Case 8: TORONTO REHAB

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Outline

- Problem Statement
- Assumptions
- Model Development
- Base Solution
- Sensitivity Analysis
- Conclusion

Problem Statement

- **Gillian Bone**, the project manager of the Spinal Cord Rehabilitation Program was required to reduce costs of the program for 2009.
- The problem was to determine what staffing levels and **staff schedule** and what the appropriate **target levels** might be. In this case, the schedule of the nurses. The nurses were divided into registered nurses (RNs) and registered practical nurses (RPNs).
- The objective was to maximize the savings. In another word, to **reduce the costs for RNs and RPNs**.

Assumptions

- Out-patients do not occupy unit beds.
- The nurses' capabilities are assumed to be at the **upper bounds**. That is, 1 nurse for every 3 patients in day shifts, 1 nurse for every 5 patients in evening shifts and 1 nurse for every 10 patients in night shifts.
- The number of nurses needed is assumed to be **rounded down** when it is calculated (similar with original plan). Other cases will be discussed in the sensitivity analysis.

Assumptions

- The model only focuses on the number of beds instead of the number of patients.
- The model only looks at minimizing the cost as there is no quantitative data for
- It is assumed that there could exist RPNs (practical nurses) in unit 2 since the objective was to reduce the cost. The scenario of no RPNs in unit 2 will be discussed in the sensitivity analysis.

Model Development

- This case is generally divided into 3 parts:
  - To calculate the **number of nurses** needed in different and different shifts.
  - Assign the nurses into different units based on the number of beds while maximising the number of RPNs.
  - To calculate the **amount of cost**.
- Since the scheduling of nurses includes optimization of variables, **Excel Solver** is used to address this problem through integer programming.

Model Development

- In this case, the objective function will be like the following:

$$min\ c_x(5\sum_{j=1}^{3}\sum_{k=1}^{3} x_{1jk} + 2\sum_{j=1}^{3}\sum_{k=1}^{3} x_{2jk}) + c_y\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{3} y_{ijk}$$

Where k=1..3 represents unit 1..3, j=1..3 represents shift day...night, and i=1..2 represents weekday/weekend. $\mathbf{x_{ijk}}$ represents the number of RNs working at weekday/end shift j in unit k, $\mathbf{y_{ijk}}$ represents the number of RPNs working at weekday/end shift j in unit k, c represents the wage costs for each 7.5-hour shift for RNs/RPNs.

Model Development

- The decision variables are

  x_1 - number of beds in unit 1 during weekdays

  x_2 - number of beds in unit 2 during weekdays

  x_3 - number of beds in unit 3 during weekdays

  x_4 - number of beds in unit 1 during weekends

  x_5 - number of beds in unit 2 during weekends

  x_6 - number of beds in unit 3 during weekends

Model Development

- Translations:

$$x_{111} = \left\lfloor \frac{x_1}{3} \right\rfloor - 1, x_{121} = \left\lfloor \frac{x_1}{5} \right\rfloor - 1, x_{131} = \left\lfloor \frac{x_1}{10} \right\rfloor$$

$$x_{112} = \left\lfloor \frac{x_2}{3} \right\rfloor - 1, x_{122} = \left\lfloor \frac{x_2}{5} \right\rfloor - 1, x_{132} = \left\lfloor \frac{x_2}{10} \right\rfloor$$

$$x_{113} = \left\lfloor \frac{x_3}{3} \right\rfloor - 1, x_{123} = \left\lfloor \frac{x_3}{5} \right\rfloor - 1, x_{133} = \left\lfloor \frac{x_3}{10} \right\rfloor$$

$$x_{211} = \left\lfloor \frac{x_4}{3} \right\rfloor - 1, x_{221} = \left\lfloor \frac{x_4}{5} \right\rfloor - 1, x_{231} = \left\lfloor \frac{x_4}{10} \right\rfloor$$

$$x_{212} = \left\lfloor \frac{x_5}{3} \right\rfloor - 1, x_{222} = \left\lfloor \frac{x_5}{5} \right\rfloor - 1, x_{232} = \left\lfloor \frac{x_5}{10} \right\rfloor$$

$$x_{213} = \left\lfloor \frac{x_6}{3} \right\rfloor - 1, x_{223} = \left\lfloor \frac{x_6}{5} \right\rfloor - 1, x_{233} = \left\lfloor \frac{x_6}{10} \right\rfloor$$

