AA222 - Intro to MDO
Spring Quarter 2012
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STANFORD AERONAUTICS & ASTRONAUTICS

PROBLEM SET 1 **Due Date: April 16th, 2012, 5pm**

Question 1

Let $f(x) = -x + \gamma x^2$, where $\gamma > 0$ is a parameter. For this question you will investigate three values for the parameter: $\gamma = 0.5, 10$, and 10^4.

1. Starting from $x_0 = 0$, use a backtracking line-search — with initial step length $\alpha_0 = 1$ and reduction parameter $\rho = 0.5$ — to find a step that satisfies the sufficient decrease condition with $\mu_1 = 10^{-4}$. How many function evaluations are required for the three different values of γ?

2. How does the performance of the backtracking line-search compare with Newton's method for the different values of γ?

Remark: this question illustrates the impact of poor scaling, a problem that often affects practical optimization problems.

Question 2

Consider a rectangular wing of span b and chord c. Its planform area is thus $S = bc$ and its aspect ratio is $A = b^2/S$. The drag of this wing can be approximated as,

$$C_D = kC_f \frac{S_{wet}}{S} + \frac{C_L^2}{\pi A e} \tag{1}$$

The first term corresponds to the parasite drag. C_f is the skin friction coefficient, which for a fully turbulent boundary layer can be approximated as,

$$C_f = \frac{0.074}{Re^{0.2}}. \tag{2}$$

Here, the Reynolds number ($Re = \rho V c/\mu$) is based on the wing chord. k is the form factor, which accounts for the effects of pressure drag.

The second term in Equation (1) is the induced drag, where e is the Oswald efficiency factor. The lift coefficient C_L and the wing planform area S are to be kept constant. The values for all the constants are listed in Table 1.

1. Write the total drag coefficient as a function of A.

2. Minimize C_D with respect to A using:

   (a) The golden section method

   (b) A line search method that satisfies sufficient decrease. (Bonus: A line search that satisfies the strong Wolfe conditions.)

   Converge the solutions to 6 significant digits.

3. Discuss the relative performance of these two methods. Try different starting points/intervals and compare convergence rates, number of iterations and any other metrics you find suitable.

Quantity	Value	Units	Description
ρ	1.23	$\mathrm{kg/m^3}$	density of air
μ	17.8×10^{-6}	kg/(m sec)	viscosity of air
V	35	m/s	airspeed
S	11.8	$\mathrm{m^2}$	planform area
S_{wet}	$2.05S$	$\mathrm{m^2}$	wing wetted area
k	1.2		form factor
C_L	0.3		lift coefficient
e	0.96		Oswald efficiency factor

Table 1: Flow conditions and other fixed variables