

GLOSSARY

DISCRETE MATHEMATICS

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1 A

Adjacent vertex.- [ə'ʤeisent 'vɜ:rteks]. *Vértice adyacente*. Two vertices u, v of a graph $G = (V, E)$ are adjacent if the pair (u, v) is an edge of G .

Alphabet.- [ælfəbet]. *Alfabeto*. A non-empty set used to construct more complex structures such as strings, lists, languages, automata, etc.

Ancestor.- [ænsesətər]. *Antecesoros*. In a tree, the ancestors of a vertex are those nodes found in the unique path from the root to that vertex.

Antisymmetric relation.- [æntɪ'simetrik rɪ'leɪʃn]. *Relación antisimétrica*. A relation R is called antisymmetric if $(a, b) \in R$ and $(b, a) \in R$ only happens when $a = b$.

Argument.- [ɑ:rgjumənt]. *Argumento (lógica)*. An argument is composed of a set of meaningful propositions, the premises, along with another proposition, the conclusion. If the argument is correct, then the conclusion has to be a logical consequence of the premises.

Acyclic graph.- [ə'saɪklɪk græf]. *Grafo acíclico*. A graph containing no simple circuits (cycles).

2 B

Base.- [beɪs]. *Base (sistemas de numeración)*. Given a fix positive integer b , any integer x can be represented as a unique sum of powers of b . The number b is called base (also radix). Typical bases are $b = 2$ (binary), $b = 10$ (decimal), or $b = 16$ (hexadecimal).

Basis step.- ['beɪsɪs 'step]. *Caso base (de una demostración por inducción)*. One of the steps in a inductive proof. See inductive proof.

Biconditional statement.- [baɪkən'dɪʃnl 'steɪmənt]. *Implicación (lógica)*. A logical formula of the form $p \longleftrightarrow q$.

Bijjective map.- [baɪ'dʒektɪv 'mæp]. *Aplicación biyectiva*. A bijective map is a map that is both injective and surjective. The existence of a bijective map between two finite sets implies that they have the same cardinality.

Binary connective.- ['bʌmərɪ kə'nektɪv]. *Conectivo u operador binario*. A binary connective is a connective that connects two propositions. In propositional logic, the binary connectives are $\{\wedge, \vee, \rightarrow, \leftrightarrow, \}$.

Binary relation.- ['bʌmərɪ rɪ'leɪʃn]. *Relación binaria*. Given two non-empty sets A, B , a binary relation is a subset of the Cartesian product $A \times B$.

Binomial coefficient.- [baɪ'nəʊmɪəl ,kəʊ'fɪʃnt]. *Coeficiente binómico o número combinatorio*. The k -th coefficient of the binomial expression $(a + b)^n$. It also accounts for the number of subsets of size k that can be drawn from a set of n elements. The binomial coefficient is notated by $\binom{n}{k}$, for positive integers n, k , where $k \leq n$.

Binomial theorem.- [baɪ'nəʊmɪəl 'θɪərəm]. *Binomio de Newton o teorema del binomio*. The Binomial Theorem describes the algebraic expansion of powers of the expression $(a + b)^n$.

Bipartite graph.- [baɪ'pɑːrtart græf]. *Grafo bipartito*. A bipartite graph (V, E) is a graph whose set of vertices can be split into two disjoint sets A, B such that every edge connects a vertex in A to one in B . Sets A and B are called independent sets.

Bitch.- ['bɪtʃ]. *Chungo (problema)*. A hell of a problem, normally appearing in a final exam, which apparently forms part of the professor's sense of beauty.

3 C

Canonical representative.- [kə'nɑːnɪkl ,reprɪ'zentətɪv]. *Representante canónico*. A distinguished element designated to name an equivalence class. It is also called canonical form. For example, in \mathbb{Z}_n , the canonical representatives are usually taken as the classes $\{\overline{0}, \overline{1}, \dots, \overline{n-1}\}$.

Cardinality.- [kɑːdrɪ'nælɪtɪ]. *Cardinal*. Note that cardinal is different from cardinality in English. Cardinality is the number of elements of a set. Normally, it is denoted by the same sign as the absolute value $||$. For example, the cardinality of set $A = \{1, 2, 3\}$ is $|A| = 3$.

Cartesian product.- [kɑ:r'ti:ziən]. *Producto cartesiano*. Given two non-empty sets A, B , its Cartesian product $X \times Y$ is the set of all possible *ordered* pairs where the first component is an element of X and the second component an element of B . The Cartesian product is named after French mathematician René Descartes, who introduced the concept. Obviously, $|X \times Y| = |X| \cdot |Y|$.

Center.- ['sentə(r)]. *Centro de un grafo*. A vertex of a graph is said to be the center of the graph if it minimizes the maximum distance between two vertices.

Child.- [tʃaɪld]. *Hijo*. In a tree, the child of a vertex v is a vertex incident with v whose initial vertex is also v .

Circuit.- ['sɜ:kɪt]. *Circuito*. A closed walk.

