;
61              if (u == v) break;
62          }
63      }
64  }
65
66
67  int main() {
68      WiwiHorz
69      init();
70
71      REP(i, m) {
72          char su, sv;
73          int u, v;
74          cin >> su >> u >> sv >> v;
75          if (su == '-') u = no(u);
76          if (sv == '-') v = no(v);
77          clause(u, v);
78      }
79
80      FOR(i, 1, 2*n+1, 1) {
81          if (!in[i]) dfs(i);
82      }
83
84      FOR(u, 1, n+1, 1) {
85          int du = no(u);
86          if (sccid[u] == sccid[du]) {
87              return cout << "IMPOSSIBLE\n", 0;
88          }
89      }
90
91      FOR(u, 1, n+1, 1) {
92          int du = no(u);
93          cout << (sccid[u] < sccid[du] ? '+' : '-') << '
94          ';
95      }
96      cout << endl;
97
98      return 0;
99  }
```

6.5   Eulerian Path - Undir

```cpp
1   // from 1 to n
2   #define gg return cout << "IMPOSSIBLE\n", void();
3
4   int n, m;
5   vector<int> g[maxn];
6   bitset<maxn> inodd;
7
8   void init() {
9   cin >> n >> m;
10  inodd.reset();
11  for (int i = 0; i < m; i++) {
12      int u, v; cin >> u >> v;
13      inodd[u] = inodd[u] ^ true;
14      inodd[v] = inodd[v] ^ true;
15      g[u].emplace_back(v);
16      g[v].emplace_back(u);
17  } }
```

```
18  stack<int> stk;
19  void dfs(int u) {
20      while (!g[u].empty()) {
21          int v = g[u].back();
22          g[u].pop_back();
23          dfs(v);
24      }
25  stk.push(u);}}
```

6.6   Eulerian Path - Dir

```
1   // from node 1 to node n
2   #define gg return cout << "IMPOSSIBLE\n", 0
3
4   int n, m;
5   vector<int> g[maxn];
6   stack<int> stk;
7   int in[maxn], out[maxn];
8
9   void init() {
10  cin >> n >> m;
11  for (int i = 0; i < m; i++) {
12      int u, v; cin >> u >> v;
13      g[u].emplace_back(v);
14      out[u]++, in[v]++;
15  }
16  for (int i = 1; i <= n; i++) {
17      if (i == 1 && out[i]-in[i] != 1) gg;
18      if (i == n && in[i]-out[i] != 1) gg;
19      if (i != 1 && i != n && in[i] != out[i]) gg;
20  } }
21  void dfs(int u) {
22      while (!g[u].empty()) {
23          int v = g[u].back();
24          g[u].pop_back();
25          dfs(v);
26      }
27      stk.push(u);
28  }
29  void solve() {
30      dfs(1)
31      for (int i = 1; i <= n; i++)
32          if ((int)g[i].size()) gg;
33      while (!stk.empty()) {
34          int u = stk.top();
35          stk.pop();
36          cout << u << ' ';
37  } }
```

6.7   Hamilton Path

```
1   // top down DP
2   // Be Aware Of Multiple Edges
3   int n, m;
4   ll dp[maxn][1<<maxn];
5   int adj[maxn][maxn];
6
7   void init() {
8       cin >> n >> m;
9       fill(dp[0], dp[maxn-1]+(1<<maxn), -1);
10  }
11
12  void DP(int i, int msk) {
13      if (dp[i][msk] != -1) return;
14      dp[i][msk] = 0;
15      REP(j, n) if (j != i && (msk & (1<<j)) && adj[j][i
]) {
16          int sub = msk ^ (1<<i);
17          if (dp[j][sub] == -1) DP(j, sub);
18          dp[i][msk] += dp[j][sub] * adj[j][i];
19          if (dp[i][msk] >= MOD) dp[i][msk] %= MOD;
20      }
21  }
22
23
24  int main() {
25      WiwiHorz
26      init();
27
28      REP(i, m) {
```

```
29          int u, v;
30          cin >> u >> v;
31          if (u == v) continue;
32          adj[--u][--v]++;
33      }
34
35      dp[0][1] = 1;
36      FOR(i, 1, n, 1) {
37          dp[i][1] = 0;
38          dp[i][1|(1<<i)] = adj[0][i];
39      }
40      FOR(msk, 1, (1<<n), 1) {
41          if (msk == 1) continue;
42          dp[0][msk] = 0;
43      }
44
45
46      DP(n-1, (1<<n)-1);
47      cout << dp[n-1][(1<<n)-1] << endl;
48
49      return 0;
50  }
```

6.8   Kth Shortest Path

```
1   // time: O(|E| \lg |E|+|V| \lg |V|+K)
2   // memory: O(|E| \lg |E|+|V|)
3   struct KSP{ // 1-base
4     struct nd{
5       int u,v; ll d;
6       nd(int ui=0,int vi=0,ll di=INF){ u=ui; v=vi; d=di;
}
7     };
8     struct heap{ nd* edge; int dep; heap* chd[4]; };
9     static int cmp(heap* a,heap* b)
10    { return a->edge->d > b->edge->d; }
11    struct node{
12      int v; ll d; heap* H; nd* E;
13      node(){}
14      node(ll _d,int _v,nd* _E){ d =_d; v=_v; E=_E; }
15      node(heap* _H,ll _d){ H=_H; d=_d; }
16      friend bool operator<(node a,node b)
17      { return a.d>b.d; }
18    };
19    int n,k,s,t,dst[N]; nd *nxt[N];
20    vector<nd*> g[N],rg[N]; heap *nullNd,*head[N];
21    void init(int _n,int _k,int _s,int _t){
22      n=_n; k=_k; s=_s; t=_t;
23      for(int i=1;i<=n;i++){
24        g[i].clear(); rg[i].clear();
25        nxt[i]=NULL; head[i]=NULL; dst[i]=-1;
26      }
27    }
28    void addEdge(int ui,int vi,ll di){
29      nd* e=new nd(ui,vi,di);
30      g[ui].push_back(e); rg[vi].push_back(e);
31    }
32    queue<int> dfsQ;
33    void dijkstra(){
34      while(dfsQ.size()) dfsQ.pop();
35      priority_queue<node> Q; Q.push(node(0,t,NULL));
36      while (!Q.empty()){
37        node p=Q.top(); Q.pop(); if(dst[p.v]!=-1)continue
;
38        dst[p.v]=p.d; nxt[p.v]=p.E; dfsQ.push(p.v);
39        for(auto e:rg[p.v]) Q.push(node(p.d+e->d,e->u,e))
;
40      }
41    }
42    heap* merge(heap* curNd,heap* newNd){
43      if(curNd==nullNd) return newNd;
44      heap* root=new heap;memcpy(root,curNd,sizeof(heap))
;
45      if(newNd->edge->d<curNd->edge->d){
46        root->edge=newNd->edge;
47        root->chd[2]=newNd->chd[2];
48        root->chd[3]=newNd->chd[3];
49        newNd->edge=curNd->edge;
50        newNd->chd[2]=curNd->chd[2];
51        newNd->chd[3]=curNd->chd[3];
52      }
```

