*AA222 - Intro to MDO*
Spring Quarter 2012
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STANFORD AERONAUTICS & ASTRONAUTICS

**PROBLEM SET 3** **Due Date: May 14th, 2012, 5pm**

1. Consider the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & 2x_1^3 - 3x_1^2 - x_2^2 \\ \text{w.r.t.} \quad & x_1, x_2 \\ \text{subject to} \quad & x_1^2 + x_2^2 \leq 2 \\ & x_1 + x_2 = 0 \end{aligned}$$

   (a) Write the Lagrangian for this problem.
   (b) By hand, find all the points that satisfy the first-order KKT conditions
   (c) Determine which points are minima. Distinguish local from global minima if necessary.

   Hint: For part (c), eliminate any maxima by showing the second-order necessary conditions are violated, i.e. that the Hessian of the Lagrangian is negative-definite in the feasible space. To classify the remaining KKT points, show that a perturbation along the equality constraint and away from the inequality constraint boundary increases the function (think about why the second-order sufficient conditions cannot be applied to the remaining KKT points in this problem).

2. Consider the aircraft from the previous assignments. In this assignment we also minimize the drag with respect to aspect ratio and wing area, but we add a stall constraint. This imposes an inequality constraint on the wing area that depends on the maximum lift coefficient that can be achieved, and a desired speed that is low enough for a safe landing. The constraint is as follows,

$$C_{L,\max} = \frac{W}{1/2\rho V_{\min}^2 S} \Rightarrow S \geq \frac{2W}{\rho V_{\min}^2 C_{L,\max}}. \tag{1}$$

   The values for the constants new to this assignment are listed in Table 1; refer to the previous assignment for the remaining constants and drag computation.

| Quantity | Value | Units | Description |
|---|---|---|---|
| $V_{\min}$ | 22 | m/s | desired landing speed |
| $C_{L,\max}$ | 2.0 | – | maximum lift coefficient with flaps down |

Table 1: Fixed parameters

   Minimize $D$ with respect to $A$ and $S$ subject to the stall constraint (1) by programming the following algorithms:

   (a) The logarithmic barrier method
   (b) An SQP method that handles *equality* constraints.

   Compare the relative performance of these algorithms and discuss their advantages and disadvantages.