See homework 5 solution in page 1 – 4, 6 – 7, and 12 – 13.

The other pages are for the Hamiltonian operator (page 5, 8 – 11), charge operator (page 16 – 20), and the other term of the momentum operator (page 14 – 15). Some of these content are covered in the lectures.

$$\pi(t,\vec{x}) = \frac{\partial \mathcal{L}}{\partial(\partial\phi/\partial t)} = \dot{\phi}^+ = \int_{-\infty}^{+\infty} d^3\vec{p}\, C(E_{\vec{p}})\, iE_{\vec{p}} \left(a_{\vec{p}}^+ e^{ip\cdot x} - b_{\vec{p}} e^{-ip\cdot x} \right)$$

$$\pi^+(t,\vec{x}) = \frac{\partial \mathcal{L}}{\partial(\partial\phi^+/\partial t)} = \dot{\phi} = \int_{-\infty}^{+\infty} d^3\vec{p}\, C(E_{\vec{p}})(-iE_{\vec{p}}) \left(a_{\vec{p}} e^{-ip\cdot x} - b_{\vec{p}}^+ e^{ip\cdot x} \right)$$

$$\frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty} d^3\vec{x}\, \phi(t,\vec{x}) e^{ik\cdot x} = \frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, C(E_{\vec{p}}) [a_{\vec{p}} e^{-ip\cdot x} + b_{\vec{p}}^+ e^{ip\cdot x}] e^{ik\cdot x}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, C(E_{\vec{p}}) \left(a_{\vec{p}} e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) + b_{\vec{p}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) \right)$$

$$= C(E_{\vec{k}}) \left(a_{\vec{k}} + b_{-\vec{k}}^+ e^{2iE_{\vec{k}}t} \right)$$

↑ since $E_{\vec{k}} = E_{-\vec{k}}$

$$\frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty} d^3\vec{x}\, \pi^+(t,x) e^{ik\cdot x} = \frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, C(E_{\vec{p}})(-iE_{\vec{p}}) (a_{\vec{p}} e^{ip\cdot x} - b_{\vec{p}}^+ e^{ip\cdot x}) e^{ik\cdot x}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, C(E_{\vec{p}})(-iE_{\vec{p}}) \left(a_{\vec{p}} e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{k}-\vec{p}) - b_{\vec{p}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) \right)$$

$$= C(E_{\vec{k}})(-iE_{\vec{k}}) \left(a_{\vec{k}} - b_{-\vec{k}}^+ e^{2iE_{\vec{k}}t} \right)$$

$$\Rightarrow \quad a_{\vec{k}} = \left(\frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty} d^3\vec{x}\, \phi(t,\vec{x}) e^{ik\cdot x} + \frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty} d^3\vec{x}\, \pi^+(t,\vec{x}) e^{ik\cdot x} \frac{1}{(-iE_{\vec{k}})} \right) \cdot \frac{1}{C(E_{\vec{k}})} \frac{1}{2}$$

$$= \frac{1}{2}\frac{1}{(2\pi)^3}\frac{1}{C(E_{\vec{k}})} \int_{-\infty}^{+\infty} d^3\vec{x} \left(\phi(t,\vec{x}) - \frac{1}{iE_{\vec{k}}} \pi^+(t,\vec{x}) \right) e^{ik\cdot x}$$

$$b_{-\vec{k}}^+ = \left(\frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty} d^3\vec{x}\, \phi(t,\vec{x}) e^{ik\cdot x} - \frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty} d^3\vec{x}\, \pi^+(t,\vec{x}) e^{ik\cdot x} \frac{1}{(-iE_{\vec{k}})} \right) \frac{1}{C(E_{\vec{k}})} \frac{1}{2} e^{-2iE_{\vec{k}}t}$$

$$= \frac{1}{2}\frac{1}{(2\pi)^3}\frac{1}{C(E_{\vec{k}})} \left(\int_{-\infty}^{+\infty} d^3\vec{x} \left(\phi(t,\vec{x}) + \frac{\pi^+(t,\vec{x})}{iE_{\vec{k}}} \right) e^{-iE_{\vec{k}}t - i\vec{k}\cdot\vec{x}} \right)$$

$$\Rightarrow b_{\vec{k}}^+ = \frac{1}{2}\frac{1}{(2\pi)^3}\frac{1}{C(E_{\vec{k}})} \int_{-\infty}^{+\infty} d^3\vec{x} \left(\phi(t,\vec{x}) + \frac{\pi^+(t,\vec{x})}{iE_{\vec{k}}} \right) e^{-ik\cdot x}$$

1. (checked)

$$\Rightarrow a^{+}_{\vec{k}} = \frac{1}{2}\frac{1}{(2\pi)^3}\frac{1}{C(E_{\vec{k}})}\int_{-\infty}^{+\infty} d^3\vec{x}\left(\phi^{+}(t,\vec{x}) + \frac{\pi(t,\vec{x})}{iE_{\vec{k}}}\right)e^{-ik\cdot x}$$

$$b_{\vec{k}} = \frac{1}{2}\frac{1}{(2\pi)^3}\frac{1}{C(E_{\vec{k}})}\int_{-\infty}^{+\infty} d^3\vec{x}\left(\phi^{+}(t,\vec{x}) - \frac{\pi(t,\vec{x})}{iE_{\vec{k}}}\right)e^{ik\cdot x}$$

$$\Rightarrow [a_{\vec{p}}, a^{+}_{\vec{p}'}] = \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2\frac{1}{C(E_{\vec{p}})}\frac{1}{C(E_{\vec{p}'})}\Big[\int_{-\infty}^{+\infty} d^3\vec{x}\left(\phi(t,\vec{x}) - \frac{1}{iE_{\vec{p}}}\pi^{+}(t,\vec{x})\right)e^{ip\cdot x}$$
$$\int_{-\infty}^{+\infty} d^3\vec{x}'\left(\phi^{+}(t,\vec{x}') + \frac{1}{iE_{\vec{p}'}}\pi(t,\vec{x}')\right)e^{-ip'\cdot x'}\Big]$$

$$= \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2\frac{1}{C(E_{\vec{p}})}\frac{1}{C(E_{\vec{p}'})}\int_{-\infty}^{+\infty} d^3\vec{x}\,d^3\vec{x}'\left(\frac{1}{iE_{\vec{p}'}}i\delta^3(\vec{x}-\vec{x}') + \frac{1}{iE_{\vec{p}}}i\delta^3(\vec{x}-\vec{x}')\right)$$
$$\cdot e^{ip\cdot x - i\vec{p}'\cdot\vec{x}'}$$

$$= \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2\frac{1}{C(E_{\vec{p}})}\frac{1}{C(E_{\vec{p}'})}\int_{-\infty}^{+\infty} d^3\vec{x}\left(\frac{1}{E_{\vec{p}'}} + \frac{1}{E_{\vec{p}}}\right)e^{iE_{\vec{p}}t - iE_{\vec{p}'}t'}$$
$$\cdot e^{-i(\vec{p}-\vec{p}')\cdot\vec{x}}$$

$$= \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2\frac{1}{C(E_{\vec{p}})}\frac{1}{C(E_{\vec{p}'})}\left(\frac{1}{E_{\vec{p}}} + \frac{1}{E_{\vec{p}'}}\right)(2\pi)^3\delta^3(\vec{p}-\vec{p}')e^{iE_{\vec{p}}t - iE_{\vec{p}'}t}$$

$$= \frac{1}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left(\frac{1}{C(E_{\vec{p}})}\right)^2\delta^3(\vec{p}-\vec{p}')$$

$$[b_{\vec{p}}, b^{+}_{\vec{p}'}] = \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2\frac{1}{C(E_{\vec{p}})}\frac{1}{C(E_{\vec{p}'})}\Big[\int_{-\infty}^{+\infty} d^3\vec{x}\left(\phi^{+}(t,\vec{x}) - \frac{1}{iE_{\vec{p}}}\pi(t,\vec{x})\right)e^{ip\cdot x},$$
$$\int_{-\infty}^{+\infty} d^3\vec{x}'\left(\phi(t,\vec{x}') + \frac{1}{iE_{\vec{p}'}}\pi^{+}(t,\vec{x}')\right)e^{-ip'\cdot x'}\Big]$$

