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An integrated framework for elevator traffic design under general traffic conditions using origin destination matrices, virtual interval, and the Monte Carlo simulation method

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Abstract

The conventional design methodology for elevator traffic analysis has been applied to the case of up-peak traffic (or incoming traffic conditions). The only user requirements are usually the expected arrival rate (*AR%*) expressed as a percentage of the building population requesting service in the peak 5 min and the target interval. The interval as classically used will be referred to as the physical interval in this paper as it is only relevant for the case of a single entrance and incoming traffic conditions. This paper presents an integrated methodology for the design of elevator traffic systems for the general case of mixed traffic conditions. It presents a fully integrated framework that covers the steps from user requirements to the selection of the number of required elevators. The user requirements describing the traffic conditions can be specified by the user, expressed as the *AR%*, the mix of incoming traffic, outgoing traffic, and interfloor traffic. This paper derives equations that can be used to combine the mix of traffic, the floor arrival percentages, and the floor population percentages into an origin–destination matrix. The origin–destination matrix is then adjusted and normalized in order to account for rational passenger behavior (i.e., a passenger will not travel to the same floor that he or she is at). A method is presented for the random generation of passenger origin–destination pairs using the origin–destination matrix (which is necessary when using the Monte Carlo Simulation (*MCS*) method to calculate the round trip time). A novel equation for evaluating the round trip time under the assumption of equal floor heights and top speed attained in one floor journey is derived and used. The equation is derived using a stepwise derivation and verification process. The verification is carried out against the MCS method for finding the value of the round trip time. The concept of a virtual interval (as opposed to the conventionally used physical interval usually used in elevator traffic system design) is introduced in order to allow the selection of the number of elevators

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to be carried out. The virtual interval is the average value of the time between the consecutive reversals of the elevators in the group.

Practical application The methodology presented in this paper allows the elevator traffic system designer to convert the user requirements specification (in the form of an arrival rate and the percentage floor strengths) into an origin–destination matrix. The origin–destination matrix is a more suitable tool for calculating the expected value of the round trip time and consequently carrying out an elevator traffic design. Thus, this methodology represents a vital step in the design process.

Keywords

Elevator, lift, round trip time, physical interval, virtual interval, incoming traffic, outgoing traffic, interfloor traffic, general traffic conditions, Monte Carlo Simulation, origin–destination matrix

Introduction

The conventional elevator traffic design methodology relies on designing the elevator traffic system assuming 100% incoming traffic. Under incoming traffic conditions, passengers arrive at the building entrances (e.g., lobby and car park floors) and travel to occupant floors up in the building. Designing based on 100% incoming traffic conditions is a reflection of the morning up-peak where the majority of passengers are entering the building. It is often assumed that an elevator traffic system designed to meet the morning up-peak demand will cope comfortably with the demand at any other time of the day.

However, recent work has shown that modern changes in working practices are changing the peak passenger demand. It is likely that in some buildings, the peak passenger demand is the lunchtime peak where some passengers are leaving the building for lunch (outgoing traffic), some passengers are entering the building after returning from lunch (incoming traffic), and other passengers are moving within the building (i.e., interfloor traffic). Examples of traffic mix conditions that are believed to be representative of the lunchtime peak traffic conditions in many modern office buildings include: 40%:40%: 20%;[1] 45%:45%:10%;[2] 42%, 42%, 16%;[3] incoming, outgoing, and interfloor traffic, respectively.

In addition, it is possible to calculate the round trip time for general traffic conditions under the Poisson arrival process assumption[4,5] or the plentiful passenger supply assumption.[6]

Based on these last two points, this paper thus presents an integrated framework for carrying out an elevator traffic design under general traffic conditions (i.e., any traffic mix of incoming traffic, outgoing traffic, and interfloor traffic). It presents a procedure starting from the user requirements of the arrival rate ($AR\%$) and the target interval (int_{tar}) as well as the specific traffic conditions (i.e., traffic mix) and ending with the required number of elevators and the actual interval and the handling capacity ($HC\%$). In doing this, it heavily relies on the concept of origin–destination (OD) matrix as an intermediate step for calculating the round trip time, as well as the new concept of the virtual interval (as opposed to the physical interval), which is a more appropriate variable to use under general traffic conditions (as opposed to the 100% incoming traffic conditions traditionally used).

The framework presented in this paper can then be combined with the $HARint$ plane[7] or the $HARint$ space[8] methodologies in order to be used as a comprehensive universal elevator traffic system design tool, whereby the average passenger waiting time and the average passenger traveling times can also be added as user requirements, and outputs such as the car capacity, the elevator speeds can be provided as outputs of the design.

The section entitled "Describing traffic in a building" outlines conventions and terminology for describing the types of floors in a building and types of traffic and presents the procedure for converting the floor arrival percentages, the floor percentage populations, and the traffic mix into an OD matrix. The section entitled "Random generation of passengers using the OD matrix" presents a procedure for generating passenger OD pairs, a step that is critical for evaluating the round trip time using the Monte Carlo simulation (MCS) method. A numerical example on generating passenger OD pairs is also presented in Random generation of passengers using the OD matrix section. A novel round trip time equation is derived in the section entitled "Derivation of a round trip time equation for general traffic conditions" for the case of general traffic conditions assuming that top speed is attained in one floor journey and that floor heights are equal. It has a form that is very similar to the conventional round trip time equation for the case of incoming traffic conditions, and thus can be very insightful.

Once general traffic conditions have been introduced, the physical interval ceases to be suitable and has to be redefined to suit the case of general traffic conditions. The virtual interval is introduced as an alternative to the physical interval and is defined in the section entitled "Redefining the interval for the case of multiple entrances and general traffic: The virtual interval". The virtual interval can be used with the round trip time in order to find the required number of elevators.

A complete numerical example is introduced in the section entitled "Numerical example" to illustrate how the integrated framework can be used to first evaluate the OD matrix and then calculate the round trip time using the equation derived in the section entitled "Derivation of a round trip time equation for general traffic conditions" and then finding the required number of elevators, the actual interval, and the HC. Conclusions are drawn in the section entitled "Conclusions".

Describing traffic in a building

This section develops a systematic methodology for describing the traffic in a building, by converting the traffic mix and the floor arrival and population percentages into an OD matrix.

Types of floors

This section looks at the classification of floors. Any floor in a building can either be described as an entrance floor or an occupant floor. An entrance floor (referred to in short as *ent* floor) is a floor through which passengers can either enter or exit the building. An entrance floor can also be referred to an entrance–exit floor, as it becomes an exit floor when the traffic is outgoing.

An occupant floor is a floor through which passengers cannot enter or exit the building, but where they will reside during their stay in the building (referred to in short as an *occ* floor). Based on the assumption above, all floors in the building would be denoted as either entrance floors or occupant floors, but not both. This is true in most buildings, although there might be some rare exceptions to this rule, where a floor could simultaneously be an entrance floor and an occupant floor.

An example of a building that is represented in this form is shown below in Table 1. Each floor is either an entrance–exit floor or an occupant floor. The percentage of passengers entering the building via an entrance floor is denoted as the percentage AR ($Pr_{arr}(i)$). The population of a floor expressed as a percentage of the total building population is denoted as the percentage population ($(U(i)/U)$).

Where a floor is an entrance floor, it has a nonzero value for its percentage AR and a zero-percentage population. Where a floor is an occupant floor, it has a nonzero percentage population, but a zero-percentage AR.

It is customary for the entrance floors to be contiguous and for the occupant floors to be contiguous, but this is not necessary. An example of a noncontiguous floor is where a restaurant is located on the top floor of a building and

Table 1. Representation of traffic in a building.

Floor notation	#	Floor name	Entrance/exit floor or Occupant floor	Percentage arrival rate	Population percentage
N	7	*L5*	*occ*	0	$80/480 = 0.1667$
$N - 1$	6	*L4*	*occ*	0	$80/480 = 0.1667$
...	5	*L3*	*occ*	0	$80/480 = 0.1667$
...	4	*L2*	*occ*	0	$120/480 = 0.25$
...	3	*L1*	*occ*	0	$120/480 = 0.25$
2	2	*G*	*ent*	0.8	0
1	1	*B*	*ent*	0.2	0

is classified as an entrance–exit floor (as it is functionally external to the building albeit not physically external). The design methodology presented later in this paper can cope with the general case where the entrance floors are non-contiguous and the occupant floors are noncontiguous.

