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## 1 Reminder

### 1.1 Bug List

- 沒開 long long
- 陣列戳出界 / 開不夠大 / 開太大本地 compile 噴怪 error
- 傳之前先確定選對檔案
- 寫好的函式忘記呼叫
- 變數打錯
- 0-base / 1-base
- 忘記初始化
- == 打成 =
- <= 打成 <+
- dp[i] 從 dp[i-1] 轉移時忘記特判 i > 0
- std::sort 比較運算子寫成 < 或是讓 = 的情況為 true
- 漏 case / 分 case 要好好想
- 線段樹改值懶標初始值不能設為 0
- DFS 的時候不小心覆寫到全域變數
- 浮點數誤差
- 多筆測資不能沒讀完直接 return
- 記得刪 cerr

### 1.2 OwO

- 可以構造複雜點的測資幫助思考
- 真的卡太久請跳題
- Enjoy The Contest!

## 2 Basic

### 2.1 Vimrc

```
set number relativenumber ai t_Co=256 tabstop=4
set mouse=a shiftwidth=4 encoding=utf8
set bs=2 ruler laststatus=2 cmdheight=2
set clipboard=unnamedplus showcmd autoread
set belloff=all
filetype indent on

inoremap ( (<Esc>i
inoremap " "<Esc>i
inoremap [ [<Esc>i
inoremap ' '<Esc>i
inoremap { {<CR><Esc>ko

nnoremap <tab> gt
nnoremap <S-tab> gT
inoremap <C-n> <Esc>:tabnew<CR>
nnoremap <C-n> :tabnew<CR>

inoremap <F9> <Esc>:w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>
nnoremap <F9> :w<CR>:!~/runcpp.sh %:p:t %:p:h<CR>

syntax on
colorscheme desert
set filetype=cpp
set background=dark
hi Normal ctermfg=white ctermbg=black
```

### 2.2 Runcpp.sh

```
#!/bin/bash
clear
echo "Start compiling $1..."
echo
g++ -O2 -std=c++20 -Wall -Wextra -Wshadow $2/$1 -o $2/
out
if [ "$?" -ne 0 ]
then
    exit 1
fi
echo
echo "Done compiling"
echo "===== "
echo "Input file:"
echo
cat $2/in.txt
echo
echo "===== "
echo
declare startTime=`date +%s%N`
$2/out < $2/in.txt > $2/out.txt
declare endTime=`date +%s%N`
delta=`expr $endTime - $startTime`
delta=`expr $delta / 1000000`
cat $2/out.txt
echo
echo "time: $delta ms"
```

### 2.3 PBDS

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

// map
tree<int, int, less<>, rb_tree_tag,
    tree_order_statistics_node_update> tr;
tr.order_of_key(element);
tr.find_by_order(rank);

// set
tree<int, null_type, less<>, rb_tree_tag,
    tree_order_statistics_node_update> tr;
tr.order_of_key(element);
tr.find_by_order(rank);

// hash table
gp_hash_table<int, int> ht;
ht.find(element);
```

```

17 ht.insert({key, value});
18 ht.erase(element);
19
20 // priority queue
21 __gnu_pbds::priority_queue<int, less<int>> big_q;
22 // Big First
23 __gnu_pbds::priority_queue<int, greater<int>> small_q;
24 // Small First
25 q1.join(q2); // join

```

## 2.4 Random

```

1 mt19937 gen(chrono::steady_clock::now().
   time_since_epoch().count());
2 uniform_int_distribution<int> dis(1, 100);
3 cout << dis(gen) << endl;
4 shuffle(v.begin(), v.end(), gen);

```

## 3 Data Structure

### 3.1 BIT

```

1 struct BIT {
2     int n;
3     long long bit[N];
4
5     void init(int x, vector<long long> &a) {
6         n = x;
7         for (int i = 1, j; i <= n; i++) {
8             bit[i] += a[i - 1], j = i + (i & -i);
9             if (j <= n) bit[j] += bit[i];
10        }
11    }
12
13    void update(int x, long long dif) {
14        while (x <= n) bit[x] += dif, x += x & -x;
15    }
16
17    long long query(int l, int r) {
18        if (l != 1) return query(1, r) - query(1, l - 1);
19
20        long long ret = 0;
21        while (l <= r) ret += bit[r], r -= r & -r;
22        return ret;
23    }
24 } bm;

```

### 3.2 DSU

```

1 struct DSU {
2     int h[N], s[N];
3
4     void init(int n) { iota(h, h + n + 1, 0), fill(s, s + n + 1, 1); }
5
6     int fh(int x) { return (h[x] == x ? x : h[x] = fh(h[x])); }
7
8     bool mer(int x, int y) {
9         x = fh(x), y = fh(y);
10        if (x == y) return 0;
11        if (s[x] < s[y]) swap(x, y);
12        s[x] += s[y], s[y] = 0;
13        h[y] = x;
14        return 1;
15    }
16 } bm;

```

### 3.3 Segment Tree

```

1 struct segtree {
2     int n, seg[1 << 19];
3
4     void init(int x) {
5         n = 1 << (lg(x) + 1);
6         for (int i = 1; i < 2 * n; i++)
7             seg[i] = inf;
8     }
9
10    void update(int x, int val) {

```

```

11        x += n;
12        seg[x] = val, x /= 2;
13        while (x)
14            seg[x] = min(seg[2 * x], seg[2 * x + 1]), x /= 2;
15    }
16
17    int query(int l, int r) {
18        l += n, r += n;
19        int ret = inf;
20        while (l < r) {
21            if (l & 1)
22                ret = min(ret, seg[l++]);
23            if (r & 1)
24                ret = min(ret, seg[--r]);
25            l /= 2, r /= 2;
26        }
27        return ret;
28    }
29 } bm;

```

### 3.4 Treap

```

1 mt19937 rng(random_device{}());
2 struct Treap {
3     Treap *l, *r;
4     int val, num, pri;
5     Treap(int k) {
6         l = r = NULL;
7         val = k;
8         num = 1;
9         pri = rng();
10    }
11};
12 int siz(Treap *now) { return now ? now->num : 0; }
13 void pull(Treap *&now) {
14     now->num = siz(now->l) + siz(now->r) + 1;
15 }
16 Treap *merge(Treap *a, Treap *b) {
17     if (!a || !b)
18         return a ? a : b;
19     else if (a->pri > b->pri) {
20         a->r = merge(a->r, b);
21         pull(a);
22         return a;
23     } else {
24         b->l = merge(a, b->l);
25         pull(b);
26         return b;
27     }
28 }
29 void split_size(Treap *rt, Treap *&a, Treap *&b, int val) {
30     if (!rt) {
31         a = b = NULL;
32         return;
33     }
34     if (siz(rt->l) + 1 > val) {
35         b = rt;
36         split_size(rt->l, a, b->l, val);
37         pull(b);
38     } else {
39         a = rt;
40         split_size(rt->r, a->r, b, val - siz(a->l) - 1);
41         pull(a);
42     }
43 }
44 void split_val(Treap *rt, Treap *&a, Treap *&b, int val) {
45     if (!rt) {
46         a = b = NULL;
47         return;
48     }
49     if (rt->val <= val) {
50         a = rt;
51         split_val(rt->r, a->r, b, val);
52         pull(a);
53     } else {
54         b = rt;
55         split_val(rt->l, a, b->l, val);
56         pull(b);

```

```

57     }
58 }
59 void treap_dfs(Treap *now) {
60     if (!now) return;
61     treap_dfs(now->l);
62     cout << now->val << " ";
63     treap_dfs(now->r);
64 }

```

### 3.5 Persistent Treap

```

1 struct node {
2     node *l, *r;
3     char c;
4     int v, sz;
5     node(char x = '$') : c(x), v(mt()), sz(1) {
6         l = r = nullptr;
7     }
8     node(node* p) { *this = *p; }
9     void pull() {
10         sz = 1;
11         for (auto i : {l, r})
12             if (i) sz += i->sz;
13     }
14 } arr[maxn], *ptr = arr;
15 inline int size(node* p) { return p ? p->sz : 0; }
16 node* merge(node* a, node* b) {
17     if (!a || !b) return a ? b;
18     if (a->v < b->v) {
19         node* ret = new (ptr++) node(a);
20         ret->r = merge(ret->r, b); ret->pull();
21         return ret;
22     } else {
23         node* ret = new (ptr++) node(b);
24         ret->l = merge(a, ret->l); ret->pull();
25         return ret;
26     }
27 }
28 P<node*> split(node* p, int k) {
29     if (!p) return {nullptr, nullptr};
30     if (k >= size(p->l) + 1) {
31         auto [a, b] = split(p->r, k - size(p->l) - 1);
32         node* ret = new (ptr++) node(p);
33         ret->r = a; ret->pull();
34         return {ret, b};
35     } else {
36         auto [a, b] = split(p->l, k);
37         node* ret = new (ptr++) node(p);
38         ret->l = b; ret->pull();
39         return {a, ret};
40     }
41 }

```

### 3.6 Li Chao Tree

```

1 constexpr int maxn = 5e4 + 5;
2 struct line {
3     ld a, b;
4     ld operator()(ld x) { return a * x + b; }
5 } arr[(maxn + 1) << 2];
6 bool operator<(line a, line b) { return a.a < b.a; }
7 #define m ((l + r) >> 1)
8 void insert(line x, int i = 1, int l = 0, int r = maxn) {
9     if (r - l == 1) {
10         if (x(l) > arr[i](l))
11             arr[i] = x;
12         return;
13     }
14     line a = max(arr[i], x), b = min(arr[i], x);
15     if (a(m) > b(m))
16         arr[i] = a, insert(b, i << 1, l, m);
17     else
18         arr[i] = b, insert(a, i << 1 | 1, m, r);
19 }
20 ld query(int x, int i = 1, int l = 0, int r = maxn) {
21     if (x < l || r <= x) return -numeric_limits<ld>::
22         max();
23     if (r - l == 1) return arr[i](x);
24     return max({arr[i](x), query(x, i << 1, l, m),
25         query(x, i << 1 | 1, m, r)});

```

```
25 #undef m
```

### 3.7 Sparse Table

```

1 const int lgmx = 19;
2
3 int n, q;
4 int spt[lgmx][maxn];
5
6 void build() {
7     FOR(k, 1, lgmx, 1) {
8         for (int i = 0; i + (1 << k) - 1 < n; i++) {
9             spt[k][i] = min(spt[k - 1][i], spt[k - 1][i
10                 + (1 << (k - 1))]);
11         }
12     }
13 }
14 int query(int l, int r) {
15     int ln = len(l, r);
16     int lg = __lg(ln);
17     return min(spt[lg][l], spt[lg][r - (1 << lg) + 1]);
18 }

```

### 3.8 Time Segment Tree

```

1 constexpr int maxn = 1e5 + 5;
2 V<P<int>> arr[(maxn + 1) << 2];
3 V<int> dsu, sz;
4 V<tuple<int, int, int>> his;
5 int cnt, q;
6 int find(int x) {
7     return x == dsu[x] ? x : find(dsu[x]);
8 };
9 inline bool merge(int x, int y) {
10     int a = find(x), b = find(y);
11     if (a == b) return false;
12     if (sz[a] > sz[b]) swap(a, b);
13     his.emplace_back(a, b, sz[b]), dsu[a] = b, sz[b] +=
14         sz[a];
15     return true;
16 };
17 inline void undo() {
18     auto [a, b, s] = his.back();
19     his.pop_back();
20     dsu[a] = a, sz[b] = s;
21 }
22 #define m ((l + r) >> 1)
23 void insert(int ql, int qr, P<int> x, int i = 1, int l
24     = 0, int r = q) {
25     // debug(ql, qr, x); return;
26     if (qr <= l || r <= ql) return;
27     if (ql <= l && r <= qr) {
28         arr[i].push_back(x);
29         return;
30     }
31     if (qr <= m)
32         insert(ql, qr, x, i << 1, l, m);
33     else if (m <= ql)
34         insert(ql, qr, x, i << 1 | 1, m, r);
35     else {
36         insert(ql, qr, x, i << 1, l, m);
37         insert(ql, qr, x, i << 1 | 1, m, r);
38     }
39 }
40 void traversal(V<int>& ans, int i = 1, int l = 0, int r
41     = q) {
42     int opcnt = 0;
43     // debug(i, l, r);
44     for (auto [a, b] : arr[i])
45         if (merge(a, b))
46             opcnt++, cnt--;
47     if (r - l == 1)
48         ans[l] = cnt;
49     else {
50         traversal(ans, i << 1, l, m);
51         traversal(ans, i << 1 | 1, m, r);
52     }
53     while (opcnt--)
54         undo(), cnt++;
55     arr[i].clear();

```

```

54 #undef m
55 inline void solve() {
56     int n, m;
57     cin >> n >> m >> q, q++;
58     dsu.resize(cnt = n), sz.assign(n, 1);
59     iota(dsu.begin(), dsu.end(), 0);
60     // a, b, time, operation
61     unordered_map<ll, V<int>> s;
62     for (int i = 0; i < m; i++) {
63         int a, b;
64         cin >> a >> b;
65         if (a > b) swap(a, b);
66         s[((ll)a << 32) | b].emplace_back(0);
67     }
68     for (int i = 1; i < q; i++) {
69         int op, a, b;
70         cin >> op >> a >> b;
71         if (a > b) swap(a, b);
72         switch (op) {
73             case 1:
74                 s[((ll)a << 32) | b].push_back(i);
75                 break;
76             case 2:
77                 auto tmp = s[((ll)a << 32) | b].back();
78                 s[((ll)a << 32) | b].pop_back();
79                 insert(tmp, i, P<int>{a, b});
80             }
81     }
82     for (auto [p, v] : s) {
83         int a = p >> 32, b = p & -1;
84         while (v.size()) {
85             insert(v.back(), q, P<int>{a, b});
86             v.pop_back();
87         }
88     }
89     V<int> ans(q);
90     traversal(ans);
91     for (auto i : ans)
92         cout << i << ' ';
93     cout << endl;
94 }

```

## 4 Flow / Matching

### 4.1 Dinic

