Export Dynamics of Multiproduct Firms with (Non-)Differentiated Products*

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Abstract

Canonical models with heterogeneous firms and sunk export costs predict more profitable firms export ("export sorting"). This paper shows export sorting pattern varies systematically with product differentiation. Using Slovenian administrative data, I document that the gap in value added per worker between exporters and non-exporters is smaller in more differentiated sectors, particularly among multiproduct firms. I rationalise this pattern with a model in which multiproduct firms choose product scope and export participation based on cost efficiency and market-specific product appeal, featuring a "star-product" mechanism. When export costs are shared across products, high-appeal products pull marginal product lines into export bundles, diluting measured productivity among exporters. This composition effect is especially pronounced in high-differentiation sectors, where high markups allow less appealing products to be exported. The model reproduces observed sorting patterns when estimated separately for high- and low-differentiation sectors. I then evaluate a 50 percentage point tariff increase on a single product and find large contractions with substantial spillovers in the low-differentiation sector but minimal spillovers and little aggregate response in the high-differentiation sector.

Keywords: Firm Heterogeneity, Export Dynamics, Multiproduct Firms, Product Differentiation, Sunk Costs.

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1 Introduction

What drives export sorting among firms? The seminal framework of Melitz (2003) shows that heterogeneity in firm-level efficiency, combined with sunk export-entry costs, selects the most efficient firms into foreign markets, generating gaps in observed productivity and profitability between exporters and non-exporters. Subsequent work emphasises additional heterogeneity in demand and entry costs as key drivers of market entry and export participation[1]. This paper argues that the degree of product differentiation, captured by the demand elasticity, shapes how heterogeneous firms sort into export markets across sectors, primarily through the product-selection margin in multiproduct firms.

Using administrative data on Slovenian manufacturing firms, I find that the exporter labour-productivity premium, i.e., the gap in log(value added/worker) between exporters and non-exporters, is positive on average but declines with the degree of product differentiation, indicating weaker sorting when differentiation is high. This pattern is primarily driven by multiproduct firms, highlighting the key role of the product margin in export sorting. The cross-sectoral pattern is consistent with the conventional view that, when products are less differentiated (higher demand elasticities), competition is tougher, markups are lower, and selection on productivity is stronger. In the context of firm entry, Syverson (2004) documents that greater substitutability is associated with higher minimum and average productivity and lower productivity dispersion, consistent with a left-truncated productivity distribution at entry. However, this intuition does not explain why multiproduct firms exhibit different export-sorting patterns from single-product firms.

To explain these patterns, I develop a model in which multiproduct firms choose their product mix and export participation based on their cost efficiency and market-specific product appeal. The model characterises the industry dynamics in sectors with different degrees of product differentiation, captured by the demand elasticity in domestic and foreign markets. Firms pay a per-product fixed cost of production, which induces selection on the efficiency-appeal combination required for profitable production. They also face firm-level sunk export-entry costs that are shared across products, generating economies of scope. Consequently, a high-appeal "star" product can pull marginally profitable products into the export bundle. This composition effect lowers measured firm-level labour productivity among exporters, particularly in more differentiated sectors, where higher markups weaken selection on efficiency and allow less appealing products to enter export markets.

The data reveal product-concentration patterns consistent with the star-product mechanism. Within multiproduct firms, sales are more concentrated on a few products (i) in more differentiated sectors compared to less differentiated sectors, (ii) among exporters than non-exporters, and (iii) even more in foreign markets than domestic markets within the same firm. These patterns support the model's key mechanisms.

[1] See Foster et al. (2008); Eaton et al. (2011); Roberts et al. (2018); Arkolakis (2010).

Targeting these key empirical patterns and moments for export participation and revenue dynamics, I estimate the model separately for two representative manufacturing sectors at opposite ends of the differentiation spectrum: Machinery and Equipment Manufacturing for high differentiation and Food Products Manufacturing for low differentiation. The estimates reveal clear sectoral differences: in the high-differentiation sector, demand is less elastic and fixed production costs are higher, while export-entry costs are lower and appeal shocks are more volatile. The estimated model reproduces the key patterns: the exporter labour-productivity premium is smaller in more differentiated sectors, and this pattern is driven primarily by multiproduct firms.

I use the estimated model to evaluate a trade policy shock: a single-product tariff increase of 50 percentage points applied to one treated product in both sectors. In the high-differentiation sector, aggregate outcomes change little despite substantial direct effects on the treated product's foreign sales. By contrast, in the low-differentiation sector, the numbers of active firms, exporters, and firm-product pairs drop by more than 40%, and negative spillovers extend to the treated product's domestic sales and to the sales of untreated products in both markets. These sectoral differences arise because low markups in the low-differentiation sector tighten efficiency-appeal selection. A product-specific shock thus pushes marginal product lines below their zero-profit cutoff, shrinks export scope, and triggers firm exit. The experiment shows that product differentiation shapes not only export sorting but also the transmission of trade shocks through the multiproduct scope margin.

The model generalises Melitz (2003) by making market entry a dynamic choice with sunk costs and by allowing multiproduct scope with within-firm heterogeneity. It is closest to Bernard et al. (2011) (BRS) and Arkolakis et al. (2021) (AGM), which both extend the Melitz model to multiproduct firms. In BRS, per-product export costs lead firms to drop their least attractive products after trade liberalisation, affecting measured firm productivity and profitability through a composition effect; similarly, in my framework, trade costs make product composition endogenous, so composition effects also operate on measured profitability. AGM document highly skewed product portfolios for wide-scope exporters, and their structural estimation demonstrates economies of scope from decreasing incremental market-access costs; in the same spirit, I assume firm-level market-access costs that generate economies of scope.

My framework departs from BRS and AGM in three respects. First, in contrast to BRS and AGM, I model export participation as a dynamic rather than static decision, allowing the model to match the persistence of export status. Alessandria et al. (2021) show that static trade models do not approximate the steady state of dynamic models. Second, whereas BRS and AGM study symmetric sectors in general equilibrium, I distinguish sectors by their degree of product differentiation and focus on industry dynamics in partial equilibrium. This approach highlights the role of product differentiation in export decisions, rationalising sectoral gaps in exporter labour-productivity premiums. It also helps explain why some firms respond more to demand shocks and others more to productivity shocks, as documented by Cooper et al. (2025). Third, while the BRS model is not quantified and AGM is quantified using exporter data, the rich micro-level data for both domestic and foreign markets allow me to provide a comprehensive view of heterogeneity in efficiency and product appeal across markets. This estimation is complementary to Blum et al. (2024), which estimate marginal costs at the firm-product-time level and demand at the firm-product-market-time level using an enhanced control-function approach, and show that exporters have better demand-cost combinations than non-exporters without necessarily dominating in a single dimension.

The remainder of the paper is organised as follows: Section 2 reviews related literature; Section 3 describes the data, sample construction, and empirical regularities; Section 4 presents the model and key mechanisms; Section 5 outlines the estimation strategy and reports parameter estimates and model fit; Section 6 presents counterfactual analyses of tariff shocks; and Section 7 concludes.

2 Related Literature

This paper contributes to the literature on firm heterogeneity and export dynamics[2]. Melitz (2003)'s seminal work formalises how fixed export-entry costs, combined with productivity heterogeneity, generate productivity-based sorting into export markets. This framework has been extended to incorporate richer sources of heterogeneity relevant for market entry and the cross-section of exporters, including destination-specific demand shocks (Eaton et al., 2011) and sunk export costs that shape export participation (Das et al., 2007). Roberts et al. (2018) quantify dispersion in marginal costs, fixed export costs, and demand using firm-destination data for Chinese footwear producers, showing substantial heterogeneity along all three dimensions. Cooper et al. (2025) estimate a dynamic model separately for state-controlled and private-controlled enterprises in China and find that private exporters' decisions are more responsive to demand, whereas state firms' export status is more dependent on productivity.

A complementary empirical literature separately measures physical productivity and demand/markup components. Foster et al. (2008) show that entrants have higher physical productivity than incumbents but charge lower prices, highlighting that revenue productivity conflates efficiency and demand. De Loecker and Warzynski (2012) document that exporters charge higher markups, pointing to demand-side advantages beyond cost efficiency. More recently, Blum et al. (2024) develop a framework that decomposes firm heterogeneity into demand and cost drivers and show that exporters tend to have flatter domestic demand curves and better demand-cost combinations compared to non-exporters. Exporters also charge a significantly lower markup in foreign markets compared to domestic markets.

The trade literature also shows that changes in trade costs (liberalisation or new barriers) induce within-firm reallocation via endogenous product composition for multiproduct firms. In Bernard et al. (2011) and Mayer et al. (2014), fixed exporting costs are incurred at the product level, and they show that tougher competition leads firms to reallocate towards

[2] Alessandria et al. (2021) provide a comprehensive literature review.

their best-performing products, with implications for measured productivity. Eckel and Neary (2010) develop a model with increasing marginal production costs the further products are from a firm's core competence, and analyse the ensuing within-firm reallocation under globalisation. Arkolakis (2010) likewise allow declining efficiency in supplying additional products but let product-level market-access costs decrease incrementally; empirically, they show that wide-scope exporters display very asymmetric product sales with long tails of low-selling products, consistent with economies of scope in accessing export markets.

This paper emphasises the degree of product differentiation as a key determinant of firm dynamics. It complements Syverson (2004), which shows that greater substitutability truncates the productivity distribution from below and tightens selection. Chaney (2008) develops a model emphasising the role of the elasticity of substitution in heterogeneous firms' response to trade liberalisation. On the intensive margin, a fall in trade costs induces incumbent exporters to expand sales more when the elasticity is high (a winner-take-all effect). On the extensive margin, a fall in trade costs induces entry by new—typically less productive—exporters, but their market shares remain small when the elasticity is high, so aggregate trade responds less through entry. Lower differentiation (higher elasticity) also increases within-firm cannibalisation and alters pricing behaviour (Eckel et al., 2015), as well as product and process innovation choices (Dhingra, 2013; Flach and Irlacher, 2018).

3 Empirical Patterns

I document two key empirical regularities in this section. First, exporter labour-productivity premiums are lower in more differentiated sectors, and this decline is driven by multiproduct firms. Second, multiproduct firms in more differentiated sectors concentrate their sales more on a few products compared to multiproduct firms in less differentiated sectors. Third, among multiproduct firms, exporters concentrate their sales more on a few products than non-exporters, especially in foreign markets. The empirical analysis serves two objectives: (i) to compare export sorting across sectors that differ in product differentiation, which in turn guides estimation of a multiproduct model separately by differentiation regime; and (ii) to provide evidence supporting the star-product mechanism that drives composition effects in the model.