Table 1 shows a building with two entrances with unequal percentage ARs (0.8 ground floor denoted as G; 0.2 from the basement denoted as B). There are five occupant floors denoted as $L1$–$L5$. They have unequal population percentages.

The lowest floor in the building is denoted as floor 1 and the topmost floor as floor N. The following convention is followed in describing the values of arrival percentages and populations for the floors:

> $Pr_{arr}(i)$ is used to denote the arrival percentage of the ith floor, where i runs from 1 to N.
> $U(i)/U$ is used to denote the percentage population of the ith floor, where i runs from 1 to N, where $U(i)$ is the population of the ith floor and U is the total building population.

The representation of traffic in a building as shown in Table 1 is the default format and represents pure incoming traffic into the building. Under such a traffic condition, all passengers would be entering the building from the entrance floors and heading to the occupant floors.

A general format for representing the traffic in a building is shown in Table 2. As a generalization, the term arrival can be extended to arrival–departure to cover both passengers entering the building under incoming traffic conditions and passengers leaving the building under outgoing traffic conditions. By setting a population percentage for a floor to zero, it is an indication that it is an entrance floor; by setting an arrival percentage for a floor to zero, it is an indication that it is an occupant floor, as shown in Table 3.

Table 2. General representation format for a building.

Floor	Percentage arrival/departure	Percentage population
N	$Pr_{arr}(N)$	$(U(N)/U)$
$N - 1$	$Pr_{arr}\ (N - 1)$	$(U(N - 1)/U)$
...	...	...
...	...	...
...	...	...
2	$Pr_{arr}(2)$	$(U(2)/U)$
1	$Pr_{arr}(1)$	$(U(1)/U)$

It is worth noting that the summation of the percentage arrival–departure rates is 1 and the summation of all the building percentage populations is 1 as shown in equations (1) and (2).

$$\sum_{i=1}^{N} \mathrm{Pr}_{arr}(i) = 1 \tag{1}$$

Table 3. General representation format for a building.

Type	#	Percentage arrival rate	Population percentage	Floor percentage ($R(i)$)
Occupant floors	N	–	$(U(N)/U)$	$(U(N)/U)$
	$N-1$	–	$(U(N-1)/U)$	$(U(N-1)/U)$
	...	–	...	...
	...	–	...	...
	$i+2$	–	$(U(i+2)/U)$	$(U(i+2)/U)$
	$i+1$	–	$(U(i+1)/U)$	$(U(i+1)/U)$
Entrance floors	i	$Pr_{arr}(i)$	–	$Pr_{arr}(i)$
	$i-1$	$Pr_{arr}(i-1)$	–	$Pr_{arr}(i-1)$
	...	...	–	...
	...	...	–	...
	2	$Pr_{arr}(2)$	–	$Pr_{arr}(2)$
	1	$Pr_{arr}(1)$	–	$Pr_{arr}(1)$

$$\sum_{i=1}^{N}\left(\frac{U(i)}{U}\right)=1 \qquad (2)$$

The floor percentage ($R(i)$) is the arrival percentage for an entrance floor or the population percentage for an occupant floor, as shown in Table 3.

Neither the occupant floors nor the entrance–exit floors need to be contiguous. They are shown as contiguous in Tables 2 and 3 for convenience and ease of understanding.

Types of traffic

This subsection classifies the possible types of journeys and thus the possible type of traffic. Every passenger journey must logically have an origin and a destination. Considering that any floor can either be an entrance–exit floor or an occupant floor, there can exist in theory four types of journeys depending on the classification of the origin and destination floors for each journey.

A journey that starts from an entrance–exit floor and terminates at an occupant floor is denoted as an incoming traffic journey. A journey that starts from an occupant floor and terminates at an entrance–exit floor is denoted as an outgoing traffic journey. A journey that starts from an occupant floor and terminates at an occupant floor is denoted as an interfloor journey. A journey that starts from an entrance–exit floor and terminates at an entrance–exit floor is denoted as an inter-entrance journey and will be discounted in this paper. These four types of traffic are listed in Table 4.

Description of the traffic in a building

It has become customary to describe the prevailing traffic in a building at any one point in time as a mixture of the three types of traffic described in the previous subsection.

The percentage of the traffic that is incoming at any one point in time is denoted as ic; the percentage of the traffic that is outgoing at any one point in time is denoted as og; and the traffic that is interfloor at any one point in time is denoted as if. The combination of these three numbers can be used to describe the traffic mix as shown below (where any of these parameters can vary between 0 and 1)

$$ic : og : if$$

As expected, the sum of all three numbers should add up to 1 as shown in equation (3).

Table 4. Types of traffic.

Start floor (origin)	End floor (destination)	Type of traffic	Description
Entrance/exit	Occupant	Incoming traffic	Passengers arriving into the building
Occupant	Entrance/exit	Outgoing traffic	Passengers leaving the building
Occupant	Occupant	Interfloor traffic	Passengers moving within the building (restaurants, meeting rooms)
Entrance/exit	Entrance/exit	Inter-entrance traffic	Small enough that it could be ignored

Thus, assigning values for two of these numbers automatically sets the value of the third parameter.

$$ic + og + if = 1 \tag{3}$$

As an example, one suggested composition of the lunchtime traffic conditions can be described by the following representation[2]

$$ic : og : if \text{ as } 0.45 : 0.45 : 0.10, \text{ respectively}$$

OD matrix

The OD matrix is a concise compact format that can be used to fully describe the traffic in a building. It is an N-dimensional matrix that contains the probabilities of a passenger choosing to go from an origin floor to a destination floor in a given round trip. The row index represents the origin floor; the column index represents the destination matrix. The sum of all terms in the OD matrix must add up to 1. It is worth noting all the events in the OD matrix are mutually exclusive (i.e., if one of them takes place in a round trip, the others cannot take place in the same round trip). Thus, if a passenger chooses to go from the third floor to the seventh floor in a round trip, he or she cannot also go from the eighth floor to the second floor in the same round trip.

The general format for an OD matrix is shown in equation (4). All the diagonal elements are equal to zero, as it is assumed that passengers are rational and would not travel from a floor to the same floor.

$$OD_{adj} = \begin{bmatrix} 0 & p_{1\cdot 2} & .. & .. & p_{1\cdot N-1} & p_{1\cdot N} \\ p_{2\cdot 1} & 0 & .. & .. & p_{2\cdot N-1} & p_{2\cdot N} \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ p_{N-1\cdot 1} & p_{N-1\cdot 2} & .. & .. & 0 & p_{N-1\cdot N} \\ p_{N\cdot 1} & p_{N\cdot 2} & .. & .. & p_{N\cdot N-1} & 0 \end{bmatrix} \tag{4}$$

The OD matrix is critical for the elevator traffic design process, specifically for the following two functions:

1. The OD matrix is used to generate passengers with origin and destination pairs to be used in finding the round trip time using MCS.
2. The OD matrix can also be used to derive the probabilities of any type of event taking place in a round trip (e.g., the probability of a journey taking place between the third and sixth floors without stopping at the fourth and fifth floors). Deriving these probabilities is critical to deriving equations for evaluating the round trip time.

Converting the traffic mix into an OD matrix

Elevator traffic design has usually been carried out under up-peak (incoming) traffic conditions. However, this paper attempts to deal with the elevator traffic design under general traffic conditions (which can comprise a mixture of incoming, outgoing, and interfloor traffic). For this reason and in preparation for the complete design process presented in future sections, this section provides a procedure for converting the arrival–population–table to an OD matrix.

