Homework 6
Due date: 2018.12.26

Note for these two problems:
Please don't specify a representation for the γ-matrices. You can directly use the results of the Problem 1 of Homework 3, if needed.

Problem 1. A spinor ψ can be decomposed as

$$\psi = \psi_L + \psi_R \,,$$

where $\psi_L \equiv P_L\psi$, $\psi_R \equiv P_R\psi$, $P_L = (1-\gamma^5)/2$ and $P_R = (1+\gamma^5)/2$.
1) Show that $(P_L)^2 = P_L$, $(P_R)^2 = P_R$ and $P_L P_R = P_R P_L = 0$. [1 point]
2) Show that $\bar{\psi}_L = \bar{\psi} P_R$ and $\bar{\psi}_R = \bar{\psi} P_L$. [1 point]
3) Show that $\gamma^\mu P_L = P_R \gamma^\mu$ and $\gamma^\mu P_R = P_L \gamma^\mu$ [1 point]
4) Write the Lagrangian for the spinor ψ, that is, $\mathcal{L} = \bar{\psi} i\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$, in terms of $\psi_L, \psi_R, \bar{\psi}_L$ and $\bar{\psi}_R$. [2 point]

Problem 2. Prove the Gordon identity [5 points],

$$2m\bar{u}(\vec{p}\,', s')\gamma^\mu u(\vec{p}, s) = \bar{u}(\vec{p}\,', s')\left[(p'+p)^\mu + i\sigma^{\mu\nu}(p'-p)_\nu\right] u(\vec{p}, s)\,,$$

where $m > 0$, $(\not{p} - m)u(\vec{p}, s) = 0$, $(\not{p}' - m)u(\vec{p}\,', s') = 0$, and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \equiv \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$.