Closed walk.- ['kləʊzd 'wɔ:k]. *Circuito o recorrido circular*. A closed walk is a walk that begins and ends at the same vertex. It is also called a circuit or cycle.

Codomain.- [cədə'meɪn]. *Conjunto final de una aplicación*. The codomain of a map is the set of possible output values the map can produce.

k -Combination.- [keɪkɒmbrɪ'neɪʃn]. *Combinaciones*. A k -combination is an *unordered* selection of k elements from a set. The number of k -combinations taken from an n -element set without repetitions is $\binom{n}{k}$. When repetitions are allowed, the number of k -combinations is $\binom{n+k-1}{k}$.

Complement of a graph.- ['kɑ:mplɪməntəvə'græf]. *Complementario de un grafo*. Given a graph $G = (V, E)$, the complement of G , to be denoted by \overline{G} , is the graph $\overline{G} = (V, \overline{E})$.

Complement of a set.- ['kɑ:mplɪməntəvə'set]. *Complementario de un conjunto*. Given a universal set U , the complement of a set $A \subset U$ is the set formed by those elements of U that are not in A .

Complete bipartite graph.- [kəm'plɪ:t baɪ'pɑ:rtat 'græf]. *Grafo bipartito completo*. A bipartite graph with the maximum of edges between its independent sets. It is denoted by $K_{n,m}$, for $n, m \geq 1$ natural numbers.

Complete graph.- [kəm'plɪ:t græf]. *Grafo completo*. A graph where there is an edge for every possible pair of vertices. If there is n vertices, then the complete graph, to be denoted by K_n , has $\binom{n}{2}$ edges.

Conditional statement.- [kən'dɪʃnl steɪmənt]. *Implicación (lógica)*. A logical formula of the form $p \longrightarrow q$.

Connected component of a graph.- [kə'nektɪd kəm'pəʊnəntəvə'græf]. *Componente conexa de un grafo*. A maximal connected subgraph of a graph. Here the term maximal is referred to set inclusion.

Connected graph.- [kə'nektɪd græf]. *Grafo conexo*. A graph is said to be connected if for every two vertices there is a path connecting them.

Conclusion.- [kən'kluːʒn]. *Conclusión*. In a deductive argument, the proposition that is the logical consequence of the premises.

Connective.- [kə'nektɪv]. *Conectivo (lógico)*. It is also called logical operator. A logical connective is a symbol used to connect one or more propositions. In propositional logic connective can be unary or binary, depending on how many propositions they connect. There is only one unary connective $\{\neg\}$; the set of binary connectives is $\{\wedge, \vee, \rightarrow, \leftrightarrow, \}$.

Constant.- ['kɑːnstənt]. *Constantes (lógicas)*. A constant in logic is a symbol that has the same truth value under every valuation. There are two constants in logic: \top , called the constant true, and \perp , the constant false.

Constant \perp .- ['kɑːnstənt fɔːls]. *Constante falso*. A symbol whose truth value is equal to false under every valuation.

Constant \top .- ['kɑːnstənt truː]. *Constante cierto*. A symbol whose truth value is equal to true under every valuation.

Contingency.- [kən'tɪŋdʒənsɪ]. *Contingencia*. A set of logic formulae is said to be a contingency if it is neither true nor false under every valuation.

Contradiction.- [kɑːtrə'dɪkʃn]. *Contradicción*. A set of logic formulae is said to be a contradiction if it is false under every valuation.

Contrapositive statement.- [kɑːntrə'pɑːzətɪv 'steɪmənt]. *Contraposición (lógica)*. The converse of the conditional statement $p \rightarrow q$ is $\neg q \rightarrow \neg p$.

Converse statement.- [kən'vɜːrs 'steɪmənt]. *Recíproco (lógica)*. The converse of the conditional statement $p \rightarrow q$ is $q \rightarrow p$.

Correctness.- [kə'rektnəs]. *Corrección (validez lógica)*. A deductive argument, which is composed of premises and a conclusion, is said to be correct if every model for the premises is also a model for the conclusion.

Correspondence.- [kɔːrə'spɑːndəns]. *Correspondencia*. Given two sets A, B , a correspondence is an assignment of an element in A to *some* elements in B .

Congruent modulo n .- [kɑːngruənt 'mɑːdjuləvən]. *Congruente módulo n* . An equivalence relation defined on \mathbb{Z} in the following way: Given an integer n , the modulo, a is said to be congruent with b modulo n if and only if $a - b$ is divisible by n .

Counterexample.- ['kaʊntərɪg'zæmpl]. *Contraejemplo*. In a deductive argument it is a model of the premises that is not a model for the conclusion.

***n*-Cube.-** [en'kju:b]. *Hipercubo*. An *n*-cube is a graph defined out a binary relation. Let us consider the set of *n*-bit strings. The vertex set will be these 2^n strings. As for the edge set, we will draw an edge between two strings if they only differ in exactly one position. The *n*-cube graph is denoted by Q_n .

Cycle.- ['saɪkl]. *Recorrido circular*. A closed walk.