The first step is to compile the floor percentages into a column matrix and in a row matrix and multiply them, as shown in equations (5) and (6). The floor percentage, denoted as $R(i)$, is the percentage arrival for the ith floor when the floor is an entrance–exit floor or the percentage population of the ith floor when the floor is an occupant floor.

$$R(i) = \begin{cases} \Pr_{arr}(i) \text{ for entrance floors} \\ \frac{U(i)}{U} \text{ for occupant floors} \end{cases} \tag{5}$$

The initial OD matrix can be evaluated as shown in equation (6) below

$$OD_{ini} = \begin{bmatrix} R(1) \\ R(2) \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ R(N-1) \\ R(N) \end{bmatrix} \cdot \begin{bmatrix} R(1) & R(2) & \cdots & \cdots & \cdots & \cdots & R(N-1) & R(N) \end{bmatrix} \tag{6}$$

The result is an N by N matrix that is denoted as the initial OD matrix (OD_{ini}), shown below in (equation 7)

$$OD_{ini} = \begin{bmatrix} R(1)\cdot R(1) & R(1)\cdot R(2) & .. & .. & R(1)\cdot R(N-1) & R(1)\cdot R(N) \\ R(2)\cdot R(1) & R(2)\cdot R(2) & .. & .. & R(2)\cdot R(N-1) & R(2)\cdot R(N) \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ R(N-1)\cdot R(1) & R(N-1)\cdot R(2) & .. & .. & R(N-1)\cdot R(N-1) & R(N-1)\cdot R(N) \\ R(N)\cdot R(1) & R(N)\cdot R(2) & .. & .. & R(N)\cdot R(N-1) & R(N)\cdot R(N) \end{bmatrix} \tag{7}$$

The next step is to convert the initial OD matrix as shown above by splitting it into four areas according to the type of traffic, as shown in Figure 1.

Figure 1 assumes that the entrance floors are contiguous and that all the occupant floors are contiguous. However, the approach shown in this paper does not necessarily require that to be the case and can deal with the general case where entrance floors are noncontiguous and where occupant floors are noncontiguous. At this stage, the sum of all the elements of the matrix at this stage is 4 (one from each of the traffic mode areas).

Although inter-entrance traffic might exist, it has been assumed that its magnitude is usually small enough to be ignored. In some cases, there are rational reasons why passengers would make an inter-entrance journey (e.g., people parking

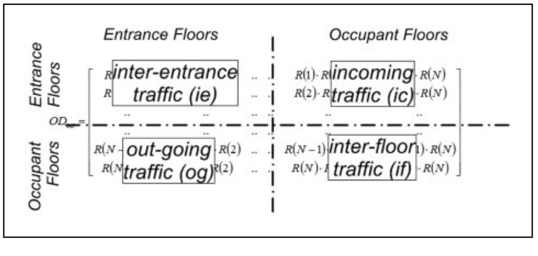

Figure 1. Split of the *OD* matrix into four traffic areas.

in basement floors who work at ground floor, or travel via ground floor). Thus, all the elements in inter-entrance traffic area of the matrix will be zeroed (unless its magnitude is too large to be ignored). All the elements in the outgoing traffic matrix will be multiplied by the percentage outgoing traffic (denoted as *og*); all the elements in the incoming traffic area of the matrix will be multiplied by the percentage incoming traffic (denoted as *ic*); all the elements in the interfloor traffic area of the matrix will be multiplied by the percentage interfloor traffic (denoted as *if*).

Once this step has been carried out, then the sum of all the elements in the original destination matrix will add up to 1. More specifically, the sum of all the elements in the inter-entrance areas will be equal to 0; the sum of all the elements in the incoming traffic area will be equal to *ic*; the sum of all the elements in the outgoing traffic area will be equal to *og*; and the sum of all the elements in the interfloor traffic areas will be equal to *if*.

The last step is to zero the nonzero diagonal elements in order to reflect passenger rational behavior. These element will be in the interfloor area (as the inter-entrance area has already been zeroed; and the incoming and outgoing areas do not cover the diagonal). This is shown in equation (8).

$$p_{ii} = 0 \text{ for } i = j \tag{8}$$

But as the diagonal items have been zeroed, the sum of all the elements in the matrix no longer adds up to 1. More specifically, the sum of all the elements in the interfloor area does not add up to the value of *if* any more. In order to correct this, the matrix has to be adjusted by dividing all the elements in the interfloor area by an adjusting factor, M, that can be evaluated as shown in equation (9).

$$M = 1 - \left(\frac{\sum_{i=1}^{N} p_{ii}}{if} \right) \tag{9}$$

where i runs for all the indices for the occupant floors. The final OD matrix (denoted as OD_{fin}) is show in equation (10), following all the amendments and adjustments. The format assumes that the upper floors are occupant floors and that the lower floors are entrance–exit floors. This is not necessarily always the case and the notation can be altered to suit each building as appropriate.

$$OD_{fin} = \begin{bmatrix} 0 & 0 & .. & .. & ic \cdot \Pr_{arr}(1) \cdot \frac{U(N-1)}{U} & ic \cdot \Pr_{arr}(1) \cdot \frac{U(N)}{U} \\ 0 & 0 & .. & .. & ic \cdot \Pr_{arr}(2) \cdot \frac{U(N-1)}{U} & ic \cdot \Pr_{arr}(1) \cdot \frac{U(N)}{U} \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ og \cdot \frac{U(N-1)}{U} \cdot \Pr_{arr}(1) & og \cdot \frac{U(N-1)}{U} \cdot \Pr_{arr}(2) & .. & .. & 0 & if \cdot \frac{U(N-1) \cdot U(N)}{M \cdot U^2} \\ og \cdot \frac{U(N)}{U} \cdot \Pr_{arr}(1) & og \cdot \frac{U(N)}{U} \cdot \Pr_{arr}(2) & .. & .. & if \cdot \frac{U(N) \cdot U(N-1)}{M \cdot U^2} & 0 \end{bmatrix} \tag{10}$$

This matrix can also be referred to as the "*normalised* origin–destination matrix." It is referred to as *normalized* as it only depends on the arrival percentages of the entrance, the population percentages, and the mix of traffic. It does not depend on the passenger arrival intensity. The matrix in this form can now be used in order to generate random passenger OD pairs for evaluating the round trip time using the MCS method. It can also be used to evaluate the round trip time using formula by calculation, as shown in Derivation of a round trip time equation for general traffic conditions section later in this paper.

Rules of the final normalized OD matrix

The final OD matrix must obey a number of rules, listed below:

1. The sum of all the elements in the OD matrix should add up to 1.
2. All the elements of the diagonal of the matrix should be equal to zero.
3. The sum of all the elements in the incoming traffic area of the OD matrix should add up to the percentage incoming traffic (i.e., ic).
4. The sum of the all the elements in the outgoing traffic area should add up to the percentage outgoing traffic (i.e., og).
5. The sum of all the elements in the interfloor traffic area in the OD matrix should add up to the percentage interfloor traffic (i.e., if).
6. The elements in the inter-entrance traffic area in the OD matrix should be zero.

It is worth noting that work has been carried out in trying to estimate the OD traffic from elevator movements in the building.[9–12] The outcome can be used in a number of ways, including generating virtual passenger traffic[13] or deciding on the suitable group control algorithm to adopt during that period of time.[14]

Random generation of passengers using the OD matrix

This section examines in detail the method of generating individual passenger OD pairs using the OD matrix. This is necessary for the evaluation of the round trip time using the MCS method.

The following is a complete set of steps that are used to generate OD pairs for passengers:

1. The initial OD matrix is generated using the floor arrival percentages and the floor percentage populations, by multiplying them and populating the matrix.
2. The four areas in the matrix are delineated in accordance with the expected three types of traffic: incoming, outgoing, and interfloor.
3. The inter-entrance area of the matrix is zeroed.
4. The elements of each part of the matrix are multiplied by the corresponding percentage traffic (i.e., the incoming traffic elements are multiplied by ic; the outgoing traffic elements are multiplied by og; the interfloor traffic elements are multiplied by if).
5. The remaining nonzero diagonal elements (which are expected to be in the interfloor area) should be zeroed. All the remaining nonzero elements in the interfloor area are divided by the adjusting factor M, in order to restore the sum of the elements in the interfloor traffic area back to the value of if.
6. The final form of the OD matrix is ready at this stage. A quick check can be carried out at this stage to ensure that the sum of all the elements in the OD matrix is 1, and that the sum of the elements in each area of the matrix corresponds to the percentage traffic type (ic; og; if).
7. A probability density function (pdf) is generated from the matrix. The number of possible values for the random variables is N^2. Each random variable is an OD pair.
8. The pdf is converted to a cumulative distribution function (cdf) by the use of integration–summation.
9. Random numbers (between 0 and 1) are generated and applied to the cdf in order to randomly generate passengers with OD pairs.