```

1 struct Dinic {
2     int n, s, t, level[N], iter[N];
3     struct edge {
4         int to, cap, rev;
5     };
6     vector<edge> path[N];
7     void init(int _n, int _s, int _t) {
8         n = _n, s = _s, t = _t;
9         FOR(i, 0, n + 1)
10             path[i].clear();
11     }
12     void add(int a, int b, int c) {
13         edge now;
14         now.to = b, now.cap = c, now.rev = sz(path[b]);
15         path[a].pb(now);
16         now.to = a, now.cap = 0, now.rev = sz(path[a]) - 1;
17         path[b].pb(now);
18     }
19     void bfs() {
20         memset(level, -1, sizeof(level));
21         level[s] = 0;
22         queue<int> q;
23         q.push(s);
24         while (q.size()) {
25             int now = q.front();
26             q.pop();
27             for (edge e : path[now]) {
28                 if (e.cap > 0 && level[e.to] == -1) {
29                     level[e.to] = level[now] + 1;
30                     q.push(e.to);
31                 }
32             }
33         }
34     }

```

```

35     int dfs(int now, int flow) {
36         if (now == t) return flow;
37         for (int &i = iter[now]; i < sz(path[now]); i++) {
38             edge &e = path[now][i];
39             if (e.cap > 0 && level[e.to] == level[now] + 1) {
40                 int res = dfs(e.to, min(flow, e.cap));
41                 if (res > 0) {
42                     e.cap -= res;
43                     path[e.to][e.rev].cap += res;
44                     return res;
45                 }
46             }
47         }
48         return 0;
49     }
50     int dinic() {
51         int res = 0;
52         while (true) {
53             bfs();
54             if (level[t] == -1) break;
55             memset(iter, 0, sizeof(iter));
56             int now = 0;
57             while ((now = dfs(s, INF)) > 0) res += now;
58         }
59         return res;
60     }
61 };

```

### 4.2 MCMF

```

1 struct MCMF {
2     int n, s, t, par[N + 5], p_i[N + 5], dis[N + 5],
3     vis[N + 5];
4     struct edge {
5         int to, cap, rev, cost;
6     };
7     vector<edge> path[N];
8     void init(int _n, int _s, int _t) {
9         n = _n, s = _s, t = _t;
10        FOR(i, 0, 2 * n + 5)
11            par[i] = p_i[i] = vis[i] = 0;
12    }
13    void add(int a, int b, int c, int d) {
14        path[a].pb({b, c, sz(path[b]), d});
15        path[b].pb({a, 0, sz(path[a]) - 1, -d});
16    }
17    void spfa() {
18        FOR(i, 0, n * 2 + 5)
19            dis[i] = INF,
20            vis[i] = 0;
21        dis[s] = 0;
22        queue<int> q;
23        q.push(s);
24        while (!q.empty()) {
25            int now = q.front();
26            q.pop();
27            vis[now] = 0;
28            for (int i = 0; i < sz(path[now]); i++) {
29                edge e = path[now][i];
30                if (e.cap > 0 && dis[e.to] > dis[now] + e.cost) {
31                    dis[e.to] = dis[now] + e.cost;
32                    par[e.to] = now;
33                    p_i[e.to] = i;
34                    if (vis[e.to] == 0) {
35                        vis[e.to] = 1;
36                        q.push(e.to);
37                    }
38                }
39            }
40        }
41    }
42    pii flow() {
43        int flow = 0, cost = 0;
44        while (true) {
45            spfa();
46            if (dis[t] == INF) break;
47            int mn = INF;
48            for (int i = t; i != s; i = par[i])

```

```

49     mn = min(mn, path[par[i]][p_i[i]].cap);
50     flow += mn;
51     cost += dis[t] * mn;
52     for (int i = t; i != s; i = par[i]) {
53         edge &now = path[par[i]][p_i[i]];
54         now.cap -= mn;
55         path[i][now.rev].cap += mn;
56     }
57     return mp(flow, cost);
58 }
59 };
60

```

### 4.3 KM

```

1 struct KM {
2     int n, mx[1005], my[1005], pa[1005];
3     int g[1005][1005], lx[1005], ly[1005], sy[1005];
4     bool vx[1005], vy[1005];
5     void init(int _n) {
6         n = _n;
7         FOR(i, 1, n + 1)
8             fill(g[i], g[i] + 1 + n, 0);
9     }
10    void add(int a, int b, int c) { g[a][b] = c; }
11    void augment(int y) {
12        for (int x, z; y; y = z)
13            x = pa[y], z = mx[x], my[y] = x, mx[x] = y;
14    }
15    void bfs(int st) {
16        FOR(i, 1, n + 1)
17            sy[i] = INF;
18        vx[st] = vy[st] = 0;
19        queue<int> q;
20        q.push(st);
21        for (;;) {
22            while (!q.empty()) {
23                int x = q.front();
24                q.pop();
25                vx[x] = 1;
26                FOR(y, 1, n + 1)
27                    if (!vy[y]) {
28                        int t = lx[x] + ly[y] - g[x][y];
29                        if (t == 0) {
30                            pa[y] = x;
31                            if (!my[y]) {
32                                augment(y);
33                                return;
34                            }
35                            vy[y] = 1, q.push(my[y]);
36                        } else if (sy[y] > t)
37                            pa[y] = x, sy[y] = t;
38                    }
39            }
40            int cut = INF;
41            FOR(y, 1, n + 1)
42                if (!vy[y] && cut > sy[y]) cut = sy[y];
43            FOR(j, 1, n + 1) {
44                if (vx[j]) lx[j] -= cut;
45                if (vy[j]) ly[j] += cut;
46            }
47            else
48                sy[j] -= cut;
49        }
50        FOR(y, 1, n + 1) {
51            if (!vy[y] && sy[y] == 0) {
52                if (!my[y]) {
53                    augment(y);
54                    return;
55                }
56                vy[y] = 1;
57                q.push(my[y]);
58            }
59        }
60    }
61    int solve() {
62        fill(mx, mx + n + 1, 0);
63        fill(my, my + n + 1, 0);
64        fill(ly, ly + n + 1, 0);
65        fill(lx, lx + n + 1, 0);
66        FOR(x, 1, n + 1)

```

```

68        FOR(y, 1, n + 1)
69            lx[x] = max(lx[x], g[x][y]);
70        FOR(x, 1, n + 1)
71            bfs(x);
72        int ans = 0;
73        FOR(y, 1, n + 1)
74            ans += g[my[y]][y];
75        return ans;
76    }
77 };

```

### 4.4 Hopcroft-Karp

```

1 struct HopcroftKarp {
2     // id: X = [1, nx], Y = [nx+1, nx+ny]
3     int n, nx, ny, m, MXCNT;
4     vector<vector<int>> g;
5     vector<int> mx, my, dis, vis;
6     void init(int nnx, int nny, int mm) {
7         nx = nnx, ny = nny, m = mm;
8         n = nx + ny + 1;
9         g.clear();
10        g.resize(n);
11    }
12    void add(int x, int y) {
13        g[x].emplace_back(y);
14        g[y].emplace_back(x);
15    }
16    bool dfs(int x) {
17        vis[x] = true;
18        Each(y, g[x]) {
19            int px = my[y];
20            if (px == -1 ||
21                (dis[px] == dis[x] + 1 &&
22                 !vis[px] && dfs(px))) {
23                mx[x] = y;
24                my[y] = x;
25                return true;
26            }
27        }
28        return false;
29    }
30    void get() {
31        mx.clear();
32        mx.resize(n, -1);
33        my.clear();
34        my.resize(n, -1);
35    }
36    while (true) {
37        queue<int> q;
38        dis.clear();
39        dis.resize(n, -1);
40        for (int x = 1; x <= nx; x++) {
41            if (mx[x] == -1) {
42                dis[x] = 0;
43                q.push(x);
44            }
45        }
46        while (!q.empty()) {
47            int x = q.front();
48            q.pop();
49            Each(y, g[x]) {
50                if (my[y] != -1 && dis[my[y]] ==
51                    -1) {
52                    dis[my[y]] = dis[x] + 1;
53                    q.push(my[y]);
54                }
55            }
56        }
57        bool brk = true;
58        vis.clear();
59        vis.resize(n, 0);
60        for (int x = 1; x <= nx; x++)
61            if (mx[x] == -1 && dfs(x))
62                brk = false;
63        if (brk) break;
64    }
65    MXCNT = 0;
66    for (int x = 1; x <= nx; x++)
67        if (mx[x] != -1) MXCNT++;
68

```

```
69 }
70 } hk;
```

## 4.5 Blossom

```
1 const int N=5e2+10;
2 struct Graph{
3     int to[N],bro[N],head[N],e;
4     int lnk[N],vis[N],stp,n;
5     void init(int _n){
6         stp=0;e=1;n=_n;
7         FOR(i,0,n+1)head[i]=lnk[i]=vis[i]=0;
8     }
9     void add(int u,int v){
10        to[e]=v,bro[e]=head[u],head[u]=e++;
11        to[e]=u,bro[e]=head[v],head[v]=e++;
12    }
13    bool dfs(int x){
14        vis[x]=stp;
15        for(int i=head[x];i;i=bro[i])
16        {
17            int v=to[i];
18            if(!lnk[v])
19            {
20                lnk[x]=v;lnk[v]=x;
21                return true;
22            }
23            else if(vis[lnk[v]]<stp)
24            {
25                int w=lnk[v];
26                lnk[x]=v,lnk[v]=x,lnk[w]=0;
27                if(dfs(w))return true;
28                lnk[w]=v,lnk[v]=w,lnk[x]=0;
29            }
30        }
31        return false;
32    }
33    int solve(){
34        int ans=0;
35        FOR(i,1,n+1){
36            if(!lnk[i]){
37                stp++;
38                ans+=dfs(i);
39            }
40        }
41        return ans;
42    }
43    void print_matching(){
44        FOR(i,1,n+1)
45            if(i<graph.lnk[i])
46                cout<<i<<" "<<graph.lnk[i]<<endl;
47    }
48 };
```

## 4.6 Weighted Blossom

```
1 struct WeightGraph { // 1-based
2     static const int inf = INT_MAX;
3     static const int maxn = 514;
4     struct edge {
5         int u, v, w;
6         edge() {}
7         edge(int u, int v, int w) : u(u), v(v), w(w) {}
8     };
9     int n, n_x;
10    edge g[maxn * 2][maxn * 2];
11    int lab[maxn * 2];
12    int match[maxn * 2], slack[maxn * 2], st[maxn * 2],
13        pa[maxn * 2];
14    int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
15        maxn * 2];
16    vector<int> flo[maxn * 2];
17    queue<int> q;
18    int e_delta(const edge &e) { return lab[e.u] + lab[
19        e.v] - g[e.u][e.v].w * 2; }
20    void update_slack(int u, int x) {
21        if (!slack[x] || e_delta(g[u][x]) < e_delta(g[
22            slack[x]][x])) slack[x] = u;
23    }
24    void set_slack(int x) {
25        slack[x] = 0;
26        for (int u = 1; u <= n; ++u)
```

```
23         if (g[u][x].w > 0 && st[u] != x && S[st[u]]
24             == 0)
25             update_slack(u, x);
26     }
27     void q_push(int x) {
28         if (x <= n)
29             q.push(x);
30         else
31             for (size_t i = 0; i < flo[x].size(); i++)
32                 q_push(flo[x][i]);
33     }
34     void set_st(int x, int b) {
35         st[x] = b;
36         if (x > n)
37             for (size_t i = 0; i < flo[x].size(); ++i)
38                 set_st(flo[x][i], b);
39     }
40     int get_pr(int b, int xr) {
41         int pr = find(flo[b].begin(), flo[b].end(), xr)
42             - flo[b].begin();
43         if (pr % 2 == 1) {
44             reverse(flo[b].begin() + 1, flo[b].end());
45             return (int)flo[b].size() - pr;
46         }
47         return pr;
48     }
49     void set_match(int u, int v) {
50         match[u] = g[u][v].v;
51         if (u <= n) return;
52         edge e = g[u][v];
53         int xr = flo_from[u][e.u], pr = get_pr(u, xr);
54         for (int i = 0; i < pr; ++i) set_match(flo[u][i
55             ], flo[u][i ^ 1]);
56         set_match(xr, v);
57         rotate(flo[u].begin(), flo[u].begin() + pr, flo
58             [u].end());
59     }
60     void augment(int u, int v) {
61         for (;;) {
62             int xnv = st[match[u]];
63             set_match(u, v);
64             if (!xnv) return;
65             set_match(xnv, st[pa[xnv]]);
66             u = st[pa[xnv]], v = xnv;
67         }
68     }
69     int get_lca(int u, int v) {
70         static int t = 0;
71         for (++t; u || v; swap(u, v)) {
72             if (u == 0) continue;
73             if (vis[u] == t) return u;
74             vis[u] = t;
75             u = st[match[u]];
76             if (u) u = st[pa[u]];
77         }
78         return 0;
79     }
80     void add_blossom(int u, int lca, int v) {
81         int b = n + 1;
82         while (b <= n_x && st[b]) ++b;
83         if (b > n_x) ++n_x;
84         lab[b] = 0, S[b] = 0;
85         match[b] = match[lca];
86         flo[b].clear();
87         flo[b].push_back(lca);
88         for (int x = u, y; x != lca; x = st[pa[y]])
89             flo[b].push_back(x), flo[b].push_back(y =
90                 st[match[x]]), q_push(y);
91         reverse(flo[b].begin() + 1, flo[b].end());
92         for (int x = v, y; x != lca; x = st[pa[y]])
93             flo[b].push_back(x), flo[b].push_back(y =
94                 st[match[x]]), q_push(y);
95         set_st(b, b);
96         for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x
97             ][b].w = 0;
98         for (int x = 1; x <= n; ++x) flo_from[b][x] =
99             0;
100        for (size_t i = 0; i < flo[b].size(); ++i) {
101            int xs = flo[b][i];
102            for (int x = 1; x <= n_x; ++x)
103                if (g[b][x].w == 0 || e_delta(g[xs][x])
104                    < e_delta(g[b][x]))
```



