Synchronization Analysis of a Nonlinear Hyperchaotic Finance System via Hybrid Control and Adaptive Projective Control

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Abstract. In this paper, the hybrid synchronization and adaptive projective synchronization of a nonlinear hyperchaotic finance system are discussed. Based on Lyapunov stability theory and Routh-Hurwitz criteria, some effective controllers are designed to achieve the hyperchaotic finance system global asymptotic synchronization on different conditions. The hybrid feedback controllers are designed for the system parameters are known. The hybrid synchronization of the hyperchaotic finance system is investigated. The synchronization error system between the response system and the drive system achieves asymptotically stable. The synchronization between two different dimensional systems is investigated. When the parameters are unknown, the adaptive control is extended for the projective synchronization between two identical hyperchaotic systems is presented. An effective adaptive controller and a parameter estimation update law are designed according to the adaptive control theory. The numerical simulation results show that proposed schemes are effective.

Keywords: Nonlinear hyperchaotic finance system, Hybrid control, Adaptive projective control

1 Introduction

Since Pecora and Carroll introduced a method to synchronize two identical chaotic systems with different initial conditions, chaos synchronization has attracted a great deal of attention from various fields during the last two decades. Furthermore, chaos synchronization has many potential applications in secure communication, chemical reactions, biomedical systems, information science, plasma technologies, and so on. Recently, the problem of control and synchronization for the chaotic (hyperchaotic) finance system has attracted increasing attention, because of its potential importance in actual applications [1-5]. Many methods have been reported to investigate chaos synchronization of some different types of chaotic (hyperchaotic) attractors [6–10], such as linear feedback control, nonlinear feedback control, adaptive control, time-delay feedback control, coupling control, active control, impulsive control, and so on.

In this paper, based on the Lyapunov stability theory and Routh-Hurwitz criteria, we propose hybrid feedback control and adaptive control for the synchronization between a new hyperchaotic finance systems and other systems. The nonlinear hyperchaotic finance system can be described as:

$$\begin{cases} \dot{x} = z + (y - a)x + w \\ \dot{y} = 1 - by - x^2 \\ \dot{z} = -x - cz \\ \dot{w} = -dxy - kw \end{cases} \tag{1}$$

where x denotes the interest rate, y denotes the investment demand, and z denotes the price exponent, w is the average profit margin. The parameter a is the saving, b is the per investment cost, c is the demands elasticity of commercials, d, k are uncertain parameters. When the parameters are $a = 0.9$, $b = 0.2$, $c = 1.5$, $d = 0.2$, and $k = 0.17$, the four Lyapunov exponents of the system (1) calculated with Wolf algorithm are L_1= 0.034432, L_2= 0.018041, L_3= 0, and L_4=−1.1499. Figure 1 shows the 3-dimensional phase portraits of hyperchaotic finance system (1). There are many experts and scholars who have conducted in-depth researches on this nonlinear hyperchaotic financial system mathematical model [11-20]. For more detailed analysis of the complex dynamics of the system, please see relative Ref. [2].

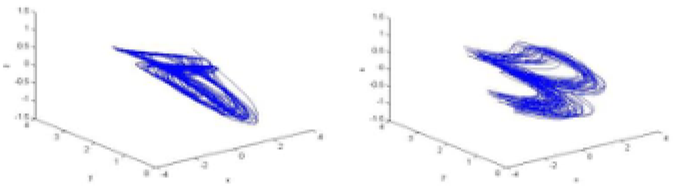

Fig. 1. Hyperchaotic attractors (a) x-y-z space; (b) x-y-w space.

2 Hybrid Synchronization Between Two Identical Systems via Hybrid Feedback Control with Known Parameters

Chaos in economics brought deep impact into western mainstream economics, because the chaos in economics-system means the inherent instability in macroeconomics. System (1) is a hyperchaotic finance system based on production, currency, securities and labor force. The authors of Ref. [2] show us the bifurcation topological structure and the global complicated character of the system (1). In this section, hybrid feedback control is applied to synchronize two identical systems (1) with known parameters. First, suppose the drive system takes the following form:

$$\begin{cases} \dot{x}_1 = z_1 + (y_1 - a)x_1 + w_1 \\ \dot{y}_1 = 1 - by_1 - x_1^2 \\ \dot{z}_1 = -x_1 - cz_1 \\ \dot{w}_1 = -dx_1 y_1 - kw_1 \end{cases} \tag{2}$$