A numerical example is presented below that illustrates the procedure of randomly generating passenger OD pairs.

Numerical example on developing the OD matrix and using it to randomly generate passengers

This section presents a practical numerical example on developing the origin–destination matrix and using it to generate passenger OD pairs.

A building has two entrance floors (B and G) and four occupant floors (1–4). The percentage ARs from the two entrances are 0.2 and 0.8 from B and G, respectively. The percentage populations of the occupant floors are: 40%, 30%, 20%, and 10% for the occupant floors 1, 2, 3, and 4, respectively. All the building details are shown in Table 5. It is required that passenger OD pairs are randomly generated based on a traffic mix of 40%:50%:10% of incoming, outgoing, and interfloor traffic, respectively.

The first step is to compile the initial OD matrix, by multiplying the percentage ARs and the percentage floor populations.

By multiplying the floor percentage $R(i)$ for each floor by the floor percentage for each floor, the initial OD matrix can be populated, as shown in Table 6.

It is worth noting that the initial OD matrix as it is currently shown has four distinct areas that are outlined in thick lines. The first area (top left) represents the inter-entrance traffic, which has been so far assumed to be rare. The second area (top right area) represents incoming traffic. The third area (bottom left area) represents outgoing traffic. The fourth area (bottom right area) represents interfloor traffic.

As it has been assumed that the inter-entrance traffic percentage is zero, the corresponding area in the matrix will be zeroed. In addition, the various traffic areas are multiplied

Table 5. Generalized representation of traffic for a building depending on the traffic mix.

Type	#	Percentage arrival	Percentage population
Occupant floors	4	0	0.1
	3	0	0.2
	2	0	0.3
	1	0	0.4
Entrance floors	G	0.8	0
	B	0.2	0

Table 6. The initial origin–destination matrix derived from percentage arrivals and percentage populations.

	Floor	B	G	1	2	3	4
Floor	Arrival percentage/ population percentage	0.2	0.8	0.4	0.3	0.2	0.1
B	0.2	0.04	0.16	0.08	0.06	0.04	0.02
G	0.8	0.16	0.64	0.32	0.24	0.16	0.08
1	0.4	0.08	0.32	0.16	0.12	0.08	0.04
2	0.3	0.06	0.24	0.12	0.09	0.06	0.03
3	0.2	0.04	0.16	0.08	0.06	0.04	0.02
4	0.1	0.02	0.08	0.04	0.03	0.02	0.01

Table 7. The initial origin–destination matrix adjusted to suit the traffic mix.

	Floor	B	G	1	2	3	4
Floor	Arrival percentage/ population percentage	0.2	0.8	0.4	0.3	0.2	0.1
B	0.2	0	0	0.032	0.024	0.016	0.008
G	0.8	0	0	0.128	0.096	0.064	0.032
1	0.4	0.04	0.16	0.016	0.012	0.008	0.004
2	0.3	0.03	0.12	0.012	0.009	0.006	0.003
3	0.2	0.02	0.08	0.008	0.006	0.004	0.002
4	0.1	0.01	0.04	0.004	0.003	0.002	0.001

Table 8. The final origin–destination matrix after the zeroing of the diagonal of the interfloor area.

	Floor	B	G	1	2	3	4
Floor	Arrival percentage/ population percentage	0.2	0.8	0.4	0.3	0.2	0.1
B	0.2	0	0	0.032	0.024	0.016	0.008
G	0.8	0	0	0.128	0.096	0.064	0.032
1	0.4	0.04	0.16	0	12/700	8/700	4/700
2	0.3	0.03	0.12	12/700	0	6/700	3/700
3	0.2	0.02	0.08	8/700	6/700	0	2/700
4	0.1	0.01	0.04	4/700	3/700	2/700	0

Table 9. Converting the *PDF OD* matrix into a *CDF OD* matrix by progressively adding the terms.

	Floor	B	G	1	2	3	4
Floor	Arrival percentage/ population percentage	0.2	0.8	0.4	0.3	0.2	0.1
B	0.2	0	0	4/125	7/125	9/125	2/25
G	0.8	2/25	2/25	26/125	38/125	46/125	2/5
1	0.4	11/25	3/5	3/5	108/175	22/35	111/175
2	0.3	93/140	549/700	561/700	561/700	81/100	57/70
3	0.2	146/175	32/35	162/175	327/350	327/350	164/175
4	0.1	663/700	691/700	139/140	349/350	1	1

Note: PDF, probability density function; OD, origin–destination; CDF, cumulative distribution function.

by their corresponding traffic percentage. The resultant matrix is shown in Table 7.

The next step is to zero the diagonal element, and then adjust the interfloor elements by the value of M. The value of M is calculated below

$$M = 1 - \left(\frac{\sum_{i=1}^{N} \left(\mathrm{Pr}_{arr}(i)\ \% \cdot \frac{U(i)}{U}\ \% \right)}{if} \right) = 1 - \frac{0.016 + 0.009 + 0.004 + 0.001}{0.1} = 0.7 \tag{11}$$

Following the zeroing of the diagonal and the division of the all the interfloor elements by M (i.e., 0.7 in this case), the final OD matrix can be seen in Table 8.

The final OD matrix shown in Table 8 is in fact the *pdf* matrix. In order for it to be used to generate passenger OD pairs, it must be converted to a *cdf* matrix. This has been done in Table 9, where each element is equal to the sum of itself and all the preceding elements. As shown by the arrows superimposed on the table, the direction of adding the terms to produce the *CDF* terms is one row at a time, moving from left to right and then down to the next row.

The values have been shown as fractions in Table 9 and as decimals to four decimal places in Table 10, in order to make it easier to generate random passenger OD pairs.

Using the *CDF*, it is now possible to generate some passengers. In order to generate three passengers, three random numbers between 0 and 1 are generated, giving the following three random numbers: 0.814, 0.308, and 0.407.

1. Taking the first random number (0.814) that lies between 0.8100 and 0.8143, and is thus assigned to 0.8143, and would represent an up passenger journey from floor 2 to floor 4.
2. The second random number (0.308) lies between 0.3040 and 0.3680 and is assigned to 0.3680 and thus represents an up passenger journey from floor G to floor 3.
3. The third random number (0.407) lies between 0.4000 and 0.4400, and is thus

Table 10. The final origin–destination matrix in the form of a *cdf* expressed to four decimal places.

	Floor	B	G	1	2	3	4
Floor	Arrival percentage/ population percentage	0.2	0.8	0.4	0.3	0.2	0.1
B	0.2	0	0	0.0320	0.0560	0.0720	0.0800
G	0.8	0.0800	0.0800	0.2080	0.3040	0.3680	0.4000
1	0.4	0.4400	0.6000	0.6000	0.6171	0.6286	0.6343
2	0.3	0.6643	0.7843	0.8014	0.8014	0.8100	0.8143
3	0.2	0.8343	0.9143	0.9257	0.9343	0.9343	0.9371
4	0.1	0.9471	0.9871	0.9929	0.9971	1	1

assigned to 0.4400. It represents a down passenger journey from floor 1 to floor B.

Derivation of a round trip time equation for general traffic conditions

Equations have been previously derived for the generalized case with the following conditions[6]:

1. Unequal floor heights.
2. Top speed not attained in one floor journey.
3. Unequal floor populations.
4. Multiple entrances.
5. General traffic conditions (i.e., traffic consisting of a mixture of incoming traffic, outgoing traffic, and interfloor traffic).

However, the calculations involved become too complicated and are impractical unless programmed in software. This section derives an equation for the round trip time for the following conditions (note that the first two conditions are different from the general case in Al-Sharif and Abu Alqumsan[6] and the last two conditions are the same as the general case in Al-Sharif and Abu Alqumsan[6]):

1. Equal floor heights.
2. Top speed attained in one floor journey.
3. Multiple entrances.
4. General traffic conditions (i.e., traffic consisting of a mixture of incoming traffic, outgoing traffic, and interfloor traffic).

The set of equations that will be derived apply to general traffic conditions and multiple entrances, but assume that the floor heights are equal and that the top speed is attained in one floor journey.