3.1 Data

Slovenian firm-level data: The analysis draws on comprehensive firm-product microdata from the Slovenian Statistical Office (SURS), combining the annual survey on *Production and Sold Production of Industrial Products and Services (IND/L)* with annual *Structural*

Business Statistics (SBS).[3] IND/L reports volumes and values of production and sales at the establishment-product-year level, separately for domestic and foreign markets, at ex-works invoiced prices that exclude VAT and transport costs. The survey universe covers establishments with 20 or more employees in sections B (Mining and Quarrying) and C (Manufacturing), and class 38.32 (Recovery of sorted materials), with limited inclusion of smaller establishments; the survey covers approximately 3,000 establishments each year. Production includes output from a firm's own materials and output produced under sub-contract; the latter is excluded from the analysis sample. Each product is identified by the Nomenclature of Industrial Products (NIP): the first eight digits correspond to the European PRODCOM code, and the first four digits correspond to the NACE code. PRODCOM codes are concorded to Harmonised System (HS) codes via EUROSTAT CN-PRODCOM tables; the first six digits of CN and HS are identical.

Because IND/L is at the establishment level while SBS is at the firm level, I aggregate IND/L to the firm level before merging; this affects few observations, as 93.8% of firms are single-plant. SBS complements IND/L with firm-level characteristics, including employment, wages, turnover, and value added for all active firms in Slovenia. A consistent firm identifier enables a precise merge, yielding a firm-product-market panel suitable for analysing the export dynamics of multiproduct firms.

Product differentiation classification: The degree of differentiation follows Rauch (1999), who classify goods into three groups: differentiated products, reference-priced products, and products traded on organised exchanges[4]. I adopt Rauch's "liberal" classification, which maximises the number of goods classified as non-differentiated by combining "reference-priced" and "traded on organised exchanges" into a single non-differentiated category. Rauch's SITC-based categories are mapped to 6-digit Harmonised System (HS) codes. I then aggregate product-level differentiation to the 2-digit NACE sector level and define four sector groups by the share of differentiated HS6 products: sectors with a share equal to 0% are classified as homogeneous (e.g., tobacco products); sectors between 10% and 32% as low-differentiation (e.g., food products); sectors between 57% and 66% as moderate-differentiation (e.g., textiles); and sectors above 77% as high-differentiation (e.g., machinery and equipment).

[3] All results have been reviewed to ensure that no confidential information is disclosed, in accordance with SURS requirements. Access to confidential firm-level microdata and the harmonised construction of variables were provided through the Micro Data Infrastructure (MDI) developed by CompNet, a secure, harmonised platform that enables researchers and policymakers to access and analyse confidential firm-level data across countries. More details are available at `https://www.comp-net.org/`.

[4] Broda and Weinstein (2006) show that Rauch's product categories correlate closely with their estimated elasticities of substitution.

3.2 Sample Construction

NACE classification of multi-activity firms: The product-market-level data allow a more precise assignment of firms' economic activities than administrative NACE codes. I therefore adopt a revenue-weighted procedure at the firm-year level: map each product to its two-digit NACE industry, aggregate product revenues to the two-digit NACE level within the firm-year, and identify (i) the main NACE as the industry with the largest revenue share and (ii) secondary NACE industries as all others that account for at least 20% of the firm's annual revenue. The estimation sample retains firms whose main or secondary NACE lies in manufacturing (NACE 10–33).

Mapping sectoral differentiation to multiproduct firms: I match each firm's main and, where applicable, secondary NACE sectors to their differentiation categories. Single-activity firms, and firms whose main and secondary activities share the same type, inherit that common differentiation category. For mixed-activity firms with different differentiation levels, I take a conservative approach and assign the lowest differentiation level present among their activities. For example, firms with both moderate and high-differentiation activities are classified as moderate-differentiation. Firms active in both high and low/homogeneous sectors are placed in an "Other" category to reflect these conflicting activities and are excluded from the analysis sample. The empirical results are robust to alternative classification rules because mixed-activity firms constitute only a small share of the sample.

Other key variables: A product is defined at the 8-digit PRODCOM level. Firms are classified as multiproduct if they report revenue from more than one product. A firm is an exporter in a given year if it reports positive foreign revenue for at least one product.

I measure labour productivity as value added per worker, computed as the reported value added divided by the number of employees. Robustness checks use total factor revenue productivity (TFPR) as alternative productivity measures[5].

I compute product-concentration metrics at the firm-year level (aggregated over markets) and at the firm-market-year level. Let ν_j denote the revenue share of product j within the relevant aggregation. I report four standard measures: the Herfindahl-Hirschman Index (HHI $= \sum_j \nu_j^2$), which lies in $[0, 1]$ and equals 1 when all sales come from a single product; the Theil (entropy) index (Theil $= \sum_j \nu_j \ln(1/\nu_j)$), which lies in $[0, \ln N]$ with N active products and equals 0 under perfect concentration; the top-product share ($\max_j \nu_j$); and the best-versus-second-best ratio ($\nu_{\text{top1}}/\nu_{\text{top2}}$). Higher HHI, top-product share, and best-versus-second-best ratios, and lower Theil values, indicate greater concentration.

[5] More details are provided in Appendix A.1.

Final estimation sample: The final sample comprises 3,645 manufacturing firms with a valid differentiation classification over the years 2009–2021. Table 1 summarises composition: exporters account for 78.9% of the firm-year observations. Multiproduct firms represent over 50% of observations, with an average product scope of 2.8 products per firm.

Table 1: Sample Summary Statistics by Product Differentiation Type

Statistic	All	Low	Moderate	High
Number of firms	3,645	635	446	2,635
Number of firm–years	23,330	4,196	2,372	16,433
Exporter share (%)	78.9	64.8	83.8	81.6
Share multiproduct firms (%)	54.8	71.4	49.5	50.6
Average product scope	2.8	4.4	2.1	2.5
Median product scope	2.0	3.0	1.0	2.0
Average employment	89	110	84	83
Average value added	4,032	4,944	7,023	3,363

Note: Statistics are computed at the firm-year level. Product scope counts PRODCOM 8-digit products with positive revenue.

3.3 Empirical Regularities

Fact 1: Exporter labour-productivity premiums are lower in differentiated sectors To investigate differences in exporter productivity premiums across sectors, I estimate a series of regressions with the following general form for firm i in sector s in year t:

$$\log\left(\frac{\text{value-added}}{\text{worker}}\right)_{i,s,t} = \mathbf{X}_{i,s,t}\boldsymbol{\beta} + \boldsymbol{\alpha}_s + \boldsymbol{\gamma}_t + \varepsilon_{i,s,t},$$

where $\boldsymbol{\alpha}_s$ and $\boldsymbol{\gamma}_t$ are sector and year fixed effects. I begin with a baseline specification in which $\mathbf{X}_{i,s,t}$ includes only exporter status:

$$\mathbf{X}_{i,s,t}\boldsymbol{\beta} = \beta_1 \, \text{Exporter}_{i,s,t}. \tag{1}$$

Here β_1 is the coefficient measuring the difference in log(value added/worker) between exporters and non-exporters after conditioning on sector and year fixed effects. $\exp(\beta_1) - 1$ is therefore the corresponding percentage premium.

To assess how premiums vary with the degree of product differentiation across sectors, I interact exporter status with differentiation categories:

$$\mathbf{X}_{i,s,t}\boldsymbol{\beta} = \sum_k \beta_k \left(\text{Exporter}_{i,s,t} \times \mathbf{1}\{\text{DiffClass}_{i,s,t} = k\}\right), \qquad k \in \{\text{Low, Moderate, High}\}. \tag{2}$$

Finally, to distinguish single- and multiproduct firms, I estimate a specification with triple interactions:

$$\mathbf{X}_{i,s,t}\boldsymbol{\beta} = \sum_k \sum_j \beta_{kj} \left(\text{Exporter}_{i,s,t} \times \mathbf{1}\{\text{DiffClass}_{i,s,t} = k\} \times \mathbf{1}\{\text{FirmType}_{i,s,t} = j\}\right), \quad k \in \{\text{Low, Moderate, High}\},\ j \in \{\text{SPF, MPF}\}. \tag{3}$$

Table 2 reports the implied exporter premiums for each differentiation-scope combination, computed as linear combinations of the coefficients from (1), (2), and (3)[6]. Premiums are positive across all cells, indicating that exporters have higher average labour productivity than non-exporters. Moreover, the premiums decline with the degree of product differentiation. In high-differentiation sectors, the labour productivity of exporters is approximately 17.1% ($\exp(0.158)-1$) higher than that of non-exporters, whereas in low-differentiation sectors the corresponding gap is about 34.7% ($\exp(0.298)-1$).

Disaggregating by firm scope shows that the cross-sector pattern is driven by multiproduct firms. The lower panel of Table 2 reports one-sided tests comparing premiums in low- versus moderate- and high-differentiation sectors. For multiproduct firms, the premium in low-differentiation sectors exceeds that in high-differentiation sectors by 0.207 log points (significant at the 1% level) and exceeds that in moderate-differentiation sectors by 0.144 log points (significant at the 10% level). In contrast, the differences are negative and statistically insignificant for single-product firms. Robustness checks using TFPR as the productivity measure yield qualitatively similar results; see Appendix A.1 for the full set of estimates.

The relationship between product differentiation and the exporter labour-productivity premium is not specific to Slovenia. Using CompNet data for 16 European countries over 1995–2023, I compute the average exporter-non-exporter productivity gap by country-sector-year. Appendix Table A2 confirms the main finding: premiums are significantly lower in moderate- and high-differentiation sectors (by 0.092 and 0.133 log points, respectively) relative to low-differentiation sectors. These results are robust to alternative specifications, with and without country and year fixed effects; see Appendix A.2 for details.

Fact 2: Firms in more differentiated sectors concentrate their sales more on a few products I examine how product sales concentration varies across sectors with different degrees of product differentiation. The analysis focuses on multiproduct firms, as single-product firms have concentration equal to one by definition. I employ a regression specification that controls for firm-level product scope (the number of products), includes sector and year fixed effects, and clusters standard errors at the firm level. Throughout the concentration analysis, I use four measures: the Herfindahl-Hirschman Index (HHI),

[6] For example, the 0.138 coefficient for MPF in high-differentiation sectors equals $\beta_{\text{exporter}} + \beta_{\text{exporter}\times\text{MPF}} + \beta_{\text{exporter}\times\text{high-diff}} + \beta_{\text{exporter}\times\text{MPF}\times\text{high-diff}}$, where each β is taken from the triple-interaction regression; standard errors are computed via the delta method.

Table 2: Exporter Productivity Premiums by Differentiation Level

Differentiation	**All Firms**	**SPF**	**MPF**
All Sectors	0.199^{a}	–	–
	(0.025)		
Low (baseline)	0.298^{b}	0.162^{c}	0.346^{c}
	(0.056)	(0.093)	(0.066)
Moderate	0.248^{b}	0.313^{c}	0.201^{c}
	(0.075)	(0.124)	(0.088)
High	0.158^{b}	0.175^{c}	0.138^{c}
	(0.029)	(0.041)	(0.041)
Low > Moderate	0.050	-0.151	$0.144^{\dagger}$
	(0.093)	(0.155)	(0.110)
Low > High	0.140^{*}	-0.012	0.207^{**}
	(0.063)	(0.101)	(0.077)

Note: Exporter premiums in log(*value-added/worker*)*. Standard errors clustered at the firm level in parentheses. Superscripts indicate the source regression for the reported linear combination:* ^{a} *= baseline exporter specification (equation 1);* ^{b} *= differentiation interaction (equation 2);* ^{c} *= triple interaction (equation 3). The lower-panel comparisons are one-sided tests of* $H_0\colon \beta_{Low}-\beta_{Moderate} \le 0$ *and* $H_0\colon \beta_{Low}-\beta_{High} \le 0$*.*

the Theil entropy index, the revenue share of the top product, and the log ratio of the top product's revenue to the second product's revenue.