```

94         g[b][x] = g[xs][x], g[x][b] = g[x][xs];
95         for (int x = 1; x <= n; ++x)
96             if (flo_from[xs][x]) flo_from[b][x] = xs;
97     }
98     set_slack(b);
99 }
100 void expand_blossom(int b) {
101     for (size_t i = 0; i < flo[b].size(); ++i)
102         set_st(flo[b][i], flo[b][i]);
103     int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b, xr);
104     for (int i = 0; i < pr; i += 2) {
105         int xs = flo[b][i], xns = flo[b][i + 1];
106         pa[xs] = g[xns][xs].u;
107         S[xs] = 1, S[xns] = 0;
108         slack[xs] = 0, set_slack(xns);
109         q_push(xns);
110     }
111     S[xr] = 1, pa[xr] = pa[b];
112     for (size_t i = pr + 1; i < flo[b].size(); ++i) {
113         int xs = flo[b][i];
114         S[xs] = -1, set_slack(xs);
115     }
116     st[b] = 0;
117 }
118 bool on_found_edge(const edge &e) {
119     int u = st[e.u], v = st[e.v];
120     if (S[v] == -1) {
121         pa[v] = e.u, S[v] = 1;
122         int nu = st[match[v]];
123         slack[v] = slack[nu] = 0;
124         S[nu] = 0, q_push(nu);
125     } else if (S[v] == 0) {
126         int lca = get_lca(u, v);
127         if (!lca)
128             return augment(u, v), augment(v, u), true;
129         else
130             add_blossom(u, lca, v);
131     }
132     return false;
133 }
134 bool matching() {
135     memset(S + 1, -1, sizeof(int) * n_x);
136     memset(slack + 1, 0, sizeof(int) * n_x);
137     q = queue<int>();
138     for (int x = 1; x <= n_x; ++x)
139         if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0, q_push(x);
140     if (q.empty()) return false;
141     for (;;) {
142         while (q.size()) {
143             int u = q.front();
144             q.pop();
145             if (S[st[u]] == 1) continue;
146             for (int v = 1; v <= n; ++v)
147                 if (g[u][v].w > 0 && st[u] != st[v]) {
148                     if (e_delta(g[u][v]) == 0) {
149                         if (on_found_edge(g[u][v]))
150                             return true;
151                     } else
152                         update_slack(u, st[v]);
153                 }
154         }
155         int d = inf;
156         for (int b = n + 1; b <= n_x; ++b)
157             if (st[b] == b && S[b] == 1) d = min(d, lab[b] / 2);
158         for (int x = 1; x <= n_x; ++x)
159             if (st[x] == x && slack[x]) {
160                 if (S[x] == -1)
161                     d = min(d, e_delta(g[slack[x]][x]));
162                 else if (S[x] == 0)
163                     d = min(d, e_delta(g[slack[x]][x]) / 2);
164             }
165         for (int u = 1; u <= n; ++u) {
166             if (S[st[u]] == 0) {
167                 if (lab[u] <= d) return 0;
168                 lab[u] -= d;
169             } else if (S[st[u]] == 1)
170                 lab[u] += d;
171         }
172         for (int b = n + 1; b <= n_x; ++b)
173             if (st[b] == b) {
174                 if (S[st[b]] == 0)
175                     lab[b] += d * 2;
176                 else if (S[st[b]] == 1)
177                     lab[b] -= d * 2;
178             }
179         q = queue<int>();
180         for (int x = 1; x <= n_x; ++x)
181             if (st[x] == x && slack[x] && st[slack[x]] != x && e_delta(g[slack[x]][x]) == 0)
182                 if (on_found_edge(g[slack[x]][x]))
183                     return true;
184         for (int b = n + 1; b <= n_x; ++b)
185             if (st[b] == b && S[b] == 1 && lab[b] == 0)
186                 expand_blossom(b);
187     }
188     return false;
189 }
190 pair<long long, int> solve() {
191     memset(match + 1, 0, sizeof(int) * n);
192     n_x = n;
193     int n_matches = 0;
194     long long tot_weight = 0;
195     for (int u = 0; u <= n; ++u) st[u] = u, flo[u].clear();
196     int w_max = 0;
197     for (int u = 1; u <= n; ++u)
198         for (int v = 1; v <= n; ++v) {
199             flo_from[u][v] = (u == v ? u : 0);
200             w_max = max(w_max, g[u][v].w);
201         }
202     for (int u = 1; u <= n; ++u) lab[u] = w_max;
203     while (matching()) ++n_matches;
204     for (int u = 1; u <= n; ++u)
205         if (match[u] && match[u] < u)
206             tot_weight += g[u][match[u]].w;
207     return make_pair(tot_weight, n_matches);
208 }
209 void add_edge(int ui, int vi, int wi) { g[ui][vi].w = g[vi][ui].w = wi; }
210 void init(int _n) {
211     n = _n;
212     for (int u = 1; u <= n; ++u)
213         for (int v = 1; v <= n; ++v)
214             g[u][v] = edge(u, v, 0);
215 }
216 };

```

## 4.7 Cover / Independent Set

1 V(E) Cover: choose some V(E) to cover all E(V)  
2 V(E) Independ: set of V(E) not adj to each other

3 M = Max Matching  
4 Cv = Min V Cover  
5 Ce = Min E Cover  
6 Iv = Max V Ind  
7 Ie = Max E Ind (equiv to M)

8  
9  
10 M = Cv (Konig Theorem)  
11 Iv = V \ Cv  
12 Ce = V - M

13 Construct Cv:  
14 1. Run Dinic  
15 2. Find s-t min cut  
16 3. Cv = {X in T} + {Y in S}

## 5 Graph

### 5.1 Heavy-Light Decomposition

1 const int N = 2e5 + 5;

```

2 int n, dfn[N], son[N], top[N], num[N], dep[N], p[N];
3 vector<int> path[N];
4 struct node {
5     int mx, sum;
6 } seg[N << 2];
7 void update(int x, int l, int r, int qx, int val) {
8     if (l == r) {
9         seg[x].mx = seg[x].sum = val;
10        return;
11    }
12    int mid = (l + r) >> 1;
13    if (qx <= mid) update(x << 1, l, mid, qx, val);
14    else update(x << 1 | 1, mid + 1, r, qx, val);
15    seg[x].mx = max(seg[x << 1].mx, seg[x << 1 | 1].mx);
16    seg[x].sum = seg[x << 1].sum + seg[x << 1 | 1].sum;
17 }
18 int big(int x, int l, int r, int ql, int qr) {
19     if (ql <= l && r <= qr) return seg[x].mx;
20     int mid = (l + r) >> 1;
21     int res = -INF;
22     if (ql <= mid) res = max(res, big(x << 1, l, mid, ql, qr));
23     if (mid < qr) res = max(res, big(x << 1 | 1, mid + 1, r, ql, qr));
24     return res;
25 }
26 int ask(int x, int l, int r, int ql, int qr) {
27     if (ql <= l && r <= qr) return seg[x].sum;
28     int mid = (l + r) >> 1;
29     int res = 0;
30     if (ql <= mid) res += ask(x << 1, l, mid, ql, qr);
31     if (mid < qr) res += ask(x << 1 | 1, mid + 1, r, ql, qr);
32     return res;
33 }
34 void dfs1(int now) {
35     son[now] = -1;
36     num[now] = 1;
37     for (auto i : path[now]) {
38         if (!dep[i]) {
39             dep[i] = dep[now] + 1;
40             p[i] = now;
41             dfs1(i);
42             num[now] += num[i];
43             if (son[now] == -1 || num[i] > num[son[now]]) son[now] = i;
44         }
45     }
46 }
47 int cnt;
48 void dfs2(int now, int t) {
49     top[now] = t;
50     cnt++;
51     dfn[now] = cnt;
52     if (son[now] == -1) return;
53     dfs2(son[now], t);
54     for (auto i : path[now])
55         if (i != p[now] && i != son[now]) dfs2(i, i);
56 }
57 int path_big(int x, int y) {
58     int res = -INF;
59     while (top[x] != top[y]) {
60         if (dep[top[x]] < dep[top[y]]) swap(x, y);
61         res = max(res, big(1, 1, n, dfn[top[x]], dfn[x]));
62         x = p[top[x]];
63     }
64     if (dfn[x] > dfn[y]) swap(x, y);
65     res = max(res, big(1, 1, n, dfn[x], dfn[y]));
66     return res;
67 }
68 int path_sum(int x, int y) {
69     int res = 0;
70     while (top[x] != top[y]) {
71         if (dep[top[x]] < dep[top[y]]) swap(x, y);
72         res += ask(1, 1, n, dfn[top[x]], dfn[x]);
73         x = p[top[x]];
74     }
75     if (dfn[x] > dfn[y]) swap(x, y);
76     res += ask(1, 1, n, dfn[x], dfn[y]);
77     return res;

```

```

78 }
79 void buildTree() {
80     FOR(i, 0, n - 1) {
81         int a, b;
82         cin >> a >> b;
83         path[a].pb(b);
84         path[b].pb(a);
85     }
86 }
87 void buildHLD(int root) {
88     dep[root] = 1;
89     dfs1(root);
90     dfs2(root, root);
91     FOR(i, 1, n + 1) {
92         int now;
93         cin >> now;
94         update(1, 1, n, dfn[i], now);
95     }
96 }

```

## 5.2 Centroid Decomposition

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const int N = 1e5 + 5;
4 vector<int> a[N];
5 int sz[N], lv[N];
6 bool used[N];
7 int f_sz(int x, int p) {
8     sz[x] = 1;
9     for (int i : a[x])
10         if (i != p && !used[i])
11             sz[x] += f_sz(i, x);
12     return sz[x];
13 }
14 int f_cen(int x, int p, int total) {
15     for (int i : a[x]) {
16         if (i != p && !used[i] && 2 * sz[i] > total)
17             return f_cen(i, x, total);
18     }
19     return x;
20 }
21 void cd(int x, int p) {
22     int total = f_sz(x, p);
23     int cen = f_cen(x, p, total);
24     lv[cen] = lv[p] + 1;
25     used[cen] = 1;
26     // cout << "cd: " << x << " " << p << " " << cen <<
27     // "\n";
28     for (int i : a[cen]) {
29         if (!used[i])
30             cd(i, cen);
31     }
32 }
33 int main() {
34     ios_base::sync_with_stdio(0);
35     cin.tie(0);
36     int n;
37     cin >> n;
38     for (int i = 0, x, y; i < n - 1; i++) {
39         cin >> x >> y;
40         a[x].push_back(y);
41         a[y].push_back(x);
42     }
43     cd(1, 0);
44     for (int i = 1; i <= n; i++)
45         cout << (char)('A' + lv[i] - 1) << " ";
46     cout << "\n";

```

## 5.3 Bellman-Ford + SPFA

```

1 int n, m;
2
3 // Graph
4 vector<vector<pair<int, ll>>> g;
5 vector<ll> dis;
6 vector<bool> negCycle;
7
8 // SPFA
9 vector<int> rlx;
10 queue<int> q;

```



```

11 vector<bool> inq;
12 vector<int> pa;
13 void SPFA(vector<int>& src) {
14     dis.assign(n + 1, LINF);
15     negCycle.assign(n + 1, false);
16     rlx.assign(n + 1, 0);
17     while (!q.empty()) q.pop();
18     inq.assign(n + 1, false);
19     pa.assign(n + 1, -1);
20
21     for (auto& s : src) {
22         dis[s] = 0;
23         q.push(s);
24         inq[s] = true;
25     }
26
27     while (!q.empty()) {
28         int u = q.front();
29         q.pop();
30         inq[u] = false;
31         if (rlx[u] >= n) {
32             negCycle[u] = true;
33         } else
34             for (auto& e : g[u]) {
35                 int v = e.first;
36                 ll w = e.second;
37                 if (dis[v] > dis[u] + w) {
38                     dis[v] = dis[u] + w;
39                     rlx[v] = rlx[u] + 1;
40                     pa[v] = u;
41                     if (!inq[v]) {
42                         q.push(v);
43                         inq[v] = true;
44                     }
45                 }
46             }
47     }
48 }
49
50 // Bellman-Ford
51 queue<int> q;
52 vector<int> pa;
53 void BellmanFord(vector<int>& src) {
54     dis.assign(n + 1, LINF);
55     negCycle.assign(n + 1, false);
56     pa.assign(n + 1, -1);
57
58     for (auto& s : src) dis[s] = 0;
59
60     for (int rlx = 1; rlx <= n; rlx++) {
61         for (int u = 1; u <= n; u++) {
62             if (dis[u] == LINF) continue; // Important
63             !!
64             for (auto& e : g[u]) {
65                 int v = e.first;
66                 ll w = e.second;
67                 if (dis[v] > dis[u] + w) {
68                     dis[v] = dis[u] + w;
69                     pa[v] = u;
70                     if (rlx == n) negCycle[v] = true;
71                 }
72             }
73         }
74     }
75
76 // Negative Cycle Detection
77 void NegCycleDetect() {
78     /* No Neg Cycle: NO
79     Exist Any Neg Cycle:
80     YES
81     v0 v1 v2 ... vk v0 */
82
83     vector<int> src;
84     for (int i = 1; i <= n; i++)
85         src.emplace_back(i);
86
87     SPFA(src);
88     // BellmanFord(src);
89
90     int ptr = -1;
91     for (int i = 1; i <= n; i++)

```

```

92         if (negCycle[i]) {
93             ptr = i;
94             break;
95         }
96
97     if (ptr == -1) {
98         return cout << "NO" << endl, void();
99     }
100
101     cout << "YES\n";
102     vector<int> ans;
103     vector<bool> vis(n + 1, false);
104
105     while (true) {
106         ans.emplace_back(ptr);
107         if (vis[ptr]) break;
108         vis[ptr] = true;
109         ptr = pa[ptr];
110     }
111     reverse(ans.begin(), ans.end());
112
113     vis.assign(n + 1, false);
114     for (auto& x : ans) {
115         cout << x << ' ';
116         if (vis[x]) break;
117         vis[x] = true;
118     }
119     cout << endl;
120 }
121
122 // Distance Calculation
123 void calcDis(int s) {
124     vector<int> src;
125     src.emplace_back(s);
126     SPFA(src);
127     // BellmanFord(src);
128
129     while (!q.empty()) q.pop();
130     for (int i = 1; i <= n; i++)
131         if (negCycle[i]) q.push(i);
132
133     while (!q.empty()) {
134         int u = q.front();
135         q.pop();
136         for (auto& e : g[u]) {
137             int v = e.first;
138             if (!negCycle[v]) {
139                 q.push(v);
140                 negCycle[v] = true;
141             }
142         }
143     }
144 }