```
53      if(root->chd[0]->dep<root->chd[1]->dep)
54        root->chd[0]=merge(root->chd[0],newNd);
55      else root->chd[1]=merge(root->chd[1],newNd);
56      root->dep=max(root->chd[0]->dep,
57              root->chd[1]->dep)+1;
58      return root;
59    }
60    vector<heap*> V;
61    void build(){
62      nullNd=new heap; nullNd->dep=0; nullNd->edge=new nd
          ;
63      fill(nullNd->chd,nullNd->chd+4,nullNd);
64      while(not dfsQ.empty()){
65        int u=dfsQ.front(); dfsQ.pop();
66        if(!nxt[u]) head[u]=nullNd;
67        else head[u]=head[nxt[u]->v];
68        V.clear();
69        for(auto&& e:g[u]){
70          int v=e->v;
71          if(dst[v]==-1) continue;
72          e->d+=dst[v]-dst[u];
73          if(nxt[u]!=e){
74            heap* p=new heap;fill(p->chd,p->chd+4,nullNd)
                ;
75            p->dep=1; p->edge=e; V.push_back(p);
76          }
77        }
78        if(V.empty()) continue;
79        make_heap(V.begin(),V.end(),cmp);
80  #define L(X) ((X<<1)+1)
81  #define R(X) ((X<<1)+2)
82        for(size_t i=0;i<V.size();i++){
83          if(L(i)<V.size()) V[i]->chd[2]=V[L(i)];
84          else V[i]->chd[2]=nullNd;
85          if(R(i)<V.size()) V[i]->chd[3]=V[R(i)];
86          else V[i]->chd[3]=nullNd;
87        }
88        head[u]=merge(head[u],V.front());
89      }
90    }
91    vector<ll> ans;
92    void first_K(){
93      ans.clear(); priority_queue<node> Q;
94      if(dst[s]==-1) return;
95      ans.push_back(dst[s]);
96      if(head[s]!=nullNd)
97        Q.push(node(head[s],dst[s]+head[s]->edge->d));
98      for(int _=1;_<k and not Q.empty();_++){
99        node p=Q.top(),q; Q.pop(); ans.push_back(p.d);
100       if(head[p.H->edge->v]!=nullNd){
101         q.H=head[p.H->edge->v]; q.d=p.d+q.H->edge->d;
102         Q.push(q);
103       }
104       for(int i=0;i<4;i++)
105         if(p.H->chd[i]!=nullNd){
106           q.H=p.H->chd[i];
107           q.d=p.d-p.H->edge->d+p.H->chd[i]->edge->d;
108           Q.push(q);
109    } } }
110   void solve(){ // ans[i] stores the i-th shortest path
111     dijkstra(); build();
112     first_K(); // ans.size() might less than k
113   }
114 } solver;
```

6.9 System of Difference Constraints

```
1 vector<vector<pair<int, ll>>> G;
2 void add(int u, int v, ll w) {
3     G[u].emplace_back(make_pair(v, w));
4 }
```

- $x_u - x_v \le c \Rightarrow$ add(v, u, c)

- $x_u - x_v \ge c \Rightarrow$ add(u, v, -c)

- $x_u - x_v = c \Rightarrow$ add(v, u, c), add(u, v -c)

- $x_u \ge c \Rightarrow$ add super vertex $x_0 = 0$, then $x_u - x_0 \ge c \Rightarrow$ add(u, 0, -c)

- Don't for get non-negative constraints for every variable if specified implicitly.

- Interval sum $\Rightarrow$ Use prefix sum to transform into differential constraints. Don't for get $S_{i+1} - S_i \ge 0$ if x_i needs to be non-negative.

- $\frac{x_u}{x_v} \le c \Rightarrow \log x_u - \log x_v \le \log c$

7 String

7.1 Rolling Hash

```
1  const ll C = 27;
2  inline int id(char c) {return c-'a'+1;}
3  struct RollingHash {
4      string s; int n; ll mod;
5      vector<ll> Cexp, hs;
6      RollingHash(string& _s, ll _mod):
7          s(_s), n((int)_s.size()), mod(_mod)
8      {
9          Cexp.assign(n, 0);
10         hs.assign(n, 0);
11         Cexp[0] = 1;
12         for (int i = 1; i < n; i++) {
13             Cexp[i] = Cexp[i-1] * C;
14             if (Cexp[i] >= mod) Cexp[i] %= mod;
15         }
16         hs[0] = id(s[0]);
17         for (int i = 1; i < n; i++) {
18             hs[i] = hs[i-1] * C + id(s[i]);
19             if (hs[i] >= mod) hs[i] %= mod;
20     } }
21     inline ll query(int l, int r) {
22         ll res = hs[r] - (l ? hs[l-1] * Cexp[r-l+1] :
              0);
23         res = (res % mod + mod) % mod;
24         return res; }
25 };
```

7.2 Trie

```
1  struct node {
2      int c[26]; ll cnt;
3      node(): cnt(0) {memset(c, 0, sizeof(c));}
4      node(ll x): cnt(x) {memset(c, 0, sizeof(c));}
5  };
6  struct Trie {
7      vector<node> t;
8      void init() {
9          t.clear();
10         t.emplace_back(node());
11     }
12     void insert(string s) { int ptr = 0;
13         for (auto& i : s) {
14             if (!t[ptr].c[i-'a']) {
15                 t.emplace_back(node());
16                 t[ptr].c[i-'a'] = (int)t.size()-1; }
17             ptr = t[ptr].c[i-'a']; }
18         t[ptr].cnt++; }
19 } trie;
```

7.3 KMP

```
1  int n, m;
2  string s, p;
3  vector<int> f;
4  void build() {
5      f.clear(); f.resize(m, 0);
6      int ptr = 0; for (int i = 1; i < m; i++) {
7          while (ptr && p[i] != p[ptr]) ptr = f[ptr-1];
8          if (p[i] == p[ptr]) ptr++;
9          f[i] = ptr;
10 }}
11 void init() {
12     cin >> s >> p;
13     n = (int)s.size();
14     m = (int)p.size();
```

```
15      build(); }
16 void solve() {
17     int ans = 0, pi = 0;
18     for (int si = 0; si < n; si++) {
19         while (pi && s[si] != p[pi]) pi = f[pi-1];
20         if (s[si] == p[pi]) pi++;
21         if (pi == m) ans++, pi = f[pi-1];
22     }
23 cout << ans << endl; }
```

7.4   Z Value

```
1 string is, it, s;
2 int n; vector<int> z;
3 void init() {
4     cin >> is >> it;
5     s = it+'0'+is;
6     n = (int)s.size();
7     z.resize(n, 0); }
8 void solve() {
9     int ans = 0; z[0] = n;
10    for (int i = 1, l = 0, r = 0; i < n; i++) {
11        if (i <= r) z[i] = min(z[i-l], r-i+1);
12        while (i+z[i] < n && s[z[i]] == s[i+z[i]]) z[i
            ]++;
13        if (i+z[i]-1 > r) l = i, r = i+z[i]-1;
14        if (z[i] == (int)it.size()) ans++;
15    }
16    cout << ans << endl; }
```

7.5   Manacher

```
1 int n; string S, s;
2 vector<int> m;
3 void manacher() {
4 s.clear(); s.resize(2*n+1, '.');
5 for (int i = 0, j = 1; i < n; i++, j += 2) s[j] = S[i];
6 m.clear(); m.resize(2*n+1, 0);
7 // m[i] := max k such that s[i-k, i+k] is palindrome
8 int mx = 0, mxk = 0;
9 for (int i = 1; i < 2*n+1; i++) {
10    if (mx-(i-mx) >= 0) m[i] = min(m[mx-(i-mx)], mx+mxk
        -i);
11    while (0 <= i-m[i]-1 && i+m[i]+1 < 2*n+1 &&
12        s[i-m[i]-1] == s[i+m[i]+1]) m[i]++;
13    if (i+m[i] > mx+mxk) mx = i, mxk = m[i];
14 } }
15 void init() { cin >> S; n = (int)S.size(); }
16 void solve() {
17    manacher();
18    int mx = 0, ptr = 0;
19    for (int i = 0; i < 2*n+1; i++) if (mx < m[i])
20        { mx = m[i]; ptr = i; }
21    for (int i = ptr-mx; i <= ptr+mx; i++)
22        if (s[i] != '.') cout << s[i];
23 cout << endl; }
```

7.6   Suffix Array

```
1 #define F first
2 #define S second
3 struct SuffixArray {  // don't forget s += "$";
4     int n; string s;
5     vector<int> suf, lcp, rk;
6     vector<int> cnt, pos;
7     vector<pair<pii, int> > buc[2];
8     void init(string _s) {
9         s = _s; n = (int)s.size();
10 // resize(n): suf, rk, cnt, pos, lcp, buc[0~1]
11    }
12    void radix_sort() {
13        for (int t : {0, 1}) {
14            fill(cnt.begin(), cnt.end(), 0);
15            for (auto& i : buc[t]) cnt[ (t ? i.F.F : i.
                F.S) ]++;
16            for (int i = 0; i < n; i++)
17                pos[i] = (!i ? 0 : pos[i-1] + cnt[i-1])
                    ;
18            for (auto& i : buc[t])
```