Cycle graph.- ['saɪkl græf]. *Grafo circular*. The graph consisting of *n* vertices v_1, \dots, v_n and *n* edges $\{(v_1, v_2), \dots, (v_{n-1}, v_n), (v_n, v_1)\}$. It is denoted by C_n .

4 D

Deductive argument.- [dɪ'dʌktɪv 'ɑ:rgjʊmənt]. *Estructura deductiva*. A deductive argument is a formal representation of a line of argument. A deductive argument is composed of a set of propositions, called premises, along with another proposition, called the conclusion, which has to be a logical consequence of the premises.

Degree.- [dɪ'grɪ:]. *Grado*. The degree of a vertex is the number of edges incident with the vertex.

De Morgan laws.- [də'mɑ:gən 'lɔ:z]. *Leyes de De Morgan*. A set of very well-known logical equivalences. For example, a De Morgan law is $\overline{p \wedge q} \equiv \overline{p} \vee \overline{q}$.

Dependence relation (task planing).- [dɪ'pendəns rɪ'leɪʃn]. *Relación de dependencia (planificación de tareas)*. See precedence relation.

Dependency tree.- [dɪ'pendənsɪ trɪ:]. *Árbol de dependencia*. Give a recursive correspondence $f : A \longrightarrow B$, a dependency tree $T_f(a)$, $a \in A$, is a tree showing all values necessary to evaluate $f(a)$. The tree is recursively built by following the formal definition of the recursive correspondence f . The root is labelled with value a . Children of the root are the calling values of $f(a)$, that is, those values $f(a_1), \dots, f(a_k)$ required to compute $f(a)$. The tree is then recursively constructed.

Descendant.- [dɪ'sendənt]. *Descendiente*. The descendants of a vertex v are those vertices that have v as ancestor.

Depth of the structural tree of a list.- ['depθəvə'strʌktʃərəl 'trɪ:]. *Profundidad del árbol estructural de una lista (recursión)*. The length of the longest path from the root to the leaves in a structural tree associated with a given list.

Difference of sets.- ['dɪfrənsəv 'sets]. *Diferencia de conjuntos*. Given two sets A, B , the difference $A - B$ is defined as those elements in A that are not in B .

Digraph.- [daɪ'græf]. *Digrafo*. A digraph is a directed pseudograph holding the following properties: (1) It may have one loop per vertex at most; (2) It may have two edges between two distinct vertices as long as they have opposite directions. According to this definition, a digraph is a subclass of pseudographs. Digraphs are used to model binary relations.

Dijkstra's algorithm.- ['deɪkstrə'ælgərɪðəm]. *Algoritmo de Dijkstra*. A graph search algorithm that solves the problem of finding all minimum paths for a weighted graph.

Directed graph.- ['daɪrɛktɪd 'græf]. *Grafo dirigido*. A directed graph is a graph whose edges are formed by ordered pair of vertices.

Disjoint sets.- [dɪs'ʤɔɪnt 'sets]. *Conjuntos disjuntos*. Two sets that has no element in common.

Domain.- [də'mem]. *Dominio de una aplicación*. The domain of a map is its set of input values, also called argument values.

5 E

Edge.- [eɖʒ]. *Arista*. A pair of adjacent vertices in a graph.

Elementary function.- [elɪ'mentɪ 'fʌŋkʃn]. *Funciones elementales (recursividad en listas)*. In the context of recursion on lists, elementary functions are certain simple functions used to handle lists. They will be used to define more complex functions. The elementary functions are the following:

- $CAB : LIST(\Sigma) - \{[,]\} \longrightarrow CAB \cup LIST(\Sigma)$. It returns the first element of a list.
- $REST : LIST(\Sigma) - \{[,]\} \longrightarrow CAB \cup LIST(\Sigma)$. It removes the head of the list and outputs the remaining list.
- $CONC : LIST(\Sigma) \times LIST(\Sigma) \longrightarrow LIST(\Sigma)$. $CONC(L_1, L_2)$ returns the list obtained by placing L_1 followed by L_2 in their order.
- $LISTA(L)$. A predicate that returns 1 if L is a list, and 0 otherwise.

Empty list.- ['emptɪlɪst]. *Lista vacía (recursión)*. The list $[]$, the list containing no elements.

Empy set.- ['emptɪset]. *Conjunto vacío*. The set that has no element.

Equivalence relation.- [ɪ'kwɪvələns rɪ'leɪʃn]. *Relación de equivalencia*. An equivalence relation is a binary relation that is reflexive, symmetric and transitive.

Eulerian circuit.- [ɔɪ'lɪəriən 'sɜ:kɪt]. *circuito euleriano*. A circuit is called Eulerian if it traverses each edge exactly once.

Eulerian graph.- [ɔɪ'lɪərɪən græf]. *Grafo euleriano*. A graph is called an Eulerian graph if it admits an Eulerian circuit.

Eulerian path.- [ɔɪ'lɪərɪən 'pæθ]. *Recorrido euleriano*. A path is called Eulerian if it traverses each edge exactly once. Note that the path may not be closed.

Event.- [ɪ'vent]. *Suceso*. In the context of probability, an event is a set of outcomes to which a probability is assigned.