```

## 5.4 BCC - AP

```

1 int n, m;
2 int low[maxn], dfn[maxn], instp;
3 vector<int> E, g[maxn];
4 bitset<maxn> isap;
5 bitset<maxn> vis;
6 stack<int> stk;
7 int bccnt;
8 vector<int> bcc[maxn];
9 inline void popout(int u) {
10     bccnt++;
11     bcc[bccnt].emplace_back(u);
12     while (!stk.empty()) {
13         int v = stk.top();
14         if (u == v) break;
15         stk.pop();
16         bcc[bccnt].emplace_back(v);
17     }
18 }
19 void dfs(int u, bool rt = 0) {
20     stk.push(u);
21     low[u] = dfn[u] = ++instp;
22     int kid = 0;
23     Each(e, g[u]) {
24         if (vis[e]) continue;
25         vis[e] = true;
26         int v = E[e] ^ u;

```

```

27     if (!dfn[v]) {
28         // tree edge
29         kid++;
30         dfs(v);
31         low[u] = min(low[u], low[v]);
32         if (!rt && low[v] >= dfn[u]) {
33             // bcc found: u is ap
34             isap[u] = true;
35             popout(u);
36         }
37     } else {
38         // back edge
39         low[u] = min(low[u], dfn[v]);
40     }
41 }
42 // special case: root
43 if (rt) {
44     if (kid > 1) isap[u] = true;
45     popout(u);
46 }
47 }
48 void init() {
49     cin >> n >> m;
50     fill(low, low + maxn, INF);
51     REP(i, m) {
52         int u, v;
53         cin >> u >> v;
54         g[u].emplace_back(i);
55         g[v].emplace_back(i);
56         E.emplace_back(u ^ v);
57     }
58 }
59 void solve() {
60     FOR(i, 1, n + 1, 1) {
61         if (!dfn[i]) dfs(i, true);
62     }
63     vector<int> ans;
64     int cnt = 0;
65     FOR(i, 1, n + 1, 1) {
66         if (isap[i]) cnt++, ans.emplace_back(i);
67     }
68     cout << cnt << endl;
69     Each(i, ans) cout << i << ' ';
70     cout << endl;
71 }

```

## 5.5 BCC - Bridge

```

1  int n, m;
2  vector<int> g[maxn], E;
3  int low[maxn], dfn[maxn], instp;
4  int bccnt, bccid[maxn];
5  stack<int> stk;
6  bitset<maxn> vis, isbrg;
7  void init() {
8      cin >> n >> m;
9      REP(i, m) {
10         int u, v;
11         cin >> u >> v;
12         E.emplace_back(u ^ v);
13         g[u].emplace_back(i);
14         g[v].emplace_back(i);
15     }
16     fill(low, low + maxn, INF);
17 }
18 void popout(int u) {
19     bccnt++;
20     while (!stk.empty()) {
21         int v = stk.top();
22         if (v == u) break;
23         stk.pop();
24         bccid[v] = bccnt;
25     }
26 }
27 void dfs(int u) {
28     stk.push(u);
29     low[u] = dfn[u] = ++instp;
30
31     Each(e, g[u]) {
32         if (vis[e]) continue;
33         vis[e] = true;
34     }

```

```

35     int v = E[e] ^ u;
36     if (dfn[v]) {
37         // back edge
38         low[u] = min(low[u], dfn[v]);
39     } else {
40         // tree edge
41         dfs(v);
42         low[u] = min(low[u], low[v]);
43         if (low[v] == dfn[v]) {
44             isbrg[e] = true;
45             popout(u);
46         }
47     }
48 }
49 }
50 void solve() {
51     FOR(i, 1, n + 1, 1) {
52         if (!dfn[i]) dfs(i);
53     }
54     vector<pii> ans;
55     vis.reset();
56     FOR(u, 1, n + 1, 1) {
57         Each(e, g[u]) {
58             if (!isbrg[e] || vis[e]) continue;
59             vis[e] = true;
60             int v = E[e] ^ u;
61             ans.emplace_back(mp(u, v));
62         }
63     }
64     cout << (int)ans.size() << endl;
65     Each(e, ans) cout << e.F << ' ' << e.S << endl;
66 }

```

## 5.6 SCC - Tarjan

```

1  // 2-SAT
2  vector<int> E, g[maxn]; // 1~n, n+1~2n
3  int low[maxn], in[maxn], instp;
4  int scnt, sccid[maxn];
5  stack<int> stk;
6  bitset<maxn> ins, vis;
7  int n, m;
8  void init() {
9      cin >> m >> n;
10     E.clear();
11     fill(g, g + maxn, vector<int>());
12     fill(low, low + maxn, INF);
13     memset(in, 0, sizeof(in));
14     instp = 1;
15     scnt = 0;
16     memset(sccid, 0, sizeof(sccid));
17     ins.reset();
18     vis.reset();
19 }
20 inline int no(int u) {
21     return (u > n ? u - n : u + n);
22 }
23 int ecnt = 0;
24 inline void clause(int u, int v) {
25     E.eb(no(u) ^ v);
26     g[no(u)].eb(ecnt++);
27     E.eb(no(v) ^ u);
28     g[no(v)].eb(ecnt++);
29 }
30 void dfs(int u) {
31     in[u] = instp++;
32     low[u] = in[u];
33     stk.push(u);
34     ins[u] = true;
35
36     Each(e, g[u]) {
37         if (vis[e]) continue;
38         vis[e] = true;
39
40         int v = E[e] ^ u;
41         if (ins[v])
42             low[u] = min(low[u], in[v]);
43         else if (!in[v]) {
44             dfs(v);
45             low[u] = min(low[u], low[v]);
46         }
47     }

```

```

48     if (low[u] == in[u]) {
49         sccnt++;
50         while (!stk.empty()) {
51             int v = stk.top();
52             stk.pop();
53             ins[v] = false;
54             sccid[v] = sccnt;
55             if (u == v) break;
56         }
57     }
58 }
59 int main() {
60     init();
61     REP(i, m) {
62         char su, sv;
63         int u, v;
64         cin >> su >> u >> sv >> v;
65         if (su == '-') u = no(u);
66         if (sv == '-') v = no(v);
67         clause(u, v);
68     }
69     FOR(i, 1, 2 * n + 1, 1) {
70         if (!in[i]) dfs(i);
71     }
72     FOR(u, 1, n + 1, 1) {
73         int du = no(u);
74         if (sccid[u] == sccid[du]) {
75             return cout << "IMPOSSIBLE\n", 0;
76         }
77     }
78     FOR(u, 1, n + 1, 1) {
79         int du = no(u);
80         cout << (sccid[u] < sccid[du] ? '+' : '-') << '
81         ';
82     }
83     cout << endl;
84 }

```

## 5.7 SCC - Kosaraju

```

1 const int N = 1e5 + 10;
2 vector<int> ed[N], ed_b[N]; // 反邊
3 vector<int> SCC(N); // 最後SCC的分組
4 bitset<N> vis;
5 int SCC_cnt;
6 int n, m;
7 vector<int> pre; // 後序遍歷
8
9 void dfs(int x) {
10     vis[x] = 1;
11     for (int i : ed[x]) {
12         if (vis[i]) continue;
13         dfs(i);
14     }
15     pre.push_back(x);
16 }
17
18 void dfs2(int x) {
19     vis[x] = 1;
20     SCC[x] = SCC_cnt;
21     for (int i : ed_b[x]) {
22         if (vis[i]) continue;
23         dfs2(i);
24     }
25 }
26
27 void kosaraju() {
28     for (int i = 1; i <= n; i++) {
29         if (!vis[i]) {
30             dfs(i);
31         }
32     }
33     SCC_cnt = 0;
34     vis = 0;
35     for (int i = n - 1; i >= 0; i--) {
36         if (!vis[pre[i]]) {
37             SCC_cnt++;
38             dfs2(pre[i]);
39         }
40     }
41 }

```

## 5.8 Eulerian Path - Undir

```

1 // from 1 to n
2 #define gg return cout << "IMPOSSIBLE\n", void();
3
4 int n, m;
5 vector<int> g[maxn];
6 bitset<maxn> inodd;
7
8 void init() {
9     cin >> n >> m;
10    inodd.reset();
11    for (int i = 0; i < m; i++) {
12        int u, v;
13        cin >> u >> v;
14        inodd[u] = inodd[u] ^ true;
15        inodd[v] = inodd[v] ^ true;
16        g[u].emplace_back(v);
17        g[v].emplace_back(u);
18    }
19 }
20 stack<int> stk;
21 void dfs(int u) {
22     while (!g[u].empty()) {
23         int v = g[u].back();
24         g[u].pop_back();
25         dfs(v);
26     }
27     stk.push(u);
28 }

```

## 5.9 Eulerian Path - Dir

```

1 // from node 1 to node n
2 #define gg return cout << "IMPOSSIBLE\n", 0;
3
4 int n, m;
5 vector<int> g[maxn];
6 stack<int> stk;
7 int in[maxn], out[maxn];
8
9 void init() {
10     cin >> n >> m;
11     for (int i = 0; i < m; i++) {
12         int u, v;
13         cin >> u >> v;
14         g[u].emplace_back(v);
15         out[u]++, in[v]++;
16     }
17     for (int i = 1; i <= n; i++) {
18         if (i == 1 && out[i] - in[i] != 1) gg;
19         if (i == n && in[i] - out[i] != 1) gg;
20         if (i != 1 && i != n && in[i] != out[i]) gg;
21     }
22 }
23 void dfs(int u) {
24     while (!g[u].empty()) {
25         int v = g[u].back();
26         g[u].pop_back();
27         dfs(v);
28     }
29     stk.push(u);
30 }
31 void solve() {
32     dfs(1) for (int i = 1; i <= n; i++) if ((int)g[i].
33         size()) gg;
34     while (!stk.empty()) {
35         int u = stk.top();
36         stk.pop();
37         cout << u << ' ';
38     }
39 }

```

## 5.10 Hamilton Path

```

1 // top down DP
2 // Be Aware Of Multiple Edges
3 int n, m;
4 ll dp[maxn][1<maxn];
5 int adj[maxn][maxn];
6
7 void init() {

```

```

8   cin >> n >> m;
9   fill(dp[0], dp[maxn-1]+(1<<maxn), -1);
10 }
11
12 void DP(int i, int msk) {
13     if (dp[i][msk] != -1) return;
14     dp[i][msk] = 0;
15     REP(j, n) if (j != i && (msk & (1<<j)) && adj[j][i]) {
16         int sub = msk ^ (1<<i);
17         if (dp[j][sub] == -1) DP(j, sub);
18         dp[i][msk] += dp[j][sub] * adj[j][i];
19         if (dp[i][msk] >= MOD) dp[i][msk] %= MOD;
20     }
21 }
22
23
24 int main() {
25     WiwiHorz
26     init();
27
28     REP(i, m) {
29         int u, v;
30         cin >> u >> v;
31         if (u == v) continue;
32         adj[--u][--v]++;
33     }
34
35     dp[0][1] = 1;
36     FOR(i, 1, n, 1) {
37         dp[i][1] = 0;
38         dp[i][1|(1<<i)] = adj[0][i];
39     }
40     FOR(msk, 1, (1<<n), 1) {
41         if (msk == 1) continue;
42         dp[0][msk] = 0;
43     }
44
45     DP(n-1, (1<<n)-1);
46     cout << dp[n-1][(1<<n)-1] << endl;
47
48     return 0;
49 }
50 }

```

## 5.11 Kth Shortest Path

```

1 // time: O(|E| \lg |E|+|V| \lg |V|+K)
2 // memory: O(|E| \lg |E|+|V|)
3 struct KSP { // 1-base
4     struct nd {
5         int u, v;
6         ll d;
7         nd(int ui = 0, int vi = 0, ll di = INF) {
8             u = ui;
9             v = vi;
10            d = di;
11        }
12    };
13    struct heap {
14        nd* edge;
15        int dep;
16        heap* chd[4];
17    };
18    static int cmp(heap* a, heap* b) { return a->edge->
19        d > b->edge->d; }
20    struct node {
21        int v;
22        ll d;
23        heap* H;
24        nd* E;
25        node() {}
26        node(ll _d, int _v, nd* _E) {
27            d = _d;
28            v = _v;
29            E = _E;
30        }
31        node(heap* _H, ll _d) {
32            H = _H;
33            d = _d;
34        }
35    };