```
19                buc[t^1][pos[ (t ? i.F.F : i.F.S) ]++]
                    = i;
20    }}
21    bool fill_suf() {
22        bool end = true;
23        for (int i = 0; i < n; i++) suf[i] = buc[0][i].
            S;
24        rk[suf[0]] = 0;
25        for (int i = 1; i < n; i++) {
26            int dif = (buc[0][i].F != buc[0][i-1].F);
27            end &= dif;
28            rk[suf[i]] = rk[suf[i-1]] + dif;
29        } return end;
30    }
31    void sa() {
32        for (int i = 0; i < n; i++)
33            buc[0][i] = make_pair(make_pair(s[i], s[i])
                , i);
34        sort(buc[0].begin(), buc[0].end());
35        if (fill_suf()) return;
36        for (int k = 0; (1<<k) < n; k++) {
37            for (int i = 0; i < n; i++)
38                buc[0][i] = make_pair(make_pair(rk[i],
                    rk[(i + (1<<k)) % n]), i);
39            radix_sort();
40            if (fill_suf()) return;
41    }}
42    void LCP() {  int k = 0;
43        for (int i = 0; i < n-1; i++) {
44            if (rk[i] == 0) continue;
45            int pi = rk[i];
46            int j = suf[pi-1];
47            while (i+k < n && j+k < n && s[i+k] == s[j+
                k]) k++;
48            lcp[pi] = k;
49            k = max(k-1, 0);
50    }}
51 };
52 SuffixArray suffixarray;
```

7.7   SA-IS

```
1 const int N=300010;
2 struct SA{
3 #define REP(i,n) for(int i=0;i<int(n);i++)
4 #define REP1(i,a,b) for(int i=(a);i<=int(b);i++)
5   bool _t[N*2]; int _s[N*2],_sa[N*2];
6   int _c[N*2],x[N],_p[N],_q[N*2],hei[N],r[N];
7   int operator [](int i){ return _sa[i]; }
8   void build(int *s,int n,int m){
9     memcpy(_s,s,sizeof(int)*n);
10    sais(_s,_sa,_p,_q,_t,_c,n,m); mkhei(n);
11  }
12  void mkhei(int n){
13    REP(i,n) r[_sa[i]]=i;
14    hei[0]=0;
15    REP(i,n) if(r[i]) {
16      int ans=i>0?max(hei[r[i-1]]-1,0):0;
17      while(_s[i+ans]==_s[_sa[r[i]-1]+ans]) ans++;
18      hei[r[i]]=ans;
19    }
20  }
21  void sais(int *s,int *sa,int *p,int *q,bool *t,int *c
        ,int n,int z){
22    bool uniq=t[n-1]=true,neq;
23    int nn=0,nmxz=-1,*nsa=sa+n,*ns=s+n,lst=-1;
24 #define MS0(x,n) memset((x),0,n*sizeof(*(x)))
25 #define MAGIC(XD) MS0(sa,n);\
26 memcpy(x,c,sizeof(int)*z); XD;\
27 memcpy(x+1,c,sizeof(int)*(z-1));\
28 REP(i,n) if(sa[i]&&!t[sa[i]-1]) sa[x[s[sa[i]-1]]++]=sa[
    i]-1;\
29 memcpy(x,c,sizeof(int)*z);\
30 for(int i=n-1;i>=0;i--) if(sa[i]&&t[sa[i]-1]) sa[--x[s[
    sa[i]-1]]]=sa[i]-1;
31    MS0(c,z); REP(i,n) uniq&=++c[s[i]]<2;
32    REP(i,z-1) c[i+1]+=c[i];
33    if(uniq) { REP(i,n) sa[--c[s[i]]]=i; return; }
34    for(int i=n-2;i>=0;i--)
35      t[i]=(s[i]==s[i+1]?t[i+1]:s[i]<s[i+1]);
```

```
36    MAGIC(REP1(i,1,n-1) if(t[i]&&!t[i-1]) sa[--x[s[i
         ]]]=p[q[i]=nn++]=i);
37    REP(i,n) if(sa[i]&&t[sa[i]]&&!t[sa[i]-1]){
38      neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
           [i])*sizeof(int));
39      ns[q[lst=sa[i]]]=nmxz+=neq;
40    }
41    sais(ns,nsa,p+nn,q+n,t+n,c+z,nn,nmxz+1);
42    MAGIC(for(int i=nn-1;i>=0;i--) sa[--x[s[p[nsa[i
         ]]]]]=p[nsa[i]]);
43  }
44 }sa;
45 int H[N],SA[N],RA[N];
46 void suffix_array(int* ip,int len){
47   // should padding a zero in the back
48   // ip is int array, len is array length
49   // ip[0..n-1] != 0, and ip[len]=0
50   ip[len++]=0; sa.build(ip,len,128);
51   memcpy(H,sa.hei+1,len<<2); memcpy(SA,sa._sa+1,len<<2)
        ;
52   for(int i=0;i<len;i++) RA[i]=sa.r[i]-1;
53   // resulting height, sa array \in [0,len)
54 }
```

7.8   Minimum Rotation

```
1 //rotate(begin(s), begin(s)+minRotation(s), end(s))
2 int minRotation(string s) {
3 int a = 0, n = s.size(); s += s;
4 for(int b = 0; b < n; b++) for(int k = 0; k < n; k++) {
5     if(a + k == b ||| s[a + k] < s[b + k]) {
6         b += max(0, k - 1);
7         break; }
8     if(s[a + k] > s[b + k]) {
9         a = b;
10        break;
11    } }
12 return a; }
```

7.9   Aho Corasick

```
1 struct ACautomata{
2   struct Node{
3     int cnt;
4     Node *go[26], *fail, *dic;
5     Node (){
6       cnt = 0; fail = 0; dic=0;
7       memset(go,0,sizeof(go));
8     }
9   }pool[1048576],*root;
10  int nMem;
11  Node* new_Node(){
12    pool[nMem] = Node();
13    return &pool[nMem++];
14  }
15  void init() { nMem = 0; root = new_Node(); }
16  void add(const string &str) { insert(root,str,0); }
17  void insert(Node *cur, const string &str, int pos){
18    for(int i=pos;i<str.size();i++){
19      if(!cur->go[str[i]-'a'])
20        cur->go[str[i]-'a'] = new_Node();
21      cur=cur->go[str[i]-'a'];
22    }
23    cur->cnt++;
24  }
25  void make_fail(){
26    queue<Node*> que;
27    que.push(root);
28    while (!que.empty()){
29      Node* fr=que.front(); que.pop();
30      for (int i=0; i<26; i++){
31        if (fr->go[i]){
32          Node *ptr = fr->fail;
33          while (ptr && !ptr->go[i]) ptr = ptr->fail;
34          fr->go[i]->fail=ptr=(ptr?ptr->go[i]:root);
35          fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
36          que.push(fr->go[i]);
37  } } } }
38 }AC;
```

8   Geometry

8.1   Basic Operations