Expansion.- [ɪks 'pænjən]. *Desarrollo (de una expresión)*. The expansion of an expression consists of performing all the operations in the given expression. For example, the expansion of $(a + b)^3$ is $a^3 + 3a^2b + 3ab^2 + b^3$.

Explicit function.- [ɪk'splɪsɪt 'fʌŋkʃn]. *Función explícita*. A function defined without any reference to itself.

6 F

Floor function.- ['flɔːr 'fʌŋkʃn]. *Parte entera*. The floor of a real number x is the largest integer that is less or equal to x . It is also called integer part.

Forest.- ['fɔːrɪst]. *Bosque (grafos)*. A set of acyclic graphs.

Formalization.- ['fɔːrmələ'zeɪʃn]. *Formalización*. To describe sentences of natural languages, such as English or Spanish, in a formal language, such as that of propositional logic.

Formula.- ['fɔːrmjələ]. *Fórmula*. A word in propositional logic formed according to certain rules. A formula is composed of proposition variables, logical constant, connectives, and parentheses. The plural of formula is formulae (['fɔːrmjəli:]).

Function.- ['fʌŋkʃn]. *Función*. A function is a map between two sets of numbers. The term map is reserved for a more general assignment between arbitrary sets, not necessarily sets of numbers.

Function composition.- ['fʌŋkʃn ,kɑːmpə'zɪʃn]. *Composición de aplicaciones*. Let $g : A \longrightarrow B$ and $f : B \longrightarrow C$ two functions. The function composition of f and g , to be denoted by $f \circ g$, is a map $f \circ g : A \longrightarrow C$ defined by $f(g(a))$, for all $a \in A$.

7 G

Graph.- [græf]. *Grafo*. A graph is a pair (V, U) composed of two sets: the set V of vertices, which must be non-empty, and the set U of edges, which consists of pair of non-repeated vertices. A graph is an abstract representation used to model a set and certain relations

between its elements. Sometimes, this kind of graph is called simple graph as opposed to multigraphs, pseudographs, or directed graphs.

8 H

Hasse diagram.- ['heɪs 'daɪəgræm]. *Diagrama de Hasse*. A Hasse diagram is a diagram used to represent partially ordered sets. It is a simplified version of the full diagram (graph) of the order relation under certain conventions.

Hamiltonian graph.- [hə'mɪl'tɔ:nɪən 'græf]. *Grafo hamiltoniano*. A graph is called Hamiltonian if it admits a Hamiltonian path.

Hamiltonian path.- [hə'mɪl'tɔ:nɪən 'pæθ]. *Camino hamiltoniano*. A path is called Hamiltonian if it visits each vertex exactly once.

Head of a list.- ['hedəvə'list]. *Cabeza de una lista*. The first element of a list.

Height (of a tree).- [haɪt]. *Altura (de un árbol)*. The length of the longest path from the root to any of its leaves.

9 I

Image of a map.- ['ɪmɪdʒəvə'mæp]. *Imagen de una aplicación*. Given a map $f : A \longrightarrow B$, we say that $b \in B$ is the image of $a \in A$ if $f(a) = b$. By image we also refer to set $f(A) = \{b \in B \mid \exists a \in A \text{ such that } f(a) = b\}$. In this latter sense, image is also called range.

Incident edge.- ['ɪnsɪdənt eɪdʒ]. *Arista incidente*. An edge e is called incident with vertex u, v if $e = (u, v)$.

Inclusion-exclusion principle.- [ɪn'klʊ:ʒn ɪk'sklʊ:ʒn 'prɪnsəpl]. *Principio de exclusión-inclusión*. It is a counting technique to correctly count subsets of a set without over-counting.

In-degree.- [ɪn'dri'grɪ:]. *Grado de entrada*. In a directed graph, the in-degree of a vertex v is the number of edges whose terminal vertex is v . It is denoted by $\deg^-(v)$.

Independent sets.- [ɪn'drɪ'pendənt 'sets]. *Conjuntos independientes (grafos)*. See bipartite graph.

Induced subgraph.- [ɪndu:st sʌb'græf]. *Subgrafo inducido*. Given a graph $G = (V, E)$ and a non-empty subset W of V , the subgraph induced by W has W as vertex set and those edges in G whose endpoints are in W as edge set. It is denoted by $\langle W \rangle$.

Inductive proof.- [m'dʌkfɪn pru:f]. *Prueba por inducción.* An inductive proof is used to prove a proposition $P(k)$ depending on a infinite number of natural values (it can be \mathbb{N} or a subset). Two steps must be carried out in a inductive proof: (1) the basis step, where the truth of proposition $P(k_0)$ is proved, for some low value k_0 ; (2) the inductive step, where the implication $P(k) \implies P(k+1)$ is proved to be correct. Then, $P(k), k \geq k_0$ is proved. For example, the proposition

$$P(k) \equiv 1 + 2 + \dots + k = \frac{k(k+1)}{2},$$

for $k \in \mathbb{N} - \{0\}$, can be proved by induction.