The specification compares concentration across differentiation categories, taking Low as the omitted baseline:

$$\text{Concentration}_{i,t} = \sum_k \beta_k \left(\mathbf{1}\{\text{DiffClass}_{i,t} = k\}\right) \; + \; \beta_z \, \log(\text{scope})_{i,t} \; + \; \boldsymbol{\alpha}_s \; + \; \boldsymbol{\gamma}_t \; + \; \varepsilon_{i,s,t}, \quad k \in \{\text{Low, Moderate, High}\}. \tag{4}$$

Table 3 Panel A shows that product sales are significantly more concentrated in moderate- and high-differentiation sectors relative to low-differentiation sectors across all four concentration measures.

Fact 3: Exporters concentrate sales more on a few products, especially in foreign markets Building on the same regression framework with controls for product scope, I conduct two complementary comparisons. First, I compare concentration between exporters and non-exporters within a sector using specification (5), where β_1 captures the exporter-non-exporter difference after controlling for sector and year fixed effects. Second, for multiproduct firms active in both markets, I compare concentration across domestic and foreign markets within the same firm using specification (6), where β_1 captures the within-firm domestic-foreign difference after controlling for firm and year fixed effects.

$$\text{Concentration}_{i,t} \;=\; \beta_1\,\text{Exporter}_{i,t} \;+\; \beta_2\,\log(\text{scope})_{i,t} \;+\; \boldsymbol{\alpha}_s \;+\; \boldsymbol{\gamma}_t \;+\; \varepsilon_{i,s,t}. \tag{5}$$

$$\text{Concentration}_{i,m,t} \;=\; \beta_1\,\text{Foreign}_{i,m,t} \;+\; \beta_2\,\log(\text{scope})_{i,m,t} \;+\; \boldsymbol{\alpha}_i \;+\; \boldsymbol{\gamma}_t \;+\; \varepsilon_{i,m,t}. \tag{6}$$

Table 3 Panels B and C display the results. Exporters exhibit significantly higher concentration than non-exporters, and within firms selling in both markets, foreign sales are significantly more concentrated than domestic sales.

Table 3: Product Concentration Analysis

Variable	**HHI**	**Theil (Entropy)**	**Top Share**	**Log Ratio Top2**
Panel A: By differentiation level				
Moderate	0.222	-0.492	0.211	1.111
	(0.069)	(0.167)	(0.059)	(0.350)
High	0.351	-0.700	0.344	1.842
	(0.071)	(0.170)	(0.062)	(0.361)
log(scope)	-0.275	0.777	-0.228	-0.611
	(0.009)	(0.017)	(0.009)	(0.049)
Panel B: By export status				
Exporter	0.021	-0.028	0.021	0.136
	(0.009)	(0.015)	(0.009)	(0.054)
log(scope)	-0.278	0.781	-0.232	-0.628
	(0.009)	(0.017)	(0.009)	(0.049)
Panel C: By market				
Foreign	0.031	-0.058	0.026	0.187
	(0.003)	(0.006)	(0.003)	(0.023)
log(scope)	-0.267	0.724	-0.218	-0.643
	(0.011)	(0.022)	(0.011)	(0.070)

Note: Higher HHI/Top Share/Log Ratio Top2 and lower Theil indicate greater concentration. Panel A compares concentration across differentiation categories (Moderate and High relative to Low baseline) for multiproduct firms, with sector and year fixed effects. Panel B compares exporters versus non-exporters within multiproduct firms, with sector and year fixed effects. Panel C compares foreign versus domestic sales within multiproduct firms active in both markets, with firm and year fixed effects. Standard errors in parentheses are clustered at the firm level across all specifications.

4 Structural Model

I introduce the model of multiproduct firms in this section. The model links sectoral primitives to observable outcomes such as exporter productivity premiums and product concentration, thereby explaining why sorting is weaker in some sectors and why the pattern is driven by multiproduct rather than single-product firms. Moreover, the estimated model is a laboratory to study product-specific tariff shocks.

4.1 Model Environment

Sectors differ in their degree of horizontal product differentiation. Higher product differentiation corresponds to a lower elasticity of substitution across products within a sector, and vice versa. Under CES consumer preferences, the elasticity of substitution equals the demand elasticity faced by firms.

Firms operate within a sector. They are multiproduct and can sell in domestic and foreign markets, endogenously choosing which products to produce and whether to export. There is firm heterogeneity in both cost efficiency and product appeal. Firm-level efficiency ε_i affects marginal costs across all products in a firm i. Firm-product-market-specific appeal shocks $\phi_{i,j}^m$ shift the demand curve for product j in market $m \in \{d, f\}$ (domestic or foreign). These appeal shocks form two vectors of length J, the maximum number of products: $\boldsymbol{\phi}_i^d = (\phi_{i,1}^d, \ldots, \phi_{i,J}^d)$ and $\boldsymbol{\phi}_i^f = (\phi_{i,1}^f, \ldots, \phi_{i,J}^f)$.

Labour is the only production input. The labour demand for firm i is:

$$l_i = \sum_{j=1}^{J} I_{i,j}^P \left(f_p + \frac{q_{i,j}}{\varepsilon_i} \right). \tag{7}$$

$q_{i,j}$ is the quantity of product j and $q_{i,j}/\varepsilon$ is the variable production cost. f_p is the fixed cost of producing each product. $I_{i,j}^P \in \{0, 1\}$ indicates whether firm i produces product j, which is determined endogenously given the realisation of the shocks $\varepsilon_i, \boldsymbol{\phi}_i^d, \boldsymbol{\phi}_i^f$ and its market strategy.

4.2 Product-Level Pricing and Selection

Under monopolistic competition, firms set prices to maximise product-level profits. For firm i's product j in market m, the pricing problem is:

$$\max_{p^m_{i,j}} \left(p^m_{i,j} - \frac{w}{\varepsilon_i} \right) q^m_{i,j} \quad \text{subject to} \quad q^m_{i,j} = \phi^m_{i,j} (p^m_{i,j})^{-\eta^m},$$

where w is the wage rate, and η^m is the demand elasticity in market m. The degree of product differentiation corresponds to the demand elasticities η_d, η_f in domestic and foreign markets, respectively.

The optimal pricing strategy yields constant markups $\mu^m = \eta^m/(\eta^m - 1)$. Price is thus determined as $p^m_{i,j} = \mu^m w/\varepsilon_i$. The product-market revenue is:

$$r^m_{i,j} = p^m_{i,j} q^m_{i,j} = \phi^m_{i,j} \varepsilon_i^{\eta^m - 1} (\mu^m)^{1-\eta^m} w^{1-\eta^m}. \tag{8}$$

The optimal $q^m_{i,j}$ determines the variable labour input as $q^m_{i,j}/\varepsilon_i = r^m_{ij}/\mu^m$, and the profit net of variable costs is $r^m_{i,j}/\eta^m$. Crucially, the product-market revenue is proportional to the appeal shock $\phi^m_{i,j}$, which captures the shift of demand due to consumer taste, iceberg transportation costs, or tariffs. At the product level, net profit is obtained by summing across markets and deducting the fixed production cost:

$$\pi_{i,j} = \sum_m I_i^M \frac{r^m_{i,j}}{\eta^m} - f_p, \tag{9}$$

where $I_i^M \in \{0, 1\}$ indicates whether firm i operates in market m.

This structure generates zero-profit cutoffs that determine which products are produced: for a given efficiency level, only products with sufficiently high appeal can generate sufficient profit to cover the fixed production cost. More formally, for a firm selling exclusively in market m, product j is produced if and only if $\phi^m_{i,j} \geq \phi^{m*}(\varepsilon_i, \eta^m)$, where the cutoff ϕ^{m*} satisfies $r^m_{i,j}/\eta^m = f_p$, i.e.,

$$\phi^{m*} \varepsilon_i^{\eta^m - 1} \frac{1}{\eta^m} \left(\frac{\eta^m}{\eta^m - 1} \right)^{1-\eta^m} w^{1-\eta^m} = f_p. \tag{10}$$

For firms selling in both markets, the condition becomes $\phi^d_{i,j}/\phi^{d*} + \phi^f_{i,j}/\phi^{f*} \geq 1$. The zero-profit cutoffs summarise the endogenous product scope decision, which is a selection of efficiency and appeal combinations for each product $j \in \{1, \ldots, J\}$. Moreover, combining equation (8) and equation (10), the product-market revenue can be rewritten as

$$r^m_{i,j} = \eta^m f_p \frac{\phi^m_{i,j}}{\phi^{m*}(\varepsilon_i, \eta^m)}. \tag{11}$$

The firm-level profit, net of all variable costs and fixed costs for production, is obtained by aggregating profit across all products and markets:

$$\pi_i = \sum_{j=1}^{J} I^P_{i,j} \pi_{i,j} = \sum_{j=1}^{J} I^P_{i,j} \left(\sum_m I_i^M \frac{r^m_{i,j}}{\eta^m} - f_p \right) \tag{12}$$

The market participation indicators I_i^M are determined endogenously through the dynamic market participation problem presented in the next section.

4.3 Dynamic Market Participation Problem

Firms make dynamic market participation decisions considering both current profits and future value, accounting for sunk entry costs, efficiency shocks, and appeal shocks. Firms can choose among four market participation strategies: $x \in \{d, f, df, 0\}$, where d represents domestic-only operation, f represents foreign-only operation, df represents operation in both domestic and foreign markets, and 0 represents exit.

Incumbent firms solve the dynamic programming problem:

$$V(\varepsilon, \boldsymbol{\phi}^d, \boldsymbol{\phi}^f, x_{-1}) = \max_{x\in\{d,f,df,0\}} \left\{ \pi_x(\varepsilon, \boldsymbol{\phi}^d, \boldsymbol{\phi}^f) - f_x - \kappa(x_{-1}, x) + \beta \mathbb{E}[V(\varepsilon', \boldsymbol{\phi}^{d'}, \boldsymbol{\phi}^{f'}, x)] \right\} \tag{13}$$

where $\pi_x(\varepsilon, \boldsymbol{\phi}^d, \boldsymbol{\phi}^f)$ is the flow profit from equation (12). f_x represents market-specific overhead costs, and the market accessing costs take the form of sunk costs $\kappa(x_{-1}, x) = k_d \mathbf{1}\{d \notin x_{-1}, d \in x\} + k_f \mathbf{1}\{f \notin x_{-1}, f \in x\}$, capturing the costs of entering new markets, where k_d and k_f are the sunk costs for domestic and foreign market entry, respectively.

All shocks evolve as AR(1) processes in logs:

$$\begin{aligned} \log \varepsilon_{t+1} &= \rho_\varepsilon \log \varepsilon_t + \xi_{t+1}, \quad \xi_{t+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2), \\ \log \phi_{j,t+1}^d &= \rho_d \log \phi_{j,t}^d + \zeta_{j,t+1}^d, \quad \zeta_{j,t+1}^d \sim \mathcal{N}(0, \sigma_d^2), \\ \log \phi_{j,t+1}^f &= \rho_f \log \phi_{j,t}^f + \zeta_{j,t+1}^f, \quad \zeta_{j,t+1}^f \sim \mathcal{N}(0, \sigma_f^2), \end{aligned}$$

for all $j = 1, \ldots, J$. All shocks are mutually independent across products and shock types.