$$= \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2\frac{1}{C(E_{\vec{p}})}\frac{1}{C(E_{\vec{p}'})}\int_{-\infty}^{+\infty} d^3\vec{x}\,d^3\vec{x}'\left(\frac{1}{iE_{\vec{p}'}}i\delta^3(\vec{x}-\vec{x}') + \frac{1}{iE_{\vec{p}}}i\delta^3(\vec{x}-\vec{x}')\right)$$
$$e^{ip\cdot x - ip'\cdot x'}$$

$$= \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2\frac{1}{C(E_{\vec{p}})}\frac{1}{C(E_{\vec{p}'})}\int_{-\infty}^{+\infty} d^3\vec{x}\left(\frac{1}{E_{\vec{p}'}} + \frac{1}{E_{\vec{p}}}\right)e^{iE_{\vec{p}}t - iE_{\vec{p}'}t}e^{-i(\vec{p}-\vec{p}')\cdot\vec{x}}$$

$$= \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2\frac{1}{C(E_{\vec{p}})}\frac{1}{C(E_{\vec{p}'})}(2\pi)^3\delta^3(\vec{p}-\vec{p}')\left(\frac{1}{E_{\vec{p}}} + \frac{1}{E_{\vec{p}'}}\right)e^{iE_{\vec{p}}t - iE_{\vec{p}'}t}$$

$$= \frac{1}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left(\frac{1}{C(E_{\vec{p}})}\right)^2\delta^3(\vec{p}-\vec{p}')$$

 (checked)

$$[a_{\vec p}, a_{\vec p'}] = \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2 \frac{1}{C(E_{\vec p})}\frac{1}{C(E_{\vec p'})}\left[\int_{-\infty}^{+\infty} d^3\vec x \left(\phi(t,\vec x) - \frac{1}{iE_{\vec p}}\pi^+(t,\vec x)\right)e^{ip\cdot x},\right.$$

$$\left.\int_{-\infty}^{+\infty} d^3\vec x' \left(\phi(t,\vec x') - \frac{1}{iE_{\vec p'}}\pi^+(t,\vec x')\right)e^{ip'\cdot x'}\right]$$

$$= \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2 \frac{1}{C(E_{\vec p})}\frac{1}{C(E_{\vec p'})}\int_{-\infty}^{+\infty} d^3\vec x\, d^3\vec x' \,(\ 0\ )\, e^{ip\cdot x + ip'\cdot x'}$$

$$= 0$$

$$\Rightarrow [a^+_{\vec p}, a^+_{\vec p'}] = 0$$

$$[b_{\vec p}, b_{\vec p'}] = \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2 \frac{1}{C(E_{\vec p})}\frac{1}{C(E_{p'})}\left[\int_{-\infty}^{+\infty} d^3\vec x \left(\phi^+(t,\vec x) - \frac{1}{iE_{\vec p}}\pi(t,\vec x)\right) e^{ip\cdot x},\right.$$

$$\left.\int_{-\infty}^{+\infty} d^3\vec x' \left(\phi^+(t,\vec x') - \frac{1}{iE_{\vec p'}}\pi(t,\vec x')\right) e^{ip'\cdot x'}\right]$$

$$= 0$$

$$\Rightarrow [\ b^+_{\vec p}, b^+_{\vec p'}] = 0$$

$$[a_{\vec p}, b^+_{\vec p'}] = \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2 \frac{1}{C(E_{\vec p})}\frac{1}{C(E_{\vec p'})}\left[\int_{-\infty}^{+\infty} d^3\vec x \left(\phi(t,\vec x) - \frac{1}{iE_{\vec p}}\pi^+(t,\vec x)\right)e^{ip\cdot x},\right.$$

$$\left.\int_{-\infty}^{+\infty} d^3\vec x' \left(\phi(t,\vec x') + \frac{1}{iE_{\vec p'}}\pi^+(t,\vec x')\right)e^{-ip'\cdot x'}\right]$$

$$= 0$$

$$\Rightarrow [b_{\vec p}, a^+_{\vec p'}] = 0$$

$$[a_{\vec p}, b_{\vec p'}] = \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2 \frac{1}{C(E_{\vec p})}\frac{1}{C(E_{\vec p'})}\left[\int_{-\infty}^{+\infty} d^3\vec x \left(\phi(t,\vec x) - \frac{1}{iE_{\vec p}}\pi^+(t,\vec x)\right)e^{ip\cdot x},\right.$$

$$\left.\int_{-\infty}^{+\infty} d^3\vec x' \left(\phi^+(t,\vec x') - \frac{1}{iE_{\vec p'}}\pi(t,\vec x')\right)e^{ip'\cdot x'}\right]$$

$$= \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2 \frac{1}{C(E_{\vec p})}\frac{1}{C(E_{\vec p'})}\int_{-\infty}^{+\infty} d^3\vec x\, d^3\vec x' \left(-\frac{1}{iE_{\vec p'}} i\delta^3(\vec x - \vec x') + \frac{1}{iE_{\vec p}} i\delta^3(\vec x - \vec x')\right) \cdot e^{ip\cdot x + ip'\cdot x'}$$

$$= \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2 \frac{1}{C(E_{\vec p})}\frac{1}{C(E_{p'})}\int_{-\infty}^{+\infty} d^3\vec x \left(-\frac{1}{E_{\vec p'}} + \frac{1}{E_{\vec p}}\right) e^{+iE_{\vec p}t + iE_{\vec p'}t}\, e^{-i(\vec p + \vec p')\cdot\vec x}$$

$$= \left(\frac{1}{2}\frac{1}{(2\pi)^3}\right)^2 \frac{1}{c(E_{\vec{p}})}\frac{1}{c(E_{\vec{p}'})}(2\pi)^3 \delta^3(\vec{p}+\vec{p}')\left(-\frac{1}{E_{\vec{p}}}+\frac{1}{E_{\vec{p}'}}\right) e^{i(E_{\vec{p}}+E_{\vec{p}'})t}$$

$$= 0$$

since $E_{\vec{p}} = E_{-\vec{p}}$

$$\Rightarrow [a^{+}_{\vec{p}}, b^{+}_{\vec{p}'}] = 0$$

$$H = \int_{-\infty}^{+\infty} d^3\vec{x}\ \mathcal{H}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{x} \left(\dot{\phi}\dot{\phi}^{+} + (\vec{\nabla}\phi)\cdot(\nabla\phi^{+}) + m^2 \phi\phi^{+} \right)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})$$

$$\times \begin{Bmatrix} (-iE_{\vec{p}})(iE_{\vec{k}})\,(a_{\vec{p}}\, e^{-ip\cdot x} - b_{\vec{p}}^{+}\, e^{ip\cdot x})(a_{\vec{k}}^{+}\, e^{ik\cdot x} - b_{\vec{k}}\, e^{-ik\cdot x}) \\ +(i\vec{p})\cdot(-i\vec{k})\,(a_{\vec{p}}\, e^{-ip\cdot x} - b_{\vec{p}}^{+}\, e^{ip\cdot x})(a_{\vec{k}}^{+}\, e^{ik\cdot x} - b_{\vec{k}}\, e^{-ik\cdot x}) \\ +m^2 \quad (a_{\vec{p}}\, e^{-ip\cdot x} + b_{\vec{p}}^{+}\, e^{ip\cdot x})(a_{\vec{k}}^{+}\, e^{ik\cdot x} + b_{\vec{k}}\, e^{-ik\cdot x}) \end{Bmatrix}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ (2\pi)^3\, C(E_{\vec{p}})\, C(E_{\vec{k}})$$