```
1 typedef long long T;
2 // typedef long double T;
3 const long double eps = 1e-8;
4
5 short sgn(T x) {
6     if (abs(x) < eps) return 0;
7     return x < 0 ? -1 : 1;
8 }
9
10 struct Pt {
11 T x, y;
12 Pt(T _x=0, T _y=0):x(_x), y(_y) {}
13 Pt operator+(Pt a) { return Pt(x+a.x, y+a.y); }
14 Pt operator-(Pt a) { return Pt(x-a.x, y-a.y); }
15 Pt operator*(T a)  { return Pt(x*a, y*a); }
16 Pt operator/(T a)  { return Pt(x/a, y/a); }
17 T operator*(Pt a)  { return x*a.x + y*a.y; }
18 T operator^(Pt a)  { return x*a.y - y*a.x; }
19 bool operator<(Pt a)
20     { return x < a.x || (x == a.x && y < a.y); }
21 //return sgn(x-a.x) < 0 || (sgn(x-a.x) == 0 && sgn(y-a.
       y) < 0); }
22 bool operator==(Pt a)
23     { return sgn(x-a.x) == 0 && sgn(y-a.y) == 0; }
24 };
25
26 Pt mv(Pt a, Pt b) { return b-a; }
27 T len2(Pt a) { return a*a; }
28 T dis2(Pt a, Pt b) { return len2(b-a); }
29
30 short ori(Pt a, Pt b) { return ((a^b)>0) - ((a^b)<0); }
31 bool onseg(Pt p, Pt l1, Pt l2) {
32     Pt a = mv(p, l1), b = mv(p, l2);
33     return ((a^b) == 0) && ((a*b) <= 0);
34 }
```

8.2   InPoly

```
1 short inPoly(Pt p) {
2 // 0=Bound 1=In -1=Out
3 REP(i, n) if (onseg(p, E[i], E[(i+1)%n])) return 0;
4 int cnt = 0;
5 REP(i, n) if (banana(p, Pt(p.x+1, p.y+2e9),
6                     E[i], E[(i+1)%n])) cnt ^= 1;
7 return (cnt ? 1 : -1);
8 }
```

8.3   Sort by Angle

```
1 int ud(Pt a) {  // up or down half plane
2     if (a.y > 0) return 0;
3     if (a.y < 0) return 1;
4     return (a.x >= 0 ? 0 : 1);
5 }
6 sort(ALL(E), [&](const Pt& a, const Pt& b){
7     if (ud(a) != ud(b)) return ud(a) < ud(b);
8     return (a^b) > 0;
9 });
```

8.4   Line Intersect Check

```
1 inline bool banana(Pt p1, Pt p2, Pt q1, Pt q2) {
2 if (onseg(p1, q1, q2) || onseg(p2, q1, q2) ||
3     onseg(q1, p1, p2) || onseg(q2, p1, p2)) {
4     return true;
5 }
6 Pt p = mv(p1, p2), q = mv(q1, q2);
7 return (ori(p, mv(p1, q1)) * ori(p, mv(p1, q2)) < 0 &&
8         ori(q, mv(q1, p1)) * ori(q, mv(q1, p2)) < 0);
9 }
```

8.5   Line Intersection

```cpp
// T: long double
Pt bananaPoint(Pt p1, Pt p2, Pt q1, Pt q2) {
if (onseg(q1, p1, p2)) return q1;
if (onseg(q2, p1, p2)) return q2;
if (onseg(p1, q1, q2)) return p1;
if (onseg(p2, q1, q2)) return p2;
double s = abs(mv(p1, p2) ^ mv(p1, q1));
double t = abs(mv(p1, p2) ^ mv(p1, q2));
return q2 * (s/(s+t)) + q1 * (t/(s+t));
}
```

8.6   Convex Hull

```cpp
vector<Pt> hull;
void convexHull() {
hull.clear(); sort(ALL(E));
REP(t, 2) {
    int b = SZ(hull);
    Each(ei, E) {
        while (SZ(hull) - b >= 2 &&
                ori(mv(hull[SZ(hull)-2], hull.back()),
                    mv(hull[SZ(hull)-2], ei)) == -1) {
            hull.pop_back();
        }
        hull.eb(ei);
    }
    hull.pop_back();
    reverse(ALL(E));
} }
```

8.7   Lower Concave Hull

```cpp
struct Line {
  mutable ll m, b, p;
  bool operator<(const Line& o) const { return m < o.m;
        }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = inf; return false; }
    if (x->m == y->m) x->p = x->b > y->b ? inf : -inf;
    else x->p = div(y->b - x->b, x->m - y->m);
    return x->p >= y->p;
  }
  void add(ll m, ll b) {
    auto z = insert({m, b, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y =
        erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.m * x + l.b;
  }
};
```

8.8   Polygon Area

```cpp
T dbarea(vector<Pt>& e) {
ll res = 0;
REP(i, SZ(e)) res += e[i]^e[(i+1)%SZ(e)];
return abs(res);
}
```

8.9   Pick's Theorem

Consider a polygon which vertices are all lattice points.
Let i = number of points inside the polygon.
Let b = number of points on the boundary of the polygon.

Then we have the following formula:

$$Area = i + \frac{b}{2} - 1$$

8.10   Minimum Enclosing Circle

```cpp
Pt circumcenter(Pt A, Pt B, Pt C) {
// a1(x-A.x) + b1(y-A.y) = c1
// a2(x-A.x) + b2(y-A.y) = c2
// solve using Cramer's rule
T a1 = B.x-A.x, b1 = B.y-A.y, c1 = dis2(A, B)/2.0;
T a2 = C.x-A.x, b2 = C.y-A.y, c2 = dis2(A, C)/2.0;
T D  = Pt(a1, b1) ^ Pt(a2, b2);
T Dx = Pt(c1, b1) ^ Pt(c2, b2);
T Dy = Pt(a1, c1) ^ Pt(a2, c2);
if (D == 0) return Pt(-INF, -INF);
return A + Pt(Dx/D, Dy/D);
}
Pt center; T r2;
void minEncloseCircle() {
mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count());
shuffle(ALL(E), gen);
center = E[0], r2 = 0;

for (int i = 0; i < n; i++) {
    if (dis2(center, E[i]) <= r2) continue;
    center = E[i], r2 = 0;
    for (int j = 0; j < i; j++) {
        if (dis2(center, E[j]) <= r2) continue;
        center = (E[i] + E[j]) / 2.0;
        r2 = dis2(center, E[i]);
        for (int k = 0; k < j; k++) {
            if (dis2(center, E[k]) <= r2) continue;
            center = circumcenter(E[i], E[j], E[k]);
            r2 = dis2(center, E[i]);
        }
    }
} }
```

8.11   PolyUnion

```cpp
struct PY{
  int n; Pt pt[5]; double area;
  Pt& operator[](const int x){ return pt[x]; }
  void init(){ //n,pt[0~n-1] must be filled
    area=pt[n-1]^pt[0];
    for(int i=0;i<n-1;i++) area+=pt[i]^pt[i+1];
    if((area/=2)<0)reverse(pt,pt+n),area=-area;
  }
};
PY py[500]; pair<double,int> c[5000];
inline double segP(Pt &p,Pt &p1,Pt &p2){
  if(dcmp(p1.x-p2.x)==0) return (p.y-p1.y)/(p2.y-p1.y);
  return (p.x-p1.x)/(p2.x-p1.x);
}
double polyUnion(int n){ //py[0~n-1] must be filled
  int i,j,ii,jj,ta,tb,r,d; double z,w,s,sum=0,tc,td;
  for(i=0;i<n;i++) py[i][py[i].n]=py[i][0];
  for(i=0;i<n;i++){
    for(ii=0;ii<py[i].n;ii++){
      r=0;
      c[r++]=make_pair(0.0,0); c[r++]=make_pair(1.0,0);
      for(j=0;j<n;j++){
        if(i==j) continue;
        for(jj=0;jj<py[j].n;jj++){
          ta=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj]))
              ;
          tb=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj
              +1]));
          if(ta==0 && tb==0){
            if((py[j][jj+1]-py[j][jj])*(py[i][ii+1]-py[
                i][ii])>0&&j<i){
              c[r++]=make_pair(segP(py[j][jj],py[i][ii
                  ],py[i][ii+1]),1);
              c[r++]=make_pair(segP(py[j][jj+1],py[i][
                  ii],py[i][ii+1]),-1);
            }
          }else if(ta>=0 && tb<0){
```

```cpp
                 tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
                 td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
                 c[r++]=make_pair(tc/(tc-td),1);
             }else if(ta<0 && tb>=0){
                 tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
                 td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
                 c[r++]=make_pair(tc/(tc-td),-1);
         } } }
         sort(c,c+r);
         z=min(max(c[0].first,0.0),1.0); d=c[0].second; s
            =0;
         for(j=1;j<r;j++){
           w=min(max(c[j].first,0.0),1.0);
           if(!d) s+=w-z;
           d+=c[j].second; z=w;
         }
         sum+=(py[i][ii]^py[i][ii+1])*s;
      }
   }
   return sum/2;
}
```

8.12 Minkowski Sum