The market participation indicators I_i^M are determined endogenously through the dynamic market participation problem (13):

$$I_i^M(\varepsilon_i, \boldsymbol{\phi}_i^d, \boldsymbol{\phi}_i^f, x_{i,-1}) = \mathbf{1}\{m \in x^*(\varepsilon_i, \boldsymbol{\phi}_i^d, \boldsymbol{\phi}_i^f, x_{i,-1})\}$$

where $x^*(\varepsilon_i, \boldsymbol{\phi}_i^d, \boldsymbol{\phi}_i^f, x_{i,-1})$ is the optimal market participation strategy. In other words, firm i participates in market m if and only if market m is included in the optimal strategy x^* given the current state.

Finally, potential entrants first draw efficiency and product-market-specific demand shocks before deciding whether to enter. The state upon entry is $\Omega_0 = \big(\varepsilon_i, \{\phi_{i,j}^m\}, x_{i,-1} = \emptyset\big)$. A firm enters if and only if its expected value exceeds the fixed entry cost f_e, i.e., $V(\Omega_0) \geq f_e$.

The law of motion for the distribution of firms characterises how the joint distribution of states $\mu_t(\varepsilon, \boldsymbol{\phi}^d, \boldsymbol{\phi}^f, x_{-1})$ evolves from period t to $t+1$. Let $\Omega = (\varepsilon, \boldsymbol{\phi}^d, \boldsymbol{\phi}^f)$ and the combined transition kernel can be written as $P(\Omega'|\Omega) = P_\varepsilon(\varepsilon'|\varepsilon) P_{\boldsymbol{\phi}^d}(\boldsymbol{\phi}^{d'}|\boldsymbol{\phi}^d) P_{\boldsymbol{\phi}^f}(\boldsymbol{\phi}^{f'}|\boldsymbol{\phi}^f)$. The distribution evolves according to:

$$\begin{aligned} \mu_{t+1}(\Omega', x) = & \sum_{\Omega, x_{-1}} P(\Omega'|\Omega)\, \mathbf{1}\{x = x^*(\Omega, x_{-1})\}\, \mu_t(\Omega, x_{-1}) \\ & + \; m \sum_{\Omega_0 : V(\Omega_0) \geq f_e} P(\Omega'|\Omega_0)\, \mathbf{1}\{x = x_0^*(\Omega_0)\}\, \mu_0(\Omega_0) \end{aligned} \tag{14}$$

The first term accounts for incumbent firms transitioning according to the AR(1) processes for efficiency and appeal shocks, with optimal market choices x^*. The second term captures the inflow of new entrants with mass m who draw from the initial distribution μ_0 of efficiency and appeal shocks, subject to the free entry condition. A stationary equilibrium is characterised by a time-invariant distribution μ^* that satisfies $\mu_{t+1} = \mu_t = \mu^*$. I assume the initial distributions for ε, ϕ^d, and ϕ^f are the stationary distributions implied by their respective AR(1) processes.

4.4 Key Mechanisms

This subsection explains how product differentiation shapes product selection, export participation, and measured firm-level labour productivity in a firm. To clarify the mechanisms, I consider a simplified two-product static model with export sales only. I also discuss two key modeling assumptions, including the treatment of within-firm cost heterogeneity and the constant-markup structure, and their implications for the mechanisms.

Simplified two-product model. Fix firm efficiency ε and suppress subscript i. Normalise the wage $w = 1$. Define the normalised appeal $x_j \equiv \phi_j / \phi^*(\varepsilon, \eta)$, where $\phi^*(\varepsilon, \eta)$ is the product-level appeal cutoff given by equation (10). A firm has two products. Product j is active if and only if $x_j \geq 1$, and its per-product operating profit net of the fixed cost f_p is

$$\pi_j(x_j) = \left(\frac{r_j}{\eta} - f_p \right) \mathbf{1}(x_j \geq 1) = f_p(x_j - 1)\, \mathbf{1}(x_j \geq 1).$$

The first equality follows equation (9), and the second equality follows equation (11).

Exporting entails a firm-level fixed cost κ, independent of the number of products exported. Let $e \in \{0, 1\}$ denote the export decision. Conditional on exporting, the firm exports exactly the products with $x_j \geq 1$. Total export profit is

$$\pi(x_1, x_2; e) = \big(\pi_1(x_1) + \pi_2(x_2)\big)e - \kappa e,$$

so the firm exports if and only if

$$\pi_1(x_1) + \pi_2(x_2) \geq \kappa. \tag{15}$$

(i) Product selection thresholds. From equation (10), the product-level appeal cutoff is

$$\phi^*(\varepsilon, \eta) = f_p\, \eta \left(\frac{\eta}{\eta - 1}\right)^{\eta-1} \varepsilon^{1-\eta},$$

which is decreasing in firm efficiency with elasticity $\partial \ln \phi^* / \partial \ln \varepsilon = 1 - \eta < 0$. Higher cost efficiency lowers the appeal threshold required for profitable production. Moreover, when products are more differentiated (lower η and hence higher markup $\mu = \eta/(\eta - 1)$), the cutoff is lower. Figure 1 illustrates how the production region expands with ε and how lower η relaxes the selection threshold.

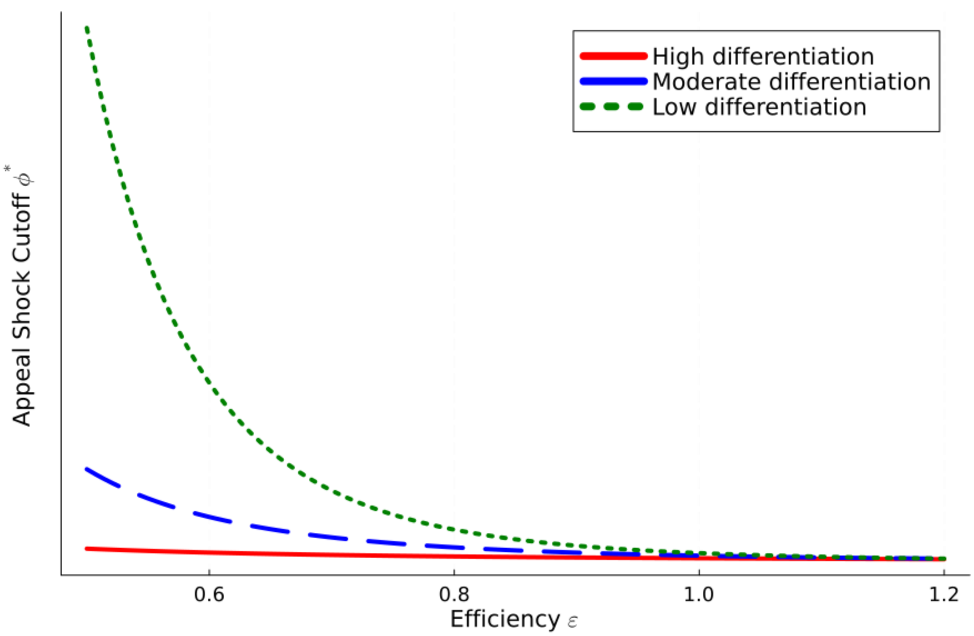

Figure 1: Product selection under varying degrees of product differentiation.

(ii) Economies of scope in export participation. The star-product mechanism arises because firm-level export costs are shared across products. Following the export participation condition (15), if both products are export-active ($x_1 \geq 1, x_2 \geq 1$), exporting is optimal if and only if

$$\frac{x_1 + x_2}{2} \geq 1 + \frac{\kappa}{2f_p}. \tag{16}$$

If only product 1 is export-active ($x_1 \geq 1, x_2 < 1$), exporting is optimal if and only if

$$x_1 \geq 1 + \frac{\kappa}{f_p}. \tag{17}$$

Since $1+\kappa/(2f_p) < 1+\kappa/f_p$, exporting can occur at a lower average appeal when two products are exported together. This is the star-product mechanism: a sufficiently high appeal in one product can pull a second, marginally profitable product into the export bundle.

(iii) Composition effect on measured labour productivity. Carrying along marginal products affects measured productivity through composition. In the simplified export-only model, firm-level value-added labour productivity can be expressed as

$$\theta^{VA}(\bar{x};\eta) \equiv \frac{VA}{L} = \frac{\bar{x}-1}{1+(\eta-1)\bar{x}}, \qquad \bar{x} \geq 1, \tag{18}$$

where $\bar{x} \equiv \frac{1}{N}\sum_{j:x_j\geq 1} x_j$ is the average normalised appeal across $N \in \{1,2\}$ active products. One can show that $\log\theta^{VA}$ is strictly increasing in $\bar{x}$ for $\bar{x} > 1$.The detailed derivation is provided in Appendix B.

The composition effect arises when a star product pulls along a second product with lower appeal. Specifically, fix a star product with $x_1 \geq 1+\kappa/f_p$. Then any second-product appeal x_2 satisfying

$$x_1 > x_2 > \max\left\{1,\, 2\left(1+\frac{\kappa}{2f_p}\right) - x_1\right\} \tag{19}$$

ensures: (i) the firm exports with two products by inequality (16); (ii) average appeal satisfies $\bar{x} = (x_1+x_2)/2 < x_1$; and (iii) measured productivity falls: $\theta^{VA}(\bar{x};\eta) < \theta^{VA}(x_1;\eta)$. The set is non-empty because the star-product condition $x_1 \geq 1+\kappa/f_p$ implies both $x_1 > 1$ and $x_1 > 2(1+\kappa/(2f_p)) - x_1$.

Within a firm, ε and η are fixed, so $\phi^*(\varepsilon,\eta)$ is fixed. Using $x_j = \phi_j/\phi^*(\varepsilon,\eta)$, condition (19) can be written in terms of appeal shocks as

$$\phi_1 \geq \phi_2 \geq \max\left\{\phi^*(\varepsilon,\eta),\, 2\left(1+\frac{\kappa}{2f_p}\right)\phi^*(\varepsilon,\eta) - \phi_1\right\}. \tag{20}$$

This representation makes clear how η interacts with the product selection, thus affecting exporters' measured productivity through a composition effect. Since $\phi^*(\varepsilon,\eta)$ is increasing in η for firms with moderate efficiency[7], lower η (higher differentiation) reduces $\phi^*(\varepsilon,\eta)$ and

[7] Specifically, $\partial\log\phi^*/\partial\eta = \log(\eta/(\eta-1)) - \log\varepsilon$, which is positive when $\varepsilon < \eta/(\eta-1)$. Actually, the sensitivity weakens as efficiency increases, since

$$\frac{\partial}{\partial\varepsilon}\left[\frac{\partial}{\partial\eta}\log\phi^*(\varepsilon,\eta)\right] = -\frac{1}{\varepsilon} < 0.$$

So ϕ^* moves with η meaningfully only when ε is sufficiently low. This relationship is clear in Figure 1.

therefore lowers the lower bound in inequality (20), expanding the set of appeal combinations that trigger scope-driven composition effects.