```

```

34     friend bool operator<(node a, node b) { return
35         a.d > b.d; }
36 };
37 int n, k, s, t, dst[N];
38 nd* nxt[N];
39 vector<nd*> g[N], rg[N];
40 heap *nullNd, *head[N];
41 void init(int _n, int _k, int _s, int _t) {
42     n = _n;
43     k = _k;
44     s = _s;
45     t = _t;
46     for (int i = 1; i <= n; i++) {
47         g[i].clear();
48         rg[i].clear();
49         nxt[i] = NULL;
50         head[i] = NULL;
51         dst[i] = -1;
52     }
53 }
54 void addEdge(int ui, int vi, ll di) {
55     nd* e = new nd(ui, vi, di);
56     g[ui].push_back(e);
57     rg[vi].push_back(e);
58 }
59 queue<int> dfsQ;
60 void dijkstra() {
61     while (dfsQ.size()) dfsQ.pop();
62     priority_queue<node> Q;
63     Q.push(node(0, t, NULL));
64     while (!Q.empty()) {
65         node p = Q.top();
66         Q.pop();
67         if (dst[p.v] != -1) continue;
68         dst[p.v] = p.d;
69         nxt[p.v] = p.E;
70         dfsQ.push(p.v);
71         for (auto e : rg[p.v]) Q.push(node(p.d + e
72             ->d, e->u, e));
73     }
74 }
75 heap* merge(heap* curNd, heap* newNd) {
76     if (curNd == nullNd) return newNd;
77     heap* root = new heap;
78     memcpy(root, curNd, sizeof(heap));
79     if (newNd->edge->d < curNd->edge->d) {
80         root->edge = newNd->edge;
81         root->chd[2] = newNd->chd[2];
82         root->chd[3] = newNd->chd[3];
83         newNd->edge = curNd->edge;
84         newNd->chd[2] = curNd->chd[2];
85         newNd->chd[3] = curNd->chd[3];
86     }
87     if (root->chd[0]->dep < root->chd[1]->dep)
88         root->chd[0] = merge(root->chd[0], newNd);
89     else
90         root->chd[1] = merge(root->chd[1], newNd);
91     root->dep = max(root->chd[0]->dep,
92         root->chd[1]->dep) +
93         1;
94     return root;
95 }
96 vector<heap*> V;
97 void build() {
98     nullNd = new heap;
99     nullNd->dep = 0;
100    nullNd->edge = new nd;
101    fill(nullNd->chd, nullNd->chd + 4, nullNd);
102    while (not dfsQ.empty()) {
103        int u = dfsQ.front();
104        dfsQ.pop();
105        if (!nxt[u])
106            head[u] = nullNd;
107        else
108            head[u] = head[nxt[u]->v];
109        V.clear();
110        for (auto& e : g[u]) {
111            int v = e->v;
112            if (dst[v] == -1) continue;
113            e->d += dst[v] - dst[u];
114            if (nxt[u] != e) {
115                heap* p = new heap;

```

```

114         fill(p->chd, p->chd + 4, nullNd);
115         p->dep = 1;
116         p->edge = e;
117         V.push_back(p);
118     }
119 }
120 if (V.empty()) continue;
121 make_heap(V.begin(), V.end(), cmp);
122 #define L(X) ((X << 1) + 1)
123 #define R(X) ((X << 1) + 2)
124 for (size_t i = 0; i < V.size(); i++) {
125     if (L(i) < V.size())
126         V[i]->chd[2] = V[L(i)];
127     else
128         V[i]->chd[2] = nullNd;
129     if (R(i) < V.size())
130         V[i]->chd[3] = V[R(i)];
131     else
132         V[i]->chd[3] = nullNd;
133 }
134 head[u] = merge(head[u], V.front());
135 }
136 }
137 vector<ll> ans;
138 void first_K() {
139     ans.clear();
140     priority_queue<node> Q;
141     if (dst[s] == -1) return;
142     ans.push_back(dst[s]);
143     if (head[s] != nullNd)
144         Q.push(node(head[s], dst[s] + head[s]->edge
145                     ->d));
146     for (int _ = 1; _ < k and not Q.empty(); _++) {
147         node p = Q.top(), q;
148         Q.pop();
149         ans.push_back(p.d);
150         if (head[p.H->edge->v] != nullNd) {
151             q.H = head[p.H->edge->v];
152             q.d = p.d + q.H->edge->d;
153             Q.push(q);
154         }
155         for (int i = 0; i < 4; i++)
156             if (p.H->chd[i] != nullNd) {
157                 q.H = p.H->chd[i];
158                 q.d = p.d - p.H->edge->d + p.H->chd
159                     [i]->edge->d;
160                 Q.push(q);
161             }
162     }
163     void solve() { // ans[i] stores the i-th shortest
164         path
165         dijkstra();
166         build();
167         first_K(); // ans.size() might less than k
168     }
169 } solver;

```

## 5.12 System of Difference Constraints

```

1 vector<vector<pair<int, ll>>> G;
2 void add(int u, int v, ll w) {
3     G[u].emplace_back(make_pair(v, w));
4 }

```

- $x_u - x_v \leq c \Rightarrow \text{add}(v, u, c)$
- $x_u - x_v \geq c \Rightarrow \text{add}(u, v, -c)$
- $x_u - x_v = c \Rightarrow \text{add}(v, u, c), \text{add}(u, v, -c)$
- $x_u \geq c \Rightarrow \text{add super vertex } x_0 = 0, \text{ then } x_u - x_0 \geq c \Rightarrow \text{add}(u, 0, -c)$
- Don't forget non-negative constraints for every variable if specified implicitly.
- Interval sum  $\Rightarrow$  Use prefix sum to transform into differential constraints. Don't forget  $S_{i+1} - S_i \geq 0$  if  $x_i$  needs to be non-negative.

$$\bullet \frac{x_u}{x_v} \leq c \Rightarrow \log x_u - \log x_v \leq \log c$$

## 6 String

### 6.1 Aho Corasick

```

1 struct AAutomata {
2     struct Node {
3         int cnt;
4         Node *go[26], *fail, *dic;
5         Node() {
6             cnt = 0;
7             fail = 0;
8             dic = 0;
9             memset(go, 0, sizeof(go));
10        }
11    } pool[1048576], *root;
12    int nMem;
13    Node *new_Node() {
14        pool[nMem] = Node();
15        return &pool[nMem++];
16    }
17    void init() {
18        nMem = 0;
19        root = new_Node();
20    }
21    void add(const string &str) { insert(root, str, 0);
22    }
23    void insert(Node *cur, const string &str, int pos)
24    {
25        for (int i = pos; i < str.size(); i++) {
26            if (!cur->go[str[i] - 'a'])
27                cur->go[str[i] - 'a'] = new_Node();
28            cur = cur->go[str[i] - 'a'];
29        }
30        cur->cnt++;
31    }
32    void make_fail() {
33        queue<Node *> que;
34        que.push(root);
35        while (!que.empty()) {
36            Node *fr = que.front();
37            que.pop();
38            for (int i = 0; i < 26; i++) {
39                if (fr->go[i]) {
40                    Node *ptr = fr->fail;
41                    while (ptr && !ptr->go[i]) ptr = ptr->fail;
42                    fr->go[i]->fail = ptr = (ptr ? ptr->go[i] : root);
43                    fr->go[i]->dic = (ptr->cnt ? ptr : ptr->dic);
44                    que.push(fr->go[i]);
45                }
46            }
47        } AC;
48    }
49 } AC;

```

### 6.2 KMP

```

1 vector<int> f;
2 void buildFailFunction(string &s) {
3     f.resize(s.size(), -1);
4     for (int i = 1; i < s.size(); i++) {
5         int now = f[i - 1];
6         while (now != -1 and s[now + 1] != s[i]) now = f[now];
7         if (s[now + 1] == s[i]) f[i] = now + 1;
8     }
9 }
10 void KMPmatching(string &a, string &b) {
11     for (int i = 0, now = -1; i < a.size(); i++) {
12         while (a[i] != b[now + 1] and now != -1) now = f[now];
13         if (a[i] == b[now + 1]) now++;
14         if (now + 1 == b.size()) {
15             cout << "found a match start at position "
16                  << i - now << endl;
17             now = f[now];
18         }
19     }
20 }

```

```

18     }
19 }
20 }

```

### 6.3 Z Value

```

1 string is, it, s;
2 int n;
3 vector<int> z;
4 void init() {
5     cin >> is >> it;
6     s = it + '0' + is;
7     n = (int)s.size();
8     z.resize(n, 0);
9 }
10 void solve() {
11     int ans = 0;
12     z[0] = n;
13     for (int i = 1, l = 0, r = 0; i < n; i++) {
14         if (i <= r) z[i] = min(z[i - 1], r - i + 1);
15         while (i + z[i] < n && s[z[i]] == s[i + z[i]])
16             z[i]++;
17         if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
18         if (z[i] == (int)it.size()) ans++;
19     }
20     cout << ans << endl;

```

### 6.4 Manacher

```

1 int n;
2 string S, s;
3 vector<int> m;
4 void manacher() {
5     s.clear();
6     s.resize(2 * n + 1, '.');
7     for (int i = 0, j = 1; i < n; i++, j += 2) s[j] = S[i];
8     m.clear();
9     m.resize(2 * n + 1, 0);
10    // m[i] := max k such that s[i-k, i+k] is
11    // palindrome
12    int mx = 0, mxk = 0;
13    for (int i = 1; i < 2 * n + 1; i++) {
14        if (mx - (i - mx) >= 0) m[i] = min(m[mx - (i - mx)], mx + mxk - i);
15        while (0 <= i - m[i] - 1 && i + m[i] + 1 < 2 * n + 1 && s[i - m[i] - 1] == s[i + m[i] + 1]) m[i]++;
16        if (i + m[i] > mx + mxk) mx = i, mxk = m[i];
17    }
18 }
19 void init() {
20     cin >> S;
21     n = (int)S.size();
22 }
23 void solve() {
24     manacher();
25     int mx = 0, ptr = 0;
26     for (int i = 0; i < 2 * n + 1; i++)
27         if (mx < m[i]) {
28             mx = m[i];
29             ptr = i;
30         }
31     for (int i = ptr - mx; i <= ptr + mx; i++)
32         if (s[i] != '.') cout << s[i];
33     cout << endl;
34 }

```

### 6.5 Suffix Array

```

1 #define F first
2 #define S second
3 struct SuffixArray { // don't forget s += "$";
4     int n;
5     string s;
6     vector<int> suf, lcp, rk;
7     vector<int> cnt, pos;
8     vector<pair<pii, int>> buc[2];
9     void init(string _s) {

```

```

10     s = _s;
11     n = (int)s.size();
12     // resize(n): suf, rk, cnt, pos, lcp, buc[0~1]
13 }
14 void radix_sort() {
15     for (int t : {0, 1}) {
16         fill(cnt.begin(), cnt.end(), 0);
17         for (auto& i : buc[t]) cnt[(t ? i.F.F : i.F.S)++]++;
18         for (int i = 0; i < n; i++)
19             pos[i] = (!i ? 0 : pos[i - 1] + cnt[i - 1]);
20         for (auto& i : buc[t])
21             buc[t ^ 1][pos[(t ? i.F.F : i.F.S)++] + 1] = i;
22     }
23 }
24 bool fill_suf() {
25     bool end = true;
26     for (int i = 0; i < n; i++) suf[i] = buc[0][i].S;
27     rk[suf[0]] = 0;
28     for (int i = 1; i < n; i++) {
29         int dif = (buc[0][i].F != buc[0][i - 1].F);
30         end &= dif;
31         rk[suf[i]] = rk[suf[i - 1]] + dif;
32     }
33     return end;
34 }
35 void sa() {
36     for (int i = 0; i < n; i++)
37         buc[0][i] = make_pair(make_pair(s[i], s[i]), i);
38     sort(buc[0].begin(), buc[0].end());
39     if (fill_suf()) return;
40     for (int k = 0; (1 << k) < n; k++) {
41         for (int i = 0; i < n; i++)
42             buc[0][i] = make_pair(make_pair(rk[i], rk[(i + (1 << k)) % n]), i);
43         radix_sort();
44         if (fill_suf()) return;
45     }
46 }
47 void LCP() {
48     int k = 0;
49     for (int i = 0; i < n - 1; i++) {
50         if (rk[i] == 0) continue;
51         int pi = rk[i];
52         int j = suf[pi - 1];
53         while (i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
54         lcp[pi] = k;
55         k = max(k - 1, 0);
56     }
57 }
58 };
59 SuffixArray suffixarray;

```

### 6.6 Minimum Rotation

```

1 // rotate(begin(s), begin(s)+minRotation(s), end(s))
2 int minRotation(string s) {
3     int a = 0, n = s.size();
4     s += s;
5     for (int b = 0; b < n; b++)
6         for (int k = 0; k < n; k++) {
7             if (a + k == b || s[a + k] < s[b + k]) {
8                 b += max(0, k - 1);
9                 break;
10            }
11            if (s[a + k] > s[b + k]) {
12                a = b;
13                break;
14            }
15        }
16     return a;
17 }

```

### 6.7 Lyndon Factorization

```

1 vector<string> duval(string const& s) {
2     int n = s.size();

```



```

3   int i = 0;
4   vector<string> factorization;
5   while (i < n) {
6       int j = i + 1, k = i;
7       while (j < n && s[k] <= s[j]) {
8           if (s[k] < s[j])
9               k = j;
10          else
11              k++;
12          j++;
13      }
14      while (i <= k) {
15          factorization.push_back(s.substr(i, j - k));
16          i += j - k;
17      }
18  }
19  return factorization; // O(n)
20 }

```

## 6.8 Rolling Hash

```

1  const ll C = 27;
2  inline int id(char c) { return c - 'a' + 1; }
3  struct RollingHash {
4      string s;
5      int n;
6      ll mod;
7      vector<ll> Cexp, hs;
8      RollingHash(string& _s, ll _mod) : s(_s), n((int)_s
9          .size()), mod(_mod) {
10         Cexp.assign(n, 0);
11         hs.assign(n, 0);
12         Cexp[0] = 1;
13         for (int i = 1; i < n; i++) {
14             Cexp[i] = Cexp[i - 1] * C;
15             if (Cexp[i] >= mod) Cexp[i] %= mod;
16         }
17         hs[0] = id(s[0]);
18         for (int i = 1; i < n; i++) {
19             hs[i] = hs[i - 1] * C + id(s[i]);
20             if (hs[i] >= mod) hs[i] %= mod;
21         }
22     }
23     inline ll query(int l, int r) {
24         ll res = hs[r] - (l ? hs[l - 1] * Cexp[r - l +
25             1] : 0);
26         res = (res % mod + mod) % mod;
27         return res;
28     }
29 };

```

## 6.9 Trie

```

1  pii a[N][26];
2
3  void build(string &s) {
4      static int idx = 0;
5      int n = s.size();
6      for (int i = 0, v = 0; i < n; i++) {
7          pii &now = a[v][s[i] - 'a'];
8          if (now.first != -1)
9              v = now.first;
10         else
11             v = now.first = ++idx;
12         if (i == n - 1)
13             now.second++;
14     }
15 }

```

# 7 Geometry

## 7.1 Basic Operations

```

1  typedef long long T;
2  // typedef long double T;
3  const long double eps = 1e-8;
4  short sgn(T x) {
5      if (abs(x) < eps) return 0;
6      return x < 0 ? -1 : 1;
7  }