```cpp
/* convex hull Minkowski Sum*/
#define INF 100000000000000LL
int pos( const Pt& tp ){
  if( tp.Y == 0 ) return tp.X > 0 ? 0 : 1;
  return tp.Y > 0 ? 0 : 1;
}
#define N 300030
Pt pt[ N ], qt[ N ], rt[ N ];
LL Lx,Rx;
int dn,un;
inline bool cmp( Pt a, Pt b ){
    int pa=pos( a ),pb=pos( b );
    if(pa==pb) return (a^b)>0;
    return pa<pb;
}
int minkowskiSum(int n,int m){
    int i,j,r,p,q,fi,fj;
    for(i=1,p=0;i<n;i++){
        if( pt[i].Y<pt[p].Y ||
        (pt[i].Y==pt[p].Y && pt[i].X<pt[p].X) ) p=i; }
    for(i=1,q=0;i<m;i++){
        if( qt[i].Y<qt[q].Y ||
        (qt[i].Y==qt[q].Y && qt[i].X<qt[q].X) ) q=i; }
    rt[0]=pt[p]+qt[q];
    r=1; i=p; j=q; fi=fj=0;
    while(1){
        if((fj&&j==q) ||
        ( (!fi||i!=p) &&
          cmp(pt[(p+1)%n]-pt[p],qt[(q+1)%m]-qt[q]) ) ){
            rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
            p=(p+1)%n;
            fi=1;
        }else{
            rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
            q=(q+1)%m;
            fj=1;
        }
        if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))
            !=0) r++;
        else rt[r-1]=rt[r];
        if(i==p && j==q) break;
    }
    return r-1;
}
void initInConvex(int n){
    int i,p,q;
    LL Ly,Ry;
    Lx=INF; Rx=-INF;
    for(i=0;i<n;i++){
        if(pt[i].X<Lx) Lx=pt[i].X;
        if(pt[i].X>Rx) Rx=pt[i].X;
    }
    Ly=Ry=INF;
    for(i=0;i<n;i++){
        if(pt[i].X==Lx && pt[i].Y<Ly){ Ly=pt[i].Y; p=i;
            }
        if(pt[i].X==Rx && pt[i].Y<Ry){ Ry=pt[i].Y; q=i;
            }
    }
    for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
    qt[dn]=pt[q]; Ly=Ry=-INF;
    for(i=0;i<n;i++){
        if(pt[i].X==Lx && pt[i].Y>Ly){ Ly=pt[i].Y; p=i;
            }
        if(pt[i].X==Rx && pt[i].Y>Ry){ Ry=pt[i].Y; q=i;
            }
    }
    for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
    rt[un]=pt[q];
}
inline int inConvex(Pt p){
    int L,R,M;
    if(p.X<Lx || p.X>Rx) return 0;
    L=0;R=dn;
    while(L<R-1){ M=(L+R)/2;
        if(p.X<qt[M].X) R=M; else L=M; }
    if(tri(qt[L],qt[R],p)<0) return 0;
    L=0;R=un;
    while(L<R-1){ M=(L+R)/2;
        if(p.X<rt[M].X) R=M; else L=M; }
    if(tri(rt[L],rt[R],p)>0) return 0;
        return 1;
}
int main(){
    int n,m,i;
    Pt p;
    scanf("%d",&n);
    for(i=0;i<n;i++) scanf("%lld%lld",&pt[i].X,&pt[i].Y
        );
    scanf("%d",&m);
    for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y
        );
    n=minkowskiSum(n,m);
    for(i=0;i<n;i++) pt[i]=rt[i];
    scanf("%d",&m);
    for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y
        );
    n=minkowskiSum(n,m);
    for(i=0;i<n;i++) pt[i]=rt[i];
    initInConvex(n);
    scanf("%d",&m);
    for(i=0;i<m;i++){
        scanf("%lld %lld",&p.X,&p.Y);
        p.X*=3; p.Y*=3;
        puts(inConvex(p)?"YES":"NO");
    }
}
```

9   Number Theory

9.1   Pollard's rho

```python
from itertools import count
from math import gcd
from sys import stdin

for s in stdin:
    number, x = int(s), 2
    break2 = False
    for cycle in count(1):
        y = x
        if break2:
            break
        for i in range(1 << cycle):
            x = (x * x + 1) % number
            factor = gcd(x - y, number)
            if factor > 1:
                print(factor)
                break2 = True
                break
```

9.2   Miller Rabin

```
// n < 4,759,123,141        3 : 2, 7, 61
// n < 1,122,004,669,633    4 : 2, 13, 23, 1662803
```

```
3  // n < 3,474,749,660,383          6 :  pirmes <= 13
4  // n < 2^64                        7 :
5  // 2, 325, 9375, 28178, 450775, 9780504, 1795265022
6  bool witness(ll a,ll n,ll u,int t){
7      if(!(a%=n)) return 0;
8      ll x=mypow(a,u,n);
9      for(int i=0;i<t;i++) {
10         ll nx=mul(x,x,n);
11         if(nx==1&&x!=1&&x!=n-1) return 1;
12         x=nx;
13     }
14     return x!=1;
15 }
16 bool miller_rabin(ll n,int s=100) {
17     // iterate s times of witness on n
18     // return 1 if prime, 0 otherwise
19     if(n<2) return 0;
20     if(!(n&1)) return n == 2;
21     ll u=n-1; int t=0;
22     while(!(u&1)) u>>=1, t++;
23     while(s--){
24         ll a=randll()%(n-1)+1;
25         if(witness(a,n,u,t)) return 0;
26     }
27     return 1;
28 }
```

9.3   Fast Power

Note: $a^n \equiv a^{(n \bmod (p-1))} (mod\ p)$

9.4   Extend GCD

```
1  ll GCD;
2  pll extgcd(ll a, ll b) {
3      if (b == 0) {
4          GCD = a;
5          return pll{1, 0};
6      }
7      pll ans = extgcd(b, a % b);
8      return pll{ans.S, ans.F - a/b * ans.S};
9  }
10 pll bezout(ll a, ll b, ll c) {
11     bool negx = (a < 0), negy = (b < 0);
12     pll ans = extgcd(abs(a), abs(b));
13     if (c % GCD != 0) return pll{-LLINF, -LLINF};
14     return pll{ans.F * c/GCD * (negx ? -1 : 1),
15                ans.S * c/GCD * (negy ? -1 : 1)};
16 }
17 ll inv(ll a, ll p) {
18     if (p == 1) return -1;
19     pll ans = bezout(a % p, -p, 1);
20     if (ans == pll{-LLINF, -LLINF}) return -1;
21     return (ans.F % p + p) % p;
22 }
```

9.5   Mu + Phi

