A Comparison of Optimization Approaches to the Single Airport Ground Hold Problem

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In this paper, we present the the first survey and comparison of optimization approaches to the single airport ground hold problem, a computational approach to air traffic flow management. We introduce four models from the literature and discuss the motivation behind their development, compare their input and output variables, and document their performance on recorded data from San Francisco International Airport.

Nomenclature

c_g	cost of one unit of ground delay
c_a	cost of one unit of airborne holding
p_q	probability of scenario q
W_{qi}	number of aircraft in airborne holding in interval i under scenario q
M_{qi}	landing capacity in interval i under scenario q
S_i	number of flights planned to arrive in interval i
S_{si}	number of flights originally scheduled to depart in stage s and arrive in interval i
D_i	number of flights originally scheduled to arrive in interval i not exempt from ground delay
E_i	number of flights originally scheduled to arrive in interval i exempt from ground delay
d_f	planned departure interval of flight f
a_f	planned arrival interval of flight f
X_i	number of flights to delay in interval i
X_{ij}	number of flights originally scheduled to arrive in interval i rescheduled to arrive in interval j
X_{qsij}	number of flights originally scheduled to arrive in interval i rescheduled to arrive in interval j in stage s under scenario q
X_{qfi}	if flight f is assigned to arrive in or before interval i under scenario q
Y_{qfi}	if flight f is released to depart in or before interval i under scenario q
Q	number of landing capacity scenarios
T	number of discrete time intervals
F	number of flights
N_m	number of scenarios in branch m
b_m	beginning interval of branch m
e_m	end interval of branch m
n	number of branches that contain more than one scenario

Subscript

q	landing capacity scenario number
i	time interval
j	planned arrival interval
s	stage
f	flight number
m	branch number

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I. Introduction

Ground delay programs (GDPs) are one of the most effective tools used by the Federal Aviation Administration (FAA) in strategic air traffic flow management. In a GDP, flights are held on the ground at their origin airports when the number of arriving aircraft is expected to exceed landing capacity at the destination airport. This initiative is meant to reduce airborne holding, aircraft waiting to land in vicinity of the destination airport. The problem of finding a strategy to minimize the total cost of ground delay and airborne holding is called the ground hold problem (GHP). The GHP can be divided into two subdomains, the multi-airport ground hold problem (MAGHP) and single airport ground hold problem (SAGHP). In the MAGHP, delays are distributed across a network of airports and downstream effects must be considered. This paper addresses the SAGHP, where delays are determined for one destination airport at a time.

We will focus on stochastic formulations of the SAGHP, as forecasted landing capacity is dependent on weather and thus inherently uncertain. Deterministic formulations are addressed by Bertsimas and Stock[2] and Hoffman and Ball.[3] Stochastic models can be categorized as static, where decisions are made once at the beginning of the planning horizon, or dynamic, where decisions are made as updated information regarding landing capacity becomes available. In this paper, we introduce four models, two static and two dynamic. In Section II we introduce the models, in Section III we compare their performance on flight schedules and historical landing capacity distributions from San Francisco International Airport (SFO) and in Section IV we conclude the paper with insights gained from the comparison.

II. Optimization Approaches to the SAGHP

The goal of optimization approaches to the SAGHP is to find a strategy that minimizes a combination of ground delay and airborne holding over a discrete number of time intervals, typically 15-60 minutes in duration. Ground delays and airborne holding are weighted by constants in the objective function, c_g and c_a, where $c_a > c_g$ due to the higher relative cost of airborne holding. The dynamics of the system can be described as a problem of supply and demand. There is a demand that must be met in the number of arriving aircraft, which we assume to be deterministic, and a finite supply of landing capacity. Since landing capacity is dependent on weather and other uncertain factors, it must have a stochastic representation. We represent one possible evolution of landing capacity as a scenario, or a sequence of landing capacities for each interval over the entire time horizon, and introduce the variable p to denote the probability of its occurrence. All landing capacity scenarios can therefore be stored compactly in the variable M, where M_{qi} is the landing capacity in time interval i under scenario q. The process of constructing landing capacity scenarios from weather forecasts and historical data is an active area of research, and Liu and Hansen[5] and Provan, Cook and Cunningham[8] have introduced two possible approaches.