It is worth noting that the composition effect is not the only channel through which η affects measured labour productivity. The total effect can be decomposed as

$$\frac{d}{d\eta}\log\theta^{VA}(\bar{x}(\eta);\eta) = \frac{\partial \log\theta^{VA}}{\partial\eta} + \frac{\partial \log\theta^{VA}}{\partial\bar{x}}\cdot\frac{d\bar{x}}{d\eta}.$$

The scope-induced composition effect operates through the second term by weakening selection on $\bar{x}$ among exporters. This mechanism also naturally distinguishes multiproduct firms from single-product firms. At the same time, when holding composition fixed, lower η (higher markups) mechanically raises measured productivity through the first term: $\partial\log\theta^{VA}/\partial\eta = -\bar{x}/[1+(\eta-1)\bar{x}] < 0$. Moreover, changes in η shift the product-selection cutoff $\phi^*(\varepsilon,\eta)$, affecting the distribution of normalised appeals $x=\phi/\phi^*$ and hence $\bar{x}$ through a normalisation channel. Such normalisation depends on the value of ε and therefore on the efficiency distribution. The mapping from demand elasticity η to exporter labour-productivity premiums therefore depends on the joint distribution of efficiency and appeal shocks for exporters and non-exporters, which is conditional on the fixed costs governing product selection and export participation, as well as the unconditional distribution of these shocks. While the two-product example isolates the key scope-induced composition effect, matching the observed pattern of exporter premiums across sectors requires quantifying the full dynamic model with all channels operating jointly.

Discussion on modeling assumptions. In the model, firm-level cost efficiency shifts marginal costs uniformly across a firm's product lines. This abstracts from a core-competence structure, as in Eckel and Neary (2010), in which marginal costs tend to rise as products move further away from a firm's core product. While within-firm cost heterogeneity are empirically relevant, Arkolakis et al. (2021) show that economies of scope in market access quantitatively dominate production diseconomies in determining export scope. Moreover, A related empirical diagnostic is whether product rankings differ systematically across markets within the same firm. Imperfect correlation between domestic and foreign product rankings within firms can point to the importance of market-specific appeal heterogeneity rather than purely cost-based hierarchies.

The model also maintains constant markups by assuming CES demand and monopolistic competition. The sectoral degree of product differentiation is summarised by the elasticity of substitution, which pins down the sector-market markup and is treated as an exogenous characteristic of the competitive environment. Endogenising markups would require moving away from CES monopolistic competition, for instance, via non-CES demand systems (e.g., quadratic preferences) or oligopolistic competition with strategic interactions. In such frameworks, markups are typically heterogeneous and increasing in productivity (Melitz and Ottaviano, 2008) or market share (Atkeson and Burstein, 2008). Importantly, however, a lower within-sector elasticity of substitution, as a sectoral primitive corresponding to higher differentiation in both model classes, still implies higher average markups, which would generate weaker selection and stronger composition effects for the average firm, as discussed above.

Allowing for asymmetric markups could amplify exporter-non-exporter gaps in measured labour productivity if more productive firms also set higher markups, especially in low-differentiation sectors where the selection on efficiency can be stronger. However, by itself this channel does not naturally explain why the cross-sector differences in sorting are driven primarily by multiproduct firms. For the questions asked in this paper, the constant-markup Melitz structure therefore provides a useful abstraction and delivers a transparent mechanism linking sectoral product differentiation, economies of scope in exporting, and the observed differences in export sorting across sectors.

5 Structural Estimation

5.1 Estimation Strategy

Representative sectors. To discipline the model, I estimate it separately for two representative manufacturing sectors that sit at opposite ends of the differentiation spectrum. The selection proceeds as follows. I collapse the data to the NACE two-digit level (codes 10-33) by including firms with either main or secondary NACE in that sector. For each sector, I compute diagnostics including the number of firms and firm-years, and the shares of differentiated products and revenues. Sectors are kept only if they have at least 100 firms or 1,000 firm-years to ensure disclosure criteria are met. Among qualifying sectors, I rank them by the share of differentiated products and select the top-ranked sector as the high-differentiation representative and the bottom-ranked sector as the low-differentiation representative. On this basis, the high-differentiation sector is NACE 28 (Machinery and equipment n.e.c.) and the low-differentiation sector is NACE 10 (Food products)[8]. Note that firms can contribute to multiple sectoral diagnostics if they have main and secondary NACE codes in different sectors, though there is no overlap between the firms in the selected representative sectors. For example, the additional activities of food products producers (NACE 10) are typically in related sectors like beverages (NACE 11) and chemicals (NACE 20), which are also classified as low differentiation sectors.

Parameters pre-determined and estimated. The vector of estimated parameters is $\Theta = \{\eta_d, \eta_f, f_p, f_d, f_f, , \kappa_f, f_e, \rho_\varepsilon, \sigma_\varepsilon, , \rho_{\phi^d}, \sigma_{\phi^d}, \rho_{\phi^f}, \sigma_{\phi^f}\}$, which includes demand elasticities,

[8]NACE 28 consists of 420 firms and 2,856 firm-years; 97.1% of its products are classified as differentiated and these account for 94.3% of total revenue. NACE 10 contributes 395 firms and 2,405 firm-years; 55.1% of products are differentiated, and differentiated goods only generate 29.9% of revenue.

fixed costs, and shock process parameters[9]. All parameters are estimated separately for the high- and low-differentiation sectors. The wage rate w is normalised to 1. The discount factor β is pre-determined at 0.975 based on Slovenia's harmonised long-term interest rates (LTIR) as a proxy for the risk-free rate over the sample period 2009-2021, calculated as $\beta = 1/(1+r)$ where $r = 0.025568$ is the average monthly yield expressed in decimals. The mass of potential entrants m is determined endogenously when solving for the stationary distribution to match the observed sector-specific growth rates of count of firms: -0.4% annual growth for high-differentiation (NACE 28) and 2.7% annual growth for low-differentiation (NACE 10), resulting in $m = 0.014$ for high-differentiation and $m = 1.85$ for low-differentiation. This parameter governs the relative weight of entrants in the stationary distribution. The determination of the entrant mass parameter and its translation into discrete numbers of entrants for panel simulation follows a procedure described in Appendix D.

Simulated Method of Moments. I estimate Θ using a simulated method of moments (SMM) approach that minimises the distance between data moments and simulated moments. The estimation uses the identity matrix as the weighting matrix. The model is solved through value function iteration with several computational innovations that make the high-dimensional problem tractable. First, I use an unordered vector representation for $\boldsymbol{\phi}_i^d$ and $\boldsymbol{\phi}_i^f$. Each appeal shock state tracks only the count of products with shock realisations in each discretised bin, rather than shock realisations for each product, reducing the state space from exponential to polynomial in the number of products [10]. Second, I implement a cutoff-based solution algorithm that exploits monotonicity in the policy functions to avoid state-by-state optimisation. These methods enable efficient handling of the large state space induced by multi-dimensional firm heterogeneity. Further technical details are provided in Appendix C.

After solving the policy functions and stationary distribution, I simulate 10,000 firms for 30 periods starting from the stationary distribution, and compute moments from this simulated panel. To efficiently search the parameter space, I generate 2^{13} quasi-random parameter vectors from a Halton sequence sampling, identifying the combination that minimises the objective function. This constitutes the first stage of the multivariate global optimisation algorithm proposed by Arnoud et al. (2019). The estimates could be further refined through a local optimisation step using algorithms like Nelder-Mead Subplex. Parameter combinations with low objective function values cluster around the estimate, indicating the objective function is well-behaved.

[9] In the estimation, I focus on the restricted choice set $x \in \{d, df, 0\}$, ruling out foreign-only operation based on the empirical observation that firms rarely export without serving the domestic market. This restriction is rationalised by setting the domestic market fixed operating cost sufficiently low ($f_d \leq \min_{\varepsilon,\boldsymbol{\phi}}[\pi_d(\varepsilon, \boldsymbol{\phi}^d)]$) and no sunk cost for domestic entry ($\kappa_d = 0$), making foreign-only operation weakly dominated by serving both markets. Formally, with these parameter restrictions, $V(\cdot, df) \geq V(\cdot, f)$ state-by-state, as firms can always achieve at least the foreign-only profit while also capturing domestic profits at minimal additional cost.

[10] The drawback is that I cannot track the product dynamics for each product in the simulation.

Targeted moments. Table 4 collects the targeted moments in estimation. They capture the main features of exporter dynamics and mirror the empirical regularities documented earlier regarding exporter labour-productivity premiums and product concentration.

The data moments reveal several notable sectoral differences. High-differentiation sectors have substantially higher exporter shares (91% vs 48%) than low-differentiation sectors. Export status is more persistent in the high-differentiation sector (export to export: 98% vs 95%), while domestic-only status is more stable in low-differentiation sectors (domestic to domestic: 92% vs 73%). The mean-to-median product scope ratio is higher in the high-differentiation sector (1.68 vs 1.23), indicating a fatter right tail in the scope distribution. In both sectors, exporters maintain a broader product scope than non-exporters. Product concentration is slightly higher in high-differentiation sectors (top product share: 63% vs 61%) despite their higher average scope. The exporter labour productivity premium is notably higher in low-differentiation sectors (0.38 log points vs 0.14), aligning with the empirical finding with the full sample. Finally, market-specific revenue growth volatility (especially foreign) tends to exceed overall revenue volatility, highlighting the importance of distinguishing market-specific dynamics.

All moments are jointly determined by the full parameter vector Θ in simulation, while some moments are naturally more informative about particular parameters. The share of exporters and transition rates between domestic and export status help identify the export entry costs κ_f, overhead costs f_d and f_f, and shock persistence parameters. Product scope and concentration measures (revenue share of top product) computed overall and by export status inform the product-level fixed cost f_p, market-specific demand elasticities, and appeal shock variances through the cost-benefit tradeoff of adding marginal products. The exporter productivity premium helps pin down the balance between export entry costs, demand elasticities and shock processes. Revenue dynamics moments, including standard deviations of one-year and three-year log revenue growth computed overall and separately by market, help identify the persistence and volatility of efficiency and market-specific appeal shocks.

5.2 Estimation Results

Table 5 summarises the estimates of parameters, which reveal the differences between the two sectors. Demand elasticities are lower in the high-differentiation sector, i.e., the elasticity of substitution is lower, with $\eta_d = 2.73$ (vs 4.14 in low-diff) and $\eta_f = 1.79$ (vs 4.18). Fixed costs follow a clear pattern: operating costs and the per-product production cost are higher in the high-differentiation sector ($f_d = 1.57$ vs 0.81, $f_f = 1.43$ vs 1.10, $f_p = 0.31$ vs 0.20), consistent with greater complexity in developing and maintaining differentiated products; the entry cost is substantially higher ($f_e = 17.79$ vs 2.90); and the sunk export-entry cost is lower ($\kappa_f = 2.49$ vs 5.07). Shock processes show that appeal shocks are much more volatile than efficiency shocks, with volatility generally higher in the high-differentiation sector (e.g., $\sigma_{\phi^d} = 0.69$ vs 0.40, $\sigma_{\phi^f} = 0.68$ vs 0.64).