```
1  const int maxn = 1e6 + 5;
2  ll f[maxn];
3  vector<int> lpf, prime;
4  void build() {
5  lpf.clear(); lpf.resize(maxn, 1);
6  prime.clear();
7  f[1] = ...;  /* mu[1] = 1, phi[1] = 1 */
8  for (int i = 2; i < maxn; i++) {
9      if (lpf[i] == 1) {
10         lpf[i] = i; prime.emplace_back(i);
11         f[i] = ...;  /* mu[i] = 1, phi[i] = i-1 */
12     }
13     for (auto& j : prime) {
14         if (i*j >= maxn) break;
15         lpf[i*j] = j;
16         if (i % j == 0) f[i*j] = ...;  /* 0, phi[i]*j
                */
17         else f[i*j] = ...;  /* -mu[i], phi[i]*phi[j] */
18         if (j >= lpf[i]) break;
19 } } }
```

9.6   Other Formulas

- Inversion:
  $aa^{-1} \equiv 1 \pmod{m}$. a^{-1} exists iff $\gcd(a, m) = 1$.

- Linear inversion:
  $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod{m}$

- Fermat's little theorem:
  $a^p \equiv a \pmod{p}$ if p is prime.

- Euler function:
  $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$

- Euler theorem:
  $a^{\phi(n)} \equiv 1 \pmod{n}$ if $\gcd(a, n) = 1$.

- Extended Euclidean algorithm:
  $ax + by = \gcd(a, b) = \gcd(b, a \bmod b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$

- Divisor function:
  $\sigma_x(n) = \sum_{d|n} d^x$. $n = \prod_{i=1}^{r} p_i^{a_i}$.
  $\sigma_x(n) = \prod_{i=1}^{r} \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$ if $x \neq 0$. $\sigma_0(n) = \prod_{i=1}^{r}(a_i + 1)$.

- Chinese remainder theorem (Coprime Moduli):
  $x \equiv a_i \pmod{m_i}$.
  $M = \prod m_i$. $M_i = M/m_i$. $t_i = M_i^{-1}$.
  $x = kM + \sum a_i t_i M_i, k \in \mathbb{Z}$.

- Chinese remainder theorem:
  $x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2} \Rightarrow x = m_1 p + a_1 = m_2 q + a_2 \Rightarrow m_1 p - m_2 q = a_2 - a_1$
  Solve for (p, q) using ExtGCD.
  $x \equiv m_1 p + a_1 \equiv m_2 q + a_2 \pmod{lcm(m_1, m_2)}$

- Avoiding Overflow: $ca \bmod cb = c(a \bmod b)$

- Dirichlet Convolution: $(f * g)(n) = \sum_{d|n} f(n)g(n/d)$

- Important Multiplicative Functions + Proterties:
  1. $\epsilon(n) = [n = 1]$
  2. $1(n) = 1$
  3. $id(n) = n$
  4. $\mu(n) = 0$ if n has squared prime factor
  5. $\mu(n) = (-1)^k$ if $n = p_1 p_2 \cdots p_k$
  6. $\epsilon = \mu * 1$
  7. $\phi = \mu * id$
  8. $[n = 1] = \sum_{d|n} \mu(d)$
  9. $[gcd = 1] = \sum_{d|gcd} \mu(d)$

- Möbius inversion: $f = g * 1 \Leftrightarrow g = f * \mu$

9.7   Polynomial

```
1  const int maxk = 20;
2  const int maxn = 1<<maxk;
3  const ll LINF = 1e18;
4
5  /* P = r*2^k + 1
6  P                    r    k    g
7  998244353            119  23   3
8  1004535809           479  21   3
9
10 P                    r    k    g
11 3                    1    1    2
12 5                    1    2    2
13 17                   1    4    3
14 97                   3    5    5
15 193                  3    6    5
16 257                  1    8    3
17 7681                 15   9    17
```

```
18  12289                3    12   11
19  40961                5    13   3
20  65537                1    16   3
21  786433               3    18   10
22  5767169              11   19   3
23  7340033              7    20   3
24  23068673             11   21   3
25  104857601            25   22   3
26  167772161            5    25   3
27  469762049            7    26   3
28  1004535809           479  21   3
29  2013265921           15   27   31
30  2281701377           17   27   3
31  3221225473           3    30   5
32  75161927681          35   31   3
33  77309411329          9    33   7
34  206158430209         3    36   22
35  2061584302081        15   37   7
36  2748779069441        5    39   3
37  6597069766657        3    41   5
38  39582418599937       9    42   5
39  79164837199873       9    43   5
40  263882790666241      15   44   7
41  1231453023109121     35   45   3
42  1337006139375617     19   46   3
43  3799912185593857     27   47   5
44  4222124650659841     15   48   19
45  7881299347898369     7    50   6
46  31525197391593473    7    52   3
47  180143985094819841   5    55   6
48  1945555039024054273  27   56   5
49  4179340454199820289  29   57   3
50  9097271247288401921  505  54   6  */
51
52  const int g = 3;
53  const ll MOD = 998244353;
54
55  ll pw(ll a, ll n) { /* fast pow */ }
56
57  #define siz(x) (int)x.size()
58
59  template<typename T>
60  vector<T>& operator+=(vector<T>& a, const vector<T>& b)
        {
61      if (siz(a) < siz(b)) a.resize(siz(b));
62      for (int i = 0; i < min(siz(a), siz(b)); i++) {
63          a[i] += b[i];
64          a[i] -= a[i] >= MOD ? MOD : 0;
65      }
66      return a;
67  }
68
69  template<typename T>
70  vector<T>& operator-=(vector<T>& a, const vector<T>& b)
        {
71      if (siz(a) < siz(b)) a.resize(siz(b));
72      for (int i = 0; i < min(siz(a), siz(b)); i++) {
73          a[i] -= b[i];
74          a[i] += a[i] < 0 ? MOD : 0;
75      }
76      return a;
77  }
78
79  template<typename T>
80  vector<T> operator-(const vector<T>& a) {
81      vector<T> ret(siz(a));
82      for (int i = 0; i < siz(a); i++) {
83          ret[i] = -a[i] < 0 ? -a[i] + MOD : -a[i];
84      }
85      return ret;
86  }
87
88  vector<ll> X, iX;
89  vector<int> rev;
90
91  void init_ntt() {
92      X.clear(); X.resize(maxn, 1);  // x1 = g^((p-1)/n)
93      iX.clear(); iX.resize(maxn, 1);
94
95      ll u = pw(g, (MOD-1)/maxn);
96      ll iu = pw(u, MOD-2);
97
98      for (int i = 1; i < maxn; i++) {
99          X[i] = X[i-1] * u;
100         iX[i] = iX[i-1] * iu;
101         if (X[i] >= MOD) X[i] %= MOD;
102         if (iX[i] >= MOD) iX[i] %= MOD;
103     }
104
105     rev.clear(); rev.resize(maxn, 0);
106     for (int i = 1, hb = -1; i < maxn; i++) {
107         if (!(i & (i-1))) hb++;
108         rev[i] = rev[i ^ (1<<hb)] | (1<<(maxk-hb-1));
109 } }
110
111 template<typename T>
112 void NTT(vector<T>& a, bool inv=false) {
113
114     int _n = (int)a.size();
115     int k = __lg(_n) + ((1<<__lg(_n)) != _n);
116     int n = 1<<k;
117     a.resize(n, 0);
118
119     short shift = maxk-k;
120     for (int i = 0; i < n; i++)
121         if (i > (rev[i]>>shift))
122             swap(a[i], a[rev[i]>>shift]);
123
124     for (int len = 2, half = 1, div = maxn>>1; len <= n
        ; len<<=1, half<<=1, div>>=1) {
125         for (int i = 0; i < n; i += len) {
126             for (int j = 0; j < half; j++) {
127                 T u = a[i+j];
128                 T v = a[i+j+half] * (inv ? iX[j*div] :
                        X[j*div]) % MOD;
129                 a[i+j] = (u+v >= MOD ? u+v-MOD : u+v);
130                 a[i+j+half] = (u-v < 0 ? u-v+MOD : u-v)
                        ;
131 } } }
132
133     if (inv) {
134         T dn = pw(n, MOD-2);
135         for (auto& x : a) {
136             x *= dn;
137             if (x >= MOD) x %= MOD;
138 } } }
139
140 template<typename T>
141 inline void resize(vector<T>& a) {
142     int cnt = (int)a.size();
143     for (; cnt > 0; cnt--) if (a[cnt-1]) break;
144     a.resize(max(cnt, 1));
145 }
146
147 template<typename T>
148 vector<T>& operator*=(vector<T>& a, vector<T> b) {
149     int na = (int)a.size();
150     int nb = (int)b.size();
151     a.resize(na + nb - 1, 0);
152     b.resize(na + nb - 1, 0);
153
154     NTT(a); NTT(b);
155     for (int i = 0; i < (int)a.size(); i++) {
156         a[i] *= b[i];
157         if (a[i] >= MOD) a[i] %= MOD;
158     }
159     NTT(a, true);
160
161     resize(a);
162     return a;
163 }
164
165 template<typename T>
166 void inv(vector<T>& ia, int N) {
167     vector<T> _a(move(ia));
168     ia.resize(1, pw(_a[0], MOD-2));
169     vector<T> a(1, -_a[0] + (-_a[0] < 0 ? MOD : 0));
170
171     for (int n = 1; n < N; n<<=1) {
172         // n -> 2*n
173         // ia' = ia(2-a*ia);
174
175         for (int i = n; i < min(siz(_a), (n<<1)); i++)
```