The models introduced here can be classified as either static or dynamic. Static models assign ground delays only at the beginning of the time horizon, simulating the way GDPs are implemented in practice. Dynamic models, on the other hand, assign delays as time progresses and therefore benefit from the ability to rule out possible landing capacity scenarios. In general we can expect dynamic models to outperform static models, with the downside of being more difficult to implement in practice.

A. Richetta-Odoni 1993

The first model we present was introduced by Richetta and Odoni in 1993.[9] The model is formulated as an integer program that yields static solutions, therefore delays are assigned to flights once at the beginning of the time horizon. The decision variables are X_{ij}, the number of flights scheduled to arrive in interval i reassigned to arrive in interval j.

Objective Function

$$\min \sum_{i=1}^{T} \sum_{j=i+1}^{T+1} c_g (j-i) X_{ij} + \sum_{q=1}^{Q} p_q \sum_{i=1}^{T} c_a W_{qi} \tag{1}$$

Constraints

$$\sum_{j=i}^{T+1} X_{ij} = S_i \text{ for } i = 1, \ldots, T, \ \forall q \tag{2}$$

$$\sum_{j=1}^{i} X_{ji} - W_{qi} + W_{qi-1} \leq M_{qi} \text{ for } i = 1, \ldots, T+1, \ \forall q \tag{3}$$

$$W_{q0} = W_{qT+1} = 0 \ \forall q \tag{4}$$

$$X_{ij}, W_{qi} \in \mathbb{Z}_{\geq 0} \tag{5}$$

An advantage of the objective function in (1) is that c_g may be a function instead of a constant, $c_g = c_g(j-i)$, which would allow us to impose a higher cost for flights assigned more delay and thus increase equity, i.e. encouraging delays to be distributed across many flights as opposed to a small subset. Constraint (2) ensures that all flights rescheduled to arrive must arrive within the time horizon and constraint (3) implies that airborne holding results when arrival demand exceeds landing capacity. The constraint matrix for this model is not totally unimodular in the general case, therefore solutions obtained by linear programming relaxation are not guaranteed to be integer.

B. Richetta-Odoni 1994

Richetta and Odoni's second model for the SAGHP was introduced in 1994.[10] This model is also an integer progam and an improvement over the 1993 model. It is dynamic, and solutions are planned over Q stages in the form of a sequential optimization problem. The decision variables are X_{qsij}, the number of flights scheduled to arrive in i rescheduled to arrive in j in stage s under landing capacity scenario q.

Objective Function

$$\min \sum_{q=1}^{Q} p_q \{ \sum_{s=1}^{Q} \sum_{i=t_s+1}^{T} \sum_{j=i+1}^{T+1} c_g(j-i) X_{qsij} + \sum_{i=1}^{T} c_a W_{qi} \} \tag{6}$$

Constraints

$$\sum_{j=i}^{T+1} X_{qsij} = S_{si} \text{ for } i = t_s + 1, \ldots, T, \ \forall s, \ \forall q \tag{7}$$

$$\sum_{s:t_s<i} \sum_{j=t_s+1}^{i} X_{qsji} - W_{qi} + W_{qi-1} \leq M_{qi} \text{ for } i = 1, \ldots, T+1, \ \forall q \tag{8}$$

$$X_{ssij} = X_{s+1sij} = \ldots = X_{Qsij} \text{ for } i = t_s + 1 \ldots, T, \ i \leq j \leq T+1, \ s = 1, \ldots, Q-1 \tag{9}$$

$$W_{q0} = W_{qT+1} = 0, \tag{10}$$

$$X_{qsij}, W_{qi} \in \mathbb{Z}_{\geq 0} \tag{11}$$

The major improvement of this model over its 1993 predecessor is that this model is dynamic. Delays are assigned over Q stages, therefore the solutions take advantage of the ability to wait for updated landing capacity information. Richetta and Odoni present an illustrative example in their paper of the advantages of a dynamic model for the SAGHP.[10] This model also inherits the advantage of the 1993 model in that c_g may be a function of the length of delay assigned. Constraint (7) ensures that all flights rescheduled to arrive must arrive within the time horizon and constraint (8) implies that airborne holding results when arrival demand exceeds landing capacity. The additional constraint (9) ensures that solutions from this model only utilize the conditional probability of future landing capacities, i.e. we do not have perfect information regarding which scenario is being realized if there is more than one scenario remaining with nonzero probability.