```

```

8  struct Pt {
9      T x, y;
10     Pt(T _x = 0, T _y = 0) : x(_x), y(_y) {}
11     Pt operator+(Pt a) { return Pt(x + a.x, y + a.y); }
12     Pt operator-(Pt a) { return Pt(x - a.x, y - a.y); }
13     Pt operator*(T a) { return Pt(x * a, y * a); }
14     Pt operator/(T a) { return Pt(x / a, y / a); }
15     T operator*(Pt a) { return x * a.x + y * a.y; }
16     T operator^(Pt a) { return x * a.y - y * a.x; }
17     bool operator<(Pt a) { return x < a.x || (x == a.x
18         && y < a.y); }
19     // return sgn(x-a.x) < 0 || (sgn(x-a.x) == 0 && sgn
20         (y-a.y) < 0); }
21     bool operator==(Pt a) { return sgn(x - a.x) == 0 &&
22         sgn(y - a.y) == 0; }
23 };
24 Pt mv(Pt a, Pt b) { return b - a; }
25 T len2(Pt a) { return a * a; }
26 T dis2(Pt a, Pt b) { return len2(b - a); }
27 short ori(Pt a, Pt b) { return ((a ^ b) > 0) - ((a ^ b)
28     < 0); }
29 bool onseg(Pt p, Pt l1, Pt l2) {
30     Pt a = mv(p, l1), b = mv(p, l2);
31     return ((a ^ b) == 0) && ((a * b) <= 0);
32 }

```

## 7.2 SVG Writer

## 7.3 Sort by Angle

```

1  int ud(Pt a) { // up or down half plane
2      if (a.y > 0) return 0;
3      if (a.y < 0) return 1;
4      return (a.x >= 0 ? 0 : 1);
5  }
6  sort(pts.begin(), pts.end(), [&](const Pt& a, const Pt&
7      b) {
8      if (ud(a) != ud(b)) return ud(a) < ud(b);
9      return (a ^ b) > 0;
10 });

```

## 7.4 Line Intersection

```

1  bool line_intersect_check(Pt p1, Pt p2, Pt q1, Pt q2) {
2      if (onseg(p1, q1, q2) || onseg(p2, q1, q2) || onseg
3          (q1, p1, p2) || onseg(q2, p1, p2)) return true;
4      Pt p = mv(p1, p2), q = mv(q1, q2);
5      return (ori(p, mv(p1, q1)) * ori(p, mv(p1, q2)) <
6          0) && (ori(q, mv(q1, p1)) * ori(q, mv(q1, p2))
7          < 0);
8  }
9  // long double
10 Pt line_intersect(Pt a1, Pt a2, Pt b1, Pt b2) {
11     Pt da = mv(a1, a2), db = mv(b1, b2);
12     T det = da ^ db;
13     if (sgn(det) == 0) { // parallel
14         // return Pt(NAN, NAN);
15     }
16     T t = ((b1 - a1) ^ db) / det;
17     return a1 + da * t;
18 }

```

## 7.5 Polygon Area

```

1  // 2 * area
2  T dbPoly_area(vector<Pt>& e) {
3      T res = 0;
4      int sz = e.size();
5      for (int i = 0; i < sz; i++) {
6          res += e[i] ^ e[(i + 1) % sz];
7      }
8      return abs(res);
9  }

```

## 7.6 Convex Hull

```

1  vector<Pt> convexHull(vector<Pt> pts) {
2      vector<Pt> hull;
3      sort(pts.begin(), pts.end());
4      for (int i = 0; i < 2; i++) {
5          int b = hull.size();
6          for (auto ei : pts) {

```

```

7   while (hull.size() - b >= 2 && ori(mv(hull[
      hull.size() - 2], hull.back()), mv(hull
      [hull.size() - 2], ei)) == -1) {
8       hull.pop_back();
9   }
10  hull.emplace_back(ei);
11  }
12  hull.pop_back();
13  reverse(pts.begin(), pts.end());
14  }
15  return hull;
16  }

```

## 7.7 Point In Convex

```

1  bool point_in_convex(const vector<Pt> &C, Pt p, bool
    strict = true) {
2      // only works when no three point are collinear
3      int n = C.size();
4      int a = 1, b = n - 1, r = !strict;
5      if (n == 0) return false;
6      if (n < 3) return r && onseg(p, C[0], C.back());
7      if (ori(mv(C[0], C[a]), mv(C[0], C[b])) > 0) swap(a
        , b);
8      if (ori(mv(C[0], C[a]), mv(C[0], p)) >= r || ori(mv
        (C[0], C[b]), mv(C[0], p)) <= -r) return false;
9      while (abs(a - b) > 1) {
10         int c = (a + b) / 2;
11         if (ori(mv(C[0], C[c]), mv(C[0], p)) > 0) b = c
            ;
12         else a = c;
13     }
14     return ori(mv(C[a], C[b]), mv(C[a], p)) < r;
15 }

```

## 7.8 Point Segment Distance

```

1  double point_segment_dist(Pt q0, Pt q1, Pt p) {
2      if (q0 == q1) {
3          double dx = double(p.x - q0.x);
4          double dy = double(p.y - q0.y);
5          return sqrt(dx * dx + dy * dy);
6      }
7      T d1 = (q1 - q0) * (p - q0);
8      T d2 = (q0 - q1) * (p - q1);
9      if (d1 >= 0 && d2 >= 0) {
10         double area = fabs(double((q1 - q0) ^ (p - q0))
            );
11         double base = sqrt(double(dis2(q0, q1)));
12         return area / base;
13     }
14     double dx0 = double(p.x - q0.x), dy0 = double(p.y
        - q0.y);
15     double dx1 = double(p.x - q1.x), dy1 = double(p.y
        - q1.y);
16     return min(sqrt(dx0 * dx0 + dy0 * dy0), sqrt(dx1 *
        dx1 + dy1 * dy1));
17 }

```

## 7.9 Point in Polygon

```

1  short inPoly(vector<Pt> &pts, Pt p) {
2      // 0=Bound 1=In -1=Out
3      int n = pts.size();
4      for (int i = 0; i < pts.size(); i++) if (onseg(p,
        pts[i], pts[(i + 1) % n])) return 0;
5      int cnt = 0;
6      for (int i = 0; i < pts.size(); i++) if (
        line_intersect_check(p, Pt(p.x + 1, p.y + 2e9),
        pts[i], pts[(i + 1) % n])) cnt ^= 1;
7      return (cnt ? 1 : -1);
8  }

```

## 7.10 Minimum Euclidean Distance

```

1  long long Min_Euclidean_Dist(vector<Pt> &pts) {
2      sort(pts.begin(), pts.end());
3      set<pair<long long, long long>> s;
4      s.insert({pts[0].y, pts[0].x});
5      long long l = 0, best = LLONG_MAX;
6      for (int i = 1; i < (int)pts.size(); i++) {

```

```

Pt now = pts[i];
long long lim = (long long)ceil(sqrt1((long
double)best));
while (now.x - pts[l].x > lim) {
    s.erase({pts[l].y, pts[l].x}); l++;
}
auto low = s.lower_bound({now.y - lim,
LLONG_MIN});
auto high = s.upper_bound({now.y + lim,
LLONG_MAX});
for (auto it = low; it != high; it++) {
    long long dy = it->first - now.y;
    long long dx = it->second - now.x;
    best = min(best, dx * dx + dy * dy);
}
s.insert({now.y, now.x});
}
return best;
}

```

## 7.11 Lower Concave Hull

```

1  struct Line {
2      mutable ll m, b, p;
3      bool operator<(const Line& o) const { return m < o.m;
        }
4      bool operator<(ll x) const { return p < x; }
5  };
6  struct LineContainer : multiset<Line, less<>> {
7      // (for doubles, use inf = 1/.0, div(a,b) = a/b)
8      const ll inf = LLONG_MAX;
9      ll div(ll a, ll b) { // floored division
10         return a / b - ((a ^ b) < 0 && a % b); }
11      bool isect(iterator x, iterator y) {
12          if (y == end()) { x->p = inf; return false; }
13          if (x->m == y->m) x->p = x->b > y->b ? inf : -inf;
14          else x->p = div(y->b - x->b, x->m - y->m);
15          return x->p >= y->p;
16      }
17      void add(ll m, ll b) {
18          auto z = insert({m, b, 0}); y = z++, x = y;
19          while (isect(y, z)) z = erase(z);
20          if (x != begin() && isect(--x, y)) isect(x, y =
            erase(y));
21          while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
22      }
23      ll query(ll x) {
24          assert(!empty());
25          auto l = *lower_bound(x);
26          return l.m * x + l.b;
27      }
28  };
29  };
30  };

```

## 7.12 Pick's Theorem

Consider a polygon which vertices are all lattice points.  
Let  $i$  = number of points inside the polygon.  
Let  $b$  = number of points on the boundary of the poly-  
gon.

Then we have the following formula:

$$Area = i + \frac{b}{2} - 1$$

## 7.13 Vector In Polygon

## 7.14 Minkowski Sum

```

1  /* convex hull Minkowski Sum*/
2  #define INF 10000000000000LL
3  int pos(const Pt &tp) {
4      if (tp.Y == 0) return tp.X > 0 ? 0 : 1;
5      return tp.Y > 0 ? 0 : 1;
6  }
7  #define N 300030
8  Pt pt[N], qt[N], rt[N];
9  LL Lx, Rx;
10 int dn, un;
11 inline bool cmp(Pt a, Pt b) {

```

```

12     int pa = pos(a), pb = pos(b);
13     if (pa == pb) return (a ^ b) > 0;
14     return pa < pb;
15 }
16 int minkowskiSum(int n, int m) {
17     int i, j, r, p, q, fi, fj;
18     for (i = 1, p = 0; i < n; i++) {
19         if (pt[i].Y < pt[p].Y ||
20             (pt[i].Y == pt[p].Y && pt[i].X < pt[p].X))
21             p = i;
22     }
23     for (i = 1, q = 0; i < m; i++) {
24         if (qt[i].Y < qt[q].Y ||
25             (qt[i].Y == qt[q].Y && qt[i].X < qt[q].X))
26             q = i;
27     }
28     rt[0] = pt[p] + qt[q];
29     r = 1;
30     i = p;
31     j = q;
32     fi = fj = 0;
33     while (1) {
34         if ((fj && j == q) ||
35             ((!fi || i != p) &&
36              cmp(pt[(p + 1) % n] - pt[p], qt[(q + 1) %
37                  m] - qt[q]))) {
38             rt[r] = rt[r - 1] + pt[(p + 1) % n] - pt[p];
39             p = (p + 1) % n;
40             fi = 1;
41         } else {
42             rt[r] = rt[r - 1] + qt[(q + 1) % m] - qt[q];
43             q = (q + 1) % m;
44             fj = 1;
45         }
46         if (r <= 1 || ((rt[r] - rt[r - 1]) ^ (rt[r - 1] -
47             rt[r - 2])) != 0) r++;
48         else rt[r - 1] = rt[r];
49         if (i == p && j == q) break;
50     }
51     return r - 1;
52 }
53 void initInConvex(int n) {
54     int i, p, q;
55     LL Ly, Ry;
56     Lx = INF;
57     Rx = -INF;
58     for (i = 0; i < n; i++) {
59         if (pt[i].X < Lx) Lx = pt[i].X;
60         if (pt[i].X > Rx) Rx = pt[i].X;
61     }
62     Ly = Ry = INF;
63     for (i = 0; i < n; i++) {
64         if (pt[i].X == Lx && pt[i].Y < Ly) {
65             Ly = pt[i].Y;
66             p = i;
67         }
68         if (pt[i].X == Rx && pt[i].Y < Ry) {
69             Ry = pt[i].Y;
70             q = i;
71         }
72     }
73     for (dn = 0, i = p; i != q; i = (i + 1) % n)
74         qt[dn++] = pt[i];
75     qt[dn] = pt[q];
76     Ly = Ry = -INF;
77     for (i = 0; i < n; i++) {
78         if (pt[i].X == Lx && pt[i].Y > Ly) {
79             Ly = pt[i].Y;
80             p = i;
81         }
82         if (pt[i].X == Rx && pt[i].Y > Ry) {
83             Ry = pt[i].Y;
84             q = i;
85         }
86     }
87     for (un = 0, i = p; i != q; i = (i + n - 1) % n)
88         rt[un++] = pt[i];
89     rt[un] = pt[q];
90 }
91 inline int inConvex(Pt p) {
92     int L, R, M;
93     if (p.X < Lx || p.X > Rx) return 0;
94     L = 0;
95     R = dn;
96     while (L < R - 1) {
97         M = (L + R) / 2;
98         if (p.X < qt[M].X) R = M;
99         else L = M;
100     }
101     if (tri(qt[L], qt[R], p) < 0) return 0;
102     L = 0;
103     R = un;
104     while (L < R - 1) {
105         M = (L + R) / 2;
106         if (p.X < rt[M].X) R = M;
107         else L = M;
108     }
109     if (tri(rt[L], rt[R], p) > 0) return 0;
110     return 1;
111 }
112 int main() {
113     int n, m, i;
114     Pt p;
115     scanf("%d", &n);
116     for (i = 0; i < n; i++) scanf("%lld%lld", &pt[i].X,
117         &pt[i].Y);
118     scanf("%d", &m);
119     for (i = 0; i < m; i++) scanf("%lld%lld", &qt[i].X,
120         &qt[i].Y);
121     n = minkowskiSum(n, m);
122     for (i = 0; i < n; i++) pt[i] = rt[i];
123     scanf("%d", &m);
124     for (i = 0; i < m; i++) scanf("%lld%lld", &qt[i].X,
125         &qt[i].Y);
126     n = minkowskiSum(n, m);
127     for (i = 0; i < n; i++) pt[i] = rt[i];
128     initInConvex(n);
129     scanf("%d", &m);
130     for (i = 0; i < m; i++) {
131         scanf("%lld %lld", &p.X, &p.Y);
132         p.X *= 3;
133         p.Y *= 3;
134         puts(inConvex(p) ? "YES" : "NO");
135     }
136 }