Table 4: Targeted Moments

Moment	High Differentiation		Low Differentiation	
	Model	Data	Model	Data
Exporter share	0.91	0.91	0.55	0.48
Transition D→D	0.82	0.73	0.97	0.92
Transition E→E	0.98	0.98	0.96	0.95
Scope (overall)	0.94	1.68	0.89	1.23
Scope (exporters)	0.96	1.75	1.02	1.57
Scope (non-exporters)	0.64	1.09	0.74	0.97
Top-product share	0.47	0.63	0.38	0.61
Top-product share (exporters)	0.48	0.63	0.40	0.57
Top-product share (non-exporters)	0.28	0.65	0.35	0.65
Exporter LP premium	0.13	0.14	0.22	0.38
1y revenue growth volatility	0.41	0.44	0.50	0.44
Volatility ratio	1.40	1.34	1.48	1.20
1y revenue growth volatility (domestic)	0.49	0.44	0.56	0.45
Volatility ratio (domestic)	1.54	1.34	1.47	1.20
1y revenue growth volatility (foreign)	0.58	0.69	0.49	0.75
Volatility ratio (foreign)	1.35	1.25	1.39	1.44

Note: Model moments are computed from 10,000 simulated firms over 30 periods starting from the stationary distribution. Exporter share*: fraction of firm-years with positive foreign revenue.* Transitions*: D→D (domestic-only to domestic-only) and E→E (exporter to exporter) between t and t+1, conditional on being observed in both years.* Scope*: number of PRODCOM 8-digit products with positive revenue, normalised by the sector median (reported overall and by export status).* Top-product share*: revenue share of the largest product within firm-year (aggregated over markets) for multiproduct firms, averaged across firms; reported overall and by export status.* Exporter LP premium*: difference in median* log(*value added/worker*) *between exporters and non-exporters.* Revenue growth volatility*: standard deviation of 1-year log revenue growth (overall, and separately for domestic/foreign revenues).* Volatility ratio*: standard deviation of 3-year growth divided by the standard deviation of 1-year growth.*

Table 4 also reports the targeted moments in the simulated panel alongside their empirical counterparts. The model reproduces exporter participation and persistence well in both sectors (exporter shares and D→D, E→E transitions). On product scope and concentration, the model delivers the correct ordering, i.e., exporters are more concentrated than non-exporters, and concentration is higher in more differentiated sectors, although it modestly understates the level of top-product concentration and the dispersion of product scope[11]. Finally, the model captures the relative magnitudes of revenue-growth volatility across markets and the 3-year/1-year ratio.

The model also matches the central patterns in labour productivity. In particular, the exporter labour-productivity premium is smaller in the high-differentiation sector and attenuates with product scope among multiproduct firms, consistent with the "star-product" composition mechanism. Table 6 presents a detailed comparison between the model and the data on exporter labour productivity premiums. The decomposition by firm type reveals that the pattern of lower exporter premiums in high-differentiation sectors is primarily driven by multiproduct firms, both in the model (0.137 vs. 0.225) and in the data (0.130 vs. 0.431). This closely matches the empirical finding in Table 2.

The model generates distinct patterns in how labour productivity premiums vary with product scope across sectors. In the high-differentiation sector, the premium decreases substantially as product scope increases (from 0.637 for single-product firms to only 0.112 for firms with 5+ products). This reflects the composition effect: as high-differentiation firms add more products to export, they increasingly include marginal products that lower their average measured labour productivity. In contrast, the low-differentiation sector shows the opposite trend, with premiums slightly increasing with product scope (from 0.199 for single-product firms to 0.247 for firms with 5+ products). This pattern emerges because the stronger selection in less differentiated sectors ensures that only highly profitable products are exported, and such selection becomes even more stringent as firms expand their product scope.

The estimation allows us to disentangle the source of variation in the observed labour productivity. Let θ denote value-added labour productivity. By the law of total variance,

$$\mathrm{Var}(\theta) \;=\; \underbrace{\mathbb{E}\big[\mathrm{Var}(\theta \mid \varepsilon)\big]}_{\text{appeal (within-}\varepsilon)} \;+\; \underbrace{\mathrm{Var}\big(\mathbb{E}[\theta \mid \varepsilon]\big)}_{\text{efficiency (between-}\varepsilon)} \quad ,$$

which holds exactly under the maintained independence between efficiency and appeal processes. Appeal shocks account for 84.1% of the variance in the low-differentiation sector and 93.3% in the high-differentiation sector. This confirms that, in the high-differentiation sector, dispersion in product-market appeal is the dominant source of measured productivity differences.

[11] These gaps are consistent with the current cap on the number of products (J) and a coarse grid for appeal shocks, both of which can be relaxed in future iterations

Table 5: Parameter Estimates

Parameter	**Description**	**High Diff.**	**Low Diff.**
Panel A: Demand Elasticity			
η_d	domestic demand elasticity	2.73	4.14
η_f	foreign demand elasticity	1.79	4.18
Panel B: Fixed Costs			
f_d	domestic operating cost	1.57	0.81
f_f	foreign operating cost	1.43	1.10
f_p	product fixed cost	0.31	0.20
κ_f	export-entry cost	2.49	5.07
f_e	entry cost	17.79	2.90
Panel C: Shock Processes			
ρ_ε	productivity persistence	0.71	0.80
σ_ε	productivity volatility	0.08	0.06
ρ_{ϕ^d}	domestic appeal persistence	0.87	0.73
σ_{ϕ^d}	domestic appeal volatility	0.69	0.40
ρ_{ϕ^f}	foreign appeal persistence	0.61	0.69
σ_{ϕ^f}	foreign appeal volatility	0.68	0.64

Table 6: Exporter Labour Productivity Premiums

Moment	**High Differentiation**		**Low Differentiation**	
	Model	**Data**	**Model**	**Data**
Median Lp Exporter Premium	0.127	0.144	0.223	0.380
Mean Lp Exporter Premium*	0.137	0.156	0.218	0.414
By Firm Type:				
Mean Lp Premium SPF*	0.637	0.263	0.199	0.339
Mean Lp Premium MPF*	0.137	0.130	0.225	0.431
By Product Scope:				
Lp Premium 1-Product*	0.637		0.199	
Lp Premium 2-Product*	0.480		0.200	
Lp Premium 3-4 Products*	0.267		0.239	
Lp Premium 5+ Products*	0.112		0.247	

*Note: Moments marked with * are untargeted in the estimation. Mean LP premium is the difference in mean* $\log($*value added/worker*$)$ *between exporters and non-exporters. SPF: single-product firms; MPF: multi-product firms.*

6 Policy Experiment: Single-Product Tariff Shock

A tariff shock can be product-specific. I use the model to study how raising the tariff on a single product propagates through the multiproduct margin. Under constant markup, a tariff $\tau > 1$ on foreign sales is isomorphic to a downward shift in the foreign-appeal process with complete pass-through. I therefore model the policy as a permanent reduction in the mean of the foreign-appeal process for one product type, with the sector-specific mapping[12]

$$\Delta\mu_f = -\,\eta_f\,(1-\rho_f)\,\log\tau,$$

where η_f is the foreign-market demand elasticity and ρ_f is the persistence of the foreign-appeal AR(1) process, both taken from the sector's estimates. This mapping implies that, for a given τ, a higher η_f and a lower ρ_f translate into a larger decrease in μ_f.

Each firm has J potential products. For the experiment, I single out one treated product type and apply a 50 p.p. increase in its tariff; the remaining $J-1$ products follow the baseline appeal processes. In the simulated panel, the treated type is assigned to one physical product per firm at $t = 0$ and kept fixed thereafter. This assignment allows me to separate the direct response on the treated line from spillovers to the untreated set. I compare the pre-shock baseline steady state with the post-shock steady state. Further details are provided in Appendix E.

Table 7 reports the responses of aggregate outcomes to a single-product tariff shock as percentage changes relative to the pre-shock steady state. In both sectors, the direct effect on the treated product's foreign sales is large. The numbers of active firms, exporters, and active firm-products decline following a tariff increase.

However, in the high-differentiation sector, aggregate outcomes move little: active firms fall by only 1.2% and active firm-products by 4.2%. Fewer firms choose to export in the new steady state, which slightly reduces foreign sales of untreated products; at the same time, domestic sales rise modestly as more firms serve only the home market.

By contrast, the low-differentiation sector experiences sharp declines across all margins: the numbers of active firms, exporters, and active firm-products fall by more than 40%. The treated product's foreign sales drop by 95.9%, and its domestic sales also fall by about half. Spillovers to the untreated set are sizable as well, with both domestic and foreign sales declining by roughly 36%.

[12] Under CES demand, an ad valorem tariff $\tau \geq 1$ can be absorbed into the demand shifter. Demand is $q_t = \phi_t(\tau p_t)^{-\eta}$. Define the adjusted appeal $\tilde{\phi}_t \equiv \phi_t\,\tau^{-\eta}$. If $\log\phi_t = \mu + \rho\log\phi_{t-1} + \nu_t$ and τ is permanently set to a constant from date T onward, then for all $t \geq T$,

$$\log\tilde{\phi}_t = \mu + \rho\log\phi_{t-1} + \nu_t - \eta\log\tau = \mu + \rho(\log\tilde{\phi}_{t-1} + \eta\log\tau) + \nu_t - \eta\log\tau = [\mu - \eta(1-\rho)\log\tau] + \rho\log\tilde{\phi}_{t-1} + \nu_t.$$

Hence the post-shock AR(1) has intercept $\tilde{\mu} = \mu - \eta(1-\rho)\log\tau$, implying the mapping $\Delta\mu_f = -\,\eta_f(1-\rho_f)\log\tau$.

Part of the difference reflects the tariff-to-appeal mapping: for a given tariff increase, the implied reduction in foreign appeal is larger when η_f is higher. To gauge this, Table E1 contrasts a 50 p.p. tariff increase in the high-differentiation sector with a 24.66 p.p. increase in the low-differentiation sector that delivers the same $\Delta\mu_f$ in both sectors. The qualitative and quantitative conclusions are essentially unchanged: the low-differentiation sector remains much more responsive because the shock works primarily through extensive margins (firm exit and product dropping), which the smaller tariff cannot undo.

Intuitively, the low-differentiation sector is more responsive because selection on the efficiency-appeal combination is tighter. A decline in foreign appeal for the treated product can push that product line below its zero-profit cutoff and, via the loss of export scope, induce firm exit. These results highlight that product differentiation shapes not only export sorting patterns but also the transmission of trade shocks in a multiproduct setting.

Table 7: Steady-State Percentage Changes from One-Product Tariff Shock

Outcome	**High Diff.**	**Low Diff.**
Overall Sales	0.2%	-41.8%
Foreign Sales of Treated Product	-59.9%	-95.9%
Domestic Sales of Treated Product	1.5%	-49.8%
Foreign Sales of Untreated Products	-3.6%	-36.2%
Domestic Sales of Untreated Products	1.4%	-36.9%
Average Active Firms	-1.2%	-40.3%
Average Active Exporters	-11.7%	-44.0%
Average Active Firm-products	-4.2%	-42.6%

Note: Entries report percentage changes relative to the pre-shock steady state. High Diff. and Low Diff. are the representative sectors defined in Section 5.