```
176        a.emplace_back(-_a[i] + (-_a[i] < 0 ? MOD :
              0));

177
178        vector<T> tmp = ia;
179        ia *= a;
180        ia.resize(n<<1);
181        ia[0] = ia[0] + 2 >= MOD ? ia[0] + 2 - MOD : ia
              [0] + 2;
182        ia *= tmp;
183        ia.resize(n<<1);
184    }
185    ia.resize(N);
186 }

187
188 template<typename T>
189 void mod(vector<T>& a, vector<T>& b) {
190    int n = (int)a.size()-1, m = (int)b.size()-1;
191    if (n < m) return;

192
193    vector<T> ra = a, rb = b;
194    reverse(ra.begin(), ra.end()); ra.resize(min(n+1, n
           -m+1));
195    reverse(rb.begin(), rb.end()); rb.resize(min(m+1, n
           -m+1));

196
197    inv(rb, n-m+1);

198
199    vector<T> q = move(ra);
200    q *= rb;
201    q.resize(n-m+1);
202    reverse(q.begin(), q.end());

203
204    q *= b;
205    a -= q;
206    resize(a);
207 }

208
209 /* Kitamasa Method (Fast Linear Recurrence):
210 Find a[K] (Given a[j] = c[0]a[j-N] + ... + c[N-1]a[j
          -1])
211 Let B(x) = x^N - c[N-1]x^(N-1) - ... - c[1]x^1 - c[0]
212 Let R(x) = x^K mod B(x)    (get x^K using fast pow and
          use poly mod to get R(x))
213 Let r[i] = the coefficient of x^i in R(x)
214 => a[K] = a[0]r[0] + a[1]r[1] + ... + a[N-1]r[N-1] */
```

10   Linear Algebra

10.1   Gaussian-Jordan Elimination

```
1 int n; vector<vector<ll> > v;
2 void gauss(vector<vector<ll>>& v) {
3 int r = 0;
4 for (int i = 0; i < n; i++) {
5    bool ok = false;
6    for (int j = r; j < n; j++) {
7        if (v[j][i] == 0) continue;
8        swap(v[j], v[r]);
9        ok = true; break;
10   }
11   if (!ok) continue;
12   ll div = inv(v[r][i]);
13   for (int j = 0; j < n+1; j++) {
14       v[r][j] *= div;
15       if (v[r][j] >= MOD) v[r][j] %= MOD;
16   }
17   for (int j = 0; j < n; j++) {
18       if (j == r) continue;
19       ll t = v[j][i];
20       for (int k = 0; k < n+1; k++) {
21           v[j][k] -= v[r][k] * t % MOD;
22           if (v[j][k] < 0) v[j][k] += MOD;
23       } }
24   r++;
25 } }
```

10.2   Determinant

1. Use GJ Elimination, if there's any row consists of only 0, then det = 0, otherwise det = product of diagonal elements.

2. Properties of det:

   - Transpose: Unchanged
   - Row Operation 1 - Swap 2 rows: $-det$
   - Row Operation 2 - $k\overrightarrow{r_i}$: $k \times det$
   - Row Operation 3 - $k\overrightarrow{r_i}$ add to $\overrightarrow{r_j}$: Unchaged

11   Flow / Matching

11.1   Dinic

```
1 struct Dinic
2 {
3    int n,s,t,level[N],iter[N];
4    struct edge{int to,cap,rev;};
5    vector<edge>path[N];
6    void init(int _n,int _s,int _t)
7    {
8        n=_n,s=_s,t=_t;
9        FOR(i,0,n+1)path[i].clear();
10   }
11   void add(int a,int b,int c)
12   {
13       edge now;
14       now.to=b,now.cap=c,now.rev=sz(path[b]);
15       path[a].pb(now);
16       now.to=a,now.cap=0,now.rev=sz(path[a])-1;
17       path[b].pb(now);
18   }
19   void bfs()
20   {
21       memset(level,-1,sizeof(level));
22       level[s]=0;
23       queue<int>q;q.push(s);
24       while(q.size())
25       {
26           int now=q.front();q.pop();
27           for(edge e:path[now])
28           {
29               if(e.cap>0&&level[e.to]==-1)
30               {
31                   level[e.to]=level[now]+1;
32                   q.push(e.to);
33               }
34           }
35       }
36   }
37   int dfs(int now,int flow)
38   {
39       if(now==t)return flow;
40       for(int &i=iter[now];i<sz(path[now]);i++)
41       {
42           edge &e=path[now][i];
43           if(e.cap>0&&level[e.to]==level[now]+1)
44           {
45               int res=dfs(e.to,min(flow,e.cap));
46               if(res>0)
47               {
48                   e.cap-=res;
49                   path[e.to][e.rev].cap+=res;
50                   return res;
51               }
52           }
53       }
54       return 0;
55   }
56   int dinic()
57   {
58       int res=0;
59       while(true)
60       {
61           bfs();
62           if(level[t]==-1)break;
63           memset(iter,0,sizeof(iter));
64           int now=0;
```

```
65          while((now=dfs(s,INF))>0)res+=now;
66      }
67      return res;
68    }
69 };
```

11.2 ISAP

```
1  #define SZ(c) ((int)(c).size())
2  struct Maxflow{
3    static const int MAXV=50010;
4    static const int INF =1000000;
5    struct Edge{
6      int v,c,r;
7      Edge(int _v,int _c,int _r):v(_v),c(_c),r(_r){}
8    };
9    int s,t; vector<Edge> G[MAXV];
10   int iter[MAXV],d[MAXV],gap[MAXV],tot;
11   void init(int n,int _s,int _t){
12     tot=n,s=_s,t=_t;
13     for(int i=0;i<=tot;i++){
14       G[i].clear(); iter[i]=d[i]=gap[i]=0;
15     }
16   }
17   void addEdge(int u,int v,int c){
18     G[u].push_back(Edge(v,c,SZ(G[v])));
19     G[v].push_back(Edge(u,0,SZ(G[u])-1));
20   }
21   int DFS(int p,int flow){
22     if(p==t) return flow;
23     for(int &i=iter[p];i<SZ(G[p]);i++){
24       Edge &e=G[p][i];
25       if(e.c>0&&d[p]==d[e.v]+1){
26         int f=DFS(e.v,min(flow,e.c));
27         if(f){ e.c-=f; G[e.v][e.r].c+=f; return f; }
28       }
29     }
30     if((--gap[d[p]])==0) d[s]=tot;
31     else{ d[p]++; iter[p]=0; ++gap[d[p]]; }
32     return 0;
33   }
34   int flow(){
35     int res=0;
36     for(res=0,gap[0]=tot;d[s]<tot;res+=DFS(s,INF));
37     return res;
38   } // reset: set iter,d,gap to 0
39 } flow;
```

11.3 MCMF

