Case 13 - Maximizing Team Diversity

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Outline

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Problem Statement

- There are 96 students in our Praxis class this year
- From past experiences, teams with most diversity in terms of language, leadership style, and learning style have the optimal performance
- Students speak 16 different languages, and all of them speak English
- There are 4 different leadership styles and a combination of 16 learning styles
- Each student has two lists of students that they like and dislike, and each list has up to five students
- We need to maximize diversity of teams and accommodate as many students in terms of their preferred and disliked teammates

Assumptions

- Since 96 is divisible by 4, we assume 24 groups of 4 students. The case of 32 teams of 3 will be discussed in the sensitivity analysis. All teams have the same number of students for fairness.
- For teams with students that speak more than 1 language, the non-repetitive languages are added to the language diversity of the team; this number can exceed 4
- Our primary objective is to maximize team diversity, so each different language, learning style, and leadership style gives 1 point to the objective function
- In sensitivity analysis, we will add 0.25 for each preferred teammate in a team and deduct 0.25 for each disliked teammate to the objective function

Model Development

$Let\ x_{ij}\ be\ if\ student\ i\ is\ in\ group\ j$

$U_{kj}\ be\ if\ teammembers\ in\ team\ j\ speak\ language\ k$

$V_{lj}\ be\ if\ teammembers\ in\ team\ j\ has\ leadership\ style\ l$

$W_{mj}\ be\ if\ teammembers\ in\ team\ j\ has\ learning\ style\ m$

$A_{ik}\ be\ if\ student\ i\ speaks\ language\ k$

$B_{il}\ be\ if\ student\ i\ has\ leadership\ style\ l$

$C_{im}\ be\ if\ student\ i\ has\ learning\ style\ m$

$All\ variables\ are\ binary$

Model Development

$$\max \sum_{j=1}^{24}\sum_{k=1}^{16} U_{kj} + \sum_{j=1}^{24}\sum_{l=1}^{4} V_{lj} + \sum_{j=1}^{24}\sum_{m=1}^{16} W_{mj}$$

s.t.

$$\sum_{i=1}^{96} A_{ik} \times x_{ij} \geq U_{kj} \; \forall \, j = 1 \ldots 24, k = 1 \ldots 16$$

$$\sum_{i=1}^{96} B_{il} \times x_{ij} \geq V_{lj} \; \forall \, j = 1 \ldots 24, l = 1 \ldots 4$$

$$\sum_{i=1}^{96} C_{im} \times x_{ij} \geq W_{mj} \; \forall \, j = 1 \ldots 24, m = 1 \ldots 16$$

$$\sum_{i=1}^{96} x_{ij} = 4 \; \forall \, j = 1 \ldots 24$$

$$\sum_{j=1}^{24} x_{ij} = 1 \; \forall \, i = 1 \ldots 96$$

Base Solution

- Using AMPL, our optimal objective function value is 272, and full answer:

26,49,66,75	6,47,62,65	7,9,42,69	15,32,58,77
12,13,22,51	3,5,80,95	68,78,81,86	1,40,84,90
4,31,48,96	2,52,57,88	19,21,87,89	34,55,56,71
10,11,45,85	8,14,24,27	38,39,60,74	20,53,73,79
28,35,37,50	16,18,30,70	17,63,82,93	23,29,59,91
40,43,44,92	33,36,54,64	25,46,61,76	67,72,83,94

Sensitivity Analysis - Like & Dislikes

- We also need to accommodate students' preferred and disliked teammates, so we add more variables and implement these to the objective functions and constraints
- This is not the primary focus, so we will give each preferred teammate a 0.25 bonus and each disliked teammate a 0.25 penalty to the objective function. This number is arbitrary but we feel it is justified since we do not want the likes and dislikes to overshadow the importance of diversity while we also don't want to ignore the students' feelings