Inductive step.- [m'dʌktiv 'step]. *Caso inductivo (de una demostración por inducción).* One of the steps in a inductive proof. See inductive proof.

Inference rules.- [ɪnfərəns 'ru:lz]. *Reglas de inferencia.* We call inference rules a set of basic deductive arguments. For example, inference rules are *modus ponendo ponens* or *modus tollendo tolens*.

Initial set.- [ɪnɪʃl set]. *Conjunto de partida (recursividad).* Given a recursive correspondence $f : A \longrightarrow B$, the initial set is formed by all values in A whose definition is not recursive.

Initial vertex.- [ɪnɪʃl 'vɜ:rtɛks]. *Vértice de entrada.* In a directed graph, an edge is an ordered pair of vertices (u, v) . Vertex u is called the initial vertex of edge (u, v) .

Injective map.- [ɪn'dʒektiv mæp]. *Aplicación inyectiva.* It is also called a one-to-one map. A map $f : A \longrightarrow B$ is injective if it is true that $f(a) = f(b)$ always implies $a = b$, for all a, b in A .

Integer part.- [ɪntɪdʒər pɑ:rt]. *Parte entera.* See floor function.

Inverse map.- [ɪn'vɜ:s 'mæp]. *Aplicación inversa.* Let $f : A \longrightarrow B$ a map. By virtue of the definition, each $a \in A$ is associated with a unique element $f(a) = b$ in B . It is possible to define a correspondence from B to A by assigning to each $b \in B$ all elements in A such that $\{a \in A \mid f(a) = b\}$. Such correspondence is called the inverse correspondence of f and is denoted by f^{-1} . When f is an injective map, the inverse correspondence is actually a function, and $f^{-1}(b)$ is a unique element in A .

Intersection of sets.- [ɪntər'sekʃnəv'sets]. *Intersección de conjuntos.* Given two sets A, B , its intersection $A \cap B$ is defined as those elements that are in A and B .

Isolated vertex.- [aɪsələtəd 'vɜ:rtɛks]. *Vértice aislado.* A vertex that does not form part of the set of edges of the graph.

Isomorphic.- [aɪsəmər'fɪk]. *Isomorfo.* Adjective. Related by an isomorphism.

Isomorphism.- [aɪsəmər'fɪzm]. *Isomorfismo*. An isomorphism between two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a bijective map $f : V_1 \longrightarrow V_2$ holding the property that for every edge $e_1 = (a, b) \in E_1$ the pair $(f(a), f(b))$ is an edge of G_2 . Intuitively, this means that both graphs have the same underlying structure.

10 J, K, L

Kruskal's algorithm.- ['krʌskəlz 'ælgərɪðəm]. *Algoritmo de Kruskal*. An algorithm to compute a minimum spanning tree of a weighted connected undirected graph.

Leaf.- [lɪ:f]. *Hoja*. A vertex of a tree that has no children is called a leaf.

Length of a list.- ['lenθəvəlɪst]. *Longitud de una lista (recursión)*. The number of elements in a list.

List.- [lɪst]. *Lista (recursión)*. Given an alphabet Σ , a recursive definition of a list on a set is defined as follows: (1) If a_1, \dots, a_k are elements in Σ , then the ordered set $[a_1, \dots, a_k]$ is a list; (2) Given a set of lists L_1, \dots, L_m , the ordered set $[L_1, \dots, L_m]$ is also a list. According to this definition, $[1, 2, 3, 4]$ and $[1, 2, [3, [4]]]$ are lists on \mathbb{N} .

Lower triangular matrix.- ['ləʊə traɪ'æŋgjələ'metɪks]. *Matriz triangular inferior*. A lower triangular matrix is a matrix where the entries above the main diagonal are zero.

Logical equivalence.- ['lɔ:ʤɪkl ɪ'kwɪvələns]. *Equivalencia lógica*. Two formulae are logically equivalence if they have the same truth value for every model.

Loop.- ['lu:p]. *Bucle (grafos)*. An edge e of a graph connecting a vertex to itself, that is, an edge of the form (a, a) .

11 M

Map.- [mæp]. *Aplicación*. Given two sets A, B , a map is an assignment of an element in A to a unique element in B . Both sets can be arbitrary sets.

Matrix.- ['metɪks]. *Matriz*. A matrix is a rectangular array of numbers. The plural is matrices.

Maximal element.- ['mæksɪməl 'elɪmənt]. *Elemento maximal*. Given some partially ordered set (R, \leq) , an element a is called maximal if there is no b such that $a \leq b$, that is, if a is not less than any element.

Maximum.- ['mæksɪmən]. *Elemento máximo*. Given some partially ordered set (R, \leq) , an element a is called maximum if $b \leq a$ for all $b \in R$.

Meaningful valuation.- ['mɪ:nɪfl̩ ,vælju'eɪʃn]. *Valoración significativa*. A meaningful valuation is a valuation restricted only to the logical variables occurring in a given formula as opposed to all possible logical variables.