7 Conclusion

I develop and estimate a model in which multiproduct firms choose their product mix and export participation based on cost efficiency and product appeal. The model accounts for the export-sorting patterns observed across sectors with different degrees of product differentiation in Slovenian data. Product differentiation shapes export status through a star-product mechanism, and the endogenous product mix affects measured firm-level labour productivity through a composition effect. The estimated model allows me to quantify direct and spillover effects of a single-product tariff shock and to compare aggregate responses across low- and high-differentiation sectors. In low-differentiation sectors, the shock induces large contractions via product dropping and firm exit, with spillovers to the treated product's domestic sales and to untreated products in both markets. By contrast, in high-differentiation sectors, despite a sharp decline in the treated product's foreign sales, aggregate outcomes change little and spillovers are negligible. This implies that resilience to tariff shocks differs markedly by the degree of product differentiation. Although the analysis is conducted at the sectoral level, it has implications for evaluating aggregate responses to changes in trade frictions in economies with diverse sectoral compositions.

The paper has a few limitations. Under monopolistic competition and CES preferences, all firms in a sector charge similar markups, and firms are restricted to sector-consistent products, so they face the sector's markup on all lines. Relaxing these assumptions would require deviating from monopolistic competition or adopting non-CES preferences, and endogenising firms' choices of product lines with different demand elasticities or different markups. These are beyond the scope of this paper but are promising directions for future research. Finally, in the policy counterfactuals, the partial-equilibrium framework abstracts from general-equilibrium feedback through prices and wages arising from firm exit; incorporating these channels may attenuate the partial-equilibrium results. I leave this quantification to future work.

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Appendix

A Additional Empirical Results

A.1 Robustness: TFPR as Productivity Measure

Table A1 replicates the analysis from Table 2 using TFPR as the productivity measure instead of labour productivity. Log TFPR is calculated as the residual: $\ln(\text{TFPR}) = \ln(Y) - \alpha_k \ln(K) - \alpha_l \ln(L) - \alpha_m \ln(M)$, where Y is total revenue, K is capital stock, L is employment, and M is raw materials costs. The input elasticities $(\alpha_k, \alpha_l, \alpha_m)$ are obtained from CompNet's 10th vintage estimates at the Nace 2-digit level for Slovenia, averaged across years (2009-2021) to ensure stability. I use the cost-share specification, which derives revenue elasticities from markups and output elasticities using country-sector-year median cost shares, covering all 55 manufacturing sectors. The correlation between log TFPR and log labour productivity in the sample is approximately 0.74, confirming that both measures capture similar productivity patterns.

The results confirm the main findings: exporter TFPR premiums decline with product differentiation, and this pattern is driven primarily by multiproduct firms. The premium in low-differentiation sectors (0.158 log points) exceeds that in high-differentiation sectors (0.076 log points) by 0.082 log points, significant at the 1% level. For multiproduct firms specifically, the low-differentiation premium (0.183 log points) exceeds the high-differentiation premium (0.074 log points) by 0.109 log points (significant at the 1% level), closely mirroring the labour productivity results.

A.2 Robustness: Cross-Country Analysis

To further investigate whether the relationship between product differentiation and exporter labour productivity premiums is specific to the Slovenian context or represents a more general pattern, I conduct a cross-country analysis using CompNet data. For each country-sector-year combination, I calculate the average productivity premium as the difference in log labour productivity between exporters and non-exporters. I then regress these premiums on sector differentiation categories using the following specification:

$$\text{Premium}_{c,s,t} = \beta_0 + \beta_1 \text{ModDiff}_s + \beta_2 \text{HighDiff}_s + \boldsymbol{\alpha}_c + \boldsymbol{\gamma}_t + \varepsilon_{c,s,t}$$

where $\text{Premium}_{c,s,t}$ is the exporter labour productivity premium in country c, sector s, and year t; ModDiff_s and HighDiff_s are indicator variables for moderately and highly differenti-

Table A1: Exporter TFPR Premiums by Differentiation Level

Differentiation	**All Firms**	**SPF**	**MPF**
All Sectors	0.091[a]	–	–
	(0.009)		
Low (baseline)	0.158[b]	0.087[c]	0.183[c]
	(0.016)	(0.028)	(0.019)
Moderate	0.048[b]	0.008[c]	0.075[c]
	(0.029)	(0.059)	(0.025)
High	0.076[b]	0.076[c]	0.074[c]
	(0.011)	(0.015)	(0.015)
Low > Moderate	0.110^{***}	0.079	0.109^{***}
	(0.033)	(0.065)	(0.031)
Low > High	0.082^{***}	0.011	0.109^{***}
	(0.019)	(0.031)	(0.024)

Note: Exporter premiums in $\log(TFPR)$. *Standard errors clustered at the firm level in parentheses. Superscripts indicate the source regression for the reported linear combination:* [a] *= baseline exporter specification;* [b] *= differentiation interaction;* [c] *= triple interaction. The lower-panel comparisons are one-sided tests of* $H_0 : \beta_{Low} - \beta_{Moderate} \leq 0$ *and* $H_0 : \beta_{Low} - \beta_{High} \leq 0$. *Significance levels:* *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$, † $p < 0.10$.

ated sectors (with low differentiation as the baseline); and $\boldsymbol{\alpha}_c$ and $\boldsymbol{\gamma}_t$ represent country and year fixed effects, respectively. I estimate four variants of this model: without fixed effects, with country fixed effects only, with year fixed effects only, and with both country and year fixed effects.

The analysis uses the CompNet 10th vintage "20e" weighted sample, which includes only firms with 20 or more employees to accommodate country-specific reporting thresholds and improve cross-country comparability[13]. Population weights are applied to recover the underlying firm population, constructed using Eurostat SBS data on the number of firms by year, 2-digit industry, and size class. The sample covers 16 European countries: the Czech Republic, Germany, Denmark, Estonia, Spain, France, Croatia, Hungary, Lithuania, Malta, the Netherlands, Poland, Portugal, Romania, Slovenia, and Slovakia.

[13] More detials are available at `https://www.comp-net.org/data/10th-vintage`

Table A2: Mean Labour Productivity Premium by Product Differentiation, cross country-sector

	(1)	(2)	(3)	(4)
Intercept (Low Differentiation)	0.467***			
	(0.037)			
Moderately Differentiated	-0.181***	-0.091*	-0.132**	-0.092*
	(0.048)	(0.036)	(0.046)	(0.036)
Highly Differentiated	-0.218***	-0.133***	-0.175***	-0.133***
	(0.039)	(0.028)	(0.037)	(0.028)
country fe	No	Yes	No	Yes
year fe	No	No	Yes	Yes
Observations	3,642	3,642	3,642	3,642
R2	0.113	0.433	0.216	0.441
Adj. R2	0.112	0.431	0.211	0.435

Note: The dependent variable is the average log labour productivity premium between exporters and non-exporters within each country-sector-year cell. Standard errors are clustered at the country level. $*p < 0.05$, $**p < 0.01$, $***p < 0.001$.

B Derivation of Labour Productivity Expression

In this section, I derive the value-added labour productivity expression in equation (18). I use the notation as in the simplified two-product export-only model, but it can be easily extended to the version with J products with both domestic and foreign sales.

In the export-only setting with normalised wage $w = 1$, the revenue for an active product j is proportional to the normalised appeal $x_j \equiv \phi_j/\phi^*(\varepsilon, \eta) \geq 1$:

$$r_j = \eta f_p \, x_j, \qquad x_j \geq 1.$$

This relationship follows from the zero-profit condition at the cutoff.

Value added at the product level equals revenue minus variable and fixed production costs:

$$va_j \equiv \frac{r_j}{\eta} - f_p = \frac{\eta f_p \, x_j}{\eta} - f_p = f_p(x_j - 1), \qquad x_j \geq 1.$$

Product-level labour consists of the per-product fixed cost f_p and variable labour for production $q_j/\varepsilon = (\eta-1)r_j/\eta$, written as:

$$\ell_j = f_p + \frac{\eta-1}{\eta} r_j = f_p\big(1+(\eta-1)x_j\big).$$

For a firm with $N \in \{1,2\}$ active products (each with $x_j \geq 1$), firm-level value added aggregates as:

$$VA = \sum_{j:x_j\geq 1} va_j = \sum_{j:x_j\geq 1} f_p(x_j-1) = f_p \sum_{j:x_j\geq 1} (x_j-1) = f_p\, N\,(\bar{x}-1),$$

where $\bar{x} \equiv \frac{1}{N}\sum_{j:x_j\geq 1} x_j$ is the average normalised appeal across active products.

Similarly, firm-level labour is:

$$L = \sum_{j:x_j\geq 1} \ell_j = \sum_{j:x_j\geq 1} f_p\big(1+(\eta-1)x_j\big) = f_p\, N\big(1+(\eta-1)\bar{x}\big).$$

Combining these expressions, value-added labour productivity is:

$$\theta^{VA}(\bar{x};\eta) \equiv \frac{VA}{L} = \frac{f_p\, N\,(\bar{x}-1)}{f_p\, N\big(1+(\eta-1)\bar{x}\big)} = \frac{\bar{x}-1}{1+(\eta-1)\bar{x}}, \qquad \bar{x} \geq 1.$$

To verify that measured productivity increases with average appeal, take logs and differentiate:

$$g(\bar{x},\eta) \equiv \log\theta^{VA} = \log(\bar{x}-1) - \log\big[1+(\eta-1)\bar{x}\big].$$

For $\bar{x} > 1$:

$$\begin{aligned}\frac{\partial g}{\partial \bar{x}} &= \frac{1}{\bar{x}-1} - \frac{\eta-1}{1+(\eta-1)\bar{x}} \\ &= \frac{\eta}{(\bar{x}-1)\big[1+(\eta-1)\bar{x}\big]} > 0.\end{aligned}$$

The inequality holds because $\bar{x} > 1$ and $\eta > 1$.

C Computational Methods for Model Solution

The high-dimensional nature of the state space with firm efficiency, product-market-specific appeal shocks across multiple products, and previous market participation requires specialized techniques to make the problem tractable. I explain the computational approach used to solve the model in this section.

C.1 Efficient State Space Representation

The model assumes each firm has J products, each with a product-specific appeal shock for both domestic and foreign markets. To keep the state space compact, I use an unordered-vector representation of the product-level appeal shock distribution. Instead of tracking every product–shock combination (which would require N_ϕ^J states for J products), I track only the number of products whose shocks fall in each discretised bin. This is sufficient for determining product-selection policies, because a product is produced whenever its appeal shock exceeds the relevant cutoff. Given the distribution over bins, the product portfolio aggregates straightforwardly to firm-level revenue and profit.

C.1.1 Unordered Vector Representation

For each possible shock value bin $i = 1, \ldots, N_\phi$, I count how many products have shocks in that bin. Each state is represented as $(n_1, n_2, \ldots, n_{N_\phi})$ where n_i is the count of products with shocks in bin i, with the constraint $\sum_i n_i = J$ ensuring all products are accounted for.