Then we rewrite the drive system as follows:

$$\dot{x} = Ax + Bg$$

where

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{z}_1 \\ \dot{w}_1 \end{bmatrix},\ x = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \end{bmatrix},\ A = \begin{bmatrix} -a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ 0 & 0 & 0 & -k \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ -d & 0 \end{bmatrix},\ g = \begin{bmatrix} x_1 y_1 \\ -x_1^2 + 1 \end{bmatrix}.$$

So we can also give the response system as follows:

$$\dot{y} = Ay + Bh + u \tag{3}$$

where

$$\dot{y} = \left[\dot{x}_2, \dot{y}_2, \dot{z}_2, \dot{w}_2\right]^{\mathrm{T}},\ \ y = \left[x_2, y_2, z_2, w_2\right]^{\mathrm{T}},\ \ h = \begin{bmatrix} x_2 y_2 \\ -x_2^2 + 1 \end{bmatrix}.$$

and $u(t)=[u_1(t), u_2(t), u_3(t), u_4(t)]^{\mathrm{T}}$ are control functions to be determined later. Define the synchronization errors as follows:

$$e_1 = x_2 - x_1,\ e_2 = y_2 - y_1,\ e_3 = z_2 - z_1,\ e_4 = w_2 - w_1.$$

Then the error dynamical system can be obtained as:

$$\dot{e} = Ae + B\left[h - g\right] + u \tag{4}$$

In order to achieve the synchronization, we design the controllers as:

$$u=B\,[g\text{-}h] + Ke, \tag{5}$$

where $K = (k_1, k_2, k_3, k_4)$ denotes feedback matrix. Obviously, $B[g\text{-}h]$ is a nonlinear controller, Ke is a linear controller, so u is the hybrid controller. Add the controller (5) to (4), the error dynamical system can be rewritten as follows:

$$\dot{e} = (A + K)e \tag{6}$$

If we choose a suitable feedback matrix K which makes the matrix A+K has all eigenvalues with negative real parts, this guarantees the asymptotic stability of the system (6).

Thus, we choose

$$K = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Then

$$A + K = \begin{bmatrix} -a & 0 & 0 & 0 \\ 0 & -b & 0 & 0 \\ 0 & 0 & -c & 0 \\ 0 & 0 & 0 & -k \end{bmatrix}.$$

The characteristic equation of A+K is:

$$(\lambda + a)(\lambda + b)(\lambda + c)(\lambda + k) = 0.$$

When $a = 0.9$, $b = 0.2$, $c = 1.4$, $d = 0.2$ and $k = 0.2$, according to the Routh-Hurwitz criteria, the system (6) is asymptotically stable. That is, the response system (3) is synchronizing to the drive system (2). Numerical simulation in example 5.1 shows the result of synchronization between the system (2) and (3).

3 Increased Order Synchronization Between Two Different Dimensional Systems

In this section, the synchronization of two different dimensional systems will be introduced. Consider the three-dimensional chaotic finance system [5] as the drive system, and the system is given by

$$\begin{cases} \dot{x} = z + (y - m)x \\ \dot{y} = 1 - ny - x^2 \\ \dot{z} = -x - pz \end{cases} \tag{7}$$

When m=0.9, n=0.2, p=1.2, the system (7) shows chaotic behavior.

Let

$$\dot{X}_0 = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix},\ X_0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ A_0 = \begin{bmatrix} -m & 0 & 1 \\ 0 & -n & 0 \\ -1 & 0 & -p \end{bmatrix},\ B_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},\ f = \begin{bmatrix} xy \\ -x^2 + 1 \end{bmatrix}.$$

and the response system is system (3).

Define the synchronization errors between the systems (7) and (3) as follows:

$e_1 = x_2 - x$, $e_2 = y_2 - y$, $e_3 = z_2 - z$, $e_4 = w_2 - (x + y + z)$.