Median.- ['mɪ:dɪən]. *Mediana de un grafo*. A vertex of a a weighted graph is called the median if it minimizes the sum of all distances to the rest of the vertices.

Minimal element.- ['mɪnɪməl 'elɪmənt]. *Elemento minimal*. Given some partially ordered set (R, \leq) , an element a is called minimal if there is no b such that $b \leq a$, that is, if a is not greater than any element.

Minimum.- ['mɪnɪmən]. *Elemento mínimo*. Given some partially ordered set (R, \leq) , an element a is called minimum if $a \leq b$ for all $b \in R$.

Minimum path.- ['mɪnɪmən pæθ]. *Camino mínimo*. In a weighted graph, a path between two given vertices such that its cost is minimum.

Model.- ['mɔ:dl̩]. *Model (lógica)*. A model is a valuation that satisfies a given formula or set of formulae.

Modulo.- ['mɔ:dʒuləv̩]. *Módulo*. The number used in the definition of a congruence relation between integers.

Multigraph.- [ˌmʌltɪ'græf]. *Multigrafo*. A multigraph is a graph where multiple edges between pair of vertices are allowed. Then, the set of edges is a multiset, that is, the unordered pair of vertices can be repeated. Loops (edges of the kind (v, v)) are not allowed in multigraphs.

12 N

Node.- [nəʊd]. *Nodo*. Node is a synonym for vertex. Node is often used when referring to a vertex of a tree.

Non-plain list.- [nɒn'pleɪn list]. *Lista no plana*. A list is said to be non-plain if it contains other lists as elements. For example, the list $[1, 2, [1, 2]]$ is non-plain because it contains list $[1, 2]$ as its third element.

13 O

One-to-one function.- [ˌwʌntu:wʌn 'fʌŋkʃn]. *Función inyectiva*. See injective function.

Onto function.- [ˌɑ:ntə'fʌŋkʃn]. *Función sobreyectiva*. See surjective function.

Operator.- [ˌɑ:pəreɪtər]. *Operador o conector (lógico)*. See logical connective.

Order relation.- [ˈɔːdə rɪˈleɪʃn]. *Relación de orden.* A binary relation that is reflexive, anti-symmetric and transitive.

Out-degree.- [aʊtdɪˈɡriː]. *Grado de salida.* In a directed graph, the out-degree of a vertex v is the number of edges whose initial vertex is v . It is denoted by $\deg^+(v)$.

14 P, Q

Pascal's triangle.- [ˈpæskl traɪˈæŋɡl]. *Triángulo de Pascal.* An arrangement of binomial coefficients as a triangle. The triangle reflects the recursive properties of the binomial coefficients.

Parent.- [ˈperənt]. *Padre.* In a tree, the parent of a vertex v is the unique vertex incident with v whose terminal vertex is precisely v .

Partial ordering.- [ˈpɑːʃl ˈɔːdəmɪŋ]. *Orden parcial.* An order relation (A, \leq) is said to be partial if there exists a pair of elements $a, b \in A$ such that neither $a \leq b$ nor $b \leq a$. In other words, there are non-comparable elements.

Path.- [pæθ]. *Recorrido.* A path in an undirected graph is a set of vertices $\{v_1, v_2, \dots, v_{k-1}, v_k\}$ such that each pair $v_1v_2, \dots, v_{k-1}v_k$ is an edge. Paths are also called walks; here both terms will be used interchangeably.

r -Permutation.- [ɑːpɜːmjʊˌteɪʃn]. *Permutaciones/variaciones.* An r -permutation is an *ordered* selection of k elements from a set. Let us assume that the set has n elements. If the selection is done without repetitions, the number of r -permutations is $P(n, r) = n(n-1) \cdot \dots \cdot (n-r+1)$. When the selection is done with repetitions, the number of r -permutations is $PR(n, r) = n^r$.

Pigeon principle.- [ˈpɪdʒm ˈprɪnsəpl]. *Principio del palomar.* The pigeon principle states that if $n+1$ objects are placed into n boxes, then there is at least one box containing two objects.

Plain list.- [ˈplem ˈlɪst]. *Lista plana.* A list on a set Σ is said plain if it is only composed of elements of Σ . The set of plain lists is denoted by $\text{LIST}_p(\Sigma)$. For example, the list $[1, 2, 3, 4]$ is a plain list on \mathbb{N} , but $[1, 2, [3, 4]]$ is not, since list $[3, 4] \notin \mathbb{N}$.

Precedence relation.- [ˈpresɪdəns rɪˈleɪʃn]. *Relación de precedencia.* Given a set of tasks $\{T_1, \dots, T_k\}$, we say that T_i precedes T_j if T_i must be completed in order to complete T_j . Such a relation is called a precedence relation.

Preimage.- [prəˈɪmɪdʒ]. *Preimagen o contraimagen de una aplicación.* Given a map $f : A \longrightarrow B$, we say that $a \in A$ is the preimage of $b \in B$ if $f(a) = b$.

Premises.- ['premisɪz]. *Premisas*. Each of the propositional formulae that forms the basis of a deductive argument.