This reduces the state space from N_ϕ^J to $\binom{J+N_\phi-1}{N_\phi-1}$, which scales polynomially rather than exponentially with J. The problem is equivalent to distributing J identical objects into N_ϕ distinct bins.

C.1.2 Transition Matrix Computation

For the AR(1) product-level shock processes, we need to calculate transition probabilities between states. Computing the probability of transitioning from state $(n_1, n_2, \ldots, n_{N_\phi})$ to state $(m_1, m_2, \ldots, m_{N_\phi})$ involves three steps:

1. For each product in bin l, calculate the probability of transitioning to bin k as $P_\phi[l, k]$
2. Compute the average transition probability to each bin k, weighted by the proportion

of products in each source bin:

$$p_k = \sum_{l=1}^{N_\phi} \frac{n_l}{J} \cdot P_\phi[l, k]$$

3. Use the multinomial probability formula to compute the probability of observing the target distribution:

$$P[(n_1, n_2, ..., n_{N_\phi}) \to (m_1, m_2, ..., m_{N_\phi})] = \frac{J!}{m_1! m_2! \dots m_{N_\phi}!} \cdot p_1^{m_1} \cdot p_2^{m_2} \cdot \dots \cdot p_{N_\phi}^{m_{N_\phi}}$$

This method works identically for both domestic and foreign shock distributions, regardless of their respective grid sizes.

C.1.3 Joint Distribution Approximation

When firms can sell in both markets, we need the joint distribution of domestic and foreign product shocks to evaluate profits. The state space only tracks marginal distributions, i.e., how many products have shocks in each bin for each market separately, but not which specific products have which pair of shocks. Let $\mathbf{n}^d = (n_1^d, \dots, n_{N_{\phi_d}}^d)$ and $\mathbf{n}^f = (n_1^f, \dots, n_{N_{\phi_f}}^f)$ represent these counts, where $\sum_i n_i^d = \sum_j n_j^f = J$.

Under the assumption that domestic and foreign shocks are independent, I construct the joint array for domestic and foreign appeal shocks.

Define probability vectors $\mathbf{p}^d = \mathbf{n}^d / J$ and $\mathbf{p}^f = \mathbf{n}^f / J$. Independence implies a probability matrix given by the outer product:

$$\mathbf{P} = \mathbf{p}^d (\mathbf{p}^f)^\top, \quad P_{ij} = p_i^d p_j^f, \quad i = 1, \dots, N_{\phi_d},\ j = 1, \dots, N_{\phi_f}$$

The corresponding expected counts are $\tilde{\mathbf{M}} = J\,\mathbf{P}$, an $N_{\phi_d} \times N_{\phi_f}$ rectangular array.

Since products are indivisible units, I map $\tilde{\mathbf{M}}$ to an integer matrix $\mathbf{M}$ that sums exactly to J. A robust allocation rule handles two cases:

- *Degenerate small-mass case:* If $\max_{i,j} \tilde{M}_{ij} < 1$, I assign all J products to the cell with highest probability.
- *Normal case:* Otherwise, I start from the floor $\mathbf{M}^{(0)} = \lfloor \tilde{\mathbf{M}} \rfloor$, scale proportionally, and round: $\mathbf{M}^{(1)} = \text{round}(J\,\mathbf{M}^{(0)} / \sum_{i,j} M_{ij}^{(0)})$. Any residual difference $\Delta = J - \sum_{i,j} M_{ij}^{(1)}$ is added to the cell with largest mass.

This procedure respects independence while ensuring exactly J products are allocated. The resulting integer matrix provides the joint counts needed to compute profits when a firm serves both markets. This approach is particularly important because a product that is marginal in each market separately may still be profitable when sold in both if the sum of its normalized revenues exceeds one.

C.2 Cutoff-Based Value Function Iteration

I implement an efficient cutoff-based solution method that exploits monotonicity in efficiency and appeal shocks to avoid costly state-by-state optimization.

C.2.1 Monotonic Policy Functions

For each previous market choice x_{-1}, I define action regions that map state variables to optimal market choices:

$$\tau_{x_{-1}} : (\varepsilon, \phi^d, \phi^f) \mapsto x^* \in \{d, f, df, 0\}$$

The key economic intuition is that payoffs are monotonically increasing in efficiency (ε) for each action choice, given fixed appeal shocks and previous market participation. This monotonicity ensures that for any fixed combination (ϕ^d, ϕ^f, x_{-1}), each action region is a connected threshold region on the ε-line.

C.2.2 Storing Cutoff Points

Instead of storing the optimal action for every state, I only store the critical ε cutoff values where the firm is indifferent between different actions. For each combination of shock distributions and previous market state, I store:

1. **Cutoffs**: Grid indices where optimal action changes (plus a boundary point)
2. **Actions**: Corresponding optimal actions for each segment

The key relationship is that `actions[i]` is the optimal action for the region starting at `cutoffs[i]` (inclusive) and ending before `cutoffs[i+1]` (exclusive).

This cutoff-based representation dramatically reduces memory requirements and speeds up policy function evaluation during simulation and equilibrium computation.

D Stationary Distribution and Simulation

When computing the stationary firm distribution and simulating the panel data, the core challenges include finding the appropriate entrant mass parameter m and generating realistic firm dynamics that preserve the equilibrium properties. This sections explains the approach in details.

D.1 Determining Entrant Mass and Stationary Firm Distribution

Potential entrants make entry decisions by comparing their expected value to the fixed entry cost. For each combination of appeal shock distributions (i_{ϕ^d}, i_{ϕ^f}), there exists a efficiency threshold above which entry is profitable:

$$\varepsilon^*(i_{\phi^d}, i_{\phi^f}) = \min\{i_\varepsilon : V_{\text{entrant}}(i_\varepsilon, i_{\phi^d}, i_{\phi^f}) \geq f_e\}$$

A key parameter governing firm dynamics is the mass of potential entrants m. When solving the model, I determine the entrant mass m that delivers a target industry growth rate g_{target}. In steady state, the total mass of firms evolves according to:

$$\text{Mass}_{t+1} = \text{Mass}_t \times (1 + g_{\text{target}})$$

I use a bisection search algorithm to find the m in the range $[10^{-6}, 3.0]$ that achieves this target growth rate. The growth rate for a given m depends on the balance between entrant inflows and firm exits:

$$g = \frac{\text{Entrant Inflow} - \text{Exit Outflow}}{\text{Current Mass}}$$

where the entrant inflow is:

$$\text{Entrant Inflow} = m \sum_{\phi^d, \phi^f} \sum_{\varepsilon \geq \varepsilon^*(\phi^d, \phi^f)} \mu_0(\varepsilon) \mu_{\phi^d}(\phi^d) \mu_{\phi^f}(\phi^f)$$

This creates a monotonic relationship between m and g that enables efficient bisection search.

The stationary distribution is computed through iterative application of the transition operator until convergence. Starting from a uniform distribution over all states, the algorithm:

1. Updates the distribution using the transition kernel and policy functions as (14)
2. Tracks changes in total mass to detect steady-state mass growth
3. Checks dual convergence criteria:
   - Distribution error: $\max |\mu_{t+1} - \mu_t| < 10^{-6}$
   - Mass growth error: $|\Delta_{t+1} - \Delta_t| < 10^{-5}$ where $\Delta_t = \sum \mu_t - 1$
4. Normalizes the distribution to sum to 1 after each iteration

Both convergence criteria must be satisfied to ensure the stationary equilibrium.

D.2 Panel Data Simulation with Entrants

The converged stationary distribution provides the foundation for firm panel simulations. A key challenge is translating the continuous entrant mass m into discrete numbers of entrants each period. The simulation determines the number of entrants per period through the following steps:

Step 1: Calculate Probability of Successful Entry. First, compute the mass of potential entrants who would successfully enter given their efficiency and shock draws:

$$\text{mass above cutoff} = \sum_{i_\varepsilon} \sum_{i_{\phi^d}} \sum_{i_{\phi^f}} \mu_0(i_\varepsilon) \cdot \mu_{\phi^d}(i_{\phi^d}) \cdot \mu_{\phi^f}(i_{\phi^f}) \cdot \mathbf{1}_{i_\varepsilon \geq \varepsilon^*(i_{\phi^d}, i_{\phi^f})}$$

Step 2: Convert Growth Rate to Discrete Entrants. The number of discrete entrants per period is calculated to maintain the target growth rate:

$$\text{number of entrants} = \text{round}\left(\frac{g_{\text{mass}} \cdot N_{\text{active}} + N_{\text{exiters}}}{\text{mass above cutoff}}\right)$$

where g_{mass} is the growth rate from the stationary distribution iteration, N_{active} is the current number of active incumbent firms, and N_{exiters} is the number of firms that exited this period.

Then for each potential entrant:

1. Draw efficiency and appeal shock distributions from the entrant efficiency distribution

2. Apply the entry condition: check if $i_\varepsilon \geq \varepsilon^*(i_{\phi^d}, i_{\phi^f})$
3. If the entry condition is met, determine the market choice using $g^*_{entrant}(i_\varepsilon, i_{\phi^d}, i_{\phi^f})$

This approach preserves the economic relationships established in the stationary equilibrium while generating realistic firm dynamics for panel analysis.

E Policy Experiment: Implementation Details

The policy experiment is product-specific, so I need to track the appeal realisations of the treated product type—something the unordered-vector state representation does not record. To implement this experiment, I expand the state space to include (i) the treated product's domestic and foreign appeal states and (ii) the unordered appeal distributions of the $J-1$ untreated products, together with firm efficiency and the previous period's market status. For the baseline and post-shock regimes, I solve for the stationary distribution and simulate 10,000 firms for 30 periods, starting from the stationary distribution, with three periods of burn-in. Shock realisations are held fixed across regimes for reproducibility.

Given the simulated panel, metrics are computed year by year and then averaged. I report percentage changes of aggregate outcomes relative to the pre-shock steady state. Table E1 contrasts two policy experiments that ensure the same foreign-appeal shift $\Delta\mu_f$[14].

Table E1: Steady-State Percentage Changes from One-Product Tariff Shock

Outcome	**High Diff.**	**Low Diff.**
Overall Sales	0.2%	-38.6%
Foreign Sales of Treated Product	-59.9%	-87.1%
Domestic Sales of Treated Product	1.5%	-40.4%
Foreign Sales of Untreated Products	-3.6%	-33.9%
Domestic Sales of Untreated Products	1.4%	-34.5%
Average Active Firms	-1.2%	-37.8%
Average Active Exporters	-11.7%	-41.3%
Average Active Firm-products	-4.2%	-37.4%

Note: Entries report percentage changes relative to the pre-shock steady state. The low-differentiation sector's tariff is chosen so that the implied foreign-appeal shift $\Delta\mu_f$ matches that in the high-differentiation sector.

[14] In the high-differentiation sector, with $\eta_f = 1.79$ and $\rho_f = 0.61$, a 50 p.p. tariff rise implies a decrease in μ_f of 0.2842; in the low-differentiation sector, we therefore set $\tau = \exp\big(0.2842 \,/\, [\eta_f^{low} \times (1-\rho_f^{low})]\big) \approx 1.2466$, i.e., a 24.66 p.p. tariff rise.