```

## 7.15 Rotating SweepLine

## 7.16 Half Plane Intersection

```

1 const long double eps = 1e-9, inf = 1e9;
2 struct Point {
3     long double x, y;
4     explicit Point(long double x = 0, long double y =
5         0) : x(x), y(y) {}
6     friend Point operator+(const Point& p, const Point&
7         q) {
8         return Point(p.x + q.x, p.y + q.y);
9     }
10    friend Point operator-(const Point& p, const Point&
11        q) {
12        return Point(p.x - q.x, p.y - q.y);
13    }
14    friend Point operator*(const Point& p, const long
15        double& k) {
16        return Point(p.x * k, p.y * k);
17    }
18    friend long double dot(const Point& p, const Point&
19        q) {
20        return p.x * q.x + p.y * q.y;
21    }
22    friend long double cross(const Point& p, const
23        Point& q) {
24        return p.x * q.y - p.y * q.x;
25    }
26 };
27 struct Halfplane {
28     Point p, pq;
29     long double angle;
30     Halfplane() {}
31     Halfplane(const Point& a, const Point& b) : p(a),
32         pq(b - a) {}

```

```

26     angle = atan2l(pq.y, pq.x);
27 }
28 bool out(const Point& r) {
29     return cross(pq, r - p) < -eps;
30 }
31 bool operator<(const Halfplane& e) const {
32     return angle < e.angle;
33 }
34 friend Point inter(const Halfplane& s, const
35     Halfplane& t) {
36     long double alpha = cross((t.p - s.p), t.pq) /
37         cross(s.pq, t.pq);
38     return s.p + (s.pq * alpha);
39 };
40 vector<Point> hp_intersect(vector<Halfplane>& H) {
41     Point box[4] = { // Bounding box in CCW order
42         Point(Inf, Inf),
43         Point(-Inf, Inf),
44         Point(-Inf, -Inf),
45         Point(Inf, -Inf)};
46     for (int i = 0; i < 4; i++) { // Add bounding box
47         Halfplane aux(box[i], box[(i + 1) % 4]);
48         H.push_back(aux);
49     }
50     sort(H.begin(), H.end());
51     deque<Halfplane> dq;
52     int len = 0;
53     for (int i = 0; i < int(H.size()); i++) {
54         while (len > 1 && H[i].out(inter(dq[len - 1],
55             dq[len - 2]))) {
56             dq.pop_back();
57             --len;
58         }
59         while (len > 1 && H[i].out(inter(dq[0], dq[1]))) {
60             dq.pop_front();
61             --len;
62         }
63         if (len > 0 && fabs1(cross(H[i].pq, dq[len - 1].pq)) < eps) {
64             if (dot(H[i].pq, dq[len - 1].pq) < 0.0)
65                 return vector<Point>();
66             if (H[i].out(dq[len - 1].p)) {
67                 dq.pop_back();
68                 --len;
69             } else
70                 continue;
71         }
72         dq.push_back(H[i]);
73         ++len;
74     }
75     while (len > 2 && dq[0].out(inter(dq[len - 1], dq[
76         len - 2]))) {
77         dq.pop_back();
78         --len;
79     }
80     while (len > 2 && dq[len - 1].out(inter(dq[0], dq
81         [1]))) {
82         dq.pop_front();
83         --len;
84     }
85     if (len < 3) return vector<Point>();
86     vector<Point> ret(len);
87     for (int i = 0; i + 1 < len; i++) {
88         ret[i] = inter(dq[i], dq[i + 1]);
89     }
90     ret.back() = inter(dq[len - 1], dq[0]);
91     return ret;
92 }

```

## 7.17 Minimum Enclosing Circle

```

1 Pt circumcenter(Pt A, Pt B, Pt C) {
2     // a1(x-A.x) + b1(y-A.y) = c1
3     // a2(x-A.x) + b2(y-A.y) = c2
4     // solve using Cramer's rule
5     T a1 = B.x - A.x, b1 = B.y - A.y, c1 = dis2(A, B) /
6         2.0;
7     T a2 = C.x - A.x, b2 = C.y - A.y, c2 = dis2(A, C) /
8         2.0;

```

```

7     T D = Pt(a1, b1) ^ Pt(a2, b2);
8     T Dx = Pt(c1, b1) ^ Pt(c2, b2);
9     T Dy = Pt(a1, c1) ^ Pt(a2, c2);
10    if (D == 0) return Pt(-INF, -INF);
11    return A + Pt(Dx / D, Dy / D);
12 }
13 Pt center;
14 T r2;
15 void minEncloseCircle() {
16     mt19937 gen(chrono::steady_clock::now().
17         time_since_epoch().count());
18     shuffle(ALL(E), gen);
19     center = E[0], r2 = 0;
20
21     for (int i = 0; i < n; i++) {
22         if (dis2(center, E[i]) <= r2) continue;
23         center = E[i], r2 = 0;
24         for (int j = 0; j < i; j++) {
25             if (dis2(center, E[j]) <= r2) continue;
26             center = (E[i] + E[j]) / 2.0;
27             r2 = dis2(center, E[i]);
28             for (int k = 0; k < j; k++) {
29                 if (dis2(center, E[k]) <= r2) continue;
30                 center = circumcenter(E[i], E[j], E[k]);
31                 r2 = dis2(center, E[i]);
32             }
33         }
34     }

```

## 7.18 Heart

## 7.19 Tangents

## 7.20 Point In Circle

## 7.21 Union of Circles

## 7.22 Union of Polygons

## 7.23 Delaunay Triangulation

## 7.24 Triangulation Voronoi

## 7.25 External Bisector

## 7.26 Intersection Area of Polygon and Circle

## 7.27 3D Point

## 7.28 3D Convex Hull

# 8 Number Theory

## 8.1 FFT

```

1 typedef complex<double> cp;
2
3 const double pi = acos(-1);
4 const int NN = 131072;
5
6 struct FastFourierTransform {
7     /*
8         Iterative Fast Fourier Transform
9         How this works? Look at this
10         0th recursion 0(000) 1(001) 2(010)
11                     3(011) 4(100) 5(101) 6(110)
12                     7(111)
13         1th recursion 0(000) 2(010) 4(100)
14                     6(110) | 1(011) 3(011) 5(101)
15                     7(111)
16         2th recursion 0(000) 4(100) | 2(010)
17                     6(110) | 1(011) 5(101) | 3(011)
18                     7(111)
19         3th recursion 0(000) | 4(100) | 2(010) |
20                     6(110) | 1(011) | 5(101) | 3(011) |
21                     7(111)
22         All the bits are reversed => We can save
23         the reverse of the numbers in an array!
24     */
25     int n, rev[NN];
26     cp omega[NN], iomega[NN];
27     void init(int n_) {
28         n = n_;
29         for (int i = 0; i < n_; i++) {
30             // Calculate the nth roots of unity

```

```

22     omega[i] = cp(cos(2 * pi * i / n_), sin(2 * pi * i / n_));
23     iomega[i] = conj(omega[i]);
24 }
25 int k = __lg(n_);
26 for (int i = 0; i < n_; i++) {
27     int t = 0;
28     for (int j = 0; j < k; j++) {
29         if (i & (1 << j)) t |= (1 << (k - j - 1));
30     }
31     rev[i] = t;
32 }
33 }
34
35 void transform(vector<cp> &a, cp *xomega) {
36     for (int i = 0; i < n; i++)
37         if (i < rev[i]) swap(a[i], a[rev[i]]);
38     for (int len = 2; len <= n; len <= 1) {
39         int mid = len >> 1;
40         int r = n / len;
41         for (int j = 0; j < n; j += len)
42             for (int i = 0; i < mid; i++) {
43                 cp tmp = xomega[r * i] * a[j + mid + i];
44                 a[j + mid + i] = a[j + i] - tmp;
45                 a[j + i] = a[j + i] + tmp;
46             }
47     }
48 }
49
50 void fft(vector<cp> &a) { transform(a, omega); }
51 void ifft(vector<cp> &a) {
52     transform(a, iomega);
53     for (int i = 0; i < n; i++) a[i] /= n;
54 }
55 } FFT;
56
57 const int MAXN = 262144;
58 // (must be 2^k)
59 // 262144, 524288, 1048576, 2097152, 4194304
60 // before any usage, run pre_fft() first
61 typedef long double ld;
62 typedef complex<ld> cplx; // real() , imag()
63 const ld PI = acos(-1);
64 const cplx I(0, 1);
65 cplx omega[MAXN + 1];
66 void pre_fft() {
67     for (int i = 0; i <= MAXN; i++) {
68         omega[i] = exp(i * 2 * PI / MAXN * I);
69     }
70 }
71 // n must be 2^k
72 void fft(int n, cplx a[], bool inv = false) {
73     int basic = MAXN / n;
74     int theta = basic;
75     for (int m = n; m >= 2; m >>= 1) {
76         int mh = m >> 1;
77         for (int i = 0; i < mh; i++) {
78             cplx w = omega[inv ? MAXN - (i * theta % MAXN) : i * theta % MAXN];
79             for (int j = i; j < n; j += m) {
80                 int k = j + mh;
81                 cplx x = a[j] - a[k];
82                 a[j] += a[k];
83                 a[k] = w * x;
84             }
85         }
86         theta = (theta * 2) % MAXN;
87     }
88     int i = 0;
89     for (int j = 1; j < n - 1; j++) {
90         for (int k = n >> 1; k > (i ^= k); k >>= 1);
91         if (j < i) swap(a[i], a[j]);
92     }
93     if (inv) {
94         for (i = 0; i < n; i++) a[i] /= n;
95     }
96 }
97 cplx arr[MAXN + 1];
98 inline void mul(int _n, long long a[], int _m, long long b[], long long ans[]) {

```

```

99     int n = 1, sum = _n + _m - 1;
100     while (n < sum) n <= 1;
101     for (int i = 0; i < n; i++) {
102         double x = (i < _n ? a[i] : 0), y = (i < _m ? b[i] : 0);
103         arr[i] = complex<double>(x + y, x - y);
104     }
105     fft(n, arr);
106     for (int i = 0; i < n; i++) arr[i] = arr[i] * arr[i];
107     fft(n, arr, true);
108     for (int i = 0; i < sum; i++) ans[i] = (long long int)(arr[i].real() / 4 + 0.5);
109 }
110
111 long long a[MAXN];
112 long long b[MAXN];
113 long long ans[MAXN];
114 int a_length;
115 int b_length;

```

## 8.2 Pollard's rho

```

1 ll add(ll x, ll y, ll p) {
2     return (x + y) % p;
3 }
4 ll qMul(ll x, ll y, ll mod) {
5     ll ret = x * y - (ll)((long double)x / mod * y) * mod;
6     return ret < 0 ? ret + mod : ret;
7 }
8 ll f(ll x, ll mod) { return add(qMul(x, x, mod), 1, mod); }
9 ll pollard_rho(ll n) {
10     if (!(n & 1)) return 2;
11     while (true) {
12         ll y = 2, x = rand() % (n - 1) + 1, res = 1;
13         for (int sz = 2; res == 1; sz *= 2) {
14             for (int i = 0; i < sz && res <= 1; i++) {
15                 x = f(x, n);
16                 res = __gcd(llabs(x - y), n);
17             }
18             y = x;
19         }
20         if (res != 0 && res != n) return res;
21     }
22 }
23 vector<ll> ret;
24 void fact(ll x) {
25     if (miller_rabin(x)) {
26         ret.push_back(x);
27         return;
28     }
29     ll f = pollard_rho(x);
30     fact(f);
31     fact(x / f);
32 }

```

## 8.3 Miller Rabin

```

1 // n < 4,759,123,141          3 : 2, 7, 61
2 // n < 1,122,004,669,633     4 : 2, 13, 23, 1662803
3 // n < 3,474,749,660,383     6 : pirmes <= 13
4 // n < 2^64                  7 :
5 // 2, 325, 9375, 28178, 450775, 9780504, 1795265022
6 bool witness(ll a, ll n, ll u, int t) {
7     if (!(a % n)) return 0;
8     ll x = mypow(a, u, n);
9     for (int i = 0; i < t; i++) {
10         ll nx = mul(x, x, n);
11         if (nx == 1 && x != 1 && x != n - 1) return 1;
12         x = nx;
13     }
14     return x != 1;
15 }
16 bool miller_rabin(ll n, int s = 100) {
17     // iterate s times of witness on n
18     // return 1 if prime, 0 otherwise
19     if (n < 2) return 0;
20     if (!(n & 1)) return n == 2;
21     ll u = n - 1;
22     int t = 0;

```

```

23 while (!(u & 1)) u >>= 1, t++;
24 while (s--) {
25     ll a = randll() % (n - 1) + 1;
26     if (witness(a, n, u, t)) return 0;
27 }
28 return 1;
29 }

```

## 8.4 Fast Power

Note:  $a^n \equiv a^{(n \bmod (p-1))} \pmod{p}$

## 8.5 Extend GCD

```

1 ll gcd;
2 pll extgcd(ll a, ll b) {
3     if (b == 0) {
4         gcd = a;
5         return pll{1, 0};
6     }
7     pll ans = extgcd(b, a % b);
8     return pll{ans.S, ans.F - a / b * ans.S};
9 }
10 pll bezout(ll a, ll b, ll c) {
11     bool negx = (a < 0), negy = (b < 0);
12     pll ans = extgcd(abs(a), abs(b));
13     if (c % gcd != 0) return pll{-LLINF, -LLINF};
14     return pll{ans.F * c / gcd * (negx ? -1 : 1),
15               ans.S * c / gcd * (negy ? -1 : 1)};
16 }
17 ll inv(ll a, ll p) {
18     if (p == 1) return -1;
19     pll ans = bezout(a % p, -p, 1);
20     if (ans == pll{-LLINF, -LLINF}) return -1;
21     return (ans.F % p + p) % p;
22 }

```

## 8.6 Mu + Phi

```

1 const int maxn = 1e6 + 5;
2 ll f[maxn];
3 vector<int> lpf, prime;
4 void build() {
5     lpf.clear();
6     lpf.resize(maxn, 1);
7     prime.clear();
8     f[1] = ...; /* mu[1] = 1, phi[1] = 1 */
9     for (int i = 2; i < maxn; i++) {
10         if (lpf[i] == 1) {
11             lpf[i] = i;
12             prime.emplace_back(i);
13             f[i] = ...; /* mu[i] = 1, phi[i] = i-1 */
14         }
15         for (auto& j : prime) {
16             if (i * j >= maxn) break;
17             lpf[i * j] = j;
18             if (i % j == 0)
19                 f[i * j] = ...; /* 0, phi[i]*j */
20             else
21                 f[i * j] = ...; /* -mu[i], phi[i]*phi[j] */
22             if (j >= lpf[i]) break;
23         }
24     }
25 }

```

## 8.7 Other Formulas

- Inversion:  
 $aa^{-1} \equiv 1 \pmod{m}$ .  $a^{-1}$  exists iff  $\gcd(a, m) = 1$ .
- Linear inversion:  
 $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod{m}$
- Fermat's little theorem:  
 $a^p \equiv a \pmod{p}$  if  $p$  is prime.
- Euler function:  
 $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$
- Euler theorem:  
 $a^{\phi(n)} \equiv 1 \pmod{n}$  if  $\gcd(a, n) = 1$ .