Model Development - Decision Variables

$$Let\ x_{ij}\ be\ if\ student\ i\ is\ in\ group\ j$$

$$U_{kj}\ be\ if\ teammembers\ in\ team\ j\ speak\ language\ k$$

$$V_{lj}\ be\ if\ teammembers\ in\ team\ j\ has\ leadership\ style\ l$$

$$W_{mj}\ be\ if\ teammembers\ in\ team\ j\ has\ learning\ style\ m$$

$$Y_{ii'j}\ be\ if\ student\ i\ and\ student\ i'\ are\ both\ in\ team\ j\ (used\ for\ like)$$

$$Z_{ii'j}\ be\ if\ student\ i\ and\ student\ i'\ are\ both\ in\ team\ j\ (used\ for\ dislike)$$

$$All\ decision\ variables\ are\ binary$$

Model Development - Data Variables

A_{ik} *be if student i speaks language k*

B_{il} *be if student i has leadership style l*

C_{im} *be if student i has learning style m*

$D_{ii'}$ *be if student i prefers student i'*

$E_{ii'}$ *be if student i dislikes student i'*

All data variables are binary

Model Development - Model

$$\max \sum_{j=1}^{24}\sum_{k=1}^{16} U_{kj} + \sum_{j=1}^{24}\sum_{l=1}^{4} V_{lj} + \sum_{j=1}^{24}\sum_{m=1}^{16} W_{mj} + \frac{1}{4}\sum_{i=1}^{96}\sum_{i'=1}^{96}\left(D_{ii'} \times \sum_{j=1}^{24} Y_{ii'j}\right) - \frac{1}{4}\sum_{i=1}^{96}\sum_{i'=1}^{96}\left(E_{ii'} \times \sum_{j=1}^{24} Z_{ii'j}\right)$$

s.t.

$$\sum_{i=1}^{96} A_{ik} \times x_{ij} \geq U_{kj} \; \forall \, j = 1 \ldots 24, k = 1 \ldots 16$$

$$\sum_{i=1}^{96} B_{il} \times x_{ij} \geq V_{lj} \; \forall \, j = 1 \ldots 24, l = 1 \ldots 4$$

$$\sum_{i=1}^{96} C_{im} \times x_{ij} \geq W_{mj} \; \forall \, j = 1 \ldots 24, m = 1 \ldots 16$$

$$\frac{1}{2}x_{ij} + \frac{1}{2}x_{i'j} \geq Y_{ii'j} \; \forall \, i = 1 \ldots 96, i' = 1 \ldots 96, j = 1 \ldots 24$$

$$x_{ij} + x_{i'j} - 1 \leq Z_{ii'j} \; \forall \, i = 1 \ldots 96, i' = 1 \ldots 96, j = 1 \ldots 24$$

$$\sum_{i=1}^{96} x_{ij} = 4 \; \forall \, j = 1 \ldots 24$$

$$\sum_{j=1}^{24} x_{ij} = 1 \; \forall \, i = 1 \ldots 96$$

Solution

- Using AMPL, under 'mipgap 0.3' (anything less than 0.3 will not yield an answer in 5 minutes), optimal value = 273.75:

37,73,89,94	19,22,33,77	36,54,72,74	26,52,65,85
6,76,81,83	16,24,59,95	27,67,70,75	4,17,21,51
3,13,56,57	7,15,42,45	31,35,46,49	32,58,64,78
34,61,63,82	18,23,47,68	53,66,79,93	12,20,48,71
5,69,88,92	9,10,29,91	1,8,30,41	38,39,60,87
14,50,62,90	28,43,80,96	3,40,44,84	11,25,55,86

Sensitivity Analysis - 3 Students per Group

- We want to see if the number of students per group make a difference in terms of overall diversity
- Same model, except with some changes to the parameter j, which becomes 32 now
- Also, the student number constraint changes from 4 to 3

Solution

- Using AMPL, under 'mipgap 0.3' (anything less than 0.3 will not yield an answer in 5 minutes, optimal value = 302:

60,71,96	61,64,82	23,51,58	1,16,59
37,89,93	12,22,32	52,72,77	25,36,87
30,38,50	63,66,79	9,15,40	74,84,91
2,39,75	42,45,86	13,24,83	10,47,81
4,5,21	7,17,94	14,62,85	49,70,80
20,35,43	8,31,88	29,54,65	3,55,90
11,48,73	27,28,67	34,41,76	46,56,57
6,69,92	19,33,44	18,26,68	53,78,95

Conclusion

- If given enough time for ampl to run, we may get a more precise answer, but given the number of variables (more than 20,000) and the magnitude of operations, our current answer is good enough
- Having 3 students per group instead of 4 yields more diversity and satisfaction, but this may create more work for each student and the negative consequences of having a hitch hiker is much more significant
- The complete models are in the attached AMPL files; due to the nature of this case study, complete data/codes are difficult to copy/paste to this Powerpoint