Then the error dynamical system can be obtained as:

$$\dot{e} = Ae + AX + Bn - A_0X - B_0m + u \tag{8}$$

In order to achieve the synchronization, we design the controllers as following rules:

$$u = (A_0 - A)X - Bn + B_0m + Ke$$

The error dynamical system (8) can be rewritten as follows:

$$\dot{e} = (A + K)e \tag{9}$$

We choose

$$K = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

Then

$$A + K = \begin{bmatrix} -a & 0 & 0 & 0 \\ 0 & -b & 0 & 0 \\ 0 & 0 & -c & 0 \\ 0 & 0 & 0 & -k \end{bmatrix}.$$

The characteristic equation of A+K is:

$$(\lambda + a)(\lambda + b)(\lambda + c)(\lambda + k) = 0$$

When a = 0.9, b = 0.2, c = 1.4, d = 0. 2 and k = 0.2, according to the Routh-Hurwitz criteria, the error system (9) is asymptotically stable. That is, the response system (3) is synchronizing with the drive system (7). Numerical simulation in example 5.2 shows the result of synchronization.

4 Adaptive Projective Synchronization with Unknown Parameters

In this section, projective synchronization between two different hyperchaotic systems is given. An effective adaptive controller and a parameter estimation update law are designed according to the adaptive control theory.

Ref. [6] gives another hyperchaotic finance system, which is described as:

$$\begin{cases} \dot{x} = -\alpha(x+y) + w \\ \dot{y} = -y - \alpha xz \\ \dot{z} = \beta + \alpha xy \\ \dot{w} = -\gamma xz - \theta w \end{cases} \tag{10}$$

where x is the interest rate, y is the investment demand, z is the price index, w is unknown state variable, and $\alpha, \beta, \gamma, \theta$ are real constant parameters. When $\alpha = 3, \beta = 15, \gamma = 0.2, \theta = 0.12$, system (10) exhibits hyperchaotic behavior.

In order to achieve the synchronization between the two systems, we assume that the system (2) is the derive system, and the response system is given as follows:

$$\begin{cases} \dot{x}_2 = -\alpha(x_2 + y_2) + w_2 + u_1(t) \\ \dot{y}_2 = -y_2 - \alpha x_2 z_2 + u_2(t) \\ \dot{z}_2 = \beta + \alpha x_2 y_2 + u_3(t) \\ \dot{w}_2 = -\gamma x_2 z_2 - \theta w_2 + u_4(t) \end{cases} \tag{11}$$

Subtracting (2) from (11) yields the following error dynamical system:

$$\begin{cases} \dot{e}_1 = -\alpha(x_2 + y_2) + w_2 - t(z_1 + (y_1 - a)x_1 + w_1) + u_1(t) \\ \dot{e}_2 = -y_2 - \alpha x_2 z_2 - t(1 - by_1 - x_1^2) + u_2(t) \\ \dot{e}_3 = \beta + \alpha x_2 y_2 - t(-x_1 - cz_1) + u_3(t) \\ \dot{e}_4 = -\gamma x_2 z_2 - \theta w_2 - t(-dx_1 y_1 - kw_1) + u_4(t) \end{cases} \tag{12}$$

where $e_1 = x_2 - tx_1$, $e_2 = y_2 - ty_1$, $e_3 = z_2 - tz_1$ and $e_4 = w_2 - tw_1$. The goal of the control is to find an effective controller $u = [u_1, u_2, u_3, u_4]^T$ and a parameter estimation update law, such that the response system (11) is asymptotically synchronized with the drive system (2).

Then we obtain the following theorem, which shows that hyperchaotic systems (2) and (11) can be synchronized effectively.

Theorem 1: Take the adaptive control law as

$$\begin{cases} u_1(t) = \hat{\alpha}(x_2 + y_2) - w_2 + tz_1 + t(y_1 - \hat{a})x_1 + tw_1 - m_1 e_1 \\ u_2(t) = y_2 + \hat{\alpha} x_2 z_2 + t(1 - \hat{b} y_1 - x_1^2) - m_2 e_2 \\ u_3(t) = -\hat{\beta} - \hat{\alpha} x_2 y_2 - tx_1 - t\hat{c} z_1 - m_3 e_3 \\ u_4(t) = \hat{\gamma} x_2 z_2 + \hat{\theta} w_2 - t\hat{d} x_1 y_1 - t\hat{k} w_1 - m_4 e_4 \end{cases} \tag{13}$$