$$\times \begin{Bmatrix} (E_{\vec{p}}E_{\vec{k}} + \vec{p}\cdot\vec{k} + m^2)\left(a_{\vec{p}}a_{\vec{k}}^{+}\, e^{-iE_{\vec{p}}t + iE_{\vec{k}}t}\, \delta^3(\vec{p}-\vec{k}) + b_{\vec{p}}^{+}b_{\vec{k}}\, e^{iE_{\vec{p}}t - iE_{\vec{k}}t}\, \delta^3(\vec{p}-\vec{k}) \right) \\ +(-E_{\vec{p}}E_{\vec{k}} - \vec{p}\cdot\vec{k} + m^2)\left(a_{\vec{p}}b_{\vec{k}}\, e^{-iE_{\vec{p}}t - iE_{\vec{k}}t}\, \delta^3(\vec{p}+\vec{k}) + b_{\vec{p}}^{+}a_{\vec{k}}^{+}\, e^{iE_{\vec{p}}t + iE_{\vec{k}}t}\, \delta^3(\vec{p}+\vec{k}) \right) \end{Bmatrix}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\ (2\pi)^3 (C(E_{\vec{p}}))^2 \Big[(E_{\vec{p}}^2 + \vec{p}^{\,2} + m^2)(a_{\vec{p}}a_{\vec{p}}^{+} + b_{\vec{p}}^{+}b_{\vec{p}}) + (-E_{\vec{p}}^2 + \vec{p}^{\,2} + m^2)(a_{\vec{p}}b_{-\vec{p}}\, e^{-2iE_{\vec{p}}t} + b_{\vec{p}}^{+}a_{-\vec{p}}^{+}\, e^{2iE_{\vec{p}}t}) \Big]$$

↑ Since $E_{\vec{p}} = E_{-\vec{p}}$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, \left[(2\pi)^3 (C(E_{\vec{p}}))^2\, 2E_{\vec{p}} \right] (a_{\vec{p}}a_{\vec{p}}^{+} + b_{\vec{p}}^{+}b_{\vec{p}})\, E_{\vec{p}}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, \left[(2\pi)^3 (C(E_{\vec{p}}))^2\, 2E_{\vec{p}} \right] (a_{\vec{p}}^{+}a_{\vec{p}} + b_{\vec{p}}^{+}b_{\vec{p}})\, E_{\vec{p}}$$
$$+ \int_{-\infty}^{+\infty} d^3\vec{p}\, \left[(2\pi)^3 (C(E_{\vec{p}}))^2\, 2E_{\vec{p}} \right] \frac{1}{(2\pi)^3}\, \frac{1}{2E_{\vec{p}}} \left(\frac{1}{C(E_{\vec{p}})} \right)^2 \delta^3(0)\, E_{\vec{p}}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, \left[(2\pi)^3 (C(E_{\vec{p}}))^2\, 2E_{\vec{p}} \right] (a_{\vec{p}}^{+}a_{\vec{p}} + b_{\vec{p}}^{+}b_{\vec{p}})\, E_{\vec{p}}$$
$$+ \left(\int_{-\infty}^{+\infty} \frac{d^3\vec{p}}{(2\pi)^3}\, E_{\vec{p}} \right) \delta^3(0)\, (2\pi)^3.$$

$$:H: = \int_{-\infty}^{+\infty} d^3\vec{p}\, \left[(2\pi)^3 (C(E_{\vec{p}})^2)\, 2E_{\vec{p}} \right] (a_{\vec{p}}^{+}a_{\vec{p}} + b_{\vec{p}}^{+}b_{\vec{p}})\, E_{\vec{p}}$$

5. (checked)

$$\hat{\vec{P}} = -\int d^3\vec{x}\,(\pi \vec{\nabla}\phi + \pi^+ \vec{\nabla}\phi^+)$$

$$= -\int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\; C(E_{\vec{p}})\,C(E_{\vec{k}})$$

$$\times \left\{ \begin{aligned} &(iE_{\vec{p}})(+i\vec{k})\,(a_{\vec{p}}^+ e^{ip\cdot x} - b_{\vec{p}}\, e^{-ip\cdot x})(a_{\vec{k}}\, e^{-ik\cdot x} - b_{\vec{k}}^+ e^{ik\cdot x}) \\ &+(-iE_{\vec{p}})(-i\vec{k})\,(a_{\vec{p}}\, e^{-ip\cdot x} - b_{\vec{p}}^+ e^{ip\cdot x})(a_{\vec{k}}^+ e^{ik\cdot x} - b_{\vec{k}}\, e^{-ik\cdot x}) \end{aligned} \right\}$$

$$= -\int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\; C(E_{\vec{p}})\,C(E_{\vec{k}})\;(-E_{\vec{p}}\vec{k})\,(2\pi)^3$$

$$\times \Big[\, a_{\vec{p}}^+ a_{\vec{k}}\, e^{iE_{\vec{p}}t - iE_{\vec{k}}t}\,\delta^3(\vec{p}-\vec{k}) - a_{\vec{p}}^+ b_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t}\,\delta^3(\vec{p}+\vec{k})$$

$$- b_{\vec{p}}\, a_{\vec{k}}\, e^{-iE_{\vec{p}}t - iE_{\vec{k}}t}\,\delta^3(\vec{p}+\vec{k}) + b_{\vec{p}}\, b_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t}\,\delta^3(\vec{p}-\vec{k})$$

$$+ a_{\vec{p}}\, a_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t}\,\delta^3(\vec{p}-\vec{k}) - a_{\vec{p}}\, b_{\vec{k}}\, e^{-iE_{\vec{p}}t - iE_{\vec{k}}t}\,\delta^3(\vec{p}+\vec{k})$$

$$- b_{\vec{p}}^+ a_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t}\,\delta^3(\vec{p}+\vec{k}) + b_{\vec{p}}^+ b_{\vec{k}}\, e^{iE_{\vec{p}}t - iE_{\vec{k}}t}\,\delta^3(\vec{p}-\vec{k})\,\Big]$$

$$= -\int_{-\infty}^{+\infty} d^3\vec{p}\,(C(E_{\vec{p}}))^2\,(-E_{\vec{p}})\,(2\pi)^3$$

$$\times \Big[\, \vec{p}\,(a_{\vec{p}}^+ a_{\vec{p}} + b_{\vec{p}}\, b_{\vec{p}}^+ + a_{\vec{p}}\, a_{\vec{p}}^+ + b_{\vec{p}}^+ b_{\vec{p}})$$

$$= ** \qquad - \vec{p}\,(-a_{\vec{p}}^+ b_{-\vec{p}}^+ e^{2iE_{\vec{p}}t} - b_{\vec{p}}\, a_{-\vec{p}}\, e^{-2iE_{\vec{p}}t} - a_{\vec{p}}\, b_{-\vec{p}}\, e^{-2iE_{\vec{p}}t} - b_{\vec{p}}^+ a_{-\vec{p}}^+ e^{2iE_{\vec{p}}t})\,\Big]$$