As will be seen later, the resulting round trip time equation is much simpler than that derived in Al-Sharif and Abu Alqumsan.[6] Moreover, it is also very *intuitive and informative* as the final form of the equation bears significant resemblance to the classical equation for the round trip time for the case of incoming traffic only (still assuming unequal floor heights and top speed not attained in one floor journey).

As shown in Figure 2, the elevator movement can be visualized as a continuous ring around the building, and not necessarily attaining the highest floor in the building in each round trip, or attaining the lowest floor in the building in each round trip. The average value of the highest attained floor in a round trip will be denoted as H (also known as the highest reversal floor) and has units of floors. The average value of the lowest attained floor in a round trip will be denoted as L and will also have units of floors.

It is acknowledged that the actual values of H and L will vary from one round trip to the next, but it is the average values over a large number of round trips that will be evaluated and used in the round trip time equation. In the next two subsections, formulae are derived for H and L as a function of the OD matrix and the number of passengers.

All the formulae for the probabilities for the various events will be based on the OD matrix.

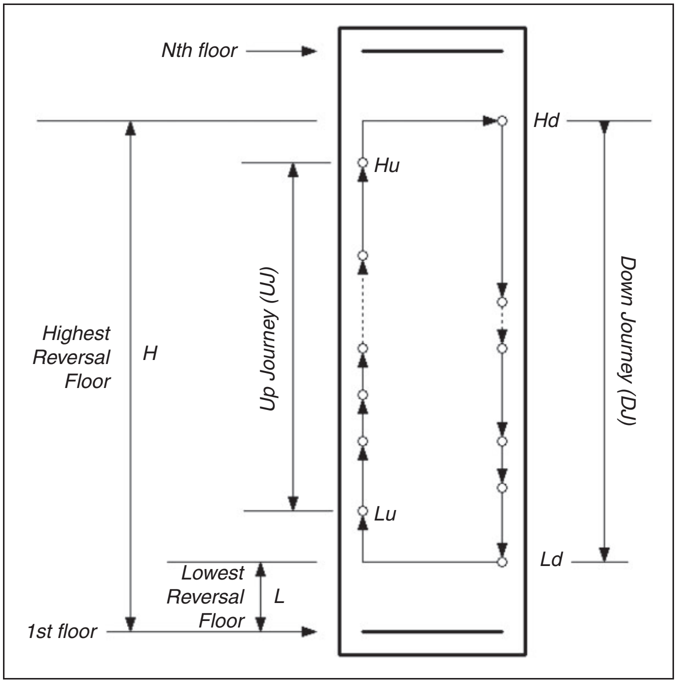

Figure 2. General overview showing the lowest reversal floor (L) and the highest reversal floor (H).

It is worth noting that all probabilities related to up traveling passenger can be found in the upper triangle of the OD matrix (i.e., all entries that are above the diagonal), and that all probabilities related to down traveling passengers can be found in the lower triangle of the OD matrix (i.e., all entries that are below the diagonal). The upper triangle of the matrix is that which is above the diagonal, and the lower triangle of the matrix is that which is below the diagonal as shown in Figure 3.

Derivation of formulae for H and L

During any round trip, the elevator will attain the highest floor, denoted as H. The highest reversal in any round trip is decided by the highest destination of the up traveling passengers or the highest origin of the down traveling passenger whichever is higher. It has unit of floors. This is consistent with the general approach, as it has been assumed that the floor heights are equal.

In order to derive a formula for the expected value of the highest reversal floor, it is necessary to derive formulae for the following two events:

The probability of the elevator not traveling to any floor above the ith floor in a round trip
The probability of the elevator not traveling to any floor above the $(i-1)$th floor in a round trip.

The difference between the probabilities of these two events is the probability of the ith floor being the highest reversal floor in a round trip.

As the traffic is general traffic in this case, there are two conditions that need to be true in a round trip for the elevator not to travel above the ith floor, and these are:

No up traveling passengers destined to (i.e., traveling to) a floor that is above the ith *floor*
AND
No down traveling passengers originating from (i.e., traveling from) a floor that is above the ith *floor*

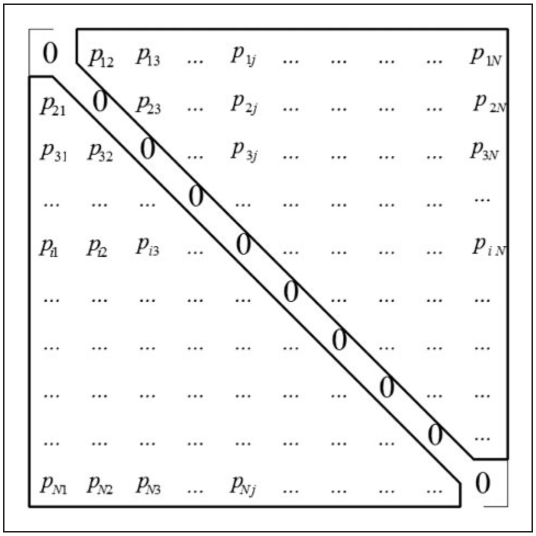

Figure 3. Upper triangle and lower triangle in the origin destination matrix.

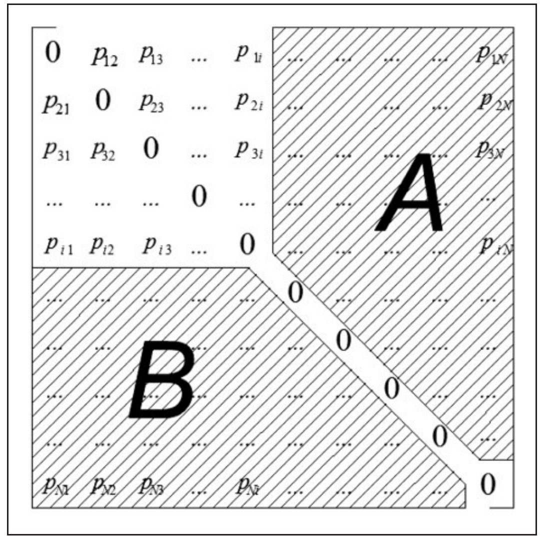

Figure 4. The shaded area *A* that represents the probability of an up traveling passenger heading for a floor above the *i*th floor.

The next step is to derive a formula for these two events. Thus, the probability of the first event can be found by excluding all the probabilities of an up traveling passenger having as a destination any of the floors above the *i*th floor. These probabilities should reside in the upper triangle (as they relate to up traveling passengers) and should be in columns with a value higher than *i*. Such an area has been denoted as A and shown as hatched in Figure 4.

As for the down traveling passengers not originating at any floors above the *i*th floor, these probabilities would reside in the lower triangle of the OD matrix (as they are down traveling passengers) and should be in rows with a value higher than *i*. Such an area has been denoted as area B and shown as hatched in Figure 4.

The event of a passenger not traveling upward to a floor above the *i*th floor in a round trip and not traveling downward from a floor above the *i*th floor in the same round trip shall be denoted as Q. The probability of the event Q can thus be found by subtracting the sum of the probabilities in areas A and B from 1

$$\Pr(Q) = 1 - \sum\sum(A) - \sum\sum(B) \tag{12}$$

Adding the limits of summation

$$\begin{aligned}\Pr(Q) = 1 &- \left(\sum_{k=1}^{i}\sum_{j=i+1}^{N}(p_{kj}) + \sum_{k=i+1}^{N-1}\sum_{j=k+1}^{N}(p_{kj})\right)\\ &- \left(\sum_{k=i+1}^{N}\sum_{j=1}^{i}(p_{kj}) + \sum_{k=i+2}^{N}\sum_{j=i+1}^{k}(p_{kj})\right)\end{aligned} \tag{13}$$

As the passenger decisions are assumed to be independent, then the probability that none of the P passengers will travel upward to a floor above the *i*th floor in a round trip and that none of the P passengers will travel downward from a floor above the *i*th floor in the same round trip can be found by raising the value found in equation (13) to the power P. Such an event is in fact the same as the highest reversal floor being smaller than i

$$\begin{aligned}\Pr(H \le i) = \Bigg(1 &- \left(\sum_{k=1}^{i}\sum_{j=i+1}^{N}(p_{kj}) + \sum_{k=i+1}^{N-1}\sum_{j=k+1}^{N}(p_{kj})\right)\\ &- \left(\sum_{k=i+1}^{N}\sum_{j=1}^{i}(p_{kj}) + \sum_{k=i+2}^{N}\sum_{j=i+1}^{k}(p_{kj})\right)\Bigg)^{P}\end{aligned} \tag{14}$$

A similar approach can be followed to find the probability of the highest reversal floor being smaller than $i-1$, by using areas C and D that are shaded in Figure 5.