C. Ball et al. 2003

The third model was introduced by Ball, Hoffman. Odoni and Rifkin in 2003.[1] This model is a simplification of the Richetta-Odoni 1993 model, introduced as a response to the Collaborative Decision Making (CDM) initiative, which encourages shared decision-making power between the FAA and the airlines. This model is static and based on integer programming. The decision variables are X_i, the number of flights allowed to arrive in interval i.

Objective Function

$$\min \sum_{i=1}^{T} c_g X_i + \sum_{q=1}^{Q} p_q \sum_{i=1}^{T} c_a W_{qi} \tag{12}$$

Constraints

$$A_i + X_i - X_{i-1} = S_i \text{ for } i = 1, \ldots, T+1 \tag{13}$$

$$A_i - W_{qi} + W_{qi-1} \leq M_{qi} \text{ for } i = 1, \ldots, T+1, \ \forall q \tag{14}$$

$$X_0 = X_{T+1} = W_{q0} = W_{qT+1} = 0 \tag{15}$$

$$X_i, W_{qi}, A_i \in \mathbb{Z}_{\geq 0} \tag{16}$$

Although this model seems to have no advantages in comparison to the Richetta-Odoni models, the motivation for its development dates back to the introduction of CDM in 1998. The consequence of this initiative for the SAGHP was that the Richetta-Odoni models were rendered obsolete, as they produce outputs not consistent with the outputs needed for new operating procedures under CDM. This model was developed to fit within the new paradigm. Since the model assumes no control over individual flights, the delay cost, c_g, cannot be a function of the number of delays assigned to flights. Constraint (13) ensures that all flights rescheduled to arrive must arrive within the time horizon and constraint (14) implies that airborne holding results when arrival demand exceeds landing capacity. The constraint matrix is totally unimodular, therefore integer solutions to the linear programming formulation are guaranteed.

1. *Ball et al. model as a special case of the Richetta-Odoni 1993 model*

The Ball et al. model can be shown to be a special case of the Richetta-Odoni 1993 model under certain modeling simplifications.[4] Equations (17) and (18) show expressions relating A_i and X_i from the Ball et al. model to the X_{ij} variables in the Richetta-Odoni 1993 model.

$$A_i = \sum_{j=1}^{i} X_{ji} \text{ for } i = 1, \ldots, T \tag{17}$$

$$X_i = \sum_{j=1}^{i} \sum_{k=i+1}^{T+1} X_{jk} \text{ for } i = 1, \ldots, T \tag{18}$$

The second term of both objective functions is the same. Assuming that c_g is a constant, the relationship between the first terms in the objective functions is shown in (19). The left hand sides of constraints (2) and (13) are shown to be equivalent in (20). Similarly, we can show that constraints (3) and (14) are equivalent by substituting (17).

$$\begin{aligned} \sum_{i=1}^{T} \sum_{j=i+1}^{T+1} c_g (j-i) X_{ij} &= \sum_{i=1}^{T} \sum_{j=1}^{k} \sum_{k=i+1}^{T+1} c_g (j-i) X_{jk} \\ &= \sum_{i=1}^{T} c_g X_i \end{aligned} \tag{19}$$

$$
\begin{aligned}
A_i + X_i - X_{i-1} &= \sum_{j=1}^{i} \sum_{k=i+1}^{T+1} X_{jk} - \sum_{j=1}^{i-1} \sum_{k=i}^{T+1} X_{jk} + \sum_{j=1}^{i} X_{ji} \\
&= \sum_{k=i+1}^{T+1} X_{ik} - \sum_{j=1}^{i-1} X_{ji} + \sum_{j=1}^{i} X_{ji} \\
&= \sum_{k=i+1}^{T+1} X_{ik} + X_{ii} \\
&= \sum_{k=i}^{T+1} X_{ik}
\end{aligned} \tag{20}
$$

By showing the equivalence of constraints in equations (19) and (20), we can therefore state that the solution to the Ball et al. model will be exactly equal to the solution found by the Richetta-Odoni 1993 model with the additional constraints in (17) and (18).