In **, for the 2nd line, use $a_{\vec{p}}\, a_{\vec{p}}^+ = a_{\vec{p}}^+ a_{\vec{p}} + \frac{1}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left[\frac{1}{C(E_{\vec{p}})}\right]^2 \delta^3(0)$

and $b_{\vec{p}}\, b_{\vec{p}}^+ = b_{\vec{p}}^+ b_{\vec{p}} + \frac{1}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left[\frac{1}{C(E_{\vec{p}})}\right]^2 \delta^3(0)$

while the $\delta^3(0)$ terms do not contribute since $\int_{-\infty}^{+\infty} d^3\vec{p}\cdot\vec{p}\times$ eigenfunction of $\vec{p}$ $=0$;

for the 3rd line, if we make $\vec{p} \to -\vec{k}$, then

$$-\int_{-\infty}^{+\infty} d^3\vec{p}\,(C(E_{\vec{p}}))^2\,(-E_{\vec{p}})\,(2\pi)^3\Big[-\vec{p}\,(-a_{\vec{p}}^+ b_{-\vec{p}}^+ e^{2iE_{\vec{p}}t} - b_{\vec{p}}\, a_{-\vec{p}}\, e^{-2iE_{\vec{p}}t} - a_{\vec{p}}\, b_{-\vec{p}}\, e^{-2iE_{\vec{p}}t} - b_{\vec{p}}^+ a_{-\vec{p}}^+ e^{2iE_{\vec{p}}t})\Big]$$

since $E_{\vec{k}} = E_{-\vec{k}}$

$$= +\int_{-\infty}^{+\infty} d^3(-\vec{k})\,(C(E_{\vec{k}}))^2\,(-E_{\vec{k}})\,(2\pi)^3\Big[\vec{k}\,(-a_{-\vec{k}}^+ b_{\vec{k}}^+ e^{2iE_{\vec{k}}t} - b_{-\vec{k}}\, a_{\vec{k}}\, e^{-2iE_{\vec{k}}t}$$

$$- a_{-\vec{k}} b_{\vec{k}} e^{-2iE_{\vec{k}}t} - b^{+}_{-\vec{k}} a^{+}_{\vec{k}} e^{2iE_{\vec{k}}t})]$$

$$\overset{k\to p}{=} \int_{-\infty}^{+\infty} d^3\vec{p} \left(C(E_{\vec{p}}) \right)^2 (-E_{\vec{p}}) (2\pi)^3 \Big[-\vec{p} \Big(-a^{+}_{-\vec{p}} b^{+}_{\vec{p}} e^{2iE_{\vec{p}}t} - b_{-\vec{p}} a_{\vec{p}} e^{-2iE_{\vec{p}}t} - a_{-\vec{p}} b_{\vec{p}} e^{-2iE_{\vec{p}}t} - b^{+}_{-\vec{p}} a^{+}_{\vec{p}} e^{2iE_{\vec{p}}t} \Big) \Big]$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p} \left(C(E_{\vec{p}}) \right)^2 (-E_{\vec{p}}) (2\pi)^3 \Big[-\vec{p} \Big(\underbrace{-a^{+}_{\vec{p}} b^{+}_{-\vec{p}} e^{2iE_{\vec{p}}t}}_{\substack{\text{4th term, and} \\ \text{use } [a^{+}_{\vec{p}},\, b^{+}_{\vec{p}'}]=0}} \underbrace{- b_{\vec{p}} a_{-\vec{p}} e^{-2iE_{\vec{p}}t}}_{\substack{\text{3rd term, and} \\ \text{use } [a_{\vec{p}},\, b_{\vec{p}'}]=0}} \underbrace{- a_{\vec{p}} b_{-\vec{p}} e^{-2iE_{\vec{p}}t}}_{\substack{\text{2nd term, and use} \\ [a_{\vec{p}},\, b_{\vec{p}'}]=0}} \underbrace{- b^{+}_{\vec{p}} a^{+}_{-\vec{p}} e^{2iE_{\vec{p}}t}}_{\substack{\text{1st term, and use} \\ [a^{+}_{\vec{p}},\, b^{+}_{\vec{p}'}]=0}} \Big) \Big]$$

$\Rightarrow$ the 3rd line of $**$ does not contribute

$$\Rightarrow \hat{\vec{P}} = - \int_{-\infty}^{+\infty} d^3\vec{p} \left(C(E_{\vec{p}}) \right)^2 (-E_{\vec{p}}) (2\pi)^3 \vec{p} \left(2a^{+}_{\vec{p}} a_{\vec{p}} + 2b^{+}_{\vec{p}} b_{\vec{p}} \right)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p} \left[(2\pi)^3 \left(C(E_{\vec{p}}) \right)^2 2E_{\vec{p}} \right] \left(a^{+}_{\vec{p}} a_{\vec{p}} + b^{+}_{\vec{p}} b_{\vec{p}} \right) \vec{p}$$

 (checked)

Let's look at the terms in H.

If write the first term as $\int_{-\infty}^{+\infty} d^3\vec{x}\ \dot{\phi}^+\dot{\phi}$, then

$$\int_{-\infty}^{+\infty} d^3\vec{x}\ \dot{\phi}^+\dot{\phi}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})$$

$$\times (iE_{\vec{p}})(-iE_{\vec{k}}) \times \left\{ (a_{\vec{p}}^+ e^{ip\cdot x} - b_{\vec{p}} e^{-ip\cdot x})(a_{\vec{k}} e^{-ik\cdot x} - b_{\vec{k}}^+ e^{ik\cdot x}) \right\}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})\, (iE_{\vec{p}})(-iE_{\vec{k}}).\,(2\pi)^3$$

$$\times \left[a_{\vec{p}}^+ a_{\vec{k}} e^{iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) + b_{\vec{p}} b_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) \right.$$

$$\left. - b_{\vec{p}} a_{\vec{k}} e^{-iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) - a_{\vec{p}}^+ b_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) \right]$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 E_{\vec{p}}^2 (2\pi)^3 \left[(a_{\vec{p}}^+ a_{\vec{p}} + b_{\vec{p}} b_{\vec{p}}^+) - (b_{\vec{p}} a_{-\vec{p}} e^{-2iE_{\vec{p}}t} + a_{\vec{p}}^+ b_{-\vec{p}}^+ e^{2iE_{\vec{p}}t}) \right]$$

while if write it as $\int_{-\infty}^{+\infty} d^3\vec{x}\ \dot{\phi}\dot{\phi}^+$, then

$$\int_{-\infty}^{+\infty} d^3\vec{x}\ \dot{\phi}\dot{\phi}^+$$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})$$

$$\times (-iE_{\vec{p}})(iE_{\vec{k}}) \times \left\{ (a_{\vec{p}} e^{-ip\cdot x} - b_{\vec{p}}^+ e^{ip\cdot x})(a_{\vec{k}}^+ e^{ik\cdot x} - b_{\vec{k}} e^{-ik\cdot x}) \right\}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})\, (-iE_{\vec{p}})(iE_{\vec{k}})\,(2\pi)^3$$

$$\times \left[a_{\vec{p}} a_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) + b_{\vec{p}}^+ b_{\vec{k}} e^{iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) \right.$$

$$\left. - b_{\vec{p}}^+ a_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) - a_{\vec{p}} b_{\vec{k}} e^{-iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) \right]$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 E_{\vec{p}}^2 (2\pi)^3 \left[(a_{\vec{p}} a_{\vec{p}}^+ + b_{\vec{p}}^+ b_{\vec{p}}) - (b_{\vec{p}}^+ a_{-\vec{p}}^+ e^{2iE_{\vec{p}}t} + a_{\vec{p}} b_{-\vec{p}} e^{-2iE_{\vec{p}}t}) \right]$$

 (checked)

Since $a_{\vec{p}} a^{+}_{\vec{p}'} + b^{+}_{\vec{p}'} b_{\vec{p}}$

$$= a^{+}_{\vec{p}'} a_{\vec{p}} + \frac{1}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \left(\frac{1}{C(E_{\vec{p}})}\right)^2 \delta^3(\vec{p}-\vec{p}\,')$$

$$+ b_{\vec{p}} b^{+}_{\vec{p}'} - \frac{1}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} \left(\frac{1}{C(E_{\vec{p}})}\right)^2 \delta^3(\vec{p}-\vec{p}\,')$$

$$= a^{+}_{\vec{p}'} a_{\vec{p}} + b_{\vec{p}} b^{+}_{\vec{p}'}$$

then $a_{\vec{p}} a^{+}_{\vec{p}} + b^{+}_{\vec{p}} b_{\vec{p}} = a^{+}_{\vec{p}} a_{\vec{p}} + b_{\vec{p}} b^{+}_{\vec{p}}$