Substituting $i-1$ in place of i in equation (14) gives

$$\Pr(H \le i-1) = \left(\begin{array}{l} 1 - \left(\sum_{k=1}^{i-1} \sum_{j=i}^{N} (p_{kj}) + \sum_{k=i}^{N-1} \sum_{j=k+1}^{N} (p_{kj}) \right) \\ \quad - \left(\sum_{k=i}^{N} \sum_{j=1}^{i-1} (p_{kj}) + \sum_{k=i+1}^{N} \sum_{j=i}^{k} (p_{kj}) \right) \end{array} \right)^{P} \tag{15}$$

Using the results from equations (14) and (15), the probability of the highest reversal floor being equal to i can be found as shown in equation (16)

$$\Pr(H = i) = \Pr(H \le i) - \Pr(H \le i-1) \tag{16}$$

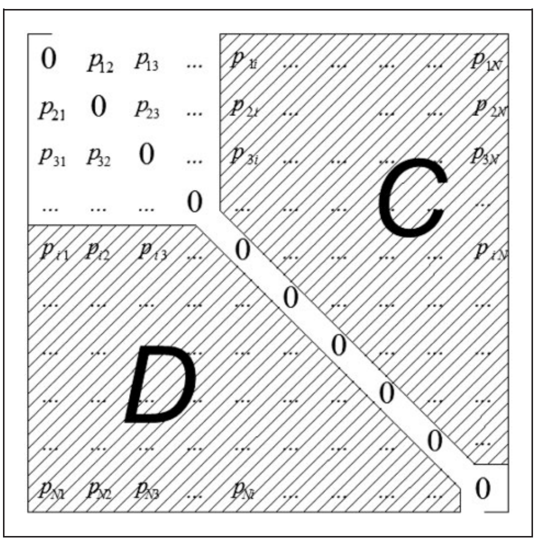

Figure 5. Areas C and D used in deriving the formula for the probability of the highest reversal floor being smaller than $i-1$.

In a similar way, the probability of the lowest reversal floor, L, being larger than or equal to i can be found by finding a formula for the probability of no down passengers traveling to a floor below the ith floor in a round trip and no up passengers traveling from a floor below the ith floor in a round trip (using areas E and F shown in Figure 6)

$$\Pr(L \ge i) = \left(1 - \sum\sum(E) - \sum\sum(F)\right)^{P} \tag{17}$$

Applying the same principle to the probability of the lowest reversal floor being larger than $i+1$, the two shaded areas G and H (shown in Figure 7) can be used as shown below

$$\Pr(L \ge i+1) = \left(1 - \sum\sum(G) - \sum\sum(H)\right)^{P} \tag{18}$$

The probability of the lowest reversal floor being equal to i is the difference between the probability of the two events shown in equations (17) and (18) above

$$\Pr(L = i) = \Pr(L \ge i) - \Pr(L \ge i+1) \tag{19}$$

It is now possible to find the formula for the expected values of the highest reversal floor, H, and the lowest reversal floor, L, as shown below as the weighted average of the floor from 1 to N, first for H

$$E(H) = \sum_{i=1}^{N} (i \cdot \Pr(H = i)) \tag{20}$$

and for L

$$E(L) = \sum_{i=1}^{N} (i \cdot \Pr(L = i)) \tag{21}$$

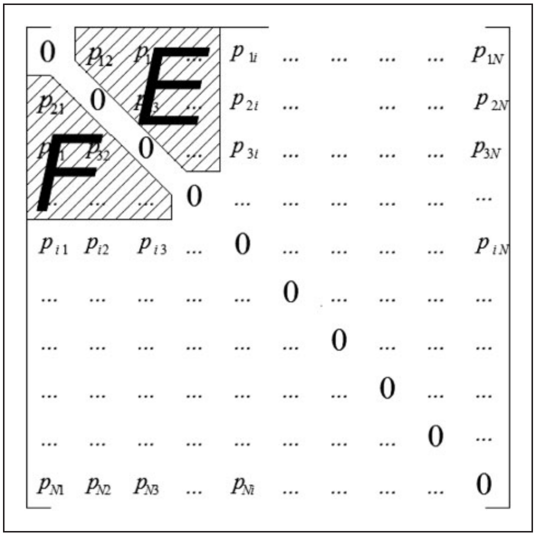

Figure 6. Shaded areas *E* and *F* used in deriving the probability of the lowest reversal floor being larger than *i*.

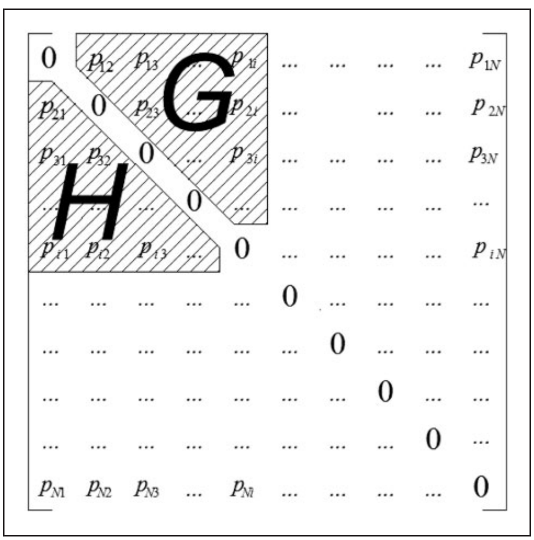

Figure 7. Shaded areas *G* and *H* used in deriving the probability of the lowest reversal floor being larger than *i* + 1.

Derivation of formulae for the number of stops

A previous paper derived a formula for the expected number of stops in a round trip for the general case of mixed traffic conditions.[6] These formulae are reproduced here without proof.

The number of stops in a round trip has two main components: the stops that take place during the up part of the journey, denoted as S_U, and the stops that take place during the down part of the journey, denoted S_D.

However, it is necessary to remember that in some journeys, there is a common stop at the top reversal of the elevator during the round trip due to the fact that the last up stop floor (where the last up passenger(s) alight(s)) happens to be the same as the first down stop floor (where the first down passenger alight(s)). Such a stop is denoted as upper coincidental stop (*UCS*), and must be subtracted from the total number of stops. The average number of *UCS* in a round trip is denoted as S_{UCS}.

A similar argument can be applied to the lower elevator reversals. In some journeys, there is also a common stop at the lower reversal of the elevator during the round trip due to the fact that the last down stop floor happens to be the same as the first up stop floor. Such a stop is denoted as a lower coincidental stop (*LCS*) and must also be subtracted from the total number of stops.

The expected number of stops in the up direction, S_U and the expected number of stops in the down direction, S_D, can be evaluated as shown in equations (22) and (23). The expected number of stops in the up direction is simply the sum of the probabilities of making a stop at all the floors from 1 to N, as shown below

$$S_U = \sum_{i=1}^{N} P(S_{Ui}) = \sum_{i=1}^{N} \left(1 - P(\overline{S}_{Ui})\right) \tag{22}$$

where the probability of stopping at a floor in the up direction can be calculated as shown below

$$P(\overline{S}_{Ui}) = \left(1 - \sum_{k=1}^{i-1} p_{ki} - \sum_{k=i+1}^{N} p_{ik}\right)^{P} \tag{23}$$

The expected number of stops in the down direction is simply the sum of the probabilities of making a stop at all the floors from 1 to N, as shown below

$$S_D = \sum_{i=1}^{N} P(S_{Di}) = \sum_{i=1}^{N} \left(1 - P\left(\overline{S}_{Di}\right)\right) \tag{24}$$

where the probability of not stopping at a floor in the down direction can be calculated as shown below

$$P\left(\overline{S}_{Di}\right) = \left(1 - \sum_{k=1}^{i-1} p_{ik} - \sum_{k=i+1}^{N} p_{ki}\right)^{P} \tag{25}$$