When $\tilde{a} = \hat{a} - a$, $\tilde{b} = \hat{b} - b$, $\tilde{c} = \hat{c} - c$, $\tilde{d} = \hat{d} - d$, $\tilde{k} = \hat{k} - k$, $\tilde{\alpha} = \hat{\alpha} - \alpha$, $\tilde{\beta} = \hat{\beta} - \beta$, $\tilde{\gamma} = \hat{\gamma} - \gamma$, $\tilde{\theta} = \hat{\theta} - \theta$, and the parameter estimation update law as:

$$\dot{\hat{a}} = tx_1 e_1, \ \dot{\hat{b}} = ty_1 e_2, \ \dot{\hat{c}} = tz_1 e_3, \ \dot{\hat{d}} = tx_1 y_1 e_4, \ \dot{\hat{k}} = tw_1 e_4, \ \dot{\hat{\alpha}} = -(x_2 + y_2) e_1, \ \dot{\hat{\beta}} = e_3,$$
$$\dot{\hat{\gamma}} = -x_2 z_2 e_4, \ \dot{\hat{\theta}} = -w_2 e_4 \tag{14}$$

Then the response system (11) will be asymptotically synchronized with drive system (2).

Proof: substituting (13) to (12) leads to the following error system:

$$\begin{cases} \dot{e}_1 = \tilde{\alpha}(x_2 + y_2) - t\tilde{a}x_1 - m_1 e_1 \\ \dot{e}_2 = \tilde{\alpha}x_2 z_2 - t\tilde{b}y_1 - m_2 e_2 \\ \dot{e}_3 = -\tilde{\beta} - \tilde{\alpha}x_2 y_2 - t\tilde{c}z_1 - m_3 e_3 \\ \dot{e}_4 = \tilde{\gamma}x_2 z_2 + \tilde{\theta}w_2 - t\tilde{d}x_1 y_1 - t\tilde{k}w_1 - m_4 e_4 \end{cases}$$

Choose the following Lyapunov function

$$V = (e_1^{\ 2} + e_2^{\ 2} + e_3^{\ 2} + e_4^{\ 2} + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{k}^2 + \tilde{\alpha}^2 + \tilde{\beta}^2 + \tilde{\gamma}^2 + \tilde{\theta}^2)/2 .$$

The time derivative of V is as follows:

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{k}\dot{\tilde{k}} + \tilde{\alpha}\dot{\tilde{\alpha}} + \tilde{\beta}\dot{\tilde{\beta}} + \tilde{\gamma}\dot{\tilde{\gamma}} + \tilde{\theta}\dot{\tilde{\theta}} \\ &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + \tilde{a}(-\dot{\hat{a}}) + \tilde{b}(-\dot{\hat{b}}) + \tilde{c}(-\dot{\hat{c}}) + \tilde{d}(-\dot{\hat{d}}) + \tilde{k}(-\dot{\hat{k}}) + \tilde{\alpha}(-\dot{\hat{\alpha}}) + \tilde{\beta}(-\dot{\hat{\beta}}) + \tilde{\gamma}(-\dot{\hat{\gamma}}) + \tilde{\theta}(-\dot{\hat{\theta}}) \end{aligned} \tag{15}$$

Substituting (13) and (14) into (15) yields

$$\dot{V} = -m_1 e_1^{\ 2} - m_2 e_2^{\ 2} - m_3 e_3^{\ 2} - m_4 e_4^{\ 2} < 0$$

Since the Lyapunov function V is positive definite and its derivative is negative definite. Based on Lyapunov stability theory, the drive system (2) and the response system (11) can be synchronized asymptotically. Numerical simulation in example 5.3 shows the result of synchronization.

5 Numerical Simulations

In the numerical simulations, the fourth-order Runge-Kutta method is used to verify and demonstrate the effectiveness of the proposed methods, we consider three numerical examples. The parameters of hyperchaotic systems are chosen as a = 0.9, b = 0.2, c = 1.5, d = 0. 2 and k = 0.17 to ensure the existence of hyperchaos of system (1) in the absence of control.

Example 5.1. We assume that the control gain (k_1, k_2, k_3, k_4) = (1, 1, 1, 1), the initial values of drive and response systems are ($x_1(0)$, $y_1(0)$, $z_1(0)$, $w_1(0)$)=(-1, -1, -1, -1), ($x_2(0)$, $y_2(0)$, $z_2(0)$, $w_2(0)$)=(1, 1, 1, 1), respectively, the initial errors of system (5) are ($e_1(0)$, $e_2(0)$, $e_3(0)$, $e_4(0)$) = (2, 2, 2, 2). Figure 2 shows the dynamics of synchronization errors for the system (2) and the system (3). The simulation shows our extending is effective.