2. Extension of Ball et al. model for sequential decision making

An extension of the Ball et al. model was proposed that takes into account flights that have departed.[7] This allows the model to be rerun sequentially, which would increase its performance in practice. The extension is formulated by replacing the constraints (13) and (14) with (21) and (22) below, where amendments to the original formulation are highlighted in bold.

$$A_i + X_i - X_{i-1} = \mathbf{D_i} \text{ for } i = 1, \dots, T+1 \tag{21}$$

$$A_i + \mathbf{E_i} - W_{qi} + W_{qi-1} \leq M_{qi} \text{ for } i = 1, \dots, T+1, \ \forall q \tag{22}$$

D. Mukherjee-Hansen 2007

The fourth model was introduced by Mukherjee and Hansen in 2007.[6] It is formulated as a binary integer program. The model is not only dynamic, but also allows for revision of assigned delays as updated capacity information becomes available. The decision variables are X_{qfi}, if flight f is planned to arrive in or before interval i under scenario q.

Objective Function

$$\sum_{q=1}^{Q} p_q \{ \sum_{f=1}^{F} \sum_{i=a_f}^{T+1} c_g (i - a_f)(X_{qfi} - X_{qfi-1}) + \sum_{i=1}^{T} c_a W_{qi} \} \tag{23}$$

where

$$X_{qfi} = \begin{cases} 1, & \text{if flight } f \text{ is assigned to arrive in or before interval } i \\ 0, & \text{otherwise} \end{cases} \quad \forall i, \forall f, \forall q \tag{24}$$

$$Y_{qfi} = \begin{cases} 1, & \text{if flight } f \text{ is released for departure in or before interval } i \\ 0, & \text{otherwise} \end{cases} \quad \forall i, \forall f, \forall q \tag{25}$$

Constraints

$$X_{qfi-1} \leq X_{qfi} \text{ for } i = 1, \dots, T+1, \ \forall f, \ \forall q \tag{26}$$

$$X_{qfT+1} = 1 \ \forall f, \ \forall q \tag{27}$$

$$\sum_{f=1}^{F}(X_{qfi} - X_{qfi-1}) - W_{qi} + W_{qi-1} \leq M_{qi} \text{ for } i = 1, \ldots, T+1, \ \forall q \tag{28}$$

$$Y_{qfi} = \begin{cases} X_{qfi+a_f-d_f}, & \text{if } i + a_f - d_f \leq T \\ 1, & \text{otherwise} \end{cases} \forall i, \forall f, \forall q \tag{29}$$

$$Y_{qfi}^{S_1^m} = \cdots = Y_{qfi}^{S_2^m} = \cdots = Y_{qfi}^{S_{N_m}^m} \text{ for } m = 1 \ldots n, \ b_m \leq i \leq e_m, \ \forall f \tag{30}$$

$$W_{q0} = W_{qT+1} = 0 \tag{31}$$

$$X_{qfi}, Y_{qfi} \in \{0,1\}, W_{qi} \in \mathbb{Z}_{\geq 0} \tag{32}$$

The Mukherjee-Hansen dynamic model has the advantages that 1) it assigns delays to individual flights, which allows for maximum control of the system dynamics, 2) it incorporates the ability to revise assigned delays using updated landing capacity information, and 3) the ground delay cost c_g can be a function of the delay length (similar to the Richetta-Odoni models) which allows for the consideration of equity. The model utilizes the idea of a scenario tree, a way to represent the evolution of landing capacity over time. An example of one such tree is shown in Figure 1. When a branching point is reached, the model acquires new information about which of the branches is being realized.