```
1  struct MCMF
2  {
3      int n,s,t,par[N+5],p_i[N+5],dis[N+5],vis[N+5];
4      struct edge{int to,cap,rev,cost;};
5      vector<edge>path[N];
6      void init(int _n,int _s,int _t)
7      {
8          n=_n,s=_s,t=_t;
9          FOR(i,0,2*n+5)par[i]=p_i[i]=vis[i]=0;
10     }
11     void add(int a,int b,int c,int d)
12     {
13         path[a].pb({b,c,sz(path[b]),d});
14         path[b].pb({a,0,sz(path[a])-1,-d});
15     }
16     void spfa()
17     {
18         FOR(i,0,n*2+5)dis[i]=INF,vis[i]=0;
19         dis[s]=0;
20         queue<int>q;q.push(s);
21         while(!q.empty())
22         {
23             int now=q.front();
24             q.pop();
25             vis[now]=0;
26             for(int i=0;i<sz(path[now]);i++)
27             {
28                 edge e=path[now][i];
29                 if(e.cap>0&&dis[e.to]>dis[now]+e.cost)
30                 {
31                     dis[e.to]=dis[now]+e.cost;
32                     par[e.to]=now;
33                     p_i[e.to]=i;
34                     if(vis[e.to]==0)
35                     {
36                         vis[e.to]=1;
37                         q.push(e.to);
38                     }
39                 }
40             }
41         }
42     }
43     pii flow()
44     {
45         int flow=0,cost=0;
46         while(true)
47         {
48             spfa();
49             if(dis[t]==INF)break;
50             int mn=INF;
51             for(int i=t;i!=s;i=par[i])
52                 mn=min(mn,path[par[i]][p_i[i]].cap);
53             flow+=mn;cost+=dis[t]*mn;
54             for(int i=t;i!=s;i=par[i])
55             {
56                 edge &now=path[par[i]][p_i[i]];
57                 now.cap-=mn;
58                 path[i][now.rev].cap+=mn;
59             }
60         }
61         return mp(flow,cost);
62     }
63 };
```

11.4 Hopcroft-Karp

```
1  struct HopcroftKarp {
2      // id: X = [1, nx], Y = [nx+1, nx+ny]
3      int n, nx, ny, m, MXCNT;
4      vector<vector<int> > g;
5      vector<int> mx, my, dis, vis;
6      void init(int nnx, int nny, int mm) {
7          nx = nnx, ny = nny, m = mm;
8          n = nx + ny + 1;
9          g.clear(); g.resize(n);
10     }
11     void add(int x, int y) {
12         g[x].emplace_back(y);
13         g[y].emplace_back(x);
14     }
15     bool dfs(int x) {
16         vis[x] = true;
17         Each(y, g[x]) {
18             int px = my[y];
19             if (px == -1 ||
20                 (dis[px] == dis[x]+1 &&
21                 !vis[px] && dfs(px))) {
22                 mx[x] = y;
23                 my[y] = x;
24                 return true;
25             }
26         }
27         return false;
28     }
29     void get() {
30         mx.clear(); mx.resize(n, -1);
31         my.clear(); my.resize(n, -1);
32
33         while (true) {
34             queue<int> q;
35             dis.clear(); dis.resize(n, -1);
36             for (int x = 1; x <= nx; x++){
37                 if (mx[x] == -1) {
38                     dis[x] = 0;
39                     q.push(x);
40                 }
41             }
42             while (!q.empty()) {
43                 int x = q.front(); q.pop();
44                 Each(y, g[x]) {
```

```
45              if (my[y] != -1 && dis[my[y]] ==
                    -1) {
46                  dis[my[y]] = dis[x] + 1;
47                  q.push(my[y]);
48              }
49          }
50      }

52      bool brk = true;
53      vis.clear(); vis.resize(n, 0);
54      for (int x = 1; x <= nx; x++)
55          if (mx[x] == -1 && dfs(x))
56              brk = false;

58      if (brk) break;
59      }
60      MXCNT = 0;
61      for (int x = 1; x <= nx; x++) if (mx[x] != -1)
            MXCNT++;
62  }
63 } hk;
```

11.5   Cover / Independent Set

```
1 V(E) Cover: choose some V(E) to cover all E(V)
2 V(E) Independ: set of V(E) not adj to each other
3
4 M = Max Matching
5 Cv = Min V Cover
6 Ce = Min E Cover
7 Iv = Max V Ind
8 Ie = Max E Ind (equiv to M)
9
10 M = Cv (Konig Theorem)
11 Iv = V \ Cv
12 Ce = V - M
13
14 Construct Cv:
15 1. Run Dinic
16 2. Find s-t min cut
17 3. Cv = {X in T} + {Y in S}
```

11.6   KM

```
1 struct KM
2 {
3     int n,mx[1005],my[1005],pa[1005];
4     int g[1005][1005],lx[1005],ly[1005],sy[1005];
5     bool vx[1005],vy[1005];
6     void init(int _n)
7     {
8         n=_n;
9         FOR(i,1,n+1)fill(g[i],g[i]+1+n,0);
10    }
11    void add(int a,int b,int c){g[a][b]=c;}
12    void augment(int y)
13    {
14        for(int x,z;y;y=z)
15            x=pa[y],z=mx[x],my[y]=x,mx[x]=y;
16    }
17    void bfs(int st)
18    {
19        FOR(i,1,n+1)sy[i]=INF,vx[i]=vy[i]=0;
20        queue<int>q;q.push(st);
21        for(;;)
22        {
23            while(!q.empty())
24            {
25                int x=q.front();q.pop();
26                vx[x]=1;
27                FOR(y,1,n+1)if(!vy[y])
28                {
29                    int t=lx[x]+ly[y]-g[x][y];
30                    if(t==0)
31                    {
32                        pa[y]=x;
33                        if(!my[y]){augment(y);return;}
34                        vy[y]=1, q.push(my[y]);
35                    }
36                    else if(sy[y]>t)pa[y]=x,sy[y]=t;
37                }
38            }
39            int cut=INF;
40            FOR(y,1,n+1)if(!vy[y]&&cut>sy[y])cut=sy[y];
41            FOR(j,1,n+1)
42            {
43                if(vx[j])lx[j]-=cut;
44                if(vy[j])ly[j]+=cut;
45                else sy[j]-=cut;
46            }
47            FOR(y,1,n+1)
48            {
49                if(!vy[y]&&sy[y]==0)
50                {
51                    if(!my[y]){augment(y);return;}
52                    vy[y]=1;q.push(my[y]);
53                }
54            }
55        }
56    }
57    int solve()
58    {
59        fill(mx,mx+n+1,0);fill(my,my+n+1,0);
60        fill(ly,ly+n+1,0);fill(lx,lx+n+1,0);
61        FOR(x,1,n+1)FOR(y,1,n+1)
62            lx[x]=max(lx[x],g[x][y]);
63        FOR(x,1,n+1)bfs(x);
64        int ans=0;
65        FOR(y,1,n+1)ans+=g[my[y]][y];
66        return ans;
67    }
68 };
```

12   Combinatorics

12.1   Catalan Number

$$C_0 = 1, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}, C_n = C_n^{2n} - C_{n-1}^{2n}$$

0	1	1	2	5
4	14	42	132	429
8	1430	4862	16796	58786
12	208012	742900	2674440	9694845

12.2   Burnside's Lemma

Let X be the original set.
Let G be the group of operations acting on X.
Let X^g be the set of x not affected by g.
Let X/G be the set of orbits.
Then the following equation holds:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

13   Special Numbers

13.1   Fibonacci Series

1	1	1	2	3
5	5	8	13	21
9	34	55	89	144
13	233	377	610	987
17	1597	2584	4181	6765
21	10946	17711	28657	46368
25	75025	121393	196418	317811
29	514229	832040	1346269	2178309
33	3524578	5702887	9227465	14930352

$f(45) \approx 10^9, f(88) \approx 10^{18}$

13.2   Prime Numbers

- First 50 prime numbers:

1	2	3	5	7	11
6	13	17	19	23	29
11	31	37	41	43	47
16	53	59	61	67	71
21	73	79	83	89	97
26	101	103	107	109	113
31	127	131	137	139	149
36	151	157	163	167	173
41	179	181	191	193	197
46	199	211	223	227	229

- Very large prime numbers:

  1000001333   1000500889   2500001909
  2000000659   900004151   850001359

- $\pi(n) \equiv$ Number of primes $\leq n \approx n/((\ln n) - 1)$
  $\pi(100) = 25, \pi(200) = 46$
  $\pi(500) = 95, \pi(1000) = 168$
  $\pi(2000) = 303, \pi(4000) = 550$
  $\pi(10^4) = 1229, \pi(10^5) = 9592$
  $\pi(10^6) = 78498, \pi(10^7) = 664579$