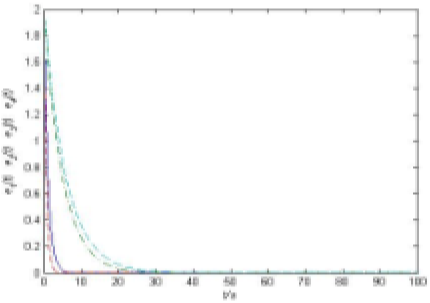

Fig. 2. The error state time response of system (2) and system (3) with the hybrid feedback method.

Example 5.2. In the numerical simulations, the parameters are chosen as m=0.9, n=0.2, p=1.2, a = 0.9, b = 0.2, c = 1.5, d= 0. 2 and k=0.17. We assume that the control gain (k_1, k_2, k_3, k_4) = (1, 1, 1, 1), the initial values o drive and response systems are ($x(0)$,

$y(0)$, $z(0)) = (-1, -2, -3)$, $(x_2(0), y_2(0), z_2(0), w_2(0)) = (1, 2, 3, 2)$, respectively, the initial errors of system (5) are $(e_1(0), e_2(0), e_3(0), e_4(0)) = (2, 4, 6, 2)$. Figure 3 displays the time evolutions of the dynamics of synchronization errors.

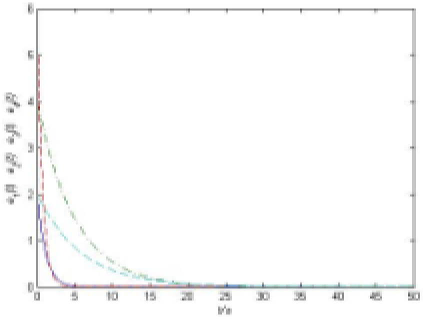

Fig. 3. Dynamics of synchronization errors between system (7) and system (3) via the hybrid feedback method.

Example 5.3. In numerical simulation, select the true values of "unknown" parameters of the drive system as $\alpha = 3, \beta = 15, \gamma = 0.2, \theta = 0.12$ $a = 0.9$, $b = 0.2$, $c = 1.5$, $d = 0.2$ and $k = 0.17$. We choose $t = 2$, the initial conditions as $(x_1(0), y_1(0), z_1(0), w_1(0)) = (-1, -1, -1, -1)$, $(x_2(0), y_2(0), z_2(0), w_2(0)) = (1, 1, 1, 1)$, $(e_1(0), e_2(0), e_3(0), e_4(0)) = (2, 2, 2, 2)$, ($(\hat{a}(0), \hat{b}(0), \hat{c}(0), \hat{d}(0), \hat{k}(0), \hat{\alpha}(0), \hat{\beta}(0), \hat{\gamma}(0), \hat{\theta}(0))$ = (1, 3, 1, 1, 1, 30, 1, 1), $m_i = 1$, $i=1,2,3,4$. The dynamics of synchronization errors for the drive system (2) and response system (11) is shown in Figure 4.

Obviously, the synchronization errors converge asymptotically to zero, hence, two different hyperchaotic systems are indeed achieved chaos synchronization.

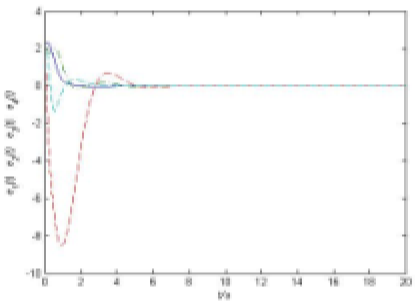

Fig. 4. Time evolution of error valuable between systems (2) and systems (11).

6 Conclusions

This paper is concerned with hybrid feedback control and adaptive feedback control to synchronize a new four-dimensional hyperchaotic finance system. Based on the Lyapunov stability theory and Routh-Hurwitz criteria, the drive and response systems could be synchronized with some effective controllers are designed for the global asymptotic synchronization on different conditions. Numerical simulations are shown to verify and illustrate the effectiveness of this controller.

Acknowledgments

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