Product rule.- ['prɒdʌkt rʊ:l]. *Regla del producto*. The product rule is a counting technique to count objects that can be expressed as the Cartesian product of two or more sets.

Propositional Logic.- [ˌprɒ:pə'zɪʃnəl lɑːdʒɪk]. *Lógica proposicional*. Propositional Logic is the study of formal reasoning. It is also called Propositional Calculus.

Propositional variable.- [ˌprɒ:pə'zɪʃnəl vəriəbl]. *Variable proposicional*. A propositional variable is a letter that can either be true or false.

Pseudograph.- [suː dʊv'græf]. *Pseudografo*. A pseudograph is a graph where multiple edges between pair of vertices and loops are allowed.

Quotient set.- ['kəʊʃnt set]. *Conjunto cociente*. The quotient set of an equivalence relation R on a set A , usually denoted by A/R , is the set of all equivalence class.

15 R

Radius.- ['reɪdiəs]. *Radio de un grafo*. The radius of a weighted graph is the maximum distance from the center to the vertices.

Range.- ['remdʒ]. *Rango de una aplicación*. See image in the second sense.

Recursion.- [rɪ'kʌ:ʃn]. *Recursión, recursividad*. Recursion is a method of defining an object in terms of itself. It is closely related to inductive proofs. For example, the factorial function $f(n) = n!$ can be defined recursively by defining $0! = 1$ and $n! = n \cdot (n - 1)!$, for $n \geq 1$.

Recursive definition.- [rɪ'kʌ:sɪv 'defɪ'nɪʃn]. *Definición recursiva*. A definition given by means of recursion. Normally, the definition is composed of two pieces: the basis case, where the definition is given explicitly for a few cases; and the recursive case, where the rule to define the object in terms of previous cases is provided.

Recursive function.- [rɪ'kʌ:sɪv 'fʌŋkʃn]. *Función recursiva*. A function that is defined in a recursive way. More technically, a recursive function f is defined by specifying two cases: (1) the explicit values, which form the initial set (see above); (2) and the recursive definition itself, where the rest of the values of f are defined in terms of themselves.

Recursive step.- [rɪ'kʌ:sɪv 'step]. *Paso inductivo*. Another word for inductive step. See inductive proof.

Regular graph.- ['regjələr 'græf]. *Grafo regular*. A graph in which all vertices have the same degree.

Rest of a list.- [restəvə'list]. *Resto de una lista*. The list obtaining by removing the head of a list.

Root.- [ru:t]. *Raíz*. A special vertex designated in a tree from which every edge is directed away from the root.

Rooted tree.- ['ru:tɪd 'tri:]. *Árbol con raíz*. A tree where a vertex has been designated as root.

16 S

Sagittal diagram.- ['sædʒɪtl 'daɪəgræm]. *Diagrama sagital*. The sagittal diagram is a graphical representation of a binary relation. Each element of the set is represented by a vertex and there is an arrow between two elements if they are related through the binary relation.

Satisfiable formula.- ['sætɪsfəəbl 'fɔ:rmjələ]. *Fórmula satisfactible*. A formula is satisfiable if there exists a valuation making the formula true.

Selections.- [sɪ'lekʃn]. *Selecciones*. They can be done with replacement (with repetitions) and without replacement (without repetitions).

Semi-Eulerian graph.- [ˌsəmiɪə'lɪəriən græf]. *Grafo semieuleriano*. A graph that admits an Eulerian path.

Siblings.- ['sɪblɪŋz]. *Hermanos*. In a tree siblings are those node whose parent is common.

Simple circuit.- ['sɪmpl'sɜ:kɪt]. *Ciclo o camino circular*. A closed walk $\{v_1, v_2, \dots, v_k\}$ such that all vertices are distinct except for v_1 and v_n , which are equal, and all edges are distinct.

Simple path.- ['sɪmpl 'pæθ]. *Camino*. A path whose edges are not repeated. The term trail is also used to refer to simple paths. Both terms will be used interchangeably.

Spanning tree.- ['spænnɪ'tri:]. *Árbol generador*. A spanning tree of a graph G is a subgraph of G that is a tree and contains every vertex of G .

Statement.- ['steɪmənt]. *Enunciado*. A meaningful declarative sentence that is either true or false .

String.- ['strɪŋ]. *Cadena*. A finite sequence drawn from some non-empty set (an alphabet). Another way to define string is as an ordered selection with repetitions from a non-empty set.

Structural recursion principle.- ['strʌtʃərəl rɪ'kɜːʃn 'prɪnsəpl]. *Principio de recursión estructural.* It is a procedure that allows to define a function on the set of well-formed logical formulae out a function only defined on propositional variables and logical constants. The principle is recursive in nature.

Structural tree of a list.- ['strʌtʃərəl'triːəvəlɪst]. *Árbol estructural de una lista (recursión).* A tree associated with list showing its internal structure.

Subgraph.- [sʌb'græf]. *Subgrafo.* A subgraph of a graph $G = (V, U)$ is a graph whose vertex set is a subset $A \subset V$, $A \neq \emptyset$, and whose edges are those of G restricted to A .