The expected number of upper coincidental stops (S_{UCS}) can be evaluated by summing the probability of an UCS taking place at each of the floors 1 to N. In fact the sum should run from 2 to N as the probability of floor 1 being an UCS is zero.

$$S_{UCS} = \sum_{i=1}^{N} P(UCS_i) = \sum_{i=1}^{N} P(UCJ_{ii}) \tag{26}$$

where the probability of an UCS taking place at the ith floor can be calculated as shown below

$$\begin{aligned} P(UCS_i) = P(UCJ_{ii}) &= P\left(\overline{S}_{Ui+1,i+2\ldots UN,DN\ldots Di+2,i+1}\right) \\ &\quad - P\left(\overline{S}_{Ui,i+1,i+2\ldots UN,DN\ldots Di+2,i+1}\right) \\ &\quad - P\left(\overline{S}_{Ui+1,i+2\ldots UN,DN\ldots Di+2,i+1,i}\right) \\ &\quad + P\left(\overline{S}_{Ui,i+1,i+2\ldots UN,DN\ldots Di+2,i+1,i}\right) \end{aligned} \tag{27}$$

Detailed equations for the probability of an up journey between two floors can be found in Ref.[6]

The expected number of lower coincidental stops (S_{LCS}) can be evaluated by summing the probabilities of *an LCS* taking place on each of the floors 1 to N. In fact the sum should run from 1 to $N-1$ as the probability of floor N being an LCS is zero.

$$S_{LCS} = \sum_{i=1}^{N} P(LCS_i) = \sum_{i=1}^{N} P(LCJ_{ii}) \tag{28}$$

where the probability of a LCS taking place at the ith floor can be evaluated by evaluating the probability of the lower connecting journey *(LCJ)* taking place between the ith floor and the ith floor

$$\begin{aligned} P(LCS_i) = P\left(LCJ_{ij}\right) = \\ &P\left(\overline{S}_{Di-1,i-2\ldots D1,U1\ldots.Ui-2,i-1}\right) \\ &- P\left(\overline{S}_{Di,i-1,i-2\ldots D1,U1\ldots.Ui-2,i-1}\right) \\ &- P\left(\overline{S}_{Di-1,i-2\ldots D1,U1\ldots.Ui-2,i-1,i}\right) \\ &+ P\left(\overline{S}_{Di,i-1,i-2\ldots D1,U1\ldots.Ui-2,i-1,i}\right) \end{aligned} \tag{29}$$

Detailed equations for the probability of a down journey between two floors can be found in Ref.[6]

Once the four components have been evaluated, the expected number of stops in a round trip (denoted as S) can be evaluated by adding the first two components and subtracting the last two components. It is worth noting that the units of S_U, S_D, S_{UCS} and S_{LCS} are stops per round trip.

$$S = S_U + S_D - S_{UCS} - S_{LCS} \tag{30}$$

The final round trip time equation

Having derived equations for the highest reversal floor, H, the lowest reversal floor, L, and the number of stops in a round trip, S, it is now possible to formulate an equation for the round trip time. The round trip time will be based on the classical format of the round trip time that is used for the case of incoming traffic condition and a single entrance, shown below.

$$RTT = 2 \times H \times \frac{d_f}{v} + (S+1) \times \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} \right) + P \times \left(t_{pi} + t_{po} \right) \quad (31)$$

It is worth noting that S in the term $(S + 1)$ is the number of stops in the up direction, and that the term 1 in the term $(S + 1)$ is the number of down stops (i.e., only one stop in the down direction).

In order to convert equation (31) to the case of multiple entrances and general traffic conditions, H will be replaced by $H-L$ and the number of stops $(S + 1)$ will be replaced by the terms in equation (30) (i.e., $S_U + S_D - S_{UCS} - S_{LCS}$), which gives the final equation shown below

$$RTT = 2 \times (H-L) \times \frac{d_f}{v} + (S_U + S_D - S_{UCS} - S_{LCS}) \times \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} \right) + P \times \left(t_{pi} + t_{po} \right) \quad (32)$$

Equation (32) is the more general form of equation (31). When equation (32) is applied to the special case of incoming traffic only and a single entrance, S_D becomes equal to 1 (only one stop in the down direction), S_{UCS} and S_{LCS} become zero (i.e., no upper or LCSs) and the value of lowest reversal floor (L) becomes 0 (as the elevator must always start from the lowest floor, which is the single entrance). Thus equation (32) reverts back to the special case represented by equation (31).

Redefining the interval for the case of multiple entrances and general traffic: The virtual interval

In buildings with one single entrance under incoming traffic conditions, the definition of the concept of the interval is straightforward. The interval is defined as the time between successive arrivals (or departures) of the elevator at the single entrance (i.e., main lobby). However, in the case where there are multiple entrances or general traffic conditions, it is no longer possible to use such a definition of the interval, as each elevator may not stop at every entrance in every round trip. This is especially true in multiple entrance cases with no one single dominant entrance. It is thus necessary to amend the definition of the interval to account for the case of multiple entrances and general traffic conditions.

In order to do so, it is necessary to introduce two new definitions of the interval and two corresponding terms: The physical interval (to be denoted as int_p) and the virtual interval (to be denoted as int_v). The physical interval is the one that has been traditionally used in incoming traffic design.

The concept of the virtual interval was first introduced by Hakonen and Siikonen.[15] The reference redefines the round trip time to suit the general traffic conditions as follows:

> *"Round Trip Time: The time from the moment the car starts up to the next time it starts up after two reversals."*

The virtual interval is defined as the time between successive reversals of the elevators in the same of sense of reversal. It is important to define the term "sense" of reversal. A reversal of direction can either take place at the lower end of the journey or the upper end of the journey. A reversal of direction at the upper end of the journey will be from up to down and will be denoted as an up–down sense of reversal. A reversal of direction at the lower end of the journey will be from down to up and will denoted as a down–up sense of reversal. Thus, the interval has to be redefined as the time between successive reversals of the elevators in the same sense of reversal. It will be more intuitive to use the lower journey reversal (i.e., down–up sense of reversal) as the basis for the virtual interval, as this is usually the point where one round trip *finishes* and the next one *starts*.

The physical interval (denoted as int_p) is similar to the original concept of the interval, where it is the time between successive arrivals of elevators at the same floor traveling in the same direction. In general, the physical interval is larger than the virtual interval. Moreover, the physical interval will be specific to each entrance. Each entrance will have its own specific physical interval. So the fact that the physical interval for a certain floor is larger than the virtual interval for the whole group of elevators is a reflection of the fact that the elevators will not stop at that floor in every round trip.

An expected question will arise at this point: Why is it important to keep the concept of interval in the case of multiple entrances or in the case of general traffic conditions? The answer is related to the quality of service. The virtual interval can still be used as the indicator of the quality of service. The virtual interval is still calculated by dividing the round trip time by the number of elevators, L. On the other hand, the physical interval is an indication of the quality of service at a certain entrance floor.

Obviously, the two intervals become equal for the case of a single entrance and incoming traffic conditions, whereby the physical entrance becomes equal to the virtual interval.

Numerical example

This section presents a numerical example to illustrate the methodology presented in this paper.

A note on car capacity

A note on car capacity is in order here. The elevator car will spend most of the time with a load less than P passengers, but the maximum possible capacity is P passengers. This happens for example in cases where all passengers in a round trip are up passengers, and all of them board the car prior to any of them alighting. In this case, the car will fill up for a period of time with P passengers. So the elevator car capacity must be equal to P passengers, as there are cases where it will fill up to that number of passengers.[6]

The example

The following conditions are true for this building:

1. Top speed attained in one floor journey.
2. Equal floor heights.
3. Unequal floor populations.
4. Multiple entrances.
5. General traffic conditions, where the traffic under which the design is carried out is a mixture of incoming traffic, outgoing traffic, and interfloor traffic.

The following are the parameters for the building:

Total number of floors: eight floors
Two entrances: B and G
Six occupant floors: 1–6
Traffic mix: 40%:40%:20% incoming, outgoing, and interfloor, respectively.
The number of passengers boarding each elevator in every round trip, P, is 10
Equal floor heights: floor height, d_f, 4.5 m
$AR\%$: 12.5% of the building population arriving in 5 min
Target interval, int_{tar}: 40 s
Rated speed, v: 1.6 m/s
Rated acceleration, a: 1 m/s^2
Rated jerk, j: 1 m/s^3
Door opening time, t_{do}: 2 s
Door closing time, t_{dc}: 3 s
Passenger boarding time, t_{pi}: 1.2 s
Passenger alighting time, t_{po}: 1.2 s

The percentage arrivals and percentage populations of the different floors in the building are shown in Table 11.