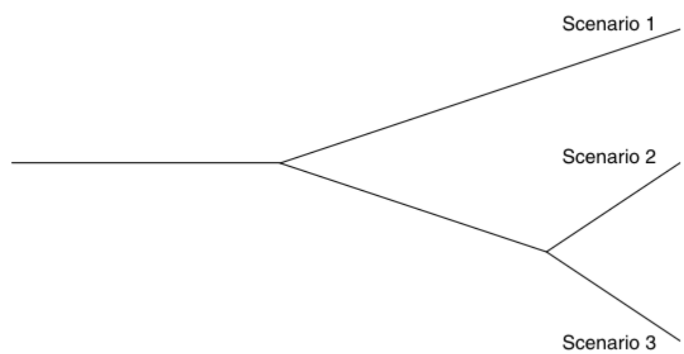

Figure 1. Scenario tree

Constraints (26) and (27) ensure that all flights rescheduled to arrive must arrive within the time horizon and constraint (28) implies that airborne holding results when arrival demand exceeds landing capacity. Constraint (29) relates the X and Y variables. Constraint (30) ensures that solutions from this model only utilize the conditional probability of future landing capacities using the scenario tree representation.

III. Experimental Results

A. Experimental Setup

The four models were tested on real data for San Francisco International Airport. The flight schedules for one week from August 2005 was taken from the Aviation System Performance Metrics database. An example arrival schedule is shown in Figure 2. The landing capacity profile distribution was constructed from historical data in the summer of 2005.[7] For SFO, the landing capacity is often reduced to 30 in the morning due to fog, then increases to 60 in the afternoon when the fog clears. An example of this behavior and a cumulative distribution of fog clearance time is shown in Figure 3. A constant value of c_g was chosen to allow comparison of all models, therefore the Richetta-Odoni and Ball et al. models performed identically. We will henceforth refer to these models as Static.

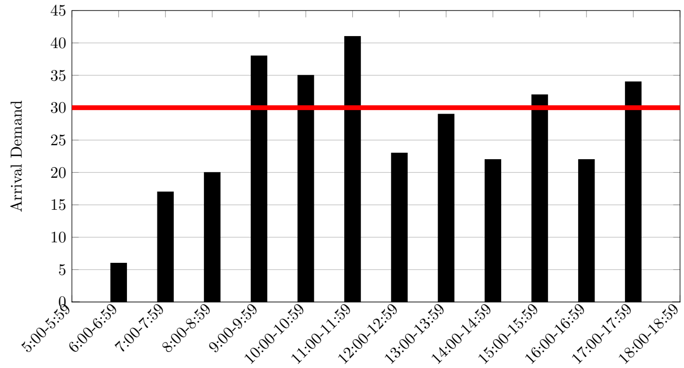

Figure 2. Arrival Demand for August 7, 2005

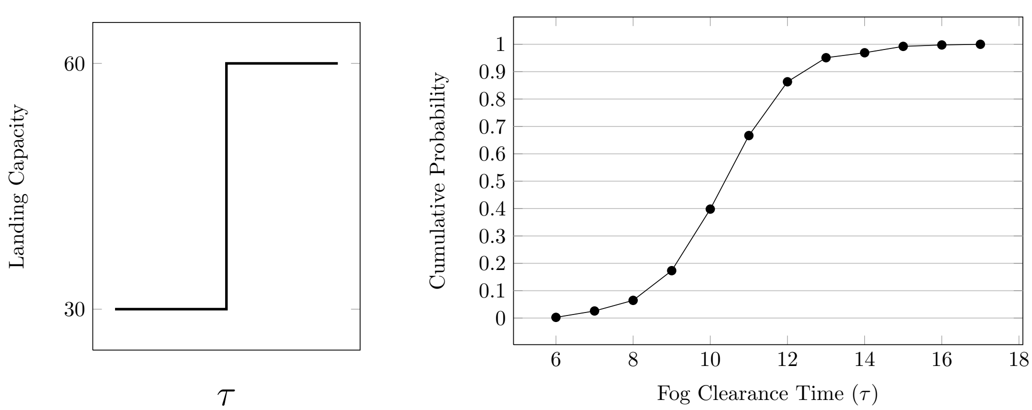

Figure 3. Landing capacity profile (left) and fog clearance time distribution (right)

B. Results

The models were tested under the following conditions:

- One day (August 7, 2005) with a varying cost ratio c_a/c_g
- Over seven days with a constant cost ratio of 2

The total units of ground delay and airborne holding for a single day, as well as the total cost as a percentage of the cost given perfect information is shown in Figure 4 (left). Figure 4 (right) shows the units of delay and holding for a single day with a varying cost ratio. Figure 5 is a comparison of total cost as a percentage of the perfect information cost for seven consecutive days in August 2005.