Also, $\int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 E_{\vec{p}}^2 (2\pi)^3 \cdot b_{\vec{p}} a_{-\vec{p}} e^{-2iE_{\vec{p}}t}$

$$\overset{\vec{p}\to-\vec{k}}{=} \int_{-\infty}^{+\infty} d^3\vec{k}\, (C(E_{\vec{k}}))^2 E_{-\vec{k}}^2 (2\pi)^3 b_{-\vec{k}} a_{\vec{k}} e^{-2iE_{-\vec{k}}t}$$

$$\overset{\vec{k}\to\vec{p}}{=} \int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 E_{\vec{p}}^2 (2\pi)^3 b_{-\vec{p}} a_{\vec{p}} e^{-2iE_{\vec{p}}t}$$

$\uparrow$ also use $E_{\vec{p}} = E_{-\vec{p}}$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 E_{\vec{p}}^2 (2\pi)^3 a_{\vec{p}} b_{-\vec{p}} e^{-2iE_{\vec{p}}t}$$

$$\int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 E_{\vec{p}}^2 (2\pi)^3 a^{+}_{\vec{p}} b^{+}_{-\vec{p}} e^{2iE_{\vec{p}}t}$$

$$\overset{\vec{p}\to\vec{k}}{=} \int_{-\infty}^{+\infty} d^3\vec{k}\, (C(E_{-\vec{k}}))^2 E_{-\vec{k}}^2 (2\pi)^3 a^{+}_{-\vec{k}} b^{+}_{\vec{k}} e^{2iE_{-\vec{k}}t}$$

$$\overset{\vec{k}\to\vec{p}}{=} \int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 E_{\vec{p}}^2 (2\pi)^3 a^{+}_{-\vec{p}} b^{+}_{\vec{p}} e^{2iE_{\vec{p}}t}$$

$\uparrow$ also use $E_{\vec{p}} = E_{-\vec{p}}$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 E_{\vec{p}}^2 (2\pi)^3 b^{+}_{\vec{p}} a^{+}_{-\vec{p}} e^{2iE_{\vec{p}}t}$$

In fact, for any even function of $\vec{p}$, denoted as $f(|\vec{p}|)$, we have

$$\int_{-\infty}^{+\infty} d^3\vec{p}\, f(|\vec{p}|)\, b_{\vec{p}} a_{-\vec{p}} = \int_{-\infty}^{+\infty} d^3\vec{p}\, f(|\vec{p}|)\, a_{\vec{p}} b_{-\vec{p}}$$

and $\int_{-\infty}^{+\infty} d^3\vec{p}\, f(|\vec{p}|)\, a^{+}_{\vec{p}} b^{+}_{-\vec{p}} = \int_{-\infty}^{+\infty} d^3\vec{p}\, f(|\vec{p}|)\, b^{+}_{\vec{p}} a^{+}_{-\vec{p}}$

Therefore $\displaystyle\int_{-\infty}^{+\infty} d^3\vec{x}\ \dot{\phi}^{+}\dot{\phi} = \int_{-\infty}^{+\infty} \dot{\phi}\,\dot{\phi}^{+}$

For the second term of H, if we write it as $\displaystyle\int_{-\infty}^{+\infty} d^3\vec{x}\,(\vec{\nabla}\phi)\cdot(\vec{\nabla}\phi^{+})$,

then $\displaystyle\int_{-\infty}^{+\infty} d^3\vec{x}\,(\vec{\nabla}\phi)\cdot(\vec{\nabla}\phi^{+})$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\,C(E_{\vec{k}})\,(i\vec{P})\cdot(-i\vec{k})$$
$$\times\left(a_{\vec{p}}\, e^{-ip\cdot x} - b_{\vec{p}}^{+} e^{ip\cdot x}\right)\left(a_{\vec{k}}^{+} e^{ik\cdot x} - b_{\vec{k}}\, e^{-ik\cdot x}\right)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\,C(E_{\vec{k}})\ \vec{p}\cdot\vec{k}\ (2\pi)^3$$
$$\times\Big(a_{\vec{p}} a_{\vec{k}}^{+} e^{-iE_{\vec{p}}t + iE_{\vec{k}}t}\,\delta^3(\vec{p}-\vec{k}) + b_{\vec{p}}^{+} b_{\vec{k}}\, e^{iE_{\vec{p}}t - iE_{\vec{k}}t}\,\delta^3(\vec{p}-\vec{k})$$
$$- a_{\vec{p}} b_{\vec{k}}\, e^{-iE_{\vec{p}}t - iE_{\vec{k}}t}\,\delta^3(\vec{p}+\vec{k}) - b_{\vec{p}}^{+} a_{\vec{k}}^{+} e^{iE_{\vec{p}}t + iE_{\vec{k}}t}\,\delta^3(\vec{p}+\vec{k})\Big)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\ \left(C(E_{\vec{p}})\right)^2 |\vec{p}|^2 (2\pi)^3$$
$$\times\Big[\left(a_{\vec{p}} a_{\vec{p}}^{+} + b_{\vec{p}}^{+} b_{\vec{p}}\right) + \left(a_{\vec{p}}\, b_{-\vec{p}}\, e^{-2iE_{\vec{p}}t} + b_{\vec{p}}^{+} a_{-\vec{p}}^{+} e^{2iE_{\vec{p}}t}\right)\Big]$$

while $\displaystyle\int_{-\infty}^{+\infty} d^3\vec{x}\,(\vec{\nabla}\phi^{+})\cdot(\vec{\nabla}\phi)$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\,C(E_{\vec{k}})\,(-i\vec{P})\cdot(i\vec{k})$$
$$\times\left(a_{\vec{p}}^{+} e^{ip\cdot x} - b_{\vec{p}}\, e^{-ip\cdot x}\right)\left(a_{\vec{k}}\, e^{-ik\cdot x} - b_{\vec{k}}^{+} e^{ik\cdot x}\right)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\,C(E_{\vec{k}})\ \vec{p}\cdot\vec{k}\ (2\pi)^3$$
$$\times\Big[a_{\vec{p}}^{+} a_{\vec{k}}\, e^{iE_{\vec{p}}t - iE_{\vec{k}}t}\,\delta^3(\vec{p}-\vec{k}) + b_{\vec{p}} b_{\vec{k}}^{+} e^{-iE_{\vec{p}}t + iE_{\vec{k}}t}\,\delta^3(\vec{p}-\vec{k})$$
$$- a_{\vec{p}}^{+} b_{\vec{k}}^{+} e^{iE_{\vec{p}}t + iE_{\vec{k}}t}\,\delta^3(\vec{p}+\vec{k}) - b_{\vec{p}} a_{\vec{k}}\, e^{-iE_{\vec{p}}t - iE_{\vec{k}}t}\,\delta^3(\vec{p}+\vec{k})\Big]$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\left(C(E_{\vec{p}})\right)^2 |\vec{p}|^2 (2\pi)^3$$
$$\times\Big[\left(a_{\vec{p}}^{+} a_{\vec{p}} + b_{\vec{p}} b_{\vec{p}}^{+}\right) + \left(a_{\vec{p}}^{+} b_{-\vec{p}}^{+} e^{2iE_{\vec{p}}t} + b_{\vec{p}}\, a_{-\vec{p}}\, e^{-2iE_{\vec{p}}t}\right)\Big]$$

10 (check 1)

Therefore $\int_{-\infty}^{+\infty} d^3\vec{x} (\vec{\nabla}\phi)\cdot(\vec{\nabla}\phi^+) = \int_{-\infty}^{+\infty} d^3\vec{x} (\vec{\nabla}\phi^+)\cdot(\vec{\nabla}\phi)$

