*AA222 - Intro to MDO*

Spring Quarter 2012

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STANFORD AERONAUTICS & ASTRONAUTICS

**PROBLEM SET 2** **Due Date: April 30th, 2012, 5pm**

## Question 1

Consider an aircraft with the rectangular wing described in the previous assignment. The drag coefficient for the aircraft is given as,

$$C_D = \frac{0.03062702}{S} + kC_f\frac{S_{\text{wet}}}{S} + \frac{C_L^2}{\pi A e} \tag{1}$$

Where first term corresponds to the parasite drag of the fuselage and the other two terms are for the wing, as given in the previous assignment.

The weight of the wing can be approximated as

$$W_w = 45.42S + 8.71\times 10^{-5}\frac{N_{\text{ult}}b^3\sqrt{W_0 W}}{S(t/c)} \tag{2}$$

where $N_{\text{ult}}$ is the ultimate load factor considered in the structural sizing, and $t/c$ is the average thickness to chord ratio. If the weight of the aircraft excluding the wing is $W_0$, then the total weight of the aircraft is,

$$W = W_0 + W_w \tag{3}$$

Note that the weight of the wing (2) depends on itself through $W$, so you will need to iterate to find the correct weights.

Once you have the aircraft weight, you can find the required lift coefficient for level flight,

$$C_L = \frac{2W}{\rho V^2 S} \tag{4}$$

Then you can compute the lift to drag ratio $L/D = C_L/C_D$ and the total drag for level flight is

$$D = \frac{W}{L/D} \tag{5}$$

The optimization problem is to minimize $D$ with respect to the aspect ratio $A$ and the wing area $S$. The values for all the constants new to this assignment are listed in Table 1; refer to the previous assignment for the remaining constants.

| Quantity | Value | Units | Description |
|---|---|---|---|
| $W_0$ | 4940 | N | weight of the aircraft excluding the wing |
| $N_{\text{ult}}$ | 2.5 | – | ultimate load factor |
| $t/c$ | 0.12 | – | average thickness to chord ratio |

Table 1: Fixed parameters

1. Write a function that computes $D$, as a function of $A$ and $S$, to the maximum numerical precision that can be achieved with double precision arithmetic.

2. Minimize $D$ with respect to $A$ and $S$ by programming the following algorithms:

   (a) Steepest descent method

   (b) Conjugate gradient method

   (c) A quasi-Newton method (BFGS, DFP, or SR1)

   Converge both the objective and design variables to 6 significant digits. Show the contour plots with the path of the major iteration points. Note the number of major iterations as well as the number of evaluations in the line search. Show a convergence plot that compares the convergence rate of the various algorithms. Discuss and explain the relative performance of these three methods.

Note: To compute the gradient of $D$ with respect to $A$ and $S$, you can use any of the methods discussed in classs. Although you are not required to use the adjoint approach here, I encourage you to think about how it could be applied.

## Question 2

Question 1 involved only 2 design variables. Study the effect of increased problem dimensionality for each of the three methods implemented in question 1 by minimizing the quadratic equation (6) for $n = 2, 10, 50$.

$$f(x) = \sum_{i=1}^{n} \frac{1}{i} x_i^2. \tag{6}$$

Discuss your results.