$$y_{111} = y_{121} = y_{112} = y_{122} = y_{113} = y_{123} = y_{211} = y_{221} = y_{212} = y_{222} = y_{213} = y_{223} = 1$$

$$y_{131} = y_{132} = y_{133} = y_{231} = y_{232} = y_{233} = 0$$

Model Development

- Constraints:

$$st. x_1 + x_2 + x_3 = 54$$

$$x_4 + x_5 + x_6 = 41$$

$$x_i \leq 20 \; \forall \; i = 1..6$$

$$x_i \geq 0 \; \forall \; i = 1..6$$

$$x_i \; is \; integer \;\; \forall \; i = 1..6$$

Base Solution

- Through optimization, the schedule for the nurses will look like this:

	Unit 1	Unit 2	Unit 3
Weekday Day Shift	6	6	6
Weekday Evening Shift	3	3	3
Weekday Night Shift	1	1	1
Weekend Day Shift	4	4	4
Weekend Evening Shift	2	2	2
Weekend Night Shift	1	1	1

Base Solution

- The actual schedule is shown as the following table:

	Unit 1 RN	Unit 1 RPN	Unit 2 RN	Unit 2 RPN	Unit 3 RN	Unit RPN
Weekday Day	**5**	**1**	**5**	**1**	**5**	**1**
Weekday Evening	**2**	**1**	**2**	**1**	**2**	**1**
Weekday Night	**1**	**0**	**1**	**0**	**1**	**0**
Weekend Day	**3**	**1**	**3**	**1**	**3**	**1**
Weekend Evening	**1**	**1**	**1**	**1**	**1**	**1**
Weekend Night	**1**	**0**	**1**	**0**	**1**	**0**

Base Solution

- As a result, the total cost would be **$50,640.75**.
- The original cost was **$66,583.125**.
- Thus the savings could be **$15,942.375**.

Sensitivity Analysis

- This sensitivity analysis will focus on 2 aspects:
  - Assume that the numbers of nurses are rounded up to see what will happen.
  - Assume that there could not exist RPNs (practical nurses) in unit 2 and see what will happen.

Sensitivity Analysis - Assume Rounding up

- In the model, we assumed the number of nurses to be rounded down when it was calculated. In the reality, it is possible that the number would be rounded up.
- The results are shown in the next slide.

Sensitivity Analysis - Assume Rounding up

	Unit 1 RN	Unit 1 RPN	Unit 2 RN	Unit 2 RPN	Unit 3 RN	Unit RPN
Weekday Day	**5**	**1**	**5**	**1**	**5**	**1**
Weekday Evening	**3**	**1**	**3**	**1**	**3**	**1**
Weekday Night	**2**	**0**	**2**	**0**	**2**	**0**
Weekend Day	**4**	**1**	**4**	**1**	**4**	**1**
Weekend Evening	**2**	**1**	**2**	**1**	**2**	**1**
Weekend Night	**2**	**0**	**2**	**0**	**2**	**0**

- Total cost = **$64,219.95**, the maximized savings = **$2,363.175.**

Sensitivity Analysis - Assume No RPNs in Unit 2

- In the model, we assumed that there could be RPNs (practical nurses) in unit 2.
- According to the original plan, there might be some reasons that did not allow us to schedule RPNs in unit 2.
- As a result in this sensitivity analysis we assume that there should not be any RPNs in unit 2. The results are illustrated in the next slide.

Sensitivity Analysis - Assume No RPNs in Unit 2

	Unit 1 - RN	Unit 1 - RPN	Unit 2 - RN	Unit 3 - RN	Unit 3 - RPN
Weekday Day Shift	5	1	6	4	1
Weekday Evening Shift	3	1	3	2	1
Weekday Night Shift	2	0	1	1	0
Weekend Day Shift	3	1	4	3	1
Weekend Evening Shift	1	1	2	1	1
Weekend Night Shift	1	0	1	1	0

- Total cost = **$53,280.6**, the maximized savings = **$13,302.525.**

Conclusion

- The number of beds in the hospital will decrease to 54 units in weekdays and 41 beds in weekends.
- According to the new schedule, Gillian can save **$15,942.375** while meeting all of the constraints.
- The new plan will ask to hire more RPNs (practical nurses) and fire some RNs (registered nurses).