For the third term of H, put the factor m^2 aside,

$$\int_{-\infty}^{+\infty} d^3\vec{x}\ \phi\phi^+$$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})C(E_{\vec{k}}) (a_{\vec{p}} e^{-ip\cdot x} + b_{\vec{p}}^+ e^{ip\cdot x})(a_{\vec{k}}^+ e^{ik\cdot x} + b_{\vec{k}} e^{-ik\cdot x})$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})C(E_{\vec{k}}) (2\pi)^3$$

$$\times \Big(a_{\vec{p}} a_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) + b_{\vec{p}}^+ b_{\vec{k}} e^{iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k})$$

$$+ a_{\vec{p}} b_{\vec{k}} e^{-iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) + b_{\vec{p}}^+ a_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k})\Big)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p} (C(E_{\vec{p}}))^2 (2\pi)^3 \Big[(a_{\vec{p}} a_{\vec{p}}^+ + b_{\vec{p}}^+ b_{\vec{p}}) + (a_{\vec{p}} b_{-\vec{p}} e^{-2iE_{\vec{p}}t} + b_{\vec{p}}^+ a_{-\vec{p}}^+ e^{2iE_{\vec{p}}t})\Big]$$

while $\int_{-\infty}^{+\infty} d^3\vec{x}\ \phi^+\phi$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})C(E_{\vec{k}}) (a_{\vec{p}}^+ e^{ip\cdot x} + b_{\vec{p}} e^{-ip\cdot x})(a_{\vec{k}} e^{-ik\cdot x} + b_{\vec{k}}^+ e^{ik\cdot x})$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})C(E_{\vec{k}}) (2\pi)^3$$

$$\times \Big(a_{\vec{p}}^+ a_{\vec{k}} e^{iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) + b_{\vec{p}} b_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k})$$

$$+ a_{\vec{p}}^+ b_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) + b_{\vec{p}} a_{\vec{k}} e^{-iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k})\Big)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p} (C(E_{\vec{p}}))^2 (2\pi)^3 \Big[(a_{\vec{p}}^+ a_{\vec{p}} + b_{\vec{p}} b_{\vec{p}}^+) + (a_{\vec{p}}^+ b_{-\vec{p}}^+ e^{2iE_{\vec{p}}t} + b_{\vec{p}} a_{-\vec{p}} e^{-2iE_{\vec{p}}t})\Big]$$

Therefore, $\int_{-\infty}^{+\infty} d^3\vec{x}\ \phi\phi^+ = \int_{-\infty}^{+\infty} d^3\vec{x}\ \phi^+\phi$

So, the order of the field operators in writting H is irrelevent.

11. (checked)

For the two terms in $\vec{P}$:

$$\int_{-\infty}^{+\infty} d^3\vec{x}\ \pi \vec{\nabla}\phi = \int_{-\infty}^{+\infty} \dot{\phi}^{+} \vec{\nabla}\phi$$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})\, (iE_{\vec{p}})\, (i\vec{k})$$

$$\times (a_{\vec{p}}^{+} e^{ip\cdot x} - b_{\vec{p}} e^{-ip\cdot x})(a_{\vec{k}} e^{-ik\cdot x} - b_{\vec{k}}^{+} e^{ik\cdot x})$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})(iE_{\vec{p}})(i\vec{k})\,(2\pi)^3$$

$$\times \Big(a_{\vec{p}}^{+} a_{\vec{k}} e^{iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) + b_{\vec{p}} b_{\vec{k}}^{+} e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k})$$

$$- b_{\vec{p}} a_{\vec{k}} e^{-iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) - a_{\vec{p}}^{+} b_{\vec{k}}^{+} e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) \Big)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\ (C(E_{\vec{p}}))^2 (-E_{\vec{p}})\,(2\pi)^3\, \vec{p}$$

$$\times \Big[(a_{\vec{p}}^{+} a_{\vec{p}} + b_{\vec{p}} b_{\vec{p}}^{+}) + (b_{\vec{p}} a_{-\vec{p}} e^{-2iE_{\vec{p}}t} + a_{\vec{p}}^{+} b_{-\vec{p}}^{+} e^{2iE_{\vec{p}}t}) \Big]$$

For an odd function of $\vec{p}$, call it $g(\vec{p})$,

we have $\displaystyle\int_{-\infty}^{+\infty} d^3\vec{p}\, g(\vec{p})\, b_{\vec{p}}\, a_{-\vec{p}} \overset{\vec{p}\to\vec{k}}{=} \int_{-\infty}^{+\infty} d^3\vec{k}\ g(-\vec{k})\, b_{-\vec{k}}\, a_{\vec{k}} = -\int_{-\infty}^{+\infty} d^3\vec{k}\, g(\vec{k}) b_{-\vec{k}} a_{\vec{k}}$

$$= -\int_{-\infty}^{+\infty} d^3\vec{p}\, g(\vec{p})\, b_{-\vec{p}}\, a_{\vec{p}} = -\int_{-\infty}^{+\infty} d^3\vec{p}\ g(\vec{p})\, a_{\vec{p}}\, b_{-\vec{p}}.$$

It is clear that $((C(E_{\vec{p}}))^2(-E_{\vec{p}})(2\pi)^3\, \vec{p}\, e^{-2iE_{\vec{p}}t}$ is an odd function of $\vec{p}$.

Also, for an odd function of $\vec{p}$, call it $h(\vec{p})$,

we have $\displaystyle\int_{-\infty}^{+\infty} d^3\vec{p}\ h(\vec{p})\, a_{\vec{p}}^{+} b_{-\vec{p}}^{+} \overset{\vec{p}\to\vec{k}}{=} \int_{-\infty}^{+\infty} d^3\vec{k}\ h(-\vec{k})\, a_{-\vec{k}}^{+}\, b_{\vec{k}}^{+} = -\int_{-\infty}^{+\infty} d^3\vec{k}\ h(\vec{k}) a_{-\vec{k}}^{+} b_{\vec{k}}^{+}$

$$= -\int_{-\infty}^{+\infty} d^3\vec{p}\, h(\vec{p})\, b_{\vec{p}}^{+}\, a_{-\vec{p}}^{+}.$$

and it is clear that $(C(E_{\vec{p}}))^2(-E_{\vec{p}})(2\pi)^3\, \vec{p}\, e^{2iE_{\vec{p}}t}$ is an odd function of $\vec{p}$.

$$\int_{-\infty}^{+\infty} (\vec{\nabla}\phi)\pi = \int_{-\infty}^{+\infty} (\vec{\nabla}\phi)\, \dot{\phi}^{+}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\; C(E_{\vec{p}})\, C(E_{\vec{k}})\, (i\vec{p})(iE_{\vec{k}})$$

$$\times \left(a_{\vec{p}} e^{-ip\cdot x} - b_{\vec{p}}^{+} e^{ip\cdot x}\right)\left(a_{\vec{k}}^{+} e^{ik\cdot x} - b_{\vec{k}} e^{-ik\cdot x}\right)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\; C(E_{\vec{p}})\, C(E_{\vec{k}})\, (i\vec{p})(iE_{\vec{k}})\, (2\pi)^3$$

$$\times \Big(a_{\vec{p}} a_{\vec{k}}^{+} e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) + b_{\vec{p}}^{+} b_{\vec{k}} e^{iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k})$$

$$- b_{\vec{p}}^{+} a_{\vec{k}}^{+} e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) - a_{\vec{p}} b_{\vec{k}} e^{-iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) \Big)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 (-E_{\vec{p}})(2\pi)^3 \cdot \vec{p}\,.$$

$$\times \left[\left(a_{\vec{p}} a_{\vec{p}}^{+} + b_{\vec{p}}^{+} b_{\vec{p}} \right) - \left(b_{\vec{p}}^{+} a_{-\vec{p}}^{+} e^{2iE_{\vec{p}}t} + a_{\vec{p}} b_{-\vec{p}} e^{-2iE_{\vec{p}}t} \right) \right]$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 (-E_{\vec{p}})(2\pi)^3\, \vec{p}$$

$$\times \left[\left(a_{\vec{p}}^{+} a_{\vec{p}} + b_{\vec{p}} b_{\vec{p}}^{+} \right) + \left(a_{\vec{p}}^{+} b_{-\vec{p}}^{+} e^{2iE_{\vec{p}}t} + b_{\vec{p}} a_{-\vec{p}} e^{-2iE_{\vec{p}}t} \right) \right]$$

