Case 12 - Craggier National Park

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Outline

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Problem Statement

- Craggier National offers trophy hunting services to tourists with 4 classes of packages that includes a total of 15 animal species
- CNP owns a facility that provides a maximum of 250 AU bulk graze, 150 AU concentrate graze, and 100 AU browse graze for the trophies
- Although the nature of trophy hunting is killing wild animals, CNP still tries to limit the number of annual trophy hunting to preserve nature and sustain the business
- The management is looking for the optimal mix of their current packages as well as possible new packages that will generate more revenues for CNP

Assumptions

- For the base model, there is no new packages
- The existing packages are the hunter packages in Exhibit 3
- There is no mating or natural death occuring when the animals are in CNP
- Each animal consumes a year of food annually, even if it is killed on Day 1
- The base model will be solving for the mix for the existing packages that generate the most profit; the sensitivity analysis will be solving for the best combination of new and existing packages that generate the most profit. Column generation will be used to solve for new package development

Model Development - Variables

Let x_i, i = 1..3 be our decision variables which represent the number of package i purchased

The data variables are:

R_j representing returns ($/AU) for animal j

a_j representing AU for animal j

F_{jk} representing required food type k for animal j

max_k representing the maximum available type k food

Δ_{ij} for the inclusion of animal j in package i

Model Development - Model

$$\max \sum_{i=1}^{3} x_i \sum_{j=1}^{15} \Delta_{ij} a_j r_j$$

$$s.t. \sum_{i=1}^{3} \sum_{j=1}^{15} \Delta_{ij} x_i \leq 300$$

$$\sum_{i=1}^{3} \sum_{j=1}^{15} x_i \Delta_{ij} a_j f_{jk} \leq max_k \;\; \forall\, k = 1..3$$

$$x_1 \leq 10$$

$$x_2 \leq 10$$

$$x_3 \leq 8$$

$$x_1, x_2, x_3 \; int$$

Base Solution

x1	x2	x3	z
10	10	5	142,077.3

- As we can see, x3 in the base solution has not reached the constraint, which is 8. Also, none of the 4 constraints are tight, so there’s definity a lot of room for improvement

Sensitivity Analysis - Column Generation

- We use column generation to find the optimal packages
- First, we solve the orginal model, but in the LP format.
- Then find the shadow prices for the base solution and implement to the subproblem model
- The subproblem model gives the most profitable package, which we will implement it back into the base model for a updated base model
- Another iteration begins and we stop when the newly generated optimal package is the same as the previous one
- The subproblem model, which was basically devised by Prof. Frances, is on the next page

Sensitivity Analysis - Column Generation

Let Δ_j be our decision variables which represent the inclusion of animal j

y_l, l = 1..4 also be our decision variables which represent the satisfied class

The data variables are:

π_m, m=1..4 representing the shadow prize for constraint m

R_j representing returns ($/AU) for animal j

a_j representing AU for animal j

F_{jk} representing required food type k for animal j

Sensitivity Analysis - Column Generation

$$\max \sum_{j=15}^{15} a_j R_j \Delta_j - \pi_1 \sum_{j=15}^{15} \Delta_j - \pi_2 \sum_{j=15}^{15} a_j F_{j1} \Delta_j - \pi_3 \sum_{j=15}^{15} a_j F_{j2} \Delta_j - \pi_4 \sum_{j=15}^{15} a_j F_{j3} \Delta_j$$

$$s.t. \sum_{j=15}^{15} \Delta_j \geq 5y_1$$

$$\sum_{j=15}^{15} \Delta_j \geq 10y_2$$

$$\sum_{j=15}^{15} \Delta_j \geq 14y_3$$

$$\Delta_{10} + \Delta_{11} + \Delta_{15} \geq 2y_4$$

$$\Delta_1 + \Delta_2 + \Delta_9 = y_1 + 2y_2 + 3y_3$$

$$y_1 + y_2 + y_3 + y_4 = 1$$

$$\Delta_j \; is \; binary \; \forall \; j = 1..15$$

Sensitivity Analysis - Column Generation

- The new package belongs to class 2
- We add this as package 4 to our original model
- The original model is slightly modified, specifically
  - The objective function includes the revenue genereated by package 4
  - Each of the constraints include food from trophies in package 4
  - The total 300 trophies constraint is unchanged
  - The maximum sales of package 4 is 10
- Solve the base model in LP, we obtain

x1	x2	x3	x4	z
10	7.142857	0	10	184,029.2

1 BIG	1
2 BIG	1
3	1
4	1
5	1
6	0
7	1
8	1
9 BIG	0
10	0
11	1
12	0
13	1
14	1
15	0

Sensitivity Analysis - Column Generation

- The new shadow price is not the same as the previous one, o we continue our iteration
- The next package belongs to Class 1, and we add it back to our base model
- Solve the base model in LP, we get

x1	x2	x3	x4	x5	z
10	3.571429	0	10	10	207,567

1 BIG	0
2 BIG	1
3	1
4	0
5	1
6	0
7	1
8	0
9 BIG	0
10	0
11	0
12	0
13	0
14	1
15	0

Sensitivity Analysis - Column Generation

- The shadow price of the 5-package model is the same as the 4-package model, so we stop. The current packages are the optimal mix
- We solve the base model as a IP, and get

x1	x2	x3	x4	x5	z
8	5	0	10	10	204,327.6

- Therefore, the most profitable package combination is the original package 1 and 2, as well as our new packages 4 and 5.

Sensitivity Analysis - Relax the Trophy Constraint

- Why was there only 2 iterations?
- Since we are allowed to sell only 300 trophies, with each package featuring roughly 10 animals, we can only sell less than 30 packages in total
- Each package can only be sold a maximum 10 times, so we would have 3 or 4 types of packages that are the most profitable, which is the case in our optimal solution
- We want to explore the difference in optimal packages if we relax the constraint from 300 to 500

Sensitivity Analysis - Relax the Trophy Constraint

- Same column generation model
- We get a similar package for the first iteration
- Also belongs to class 2, but with a different combination
- Implement this package back to the original model

x1	x2	x3	x4	z
10	9.538056	0	10	199,990.4

- The new shadow price is different from the previous one
- So we continue

1 BIG	1
2 BIG	1
3	1
4	1
5	1
6	1
7	1
8	1
9 BIG	0
10	0
11	1
12	1
13	1
14	0
15	0

Sensitivity Analysis - Relax the Trophy Constraint

- For iteration 2, we get a class 1 package
- The combination is also different
- Back to the base model,

x1	x2	x3	x4	x5	z
10	7.320538	0	10	10	234,178.4

- New shadow price = the previous one, solution optimal
- Re-run the model as an IP, we get the following:

x1	x2	x3	x4	x5	z
10	7	1	10	10	232,922.9

1 BIG	0
2 BIG	1
3	0
4	0
5	0
6	1
7	1
8	1
9 BIG	0
10	0
11	1
12	1
13	1
14	1
15	1

Conclusion

- For current packages, the optimal mix is 10 package #1, 10 package #2, and 5 package #3, which yields $142,077.3 revenue
- The most optimal packages under current constraints would be the current package #1 and #2, and a new class 2 package with animals 1-5, 7-8,11, and 13-14, and a new class 1 package with animals 2,3,5,7,14.
- The optimal mix would be 8 package #1, 5 package #2, and 10 of each new packages.
- The 300 annual trophy constraint is the most restraining one. If we can bypass that, we will be able to generate more revenues with similar new packages, but the increase in revenue is not very significant