- Extended Euclidean algorithm:

$$ax + by = \gcd(a, b) = \gcd(b, a \bmod b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$$

- Divisor function:

$$\sigma_x(n) = \sum_{d|n} d^x. n = \prod_{i=1}^r p_i^{a_i}.$$

$$\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1} \text{ if } x \neq 0. \sigma_0(n) = \prod_{i=1}^r (a_i + 1).$$

- Chinese remainder theorem (Coprime Moduli):

$$x \equiv a_i \pmod{m_i}.$$

$$M = \prod m_i. M_i = M/m_i. t_i = M_i^{-1}.$$

$$x = kM + \sum a_i t_i M_i, k \in \mathbb{Z}.$$

- Chinese remainder theorem:

$$x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2} \Rightarrow x = m_1 p + a_1 = m_2 q + a_2 \Rightarrow m_1 p - m_2 q = a_2 - a_1$$

Solve for  $(p, q)$  using ExtGCD.

$$x \equiv m_1 p + a_1 \equiv m_2 q + a_2 \pmod{\text{lcm}(m_1, m_2)}$$

- Avoiding Overflow:  $ca \bmod cb = c(a \bmod b)$

- Dirichlet Convolution:  $(f * g)(n) = \sum_{d|n} f(d)g(n/d)$

- Important Multiplicative Functions + Properties:

- $\epsilon(n) = [n = 1]$
- $1(n) = 1$
- $id(n) = n$
- $\mu(n) = 0$  if  $n$  has squared prime factor
- $\mu(n) = (-1)^k$  if  $n = p_1 p_2 \cdots p_k$
- $\epsilon = \mu * 1$
- $\phi = \mu * id$
- $[n = 1] = \sum_{d|n} \mu(d)$
- $[gcd = 1] = \sum_{d|gcd} \mu(d)$

- Möbius inversion:  $f = g * 1 \Leftrightarrow g = f * \mu$

## 8.8 Polynomial

```

1 const int maxk = 20;
2 const int maxn = 1<<maxk;
3 const ll LINF = 1e18;
4
5 /* P = r*2^k + 1
6 P          r    k    g
7 998244353    119 23    3
8 1004535809    479 21    3
9
10 P          r    k    g
11 3           1    1    2
12 5           1    2    2
13 17          3    4    3
14 97          3    5    5
15 193         3    6    5
16 257         1    8    3
17 7681        15    9   17
18 12289       3   12   11
19 40961       5   13    3
20 65537       1   16    3
21 786433      3   18   10
22 5767169     11   19    3
23 7340033     7   20    3
24 23068673    11   21    3
25 104857601   25   22    3
26 167772161   5   25    3
27 469762049   7   26    3
28 1004535809  479  21    3
29 2013265921  15  27   31
30 2281701377  17  27    3
31 3221225473   3   30    5
32 75161927681 35   31    3
33 77309411329  9   33    7
34 206158430209 3   36   22
35 2061584302081 15  37    7
36 2748779069441 5   39    3

```



```

37 6597069766657      3  41  5
38 39582418599937     9  42  5
39 79164837199873     9  43  5
40 263882790666241    15  44  7
41 1231453023109121   35  45  3
42 1337006139375617   19  46  3
43 3799912185593857   27  47  5
44 4222124650659841   15  48  19
45 7881299347898369    7  50  6
46 31525197391593473  7  52  3
47 180143985094819841  5  55  6
48 1945555039024054273 27  56  5
49 4179340454199820289 29  57  3
50 9097271247288401921 505 54  6 */
51
52 const int g = 3;
53 const ll MOD = 998244353;
54
55 ll pw(ll a, ll n) { /* fast pow */ }
56
57 #define siz(x) (int)x.size()
58
59 template<typename T>
60 vector<T>& operator+=(vector<T>& a, const vector<T>& b) {
61     {
62         if (siz(a) < siz(b)) a.resize(siz(b));
63         for (int i = 0; i < min(siz(a), siz(b)); i++) {
64             a[i] += b[i];
65             a[i] -= a[i] >= MOD ? MOD : 0;
66         }
67         return a;
68     }
69
70 template<typename T>
71 vector<T>& operator-=(vector<T>& a, const vector<T>& b) {
72     {
73         if (siz(a) < siz(b)) a.resize(siz(b));
74         for (int i = 0; i < min(siz(a), siz(b)); i++) {
75             a[i] -= b[i];
76             a[i] += a[i] < 0 ? MOD : 0;
77         }
78         return a;
79     }
80
81 template<typename T>
82 vector<T> operator-(const vector<T>& a) {
83     vector<T> ret(siz(a));
84     for (int i = 0; i < siz(a); i++) {
85         ret[i] = -a[i] < 0 ? -a[i] + MOD : -a[i];
86     }
87     return ret;
88 }
89
90 vector<ll> X, iX;
91 vector<int> rev;
92
93 void init_ntt() {
94     X.clear(); X.resize(maxn, 1); // x1 = g^((p-1)/n)
95     iX.clear(); iX.resize(maxn, 1);
96
97     ll u = pw(g, (MOD-1)/maxn);
98     ll iu = pw(u, MOD-2);
99
100     for (int i = 1; i < maxn; i++) {
101         X[i] = X[i-1] * u;
102         iX[i] = iX[i-1] * iu;
103         if (X[i] >= MOD) X[i] %= MOD;
104         if (iX[i] >= MOD) iX[i] %= MOD;
105     }
106
107     rev.clear(); rev.resize(maxn, 0);
108     for (int i = 1, hb = -1; i < maxn; i++) {
109         if (!(i & (i-1))) hb++;
110         rev[i] = rev[i ^ (1<<hb)] | (1<<(maxk-hb-1));
111     }
112
113 template<typename T>
114 void NTT(vector<T>& a, bool inv=false) {
115     int _n = (int)a.size();
116     int k = __lg(_n) + ((1<<__lg(_n)) != _n);
117     int n = 1<<k;
118
119     a.resize(n, 0);
120
121     short shift = maxk-k;
122     for (int i = 0; i < n; i++)
123         if (i > (rev[i]>>shift))
124             swap(a[i], a[rev[i]>>shift]);
125
126     for (int len = 2, half = 1, div = maxn>>1; len <= n; len<=1, half<=1, div>=1) {
127         for (int i = 0; i < n; i += len) {
128             for (int j = 0; j < half; j++) {
129                 T u = a[i+j];
130                 T v = a[i+j+half] * (inv ? iX[j*div] : X[j*div]) % MOD;
131                 a[i+j] = (u+v >= MOD ? u+v-MOD : u+v);
132                 a[i+j+half] = (u-v < 0 ? u-v+MOD : u-v);
133             }
134         }
135     }
136
137     if (inv) {
138         T dn = pw(n, MOD-2);
139         for (auto& x : a) {
140             x *= dn;
141             if (x >= MOD) x %= MOD;
142         }
143     }
144
145 template<typename T>
146 inline void resize(vector<T>& a) {
147     int cnt = (int)a.size();
148     for (; cnt > 0; cnt--) if (a[cnt-1]) break;
149     a.resize(max(cnt, 1));
150 }
151
152 template<typename T>
153 vector<T>& operator*=(vector<T>& a, vector<T> b) {
154     int na = (int)a.size();
155     int nb = (int)b.size();
156     a.resize(na + nb - 1, 0);
157     b.resize(na + nb - 1, 0);
158
159     NTT(a); NTT(b);
160     for (int i = 0; i < (int)a.size(); i++) {
161         a[i] *= b[i];
162         if (a[i] >= MOD) a[i] %= MOD;
163     }
164     NTT(a, true);
165
166     resize(a);
167     return a;
168 }
169
170 template<typename T>
171 void inv(vector<T>& ia, int N) {
172     vector<T> _a(move(ia));
173     ia.resize(1, pw(_a[0], MOD-2));
174     vector<T> a(1, -_a[0] + (-_a[0] < 0 ? MOD : 0));
175
176     for (int n = 1; n < N; n<=1) {
177         // n -> 2*n
178         // ia' = ia(2-a*ia);
179
180         for (int i = n; i < min(siz(_a), (n<<1)); i++)
181             a.emplace_back(-_a[i] + (-_a[i] < 0 ? MOD : 0));
182
183         vector<T> tmp = ia;
184         ia *= a;
185         ia.resize(n<<1);
186         ia[0] = ia[0] + 2 >= MOD ? ia[0] + 2 - MOD : ia[0] + 2;
187         ia *= tmp;
188         ia.resize(n<<1);
189     }
190     ia.resize(N);
191
192 template<typename T>
193 void mod(vector<T>& a, vector<T>& b) {
194     int n = (int)a.size()-1, m = (int)b.size()-1;
195     if (n < m) return;
196
197     vector<T> ra = a, rb = b;

```

```

194 reverse(ra.begin(), ra.end()); ra.resize(min(n+1, n
    -m+1));
195 reverse(rb.begin(), rb.end()); rb.resize(min(m+1, n
    -m+1));
196
197 inv(rb, n-m+1);
198
199 vector<T> q = move(ra);
200 q *= rb;
201 q.resize(n-m+1);
202 reverse(q.begin(), q.end());
203
204 q *= b;
205 a -= q;
206 resize(a);
207 }
208
209 /* Kitamasa Method (Fast Linear Recurrence):
210 Find a[K] (Given a[j] = c[0]a[j-N] + ... + c[N-1]a[j
    -1])
211 Let B(x) = x^N - c[N-1]x^(N-1) - ... - c[1]x^1 - c[0]
212 Let R(x) = x^K mod B(x) (get x^K using fast pow and
    use poly mod to get R(x))
213 Let r[i] = the coefficient of x^i in R(x)
214 => a[K] = a[0]r[0] + a[1]r[1] + ... + a[N-1]r[N-1] */

```

## 9 Linear Algebra

### 9.1 Gaussian-Jordan Elimination

```

1 int n;
2 vector<vector<ll>> v;
3 void gauss(vector<vector<ll>>& v) {
4     int r = 0;
5     for (int i = 0; i < n; i++) {
6         bool ok = false;
7         for (int j = r; j < n; j++) {
8             if (v[j][i] == 0) continue;
9             swap(v[j], v[r]);
10            ok = true;
11            break;
12        }
13        if (!ok) continue;
14        ll div = inv(v[r][i]);
15        for (int j = 0; j < n + 1; j++) {
16            v[r][j] *= div;
17            if (v[r][j] >= MOD) v[r][j] %= MOD;
18        }
19        for (int j = 0; j < n; j++) {
20            if (j == r) continue;
21            ll t = v[j][i];
22            for (int k = 0; k < n + 1; k++) {
23                v[j][k] -= v[r][k] * t % MOD;
24                if (v[j][k] < 0) v[j][k] += MOD;
25            }
26        }
27        r++;
28    }
29 }

```

### 9.2 Determinant

1. Use GJ Elimination, if there's any row consists of only 0, then  $\det = 0$ , otherwise  $\det = \text{product of diagonal elements}$ .

2. Properties of  $\det$ :

- Transpose: Unchanged
- Row Operation 1 - Swap 2 rows:  $-\det$
- Row Operation 2 -  $k\vec{r}_i$ :  $k \times \det$
- Row Operation 3 -  $k\vec{r}_i$  add to  $\vec{r}_j$ : Unchanged

## 10 Combinatorics

### 10.1 Catalan Number

$$C_0 = 1, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}, C_n = C_n^{2n} - C_{n-1}^{2n}$$

0	1	1	2	5
4	14	42	132	429
8	1430	4862	16796	58786
12	208012	742900	2674440	9694845

### 10.2 Burnside's Lemma

Let  $X$  be the original set.

Let  $G$  be the group of operations acting on  $X$ .

Let  $X^g$  be the set of  $x$  not affected by  $g$ .

Let  $X/G$  be the set of orbits.

Then the following equation holds:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

## 11 Special Numbers

### 11.1 Fibonacci Series

1	1	1	2	3
5	5	8	13	21
9	34	55	89	144
13	233	377	610	987
17	1597	2584	4181	6765
21	10946	17711	28657	46368
25	75025	121393	196418	317811
29	514229	832040	1346269	2178309
33	3524578	5702887	9227465	14930352

$$f(45) \approx 10^9, f(88) \approx 10^{18}$$

### 11.2 Prime Numbers

- First 50 prime numbers:

1	2	3	5	7	11
6	13	17	19	23	29
11	31	37	41	43	47
16	53	59	61	67	71
21	73	79	83	89	97
26	101	103	107	109	113
31	127	131	137	139	149
36	151	157	163	167	173
41	179	181	191	193	197
46	199	211	223	227	229

- Very large prime numbers:

1000001333	1000500889	2500001909
2000000659	900004151	850001359

- $\pi(n) \equiv \text{Number of primes} \leq n \approx n/((\ln n) - 1)$

$$\pi(100) = 25, \pi(200) = 46$$

$$\pi(500) = 95, \pi(1000) = 168$$

$$\pi(2000) = 303, \pi(4000) = 550$$

$$\pi(10^4) = 1229, \pi(10^5) = 9592$$

$$\pi(10^6) = 78498, \pi(10^7) = 664579$$