$$= \int_{-\infty}^{+\infty} \pi\, \vec{\nabla}\phi$$

For $\int_{-\infty}^{+\infty} d^3\vec{x}\, \pi^+ \vec{\nabla} \phi^+$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, \dot{\phi}\, \vec{\nabla} \phi^+$$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\; C(E_{\vec{p}})\, C(E_{\vec{k}})\; (-iE_{\vec{p}})\, (-i\vec{k})$$

$$\times \left(a_{\vec{p}} e^{-ip\cdot x} - b_{\vec{p}}^+ e^{ip\cdot x} \right) \left(a_{\vec{k}}^+ e^{ik\cdot x} - b_{\vec{k}} e^{-ik\cdot x} \right)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\; C(E_{\vec{p}})\, C(E_{\vec{k}})\, (-E_{\vec{p}})\, \vec{k}\, (2\pi)^3$$

$$\times \Big(a_{\vec{p}} a_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) + b_{\vec{p}}^+ b_{\vec{k}}\, e^{iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k})$$

$$- b_{\vec{p}}^+ a_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) - a_{\vec{p}} b_{\vec{k}}\, e^{-iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) \Big)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, \left(C(E_{\vec{p}})\right)^2 (-E_{\vec{p}})\, (2\pi)^3\, \vec{p}$$

$$\times \left[\left(a_{\vec{p}} a_{\vec{p}}^+ + b_{\vec{p}}^+ b_{\vec{p}} \right) + \left(b_{\vec{p}}^+ a_{-\vec{p}}^+ e^{2iE_{\vec{p}}t} + a_{\vec{p}} b_{-\vec{p}} e^{-2iE_{\vec{p}}t} \right) \right]$$

while $\int_{-\infty}^{+\infty} d^3\vec{x}\, (\vec{\nabla} \phi^+)\, \pi^+$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, (\vec{\nabla} \phi^+)\, \dot{\phi}$$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\; C(E_{\vec{p}})\, C(E_{\vec{k}})\, (-i\vec{p})\, (-iE_{\vec{k}})$$

$$\times \left(a_{\vec{p}}^+ e^{ip\cdot x} - b_{\vec{p}} e^{-ip\cdot x} \right) \left(a_{\vec{k}} e^{-ik\cdot x} - b_{\vec{k}}^+ e^{ik\cdot x} \right)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\; C(E_{\vec{p}})\, C(E_{\vec{k}})\, (-E_{\vec{p}})\, \vec{p}\, (2\pi)^3$$

$$\times \Big(a_{\vec{p}}^+ a_{\vec{k}}\, e^{iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) + b_{\vec{p}} b_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k})$$

$$- a_{\vec{p}}^+ b_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) - b_{\vec{p}} a_{\vec{k}}\, e^{-iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) \Big)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, \left(C(E_{\vec{p}})\right)^2 (-E_{\vec{p}})\, (2\pi)^3\, \vec{p}$$

$$\times \left[\left(a_{\vec{p}}^+ a_{\vec{p}} + b_{\vec{p}} b_{\vec{p}}^+ \right) - \left(a_{\vec{p}}^+ b_{-\vec{p}}^+ e^{2iE_{\vec{p}}t} + b_{\vec{p}} a_{-\vec{p}} e^{-2iE_{\vec{p}}t} \right) \right]$$

 (checked)

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\ \left(C(E_{\vec{p}})\right)^2 (-E_{\vec{p}})\ (2\pi)^3\ \vec{p}$$

$$\times \left[(a_{\vec{p}} a_{\vec{p}}^{+} + b_{\vec{p}}^{+} b_{\vec{p}}) + (b_{\vec{p}}^{+} a_{-\vec{p}}^{+} e^{2iE_{\vec{p}}t} + a_{\vec{p}} b_{-\vec{p}} e^{-2iE_{\vec{p}}t} \right)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\ \pi^{+} \vec{\nabla} \phi^{+}$$

Therefore, the order of the field operators in writting $\hat{\vec{P}}$ is irrelevent.

For $\hat{Q} = i\int_{-\infty}^{+\infty} d^3\vec{x}\ (\dot{\phi}\phi^+ - \dot{\phi}^+\phi)$.

we have $\int_{-\infty}^{+\infty} d^3\vec{x}\ \dot{\phi}\phi^+$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})\, (-iE_{\vec{p}})$$

$$\times (a_{\vec{p}}\, e^{-ip\cdot x} - b_{\vec{p}}^+ e^{ip\cdot x})(a_{\vec{k}}^+ e^{ik\cdot x} + b_{\vec{k}} e^{-ik\cdot x})$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})\, (-iE_{\vec{p}})\, (2\pi)^3$$

$$\times \Big(a_{\vec{p}} a_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t}\, \delta^3(\vec{p}-\vec{k}) - b_{\vec{p}}^+ b_{\vec{k}}\, e^{iE_{\vec{p}}t - iE_{\vec{k}}t}\, \delta^3(\vec{p}-\vec{k})$$

$$- b_{\vec{p}}^+ a_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t}\, \delta^3(\vec{p}+\vec{k}) + a_{\vec{p}} b_{\vec{k}}\, e^{-iE_{\vec{p}}t - iE_{\vec{k}}t}\, \delta^3(\vec{k}+\vec{p}) \Big)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 (-iE_{\vec{p}})\, (2\pi)^3$$

$$\times \Big[(a_{\vec{p}} a_{\vec{p}}^+ - b_{\vec{p}}^+ b_{\vec{p}}) - (b_{\vec{p}}^+ a_{-\vec{p}}^+ e^{2iE_{\vec{p}}t} - a_{\vec{p}}\, b_{-\vec{p}}\, e^{-2iE_{\vec{p}}t}) \Big]$$

while $\int_{-\infty}^{+\infty} d^3\vec{x}\ \phi^+\dot{\phi}$

$$= \int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})\, (-iE_{\vec{k}})$$

$$\times (a_{\vec{p}}^+ e^{ip\cdot x} + b_{\vec{p}}\, e^{-ip\cdot x})(a_{\vec{k}}\, e^{-ik\cdot x} - b_{\vec{k}}^+ e^{ik\cdot x})$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})(-iE_{\vec{k}})\, (2\pi)^3$$

$$\times \Big(a_{\vec{p}}^+ a_{\vec{k}}\, e^{iE_{\vec{p}}t - iE_{\vec{k}}t}\, \delta^3(\vec{p}-\vec{k}) - b_{\vec{p}} b_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t}\, \delta^3(\vec{p}-\vec{k})$$

$$- a_{\vec{p}}^+ b_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t}\, \delta^3(\vec{p}+\vec{k}) + b_{\vec{p}}\, a_{\vec{k}}\, e^{-iE_{\vec{p}}t - iE_{\vec{k}}t}\, \delta^3(\vec{p}+\vec{k}) \Big)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 (-iE_{\vec{p}})\, (2\pi)^3$$

$$\times \Big[(a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}} b_{\vec{p}}^+) - (a_{\vec{p}}^+ b_{-\vec{p}}^+ e^{2iE_{\vec{p}}t} - b_{\vec{p}}\, a_{-\vec{p}}\, e^{-2iE_{\vec{p}}t}) \Big]$$

16. (checked)

For an even function of $\vec{p}$, call it $f(\vec{p})$, we have

$$\int_{-\infty}^{+\infty} d^3\vec{p}\, f(\vec{p})\, a_{\vec{p}}^{+} b_{-\vec{p}}^{+} \overset{\vec{p}\to-\vec{k}}{=} \int_{-\infty}^{+\infty} d^3\vec{k}\, f(\vec{k})\, a_{-\vec{k}}^{+} b_{\vec{k}}^{+} \overset{\vec{k}\to\vec{p}}{=} \int_{-\infty}^{+\infty} d^3\vec{p}\, f(\vec{p})\, b_{\vec{p}}^{+} a_{-\vec{p}}^{+}$$

and

$$\int_{-\infty}^{+\infty} d^3\vec{p}\, f(\vec{p})\, b_{\vec{p}}\, a_{-\vec{p}} \overset{\vec{p}\to\vec{k}}{=} \int_{-\infty}^{+\infty} d^3\vec{k}\, f(\vec{k})\, b_{-\vec{k}}\, a_{\vec{k}} \overset{\vec{k}\to\vec{p}}{=} \int_{-\infty}^{+\infty} d^3\vec{p}\, f(\vec{p})\, a_{\vec{p}}\, b_{-\vec{p}}$$