Subtree.- [sʌb'triː]. *Subárbol.* A subgraph of a tree that is also a tree.

Sum rule.- ['sʌm 'rʊːl]. *Regla de la suma.* The sum rule is a counting technique to count objects that can be expressed as the disjoint union of two or more sets.

Surjective map.- [sɜː'dʒektɪv mæp]. *Aplicación sobreyectiva.* A map $f : A \longrightarrow B$ is injective if for every b in B there exists an a in A such that $f(a) = b$. A surjective map is also called an onto map.

Symmetric relation.- [sɪ'metrɪk rɪ'leɪʃn]. *Relación simétrica.* A binary relation R is said to be symmetric if $(a, b) \in R$ implies that $(b, a) \in R$.

17 T

Tableaux.- [tə'blɔː]. *Tableaux.* The whole name is the method of analytic tableaux. It is an algorithmic procedure to determine the satisfiability of finite sets of formulas. It can be used to prove if a deductive argument is correct.

Tail of a list.- ['teɪəvəlɪst]. *Cabeza de una lista.* The first element of a list.

Tautology.- [tə'tɑːlədʒɪ]. *Tautología.* A set of logic formulae is said to be a tautology if it is true under every valuation.

Terminal vertex.- ['tɜːrmɪnl vɜːrteks]. *Vértice de salida.* In a directed graph, an edge is an ordered pair of vertices (u, v) . Vertex v is called the initial vertex of edge (u, v) .

Topological ordering.- [təpə'lɑːdʒɪkl ɔːrdə(r)]. *Orden topológico.* Constructing a compatible total ordering from a partial ordering is called topological ordering. It is also called topological sorting.

Total order.- ['təʊtl 'ɔːdə(r)]. *Orden total.* An order relation (A, \leq) is said to be total if for any two elements $a, b \in A$ either $a \leq b$ or $b \leq a$. In other words, every two elements in A are comparable.

Trail.- [treil]. *Camino*. See simple path.

Tree.- [tri:]. *Árbol (grafos)*. A acyclic, connected, undirected graph.

Transitive relation.- ['trænsətɪv rɪ'leɪʃn]. *Relación transitiva*. A binary relation R is said to be transitive if $(a, b), (b, c) \in R$ implies that $(a, c) \in R$.

Triangular matrix.- [traɪ'æŋgjələ'metɪks]. *Matriz triangular*. A triangular matrix is a matrix where the entries either above or below the main diagonal are zero.

Truth value.- ['tru:θ vælju:]. *Valor veritativo*. A logical value indicating the relation of a proposition to truth. In propositional logic the two truth values are true and false.

Truth-value assignment.- ['tru:θ vælju: ə'sammənt]. *Valoración (lógica)*. See valuation.

18 U

Unary connective.- ['ju:nəri kə'nektɪv]. *Conectivo u operador unario*. It is a connective affecting only one proposition. There is only one unary connective, the negation $\{\neg\}$.

Undirected graph.- [ʌn'daɪrektɪd græf]. *Grafo no dirigido*. An undirected graph is a graph whose edges are formed by unordered pair of vertices.

Union of sets.- ['ju:nɪən əv 'sets]. *Unión de conjuntos*. Given two sets A, B , the union, to be denoted by $A \cup B$ is the set formed by all the elements in either A or B .

Upper triangular matrix.- [ʌpə traɪ'æŋgjələ'metɪks]. *Matriz triangular superior*. A upper triangular matrix is a matrix where the entries below the main diagonal are zero.

Unsatisfiable formula.- [ʌn'sætɪsfaiəbl 'fɔ:rmjələ]. *Fórmula insatisfactible*. A formula is said to be unsatisfiable if no valuation is a model for the formula.

19 V

Valuation.- [vælju'eɪʃn]. *Valoración (lógica)*. An assignment of truth values to propositional variables.

Validity.- [və'lɪdətɪ]. *Corrección o validez*. See correctness.

Vector.- ['vektə(r)]. *Vector*. A vector is a $1 \times n$ matrix or an $n \times 1$ matrix.

Vertex.- ['vɜ:rtɛks]. *Vértice*. A graph consists of a pair of two sets (V, E) . The elements of the first set V are called vertices. The singular of vertices is vertex.

20 W, X, Y , Z

Walk.- [wɔ:k]. *Recorrido*. See the definition of path.

Weight.- [wert]. *Peso*. A real function defined on the set of edges of a graph.

Weighted graph.- ['weɪtɪd græf]. *Grafo con peso*. A graph whose edges have weights associated with them.

Well-Ordering property.- ['wel 'ɔ:drɪŋ'prɔ:pəti]. *Propiedad del buen orden*. Every non-empty set of natural numbers has a minimum element.

Well-ordered set.- ['wel 'ɔ:dərd set]. *Conjunto con buen orden*. An ordered set where each non-empty set has a minimum element.

Wheel graph.- ['wi:l 'saɪkl græf]. *Grafo rueda*. The graph consisting of a C_n plus an extra vertex to which every other vertex is connected. It is denoted by W_n .