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FINAL PROJECT **Due Date: June 13th, 2012, 5pm**

Problem Definition

For this project you will optimize an aircraft wing in more detail. We consider two disciplines: aerodynamics and structures. The objective is to minimize the weight of fuel for a 9,500 nautical mile mission by changing the twist distribution (or incidence) γ and the wing spar thickness distribution t, while making sure that the structure does not fail during a 2.5g symmetric maneuver, i.e.,

$$\begin{aligned} \text{minimize} \quad & W_{\text{fuel}}(t,\gamma) = (W_s + W_0)\left[\exp\left(R\frac{c}{V}\frac{D}{L}\right) - 1\right] \\ \text{w.r.t} \quad & t, \gamma \\ \text{subject to} \quad & \sigma \leq \sigma_{\text{yield}} \quad (L_{2.5} = 2.5W_{\text{initial}}), \end{aligned} \tag{1}$$

where σ is the vector of stresses in the wing structure. The fuel weight is derived from the Breguet range equation. In this equation, R is the range, V is the cruise speed, L is the lift, and D is the drag. W_0 is the empty weight of the aircraft excluding the weight of wing structure, W_s is the weight of wing structure that depends on the sizing of the wing spar. We also assume that all the fuel is used at the end of mission.

$$\begin{aligned} W_{\text{initial}} &= W_s + W_0 + W_{\text{fuel}} \\ W_{\text{final}} &= W_s + W_0 \end{aligned}$$

N.B.: In the objective equation equation, c is the thrust specific fuel consumption (TSCF), which is the weight of fuel burned per unit thrust per unit time. Thus the units end up being 1/time if the units for weight and thrust are the same, which is not always the case. Make sure the TSFC number you get is in the correct dimensions. You should be able to do a sanity check for the fuel weight you obtain against existing aircraft data.

The aircraft must adjust its angle of attack, α, so that lift matches its weight. Thus, the angle of attack is also a variable in the aerostructural analysis that is updated so that the lift constraint is satisfied.

This aerostructural problem is highly coupled and its governing equations are:

$$R(u,\Gamma,\alpha,t,\gamma) = \begin{bmatrix} Ku - f \\ A\Gamma - v \\ L - W \end{bmatrix} = 0 \tag{2}$$

where K is the stiffness matrix, f is the vector of external forces, A is the aerodynamic influence coefficient matrix, and v is a vector of panel boundary conditions. The state variables that must be solved for by the aero-structural analysis are the finite-element displacements, u, and the panel circulations, Γ, and the angle of attack, α.

The lift must equal the weight of the aircraft during flight. To simplify the analysis, we assume that the lift is equal to the initial weight, i.e., $L - W_{\text{initial}} = 0$. The root twist is fixed at zero degrees.

The governing equations (2) are solved using Newton's method which for this coupled system can be written as:

$$\begin{bmatrix} K & -\frac{\partial f}{\partial \Gamma} & 0 \\ -\frac{\partial v}{\partial u} & A & -\frac{\partial v}{\partial \alpha} \\ 0 & \frac{\partial L}{\partial \alpha} & 0 \end{bmatrix} \begin{bmatrix} u \\ \Gamma \\ \alpha \end{bmatrix} = \begin{bmatrix} f - Ku \\ v - A\Gamma \\ W_{\text{initial}} - L \end{bmatrix}, \tag{3}$$

where an initial guess of 0 is assumed, since this system is linear, and thus it converges to the solution in one iteration.

Code Description

The provided code performs the aerostructural analysis described above and is based on the following components:

`aeopt.m` : Main routine. The problem parameters are set here and the aerostructural analysis function , `aeroln.m`, is called.

`aeroln.m` : Main aerostructural analysis routine. Computes the forces, displacements and solves the aerodynamics and structures simultaneously using a Newton method. Since both models are linear, the Newton iteration converges in a single step. You can use some of the code in this function to separate the aerodynamic and structural analysis for question 2 below.

`aeroAIC.m` : Computes the matrix of lifting line aerodynamic influence coefficients, A.

`stiffness.m` : Computes the stiffness matrix for the structure, K.

The aerodynamic analysis is an inviscid panel code: the wing (with specified planform) is divided into a number of spanwise stations or panels, `nPanel`. A horseshoe vortex of a given strength (or circulation, Γ) is associated with each panel by solving the relevant governing equations. The analysis is linear ($A\Gamma = v$). With the wing geometry, twist distribution (specified at each panel) and the total angle of attack, the method is able to compute the total lift and drag (with a fixed viscous drag markup, `Dfriction`, at the end of `totalDrag.m`).

A beam-element finite element code for the computation of both the bending and torsional deflections of the main structure of the wing, which is composed of a single, thin-walled spar of circular cross-section. By providing aerodynamic forces and moments, the resulting deflected shape can be calculated. Just like in the aerodynamics, the structure is decomposed into a number of elements, `nElem`, in the spanwise direction. For simplicity (to avoid interpolation), we use `nPanel=nElem`. The structural analysis consists in solving $Ku = f$ and then computing the stresses as a function of u.

You can consider this code a black box whose inputs are:

1. Planform geometry
2. Spanwise jig twist distribution
3. Spanwise spar diameter and thickness distribution
4. Markups for non-spar weight and friction drag

The outputs are the range, the angle of attack needed to match the target lift, the spanwise lift and elastic twist distributions, the drag, the weights, and the stress distribution on the spar.

Assignment

1. Find all the necessary parameters and range for a jet aircraft of your choosing. Make sure to use consistent units. You can look at some of the sample values in the provided code. Start by making sure that the code gives reasonable results for total weight and fuel weight and other outputs.

2. Solve the optimization problem (1) using the MDF approach. You may use `fmincon` or another off-the-shelf optimizer. Clearly state all your choices, approach, and results of the optimization with graphs of the results and the convergence history as appropriate. How do these results compare with what you have learned about optimal aerodynamics? Why is the optimum load distribution not elliptical?

3. Solve the same problem using one of the other approaches that we have discussed in class (IDF, SAND, CO, or BLISS). Discuss your approach to the decomposition, the creation of new variables and constraints, the cost, and convergence properties of your optimization. Compare with the results of the MDF approach and discuss the relative merits of the two methods.