For $-\int_{-\infty}^{+\infty} d^3\vec{x}\ \dot{\phi}^+ \phi$

$$= -\int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})\, (-iE_{\vec{p}})$$

$$\times \left(a_{\vec{p}}^+ e^{ip\cdot x} - b_{\vec{p}} e^{-ip\cdot x} \right)\left(a_{\vec{k}} e^{-ik\cdot x} + b_{\vec{k}}^+ e^{ik\cdot x} \right)$$

$$= -\int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})\ iE_{\vec{p}}\ (2\pi)^3$$

$$\times \Big(a_{\vec{p}}^+ a_{\vec{k}}\, e^{iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) - b_{\vec{p}} b_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k})$$

$$- b_{\vec{p}} a_{\vec{k}}\, e^{-iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) + a_{\vec{p}}^+ b_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) \Big)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, (C(E_{\vec{p}}))^2 (-iE_{\vec{p}})(2\pi)^3$$

$$\times \left[\left(a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}} b_{\vec{p}}^+ \right) - \left(b_{\vec{p}}\, a_{-\vec{p}}\, e^{-2iE_{\vec{p}}t} - a_{\vec{p}}^+ b_{-\vec{p}}^+ e^{2iE_{\vec{p}}t} \right) \right]$$

while $-\int_{-\infty}^{+\infty} d^3\vec{x}\ \phi\, \dot{\phi}^+$

$$= -\int_{-\infty}^{+\infty} d^3\vec{x}\, d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})\, (iE_{\vec{k}})$$

$$\times \left(a_{\vec{p}} e^{-ip\cdot x} + b_{\vec{p}}^+ e^{ip\cdot x} \right)\left(a_{\vec{k}}^+ e^{ik\cdot x} - b_{\vec{k}} e^{-ik\cdot x} \right)$$

$$= -\int_{-\infty}^{+\infty} d^3\vec{p}\, d^3\vec{k}\ C(E_{\vec{p}})\, C(E_{\vec{k}})\, (iE_{\vec{k}})\ (2\pi)^3$$

$$\times \Big(a_{\vec{p}} a_{\vec{k}}^+ e^{-iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k}) - b_{\vec{p}}^+ b_{\vec{k}}\, e^{iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}-\vec{k})$$

$$+ b_{\vec{p}}^+ a_{\vec{k}}^+ e^{iE_{\vec{p}}t + iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) - a_{\vec{p}} b_{\vec{k}}\, e^{-iE_{\vec{p}}t - iE_{\vec{k}}t} \delta^3(\vec{p}+\vec{k}) \Big)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\, \left((C(E_{\vec{p}}))^2 (-iE_{\vec{p}}) \right) (2\pi)^3$$

$$\times \left[\left(a_{\vec{p}} a_{\vec{p}}^+ - b_{\vec{p}}^+ b_{\vec{p}} \right) - \left(a_{\vec{p}}\, b_{-\vec{p}}\, e^{-2iE_{\vec{p}}t} - b_{\vec{p}}^+ a_{-\vec{p}}^+ e^{2iE_{\vec{p}}t} \right) \right]$$

 (checked)

$$\Rightarrow i\int_{-\infty}^{+\infty} d^3\vec{x}\,(\dot{\phi}\phi^+ - \dot{\phi}^+\phi)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\,(C(E_{\vec{p}}))^2 E_{\vec{p}}(2\pi)^3\left[(a_{\vec{p}}a_{\vec{p}}^+ - b_{\vec{p}}^+ b_{\vec{p}}) + (a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}} b_{\vec{p}}^+)\right]$$

$$i\int_{-\infty}^{+\infty} d^3\vec{x}\,(\dot{\phi}\phi^+ - \phi\dot{\phi}^+)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\,(C(E_{\vec{p}}))^2 E_{\vec{p}}(2\pi)^3\left[(a_{\vec{p}}a_{\vec{p}}^+ - b_{\vec{p}}^+ b_{\vec{p}}) + (a_{\vec{p}}a_{\vec{p}}^+ - b_{\vec{p}}^+ b_{\vec{p}})\right]$$

$$i\int_{-\infty}^{+\infty} d^3\vec{x}\,(\phi^+\dot{\phi} - \dot{\phi}^+\phi)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\,(C(E_{\vec{p}}))^2 E_{\vec{p}}(2\pi)^3\left[(a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}} b_{\vec{p}}^+) + (a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}} b_{\vec{p}}^+)\right]$$

$$i\int_{-\infty}^{+\infty} d^3\vec{x}\,(\phi^+\dot{\phi} - \phi\dot{\phi}^+)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\,(C(E_{\vec{p}}))^2 E_{\vec{p}}(2\pi)^3\left[(a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}} b_{\vec{p}}^+) + (a_{\vec{p}}a_{\vec{p}}^+ - b_{\vec{p}}^+ b_{\vec{p}})\right]$$

since

$$a_{\vec{p}}a_{\vec{p}}^+ - b_{\vec{p}}b_{\vec{p}}^+$$

$$= a_{\vec{p}}^+ a_{\vec{p}} + \frac{1}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\frac{1}{(C(E_{\vec{p}}))^2}\delta^3(0)$$
$$-\left(b_{\vec{p}}^+ b_{\vec{p}} + \frac{1}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left(\frac{1}{C(E_{\vec{p}})}\right)^2\delta^3(0)\right)$$

$$= a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}}^+ b_{\vec{p}}$$

$$\Rightarrow i\int_{-\infty}^{+\infty} d^3\vec{x}\,(\dot{\phi}\phi^+ - \dot{\phi}^+\phi) = i\int_{-\infty}^{+\infty} d^3\vec{x}\,(\phi^+\dot{\phi} - \phi\dot{\phi}^+)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\left[(C(E_{\vec{p}}))^2\, 2E_{\vec{p}}(2\pi)^3\right](a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}}^+ b_{\vec{p}})$$

while $i\int_{-\infty}^{+\infty} d^3\vec{x}\,(\dot{\phi}\phi^+ - \phi\dot{\phi}^+)$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\left[(C(E_{\vec{p}}))^2\, 2E_{\vec{p}}(2\pi)^3(a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}}^+ b_{\vec{p}})\right.$$
$$+ \int_{-\infty}^{+\infty} d^3\vec{p}\,(C(E_{\vec{p}}))^2\, 2E_{\vec{p}}(2\pi)^3\frac{1}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\left(\frac{1}{C(E_{\vec{p}})}\right)^2\delta^3(0)$$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\left[(C(E_{\vec{p}}))^2\, 2E_{\vec{p}}(2\pi)^3(a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}}^+ b_{\vec{p}})\right.$$
$$+ \int_{-\infty}^{+\infty}\frac{d^3\vec{p}}{(2\pi)^3}\delta^3(0)(2\pi)^3$$

and $i\int_{-\infty}^{+\infty} d^3\vec{x}\ (\phi^+\dot{\phi} - \dot{\phi}^+\phi)$

$$= \int_{-\infty}^{+\infty} d^3\vec{p}\ \left(C(E_{\vec{p}})\right)^2 2E_{\vec{p}}\ (2\pi)^3\ (a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}}^+ b_{\vec{p}})$$

$$- \int_{-\infty}^{+\infty} \frac{d^3\vec{p}}{(2\pi)^3}\ (2\pi)^3 \delta^3(0)$$

So we only have

$$:Q: = \int_{-\infty}^{+\infty} d^3\vec{p}\ \left(C(E_{\vec{p}})\right)^2 2E_{\vec{p}}\ (2\pi)^3 (a_{\vec{p}}^+ a_{\vec{p}} - b_{\vec{p}}^+ b_{\vec{p}})$$