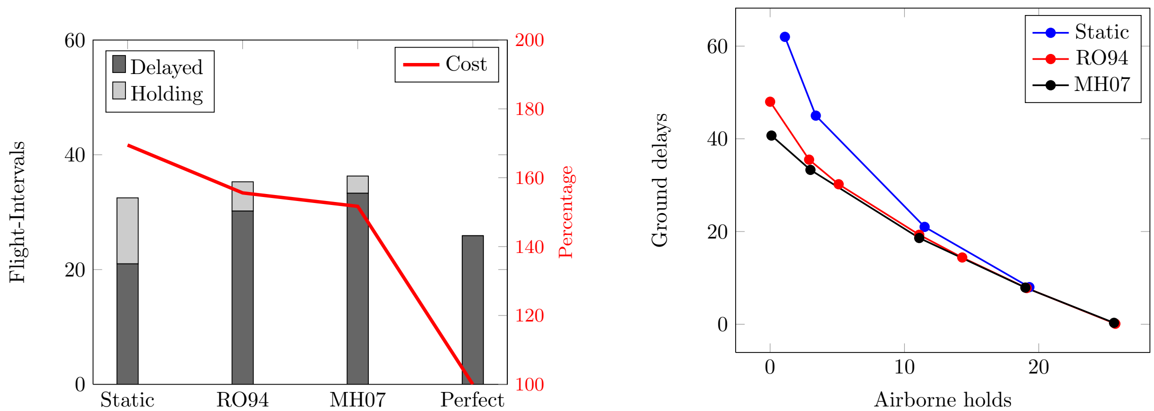

Figure 4. Holding, delayed and cost for August 7, 2005

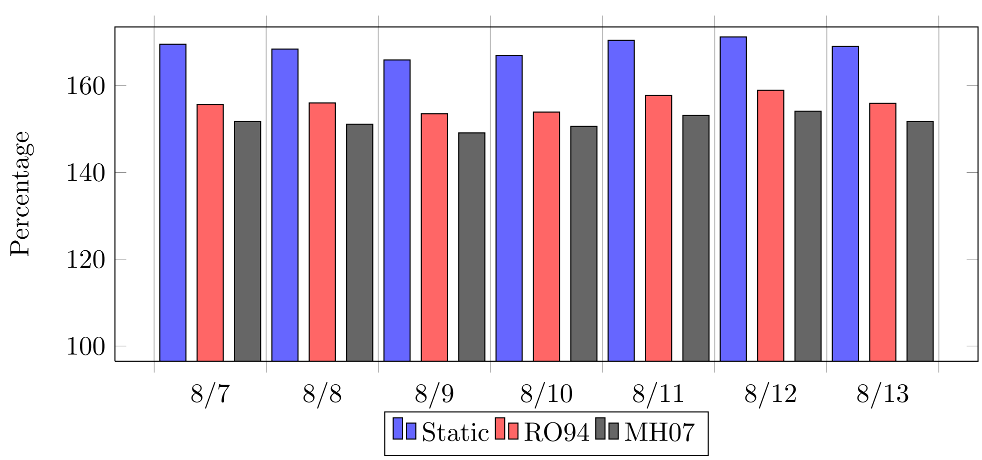

Figure 5. Cost for one week in August 2005

IV. Conclusion

Clearly, the dynamic models convey some advantage in ground delay planning. Figure 4 (left) suggests that the performance difference between the dynamic and static models is greater than that between the dynamic models themselves. Figure 4 (right) shows that this performance gap widens as the cost ratio, c_a/c_g, increases and Figure 5 shows that this advantage is not dependent on the day. We therefore present the following conclusions:

- Optimization models for the SAGHP share similar objectives and constraints
- The Ball et al. 2003 model is a simplification of the Richetta-Odoni 1993 model
- Dynamic models have a significant performance advantage over static models
- Models that move toward delaying individual flights perform better in theory, but are more difficult to implement in practice

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