The initial OD matrix is produced by multiplying the floor percentages and is shown in Table 12.

The value of M is calculated using equation (9) giving a value of 0.805556. Once the inter-entrance area has been zeroed, the incoming traffic and the outgoing traffic areas have been multiplied by 0.4, the interfloor area multiplied by 0.2, its diagonal zeroed, and the interfloor values divided by M, the final OD matrix is derived and is shown in Table 13.

Having arrived at the final OD matrix, as shown in Table 13, it is now possible to find the value of the round trip time using equations in this paper.

Using the values of the parameters shown in Table 14 above and substituting them into equation (32) gives the final value of the round trip time. It has been verified versus the general round trip time equation from reference[6] that was derived for the general case of top speed not attained in one floor journey and unequal floor heights, and against the MCS method.[16] The match between the two equation methods is excellent, as shown in Table 15. The match with the MCS method is also very good with a difference of 0.036%.

The required number of elevators, the actual interval, and the HC can be calculated as shown below in equations (33), (34), and (35), respectively.

$$E = \left\lceil \frac{RTT}{\text{int}_{tar}} \right\rceil = \left\lceil \frac{136.44}{40} \right\rceil = 4 \tag{33}$$

$$\text{int}_{act} = \frac{RTT}{E} = \frac{136.44}{4} = 34.11\,\text{s} \tag{34}$$

$$HC\ \% = \frac{300 \times E \times P}{RTT \times U} = \frac{300 \times 4 \times 10}{136.44 \times 600} = 14.66\ \% \tag{35}$$

The design is compliant as the actual interval is smaller than the target interval and the $HC\%$, is larger than the $AR\%$. The design can be

Table 11. Description of the arrival percentages and population percentages in the building.

Floor	Type of floor	Percentage arrival rate	Percentage population
B	Entrance/exit	0.3	–
G	Entrance/exit	0.7	–
1	Occupant	–	¼ (150/600)
2	Occupant	–	¼ (150/600)
3	Occupant	–	1/6 (100/600)
4	Occupant	–	1/6 (100/600)
5	Occupant	–	1/12 (50/600)
6	Occupant	–	1/12 (50/600)

Table 12. The initial origin–destination matrix resulting from the multiplication of the floor percentages.

	Floor	B	G	1	2	3	4	5	6
	Percentage	0.3	0.7	0.25	0.25	0.166667	0.166667	0.083333	0.083333
B	**0.3**	0	0	0.075	0.075	0.05	0.05	0.025	0.025
G	**0.7**	0	0	0.175	0.175	0.116667	0.116667	0.058333	0.058333
1	**0.25**	0.075	0.175	0.0625	0.0625	0.041667	0.041667	0.020833	0.020833
2	**0.25**	0.075	0.175	0.0625	0.0625	0.041667	0.041667	0.020833	0.020833
3	**0.166667**	0.05	0.116667	0.041667	0.041667	0.027778	0.027778	0.013889	0.013889
4	**0.166667**	0.05	0.116667	0.041667	0.041667	0.027778	0.027778	0.013889	0.013889
5	**0.083333**	0.025	0.058333	0.020833	0.020833	0.013889	0.013889	0.006944	0.006944
6	**0.083333**	0.025	0.058333	0.020833	0.020833	0.013889	0.013889	0.006944	0.006944

Table 13. Final origin–destination matrix.

	floor	B	G	1	2	3	4	5	6
	percentage	**0.3**	**0.7**	**0.25**	**0.25**	**0.166667**	**0.166667**	**0.083333**	**0.083333**
B	**0.3**	0	0	0.03	0.03	0.02	0.02	0.01	0.01
G	**0.7**	0	0	0.07	0.07	0.046667	0.046667	0.023333	0.023333
1	**0.25**	0.03	0.07	0	0.015517	0.010345	0.010345	0.005172	0.005172
2	**0.25**	0.03	0.07	0.015517	0	0.010345	0.010345	0.005172	0.005172
3	**0.166667**	0.02	0.046667	0.010345	0.010345	0	0.006897	0.003448	0.003448
4	**0.166667**	0.02	0.046667	0.010345	0.010345	0.006897	0	0.003448	0.003448
5	**0.083333**	0.01	0.023333	0.005172	0.005172	0.003448	0.003448	0	0.001724
6	**0.083333**	0.01	0.023333	0.005172	0.005172	0.003448	0.003448	0.001724	0

Table 14. The values of the different parameters that constitute the round trip time equation using equations and the Monte Carlo Simulation (MCS) method.

Parameter	Value from the equations in this paper	Value from the MCS method
S_U	5.4167	5.42
S_D	5.4167	5.4237
S_{UCS}	0.2775	0.2777
S_{LCS}	0.5703	0.566
S	9.9856	10
H	7.562	7.579
L	1.0643	1.05

Table 15. The resultant values of the round trip time using three methods.

Method	Value of the round trip time(s)
Calculation using equations in this paper (equation (32))	136.4403
Calculation using equations in Ref.[6] (the most general set of equations for all conditions)	136.4403
Monte Carlo Simulation method[16]	136.49

finalized using the *HARint* plane[7] and the *HARint* space.[8]

Conclusions

An integrated framework has been introduced that can be used to carry out an elevator traffic design under general traffic conditions (i.e., a mixture of incoming, outgoing, and interfloor traffic conditions). The first step comprises converting the traffic conditions (i.e., traffic mix) and the floor percentages (the floor percentage arrivals and the floor percentage populations) into an OD matrix. The OD matrix is a compact concise form for expressing the passenger movements within the building.

Once the OD matrix has been developed, it is then used to evaluate the round trip time. This can be done using one of the two methods: the MCS method that relies on the OD matrix to generate random passenger OD pairs; or using formulae for evaluating the round trip for general traffic conditions. A novel set of equations have been derived in this paper for the case of top speed attained in one floor journey and equal floor heights under general traffic conditions.

The virtual interval is presented as an alternative to the physical interval that can be used as a critical user requirements for the case of multiple entrances and general traffic conditions.

A detailed numerical example has been presented for the design of the elevator traffic system for a building under general traffic conditions, equal floor heights, and top speed attained in one floor journey. The OD matrix is produced from the floor percentages and the specified traffic conditions. The round trip time is then derived using the novel equations derived in this paper. By dividing the value of the round trip time by the virtual interval and rounding up, the required number of elevators can be found.

Although not discussed in this paper, the *HARint* space methodology can be used to complement this integrated framework. The *HARint* space allows the designer to stipulate the passenger average waiting time and the passenger average traveling time and integrate them into the user requirements. It can then recommend the elevator car capacity; the elevator rated speed as well as making recommendations on building zoning/banking as appropriate.

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Conflict of interest

None declared.

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Appendix 1

Notation

a	the rated acceleration in m/s^2
AR	arrival rate
d_f	the height of a floor in m
E	is the number of elevators in the group

H is the highest reversal floor in a round trip having units of floors
HC handling capacity
ic is the percentage of incoming traffic under the traffic mix
if is the percentage of interfloor traffic under the traffic mix
j the rated jerk in m/s^3
L is the lowest reversal floor in a round trip having units of floors
N the total number of floors in the building
MCS Monte Carlo simulation
OD origin-destination
og is the percentage of outgoing traffic under the traffic mix
P the number of passengers served in one round trip
$P_{arr}(i)$ the percentage arrival from the ith floor
p_{ij} is the passenger transition probability from the ith floor to the jth floor
RTT the round trip time in s
S is the expected number of stops in a round trip
S_D is the average number of down stops in a round trip
S_{LCS} is the average number of lower coincidental stops in a round trip
S_U is the average number of up stops in a round trip
S_{UCS} is the average number of upper coincidental stops in a round trip
t_{dc} the door closing time in s
t_{do} the door opening time in s
t_{pi} the passenger boarding time in s
t_{po} the passenger alighting time in s
U is the total building population
$U(i)$ the building population on the ith floor
v the rated speed in m/s